1 INTRODUCTION

The significance of information in various models in economics is already well-established. In the last two decades we witnessed an abundant literature demonstrating the crucial role that information plays in the decision making process of individual agents facing uncertainty and upon the existence of markets and their functioning. In many circumstances economic agents observe, prior to the occurrence of a random economic event, a signal which is correlated to the forthcoming economic event (for example, economic indicators regarding the state of the economy). Consequently, there exist information systems used to update their beliefs regarding the state of nature that will occur in the future.

In his seminal works, comparing statistical experiments, Blackwell (1951,1953) characterized several equivalent definitions of the important notion ”more informative than”. These criteria, which turned out to be important for information economics, have been introduced to economic theory by Marschak (1968), Marschak and Radner (1972) and others, and later applied in a very restricted manner, surprisingly, in various economic frameworks. Let us mention a few examples from the literature on this topic. Grossman, Kihlstrom and Mirman (1978) applied the notion of informativity to a learning by doing dynamic model. Jones and Ostroy (1984) have related ”more informative” with ”more flexible; Grossman and Hart (1980) applied this notion to a Principal-agent problem.

In our view, a main reason for the narrow applicability in economic models of Blackwell’s result, i.e., that ”more information” is advantageous, lies in the fact that once the signal is observed by the economic agents their opportunity sets may change. More specifically, following the signal agents they update the probability distribution and hence, the agents’ opportunity sets vary (for example, due to variations in prices). In other words, the signal may change the feasible set from which each agent chooses (under uncertainty). Consequently, the result stating that a more informative system will be preferred by all decision makers with monotone utilities, does not necessarily hold for a signal-dependent feasible sets.

Our main claim in this paper is that in many economic circumstances where signals can be observed by all participants in the economy, consequently the opportunity sets may vary, more information may result in a worse off situation for the risk-averse agents. We shall demonstrate this claim for two different models with uncertainty where the opportunity sets of agents are signal-dependent. In both cases it is shown that as long as there exist no risk-sharing markets the risk averse agents are better off with more information. However, when we introduce risk-sharing markets the economic circumstances change in a way that with the more informative system the risk-averse agents are worse off even though ”more information” is gained from each signal. We believe that this phenomenon exists in many economic frameworks where agents update their beliefs following the observation of a signal.

In our framework we *exclude* strategic considerations on the part of economic agents. In game theory it is much easier to demonstrate examples where more information is disadvantageous due to the effects information have on the choice of strategies (see, for example, Green and Stokey (1980)).

In our first example we consider competitive producers who export their product abroad under a random exchange rate. Thus producers make their decisions about production facing uncertain profits. In the absence of currency forward/futures markets we show that more information is preferred. However, in the presence of currency forward/futures markets, i.e., when these firms can hedge their random foreign currency proceeds, by selling part of this sum in the forward market, it might be the case that more information, conveyed by a signal correlated to the exchange rate, can be disadvantageous.

In our second example we consider individuals facing uncertain lifetime. Moreover, this random horizon results in an uncertainty about their lifetime stream of incomes. It is proved that in the absence of a life insurance market these individuals prefer more information. However, when life insurance (and annuities) markets operate, this result may be reversed. In this framework, signals correlated with the individual's lifetime are observed by the individual and by the insurance companies, thus affecting the insurance premia. We show that the individual may be worse off when ”more information” is derived from each signal.

The fact that more information can be disadvantageous is not new to economic theory (see, for example Hart (1975), Wakker (1988), Green (1981), Radner and Stiglitz (1974) and many others). However, our models (with symmetric information) demonstrate a different type of phenomena: In the absence of risk-sharing mechanism more information, gained from each signal, is preferable to the risk-averse economic agents. Once a risk-sharing market is established we may find the opposite, i.e., they are better off with less information. Moreover, we are not considering marginal circumstances (e.g., a very special type of preference, information systems) or the non-expected utility case, but rather straightforward choices of preferences, cost functions and information systems.

The paper is organized as follows. In section 2 we present a model with signals and information systems where the feasible sets are signal-dependent. We find a condition under which more information is preferable. In sections 3 and 4 we bring two economic models where introducing risk-sharing markets can be disadvantageous. In section 5 we discuss the generality of these results.

2 THE GENERAL FRAMEWORK

Consider decision makers under uncertainty. Let S = $\{s_1, ..., s_n\}$ be the set of states of nature, and $\pi = (\pi_1, ..., \pi_n)$ be a prior probability distribution over S. We assume that each decision maker

(DM) is an expected utility maximizer where her von-Neumann Morgenstern utility function is state-dependent $U = \{u(., s_i)\}$ each defined over the set of feasible actions $B \subseteq R^k$. Before taking an action the DM observes a signal y which is *correlated* to the state of nature. Denote by $Y = \{y_1, ..., y_m\}$ the set of possible signals. We take $m = n$ for simplicity.

An *information system P* is an *nxn* row stochastic matrix specifying for each state of nature a probability vector over the set of signals. In our model a DM does not observe the true state of nature but rather observes signals which are generated by those states. Upon receiving a signal the DM updates her apriori probability vector, using Bayes rule, and then chooses actions so as to maximize her expected utility.

Let P and Q be two information systems. We say that P is *more informative than* Q, denoted by $P \succeq Q$, if there exists an *nxn* row stochastic matrix R such that $Q = PR$. Multiplying by R adds some noise (randomization) to the information contained in P.

Denote by $V(P, \pi, U)$ the value function of information given the utility function U and the probabilities vector $q = (q_1, ..., q_n)$ where q_j is the probability (derived from π and P) that the signal y_j would be received. More precisely,

$$V(P, \pi, U) = \sum_{y_j \in Y} q_j \max_{a \in B} \sum_{i=1}^{n} \pi_i(y_i) U(a, s_i)$$

where $\pi_i(y_j)$ is the posterior, i.e., the updated probability of state s_i given the signal y_j.

Theorem: (Blackwell): $P \succeq Q$ if and only if $V(P, \pi, U) \geq V(Q, \pi, U)$ for all π, U.

Blackwell's theorem states that an information system P is ”more informative” than an information system Q if and only if every expected utility DM prefers (weakly) using P to using Q.

Consider the following extension of the model described above; assume now that the set B of feasible actions would not remain the same regardless of which signal was received. Instead, assume that to every signal y there corresponds a feasible set, to be denoted by $B(P, y)$; the notation $B(P, y)$ emphasizes the dependence on both elements: the information system and the specific signal.

Within this extended model the value function of information should be slightly adjusted as follows:

$$V^*(P, \pi, U) = \sum_{y_j} q_{y_j} \max_{a \in B(P, y_j)} \sum_{i=1}^{n} \pi_i(y_j) U(a, s_i) \tag{1}$$

Observing this model the question whether Blackwell's theorem still holds in such an extended framework seems natural. Put differently, given a model with signal- dependent feasible set, is it always the case that more information is advantageous. It turns out, surprisingly, that the answer is negative.

Proposition 1 *Let P, Q be two information systems and assume that $P \succ Q$. There exist two families of feasible sets $\{B(P, y_i)\}_{i=1}^n, \{B(Q, y_i)\}_{i=1}^n$, a state dependent utility function U and an apriori probability vector π, such that: $V^*(P, \pi, U) < V^*(Q, \pi, U)$.*

Proof. Let $P \succ Q$. Consider the following families of feasible sets: Choose U, π and two sets B_1 and B_2 such that $B(P, y_i) = B_1$ and $B(Q, y_i) = B_2$ for $i = 1, ..., n$, then the following inequality holds:

$$\max_{a \in B_1} \sum_{i=1}^{n} P_{ij}\pi_i u(a, s_i) < \max_{a \in B_2} \sum_{i=1}^{n} Q_{ij}\pi_i u(a, s_i) \text{ for } j = 1, ..., n. \quad (2)$$

Note that for a given U and π the existence of sets B_1 and B_2 such that (2) holds is straightforward. □

That is, allowing the feasible set to be signal dependent (as opposed to Blackwell's classical model of information) might lead the DM to prefer less information to more information. One might argue that the above reult stems mainly from the requirement in (2). Surprisingly, it can be shown that even in the extreme case where $P \succ Q$ and $B(P, y_i) \supseteq B(Q, y_i)$ for all i, it might happen (i.e., there exists some U and π) that $V^*(P, \pi, U) < V^*(Q, \pi, U)$. To demonstrate this consider the following example:

Example 1: There are two information systems: $I_n = \begin{bmatrix} 1 & 0... & 0 \\ 0 & 1... & 0 \\ 0 & 0... & 1 \end{bmatrix}$ the *full information* system

and

$E_n = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & ... & \frac{1}{n} \\ . & & & . \\ . & & & . \\ . & & & . \\ \frac{1}{n} & \frac{1}{N} & ... & \frac{1}{N} \end{bmatrix}$ the *null information* system. For $n = 2$, the relevant value functions

are:

$$V^*(I, \pi, U) = \pi_1 \max_{a \in B(I, y_1)} u(a, s_1) + \pi_2 \max_{a \in B(I, y_2)} u(a, s_2)$$

$$V^*(E, \pi, U) = \tfrac{1}{2} \max_{a \in B(E, y_1)}(\pi_1 u(a, s_1) + \pi_2 u(a, s_2)) + \tfrac{1}{2} \max_{a \in B(E, y_2)}(\pi_1 u(a, s_1) + \pi_2 u(a, s_2)).$$

Assume that $B(E,y_1), B(E,y_2), U$ and π are such that $\pi_1 u(.,s_1) + \pi_2 u(.,s_2)$ is maximized on $B(E,y_1)$ at a_1 while $\pi_1 u(.,s_1) + \pi_2 u(.,s_2)$ is maximized on $B(E,y_2)$ at a_2. Also assume that $a_2 \notin B(I,y_1)$ and $a_1 \notin B(I,y_2)$. Then, clearly, there exists a state dependent utility and a prior π such that $V^*(E,\pi,U) > V^*(I,\pi,U)$.

This example demonstrates that allowing the feasible set to vary with signals results in the collapse of Blackwell's theorem.As we shall see later, such circumstances are not rare in economic models - hence Proposition 1 is interesting. On the other hand, since in many frameworks with uncertainty and signals, we face signal-dependent feasible sets, it would be useful to find some criterion that will guarantee that more information is preferable. The next theorem provides sufficient condition for that.

Theorem 1 *Let P, Q be two information systems, $\{B(P,y_i)\}_{i=1}^n$ and $\{B(Q,y_i)\}_{i=1}^n$ are the associated families of feasible sets. Assume that $P \succ Q$, i.e., there exists a stochastic matrix R such that $PR = Q$. If the following condition holds:*

$$\text{For all } k,j: \; R_{kj} > 0 \Longrightarrow B(P,y_k) \supseteq B(Q,y_j). \tag{3}$$

Then, for all U and π, $V^*(P,\pi,U) \geq V^*(Q,\pi,U)$.

Proof. Given some π, U consider the value function associated with Q:

$$V^*(Q,\pi,U) \equiv \sum_{y_j} \max_{a\in B(Q,y_j)} \sum_{i=1}^{n} (Q_{ij}\pi_i)u(a,s_i)$$

$$= \sum_{y_j} \max_{a\in B(Q,y_j)} \sum_{i=1}^{n} (\sum_{k=1}^{n} R_{kj}P_{ik})\pi_i u(a,s_i)$$

where the second equality is a consequence of $PR = Q$. Changing the order of the summation, one gets:

$$V^*(Q,\pi,U) = \sum_{yj} \max_{a\in B(Q,y_j)} \sum_{k=1}^{n} R_{kj} \sum_{i=1}^{n} P_{ik}\pi_i u(a,s_i)$$

$$\leq \sum_{y_j} \left(\sum_{k=1}^{n} R_{kj} \left(\max_{a\in B(Q,y_k)} \sum_{i=1}^{n} P_{ik}\pi_i u(a,s_i) \right) \right)$$

where the inequality follows from the ”sub-linearity” property of the maximum operator.

Now, since condition (3) holds, it follows that:

$$V^*(Q,\pi,U) \leq \sum_{y_j} \left(\sum_{k=1}^{n} R_{kj} \left(\max_{a\in B(P,y_k)} \sum_{i=1}^{n} P_{ik}\pi_i u(a,s_i) \right) \right) = V^*(P,\pi,U)$$

where the equality is a consequence of R being a stochastic matrix. □

It is of note that condition (3) is minimal in some sense. Specifically, the requirement that $B(P, y_i) \supseteq B(Q, y_j)$ for all i, j is, of course, sufficient also. However, surprisingly as we have just shown in Example 1, the requirement that:

$$B(P, y_i) \supseteq B(Q, y_i) \qquad \text{for} \quad i = 1, ..., n$$

is sometimes insufficient. To conclude our discussion we note that by relaxing the assumption of signal-independent opportunity sets we encounter a wide range of decision problems where less information is preferred to more. To demonstrate the economic implications of this finding we bring two economic models where such a case occurs only in the presence of certain economic institutions: risk-sharing markets..

3 PRICE UNCERTAINTY, FUTURES MARKETS AND HEDGING

In recent years we observe high commodity price volatility and exporting/importing firms face significant exchange rate fluctuations in the major industrial currencies. For those risk-averse economic agents who have future commitments in foreign currency, the higher uncertainty about their profits results in adverse effects upon their economic activities such as the production level (see Sandmo (1971)). As a result, we witness rapidly developing financial risk-sharing markets, such as commodity futures or foreign currency forward markets, which are actively utilized. The economic impacts of such hedging tools on risk averse agents have been studied extensively in the literature; for example, Danthine (1978), Holthausen (1979), Baron (1976), Kawai (1981) and Kawai and Zilcha (1986). It was demonstrated in various models, that introducing forward/futures markets will increase the production level and international trade under some mild conditions about the forward/futures markets. Let us present now a specific case with uncertain price resulting from random exchange rates.

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3.1 Exporting Firm: The Case where Futures Markets are not Available

Consider an exporting firm selling its product on the international market. The firm is competitive and risk averse facing a fixed price P^* abroad, but the exchange rate $\tilde{e}$ is a random variable. Production takes place at the home country and let $C(x)$ be the cost function (in domestic currency); we assume that $C'(x) > 0$ and $C''(x) \geq 0$. Let $U(\xi)$ be the von-Neumann-Morgenstern utility function defined on profits denominated in *local* currency. Assume that $U' > 0$ and $U'' < 0$. We also take the random exchange rate $\tilde{e}$ to assume n values $\{e_1, ..., e_n\}$ and without loss of generality let $P^* = 1$.

Consider now the following order of events. At date 0 the firm observes a signal y correlated with the state of nature (it is observed by all agents in this economy) and at that time a decision regarding the production level must be made. At date $t = 1$ the firm exports its output and the exchange rate $\tilde{e}$ is observed. The firm, and other agents, use at $t = 0$ a certain information system to revise their prior about the probabilities of states of nature once the signal becomes known. Clearly, the choice of information system will have an impact on the production decisions made at date 0.

For each signal y_j and information system P the probability of state i is given by $\pi_i(P, y_j)$.A signal y_j is called *excluding signal* if for some state i $\pi_i(P, y_j) = 0$ while $\pi_i > 0$, for the given *prior* π, and as a result the feasible set of actions changes. Given y_j and P the feasible set of production levels is:

$$B(P;y_j) = \{x \geq 0 | e_i x - C(x) \geq -C^* \quad \text{for each state i}$$
$$\text{with } \pi_i(P,y_j) > 0, \quad i = 1,2,...,n\}$$

for some exogenously given nonnegative constant C^*.Thus we do not allow losses above C^* in any state of nature which can occur with positive probability. Since some signals may be *excluding signals* the feasible set is signal -dependent. It is possible to define $B(P,y_i)$ as signal independent without any significant change in the optimization by requiring that $e_i x - C(x) \geq -C^*$ for <u>all</u> states i. However, this presentation facilitates a generalization of this example to a more general framework. Now, given π and U (to be taken fixed in the sequel), the *value* of the information system P, under the above feasible actions, is given by:

$$V(P) = \sum_j q_{y_j} \left[\max_{x \in B(P,y_j)} \sum_i \pi_i(P,y_j) U(e_i x - C(x)) \right].$$

Now let us show that in the absence of a risk-sharing mechanism (such as currency futures markets) the firm is better off with the more informative system.

Theorem 2 *If $P \succ Q$ then $V(P) > V(Q)$.*

Proof. Assume that $P \succ Q$, hence by definition there exists a stochastic matrix R such that $PR = Q$. Using the result proven earlier, if we show that for all $k,j[R_{kj} > 0 \Longrightarrow B(P,y_k) \supseteq B(Q,y_j)]$ holds, then $V(P) \geq V(Q)$. Consider first a non-excluding signal y_j (hence the bottom elements of $\pi(Q,y_j)$ are not all zeroes), in this case,

$$R_{kj} > 0 \Longrightarrow B(P, y_k) \supseteq B(Q, y_j)$$

since it may be possible that the lower tail of $\pi(P, y_k)$ has zeroes. Now, consider an excluding signal y_j; in this case $R_{kj} > 0$ implies that the number of zeroes at the bottom of $\pi(P, y_k)$ are at least as much as that in $\pi(Q, y_j)$. But this implies, from definition, that $B(P, y_k) \supseteq B(Q, y_j)$,which completes the proof.

3.2 Futures Markets and Hedging

Let us now consider the above framework when currency futures markets are introduced. In this case the order of events is as follows:

Date 0: Given a certain information system, which is used economywide, some signal y is observed (by all agents). Now the firm determines its production level X and its futures contracting (sale or purchase) of foreign currency Z given the futures exchange rate e_f. This forward contract is to be executed at date 1 (where the spot market for foreign exchange operates as well).

Date 1: The exchange rate $\tilde{e}$ is revealed, hence currency spot market operates. The firm exports its product and receives foreign currency (which is used to cover part of its futures contract obligations). Note that since all transactions take place at date 1 there is no difference between forward and futures markets (see Cox, Ingersoll and Ross (1981)).

For a given information system and a signal y, the posterior probability distribution over states of nature is known to all agents. It is well founded in the literature on currency forward (futures) markets that the assumption of *unbiasedness* is a reasonable one; namely, that the forward exchange rate e_f is an unbiased estimator of the currency spot price $\tilde{e}$. Thus we assume throughout this section that

$$e_f(Q, y) = E[\tilde{e}|Q, y] \quad \text{for all y.} \tag{4}$$

Let us write the feasible set for the firm in this framework for a given information system Q and a signal y (which is observed economywide); note that due to our choice of the international price $P^* = 1$ the firm received $\tilde{e}$ for each unit it sells on this competitive market.

Denote by ξ_i the firm's *profits* in state i, i.e.:

$$\xi_i(X, Z, Q, y_j) = e_i(X - Z) + e_f(Q, y_j)Z - C(X).$$

The signal-dependent feasible sets in this case are:

$$B(Q, y_j) = \{(X, Z)|X \geq 0, \text{ and for each } i, \ \zeta_i(X, Z, Q, y_j) \geq -C^*$$
$$\text{whenever } \Pr\text{ob}\{\tilde{e} = e_i|Q, y_j\} > 0 .$$

The value function is:

$$V(Q) = \sum_j q_{y_j} \left[\max_{(X,Z)\in B(Q,y_j)} \sum_i \pi_i(Q, y_j) U(\xi_i(X, Z, Q, y_j)) \right] .$$

Now let us consider Theorem 2 when currency forward/ futures markets are available. We claim that in this case it is possible that the firm will be worse off when the more informative system prevails.

Theorem 3 *In the presence of currency forward/futures market it is possible that*

$P \succ Q$ *and* $V(P) < V(Q)$.

Proof. Let us bring a simple example to demonstrate this claim for $n = 3$, $P = I_3$ (i.e., the full information case) and $Q = E_3$ (i.e., the no information case).

Consider an exporting competitive firm with a cost function $C(x) = \frac{1}{2}\alpha x^2$ where $0 < \alpha < 1$ and a von-Neumann Morgenstern utility function $U(\theta) = \ln\theta$. Assume also that the *currency forward market is unbiased*, i.e.,

$e_f(P, y) = E[\tilde{e}|P, y]$ and $e_f(Q, y) = E[\tilde{e}|Q, y]$ for all y.

Let $\tilde{e}$ assume 3 values $\{1, 2, 3\}$ with a prior probability distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Given this prior we have $q_{y_j} = \frac{1}{3}$ for $j = 1, 2, 3$ for either P or Q. Due to the unbiasedness of the futures market the optimal hedge that the firm chooses, $Z^*(P, y)$ for each given signal, will be a full-hedge, i.e., the firm will sell forward at date 0 all its foreign currency proceeds. Hence, its profit will be risk-free (see Feder, Just and Schmitz (1980), or Kawai and Zilcha (1986)). Also, using what is known as the "separation theorem", the firm's optimal output will be *independent* of the probability distribution of the exchange rate and its attitude towards risk (see Danthine (1978) and the above two references). Moreover, its optimal production (in the case where the information system is P) is given by:

$$C'(X^*(P, y)) = e_f(P, y) \qquad \text{for all } y. \tag{5}$$

and the optimal $(X^*(P,y), Z^*(P,y))$ yield maximum profits:

$\xi_j(P,y) = e_f(P,y)X^*(P,y) - C(X^*(P,y))$ for all y.

Using the above specific form of the cost function we can compute the optimal production levels under P or Q, for each signal y_j. First let prevailing information system be $P = I_3$ then,

$$e_f(P,y_1) = 1, \quad e_f(P,y_2) = 2 \quad e_f(P,y_3) = 3.$$

Hence, by equation (5) we find that,

$$X^*(P,y_1) = 1/\alpha \quad X^*(P,y_2) = 2/\alpha, \quad X^*(P,y_3) = 3/\alpha. \tag{6}$$

By the Full-hedging Theorem (see Kawai and zilcha (1986)) we have:

$$Z^*(P,y_j) = X^*(P,y_j) \text{ for } j = 1,2,3. \tag{7}$$

Thus:

$$\xi_1(P,y_1) = \frac{1}{2\alpha}.$$

$$\xi_2(P,y_2) = \frac{2}{\alpha}.$$

$$\xi_3(P,y_3) = \frac{9}{2\alpha}.$$

$$V(P) = \frac{1}{3}\ln(\frac{1}{2\alpha}) + \frac{1}{3}\ln(\frac{2}{\alpha}) + \frac{1}{3}\ln(\frac{9}{2\alpha}).$$

Consider now the case where $Q = E_3$ prevails. The forward exchange rate in this no-information signals case is:

$e_f(Q, y_j) = \frac{1}{3}1 + \frac{1}{3}2 + \frac{1}{3}3 = 2$ for all j.

Hence:

$X^*(Q, y_j) = 2/\alpha$ for all j.

$Z^*(Q, y_j) = 2/\alpha$ for all j.

$\xi_j(Q, y_j) = \frac{4}{a} - \frac{2}{a} = \frac{2}{a}$ for all j.

$V(Q) = \ln(\frac{2}{\alpha})$

Since for $0 < \alpha < 1$ we obtained that

$$V(Q) = \ln \frac{2}{\alpha} > \frac{1}{3} \ln(\frac{9}{2\alpha^3}) = V(P)$$

which proves our claim.

To understand this result, let us demonstrate here how the feasible sets are related. Let us take $C^* = 0$ here.For the case of no-informational signals we have for all j,

$$\begin{aligned} B(Q, y_j) = B^*(Q) = \{(X, Z)|X \geq 0 \text{ and } e_i(X - Z) + 2Z - \tfrac{1}{2}\alpha X^2 \geq 0 \text{ for } i = 1, 2, 3\} = \\ \{(X, Z)|X \geq 0, Z \leq X \text{ and X} + Z - \tfrac{1}{2}\alpha X^2 \geq 0\} \bigcup \\ \bigcup\{(X, Z)|X \geq 0, Z > X \text{ and } 3X - Z - \tfrac{1}{2}\alpha X^2 \geq 0\}. \end{aligned}$$

Now

$$B(P,y_1) = \{(X,Z)|X \geq 0 \text{ and } X - \tfrac{1}{2}\alpha X^2 \geq 0\}$$
$$B(P,y_2) = \{(X,Z)|X \geq 0 \text{ and } 2X - \tfrac{1}{2}\alpha X^2 \geq 0\}.$$
$$B(P,y_3) = \{(X,Z)|X \geq 0 \text{ and } 3X - \tfrac{1}{2}\alpha X^2 \geq 0\}.$$

Thus, we find here that $B(P,y_1) \subset B^*(Q)$ while $B(P,y_3) \supset B^*(Q)$. For $Z = X$ we also find that $B(P,y_2) = B^*(Q)$. This type of relationship between the feasible sets makes it possible to achieve $V(Q) > V(P)$. In a different framework Green (1981) shows that in the presence of options markets "improved information structure is almost surely beneficial" which clearly differs from our result.

4 LIFE CYCLE MODEL WITH UNCERTAIN LIFETIME

4.1 The Model Without Life Insurance

Consider a discrete-time version of Yaari's (1965) "Marshallian model" where the lifetime utility function contains a bequest motive. Although consumers face uncertain lifetime, let us assume that they cannot purchase life insurance.This assumption will be relaxed in the next section. Denote the uncertain lifetime horizon by $\widetilde{T}$,a random variable which assumes one of the values $1, 2, ..., T$. When $\widetilde{T} = \tau$,$1 \leq \tau \leq T$, it means that he dies during or at the end of period τ.

Let $\pi = (\pi_1, ..., \pi_T)$ be the apriori probability vector related to the random variable $\widetilde{T}$. Let $u(\cdot)$ be the consumer's utility function from a one-period consumption, and let $\phi_t(b)$ be the utility function from bequest b if he dies at the end of period t. For technical reasons assume all these functions to be strictly concave. This individual income stream $I_1, ..., I_T$, where I_t denotes his non-interest income received at the beginning of the t-th period if he is alive and it is known with certainty for all t. His lifetime total income, however, is uncertain. More specifically, if $\tilde{T}$ realizes at t, then she receives the truncated sequence of incomes $\{I_1, ..., I_t\}$, while $\{I_{t+1}, ..., I_T\}$ are lost. Let $0 < \delta \leq 1$ be the coefficient of time-preference and let $r_t,\ t = 1, ..., T$, stand for the period t interest rate, which is known with certainty at the beginning of the planning. Denote $r_k(t) = \prod_{\tau=k}^{t}(1 + r_\tau)$ for $k \leq t, t = k, ..., T$ and assume $r_k(k-1) = 1$.

Let c_t stand for his planned consumption spending during the t-th period. The realized consumption stream, however, is contingent upon the realization of $\tilde{T}$. Corresponding to the *consumption plan* $(c_1, ..., c_T)$, given the interest rates, there is a contingent *bequest plan*, denoted by $(b_1, ..., b_T)$. It is given by

$$b_t = \sum_{k=1}^{t}(I_k - c_k)r_k(t), \quad \text{for } 1 \leq t \leq T.$$

Specifically, if $\tilde{T}$ realizes at t, then b_t is the bequest to his offspring.

In our framework, as distinguished from that of Yaari (1965) and Karni and Zilcha (1985), the individual's choice of optimal consumption-bequest plan is taken after she observes a signal $y \epsilon Y$, (correlated to the lifetime horizon), to be interpreted through some information system. That is, in this extended model, the choice is two-fold. In the first stage, the beginning of the planning horizon, before receiving any signal (new information), this individual chooses an information system, and later in the second stage, after receiving a signal, she chooses an optimal consumption-bequest plan. Specifically, given some information system P, the optimal consumption-bequest plan is chosen by maximizing:

$$
\begin{aligned}
E_y \max_{\{c_1,\ldots,c_T;b_1,\ldots,b_T\}} E_{\tilde{T}} \left\{ \sum_{t=1}^{\tilde{T}} \delta^t u(c_t) + \phi_{\tilde{T}}(b_{\tilde{T}}) \mid P, y \right\} \\
s.t \ \ .c_t \geq 0, \qquad t = 1, \ldots, T \\
b_t = \textstyle\sum_{k=1}^{t} (I_k - c_k) r_k(t) \quad t = 1, \ldots, T
\end{aligned}
\tag{8}
$$

where, for simplicity, the dependence of b_t and c_t on P and y was omitted.

Due to economic considerations, we will focus our discussion on individuals for which $\phi_t'(0) = \infty$ for $1 \leq t \leq T$. That is, we consider only $b_t \geq 0$ whenever $\Pr\{\tilde{T} \leq \dot{t}\} > 0$. In other words, we are dealing with individuals who have no incentives to leave *debts* to their offspring.

Let us analyze first the structure of the feasible sets. Assume that the individual's information system is P_o. Upon receiving a signal (a medical check-up results, for example) he/she re-evaluates his prior probability vector related to $\tilde{T}$, using his/her information system (his medical practitioner, for example).

The feasible consumption-bequest plans are:

$$
B(P, y_j) = \left\{ \begin{array}{c} (c_1(y_j), \ldots, c_T(y_j)) \geq 0 \ \mid \ \text{Each } \ b_t(y) \text{ is given by (8), it} \\ \text{satisfies: } \ b_t(y_j) \geq 0 \text{ whenever } P_r(\tilde{T} = t | P, y_j) > 0, \ t = 1, \ldots, T-1, \text{ and } b_T(y_{j)} \geq 0. \end{array} \right\}
$$

Clearly, given the information system P, if for each signal y_j $\text{Prob}(\tilde{T} = t \ P, y_j) > 0$, for $t = 1, \ldots, T$, then the feasible set is independent of y_j. Since the individual does not know with certainty that she will live above a certain age, higher than 1, her consumption-bequest plans must satisfy $b_t \geq 0$ for *all* t. However, if for some signal y_k (actually, an excluding signal), there exists a $\bar{t}(y_k)$ such that $\text{Prob}(\tilde{T} = t \mid P, y_k)+ = 0$ for $1 \leq t \leq \bar{t}$, then the feasible set $B(P, y_k)$, is:

$$
B(P, y_k) = \{(c_1(y_k), \ldots, c_T(y_k)) \geq \mid 0 b_\tau \geq 0 \text{ for all } \tau > \bar{t}(y_k)\}
$$

Thus, due to the possiblity that excluding signals exist, this environment has signal- dependent feasible sets. Surprisingly, although the individual faces signal- dependent opportunity sets Blackwell's theorem is not violated.

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Theorem 4 *Let P, Q be two information systems. In the absence of life insurance markets we have,*

$$P \succ Q \Rightarrow V^{*}(P, \pi, U) \geq V^{*}(Q, \pi, U) \qquad \text{for all } \pi, U.$$

The proof is omitted since it is similar to the proof of Theorem 2. That is, an individual who cannot acquire life insurance to "smooth" consumption over his uncertain lifetime, always prefers more information to less.

4.2 The Case of Life Insurance

Let us explain first the notion of actuarial note introduced by Yaari (1965). This type of interest-bearing note is equipped with the additional property that is automatically cancelled if the individual issuing (or purchasing) such a note dies prior to redemption date. If, for example, one issues a one-period actuarial note, the note stays on the books and is redeemed at the end of the period if the issuer is still alive. If, however, the issuer dies during this period, his estate is held free of any obligations arising from the note. Clearly, the rate of interest on actuarial notes exceeds that of regular notes, to reflect the extra-risk involved. Mathematically, denote by i_t that rate of interest on actuarial notes issued by the individual at age t, then it is given by:

$$1 + i_t = \left(\frac{1 + r_t}{1 - m}\right)\frac{\zeta_t}{\zeta_{t+1}} \quad t = 1, ..., T - 1$$

where m is the loading factor (for fair insurance: $m = 0$) and where

$$\zeta_t = \sum_{\tau=t}^{T} \pi_\tau, \qquad \zeta_{T+1} = 0.$$

Actually, the expression $\left(\frac{\zeta_{t+1}}{\zeta_t}\right)$ can be interpreted as a measure of "optimism". This is the conditional probability that if the individual is alive at date t he shall survive to date $t + 1$. Buying life insurance for \$Z in this case amounts to selling actuarial notes for \$Z. The insurance premium is given by $(i_t - r_t)Z$. To examine whether, in this extended model, it might be the case that the individual prefers less information to more, let us explore first the dependence of the feasible set on signals. Clearly, if there is no such dependence, Blackwell's theorem applies "automatically." When a market for actuarial notes exists, given a signal y_j, the feasible set is:

$$B(P, y_j) = \left\{ \begin{array}{c} (c_1, ..., c_t;\ b_1, ..., b_T) \mid c_t \geq 0 \text{ for } t = 1, ..., T \\ b_t \geq 0 \text{ for } t \geq \bar{t}(y_j) \text{ where} \\ b_t = b_{t+1} + I_t + z_t - (1 + i_{t-1})z_{t-1} - c_t \text{ for } t = 1, ..., T \\ \text{and } z_0 = 0 = z_T \end{array} \right\}$$

where $\bar{t}(y_j)$ is the first period with positive probability of death given y_j, and therefore Blackwell's theorem does not necessarily hold.

Let us bring now example where the individual prefers less information (in this case, no information) to more information (in this case, almost full information). Consider the following two information systems:

$$P = \begin{pmatrix} 1-2\xi & \xi & \xi \\ \xi & 1-2\xi & \xi \\ \xi & \xi & 1-2\xi \end{pmatrix} \qquad Q = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

where ξ is very small. Clearly, given either P or Q, none of the signals are excluding signals. Let $\pi = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Consider the sequences $\left\{\frac{z_t}{\zeta_{t+1}}\right\}$, as $\xi \to 0$ for each signal:

	P			Q		
	y_1	y_2	y_3	y_1	y_2	y_3
$\frac{\zeta_1}{\zeta_2}$	$\to \infty$	$\to 1$	$\to 1$	1.5	1.5	1.5
$\frac{\zeta_2}{\zeta_3}$	2	$\to \infty$	$\to 1$	2	2	2

Choose the bequest functions in the following manner: $\phi_1(.) \approx \phi_3(.) \approx 0$ while $\phi_2(.)$ is "very large". Let the one-period utility satisfy $u(.) \approx 0$. Choose the incomes such that I_2 is "small" while I_3 is "very large". Now, since I_3 was taken to be "very large", the individual desires to increase b_2, a desire which is also a consequence of $\phi_2(.)$ being a "very large" function. However, this "desire" is strongly mitigated by our choice of i_2 - the interest on actuarial notes loan from I_3, to be very large. It is easy to see that under the above assumptions (all functions but $\phi_2(.)$ being close to zero),

$$V^*(P,\pi,U) \approx \zeta_2\phi_2(b_2^*()P,\pi,U) \quad V^*(Q,\pi,U) \approx \zeta_2\phi_2(b_2^*(Q,\pi,U)) \tag{9}$$

However, under P $i_2(y_2) \to \infty$ it follows that $Z_2(y_2) \to 0$ which results in $\zeta_2\phi_2(b_2^*(P,\pi,U)$ being small. Under Q, no such problem arises, and therefore $\zeta_2\phi_2(b_2^*(Q,\pi,U))$ is large (no matter which signal is received). Hence, in the environment we definede we have that:

$V^*(P, \pi, U) < U^*(Q, \pi, U)$.

We can sum up this example by the following theorem.

Theorem 5 *In the above framework where a fair life insurance market exists it is possible that*

$$P \succ Q \text{ and } V^*(P, \pi, U) < V^*(Q, \pi, U).$$

Let us offer the following explanation to this phenomena. Introducing life insurance market results in an enlargement of the feasible sets available to this individual. However, the cost of life insurance, which is determined by the lifetime probability distribution, depends heavily upon the interpretation of each signal, i.e., the available information system (applied by the individual and the insurance company as well). Thus, it is certainly possible that for some signals, the cost of insurance under the more informative system is much higher and hence the bequest will be much lower than under the less informative system. Given that the utility from bequest, at that period, is significant to this individual it makes less information preferable.

5 DISCUSSION

Is it the case that our specific assumptions, in these two economic models, are essential to deriving the result that more information is undesirable? The answer is negative. The phenomenon that introducing risk-sharing markets may hurt risk-averse economic agents, when signals provide *more information* holds under much more general circumstances and in a wide range of competitive economies. Let us point out how few of our simplifying assumptions can be relaxed. We did not consider diversity of information systems, i.e., allowing different economic agents to have different interpretations of the signals. Our results are valid if the diversity in information systems is not too large. Also we have taken the risk-sharing mechanism to be fair (i.e., unbiased forward market and fair life insurance market). The basic result holds if the insurance premium is not ”too large”. Moreover, one can construct examples with the such adverse effects even without this requirement; however, computations will become very clumsy if we do not assume fair risk-sharing markets.

It is possible to present the signal-dependent feasible sets as a set-valued function defined on the posterior probability. However, this way of presentation restricts the scope of the model; particularly, in cases where agents may differ in their information systems and beliefs.

Let us pose here an open question. We have shown that Blackwell’s result that ”more informative” system is preferable, when the opportunity set of actions does not vary as the signals are revealed. What are the necessary and sufficient conditions under which this remains true for a signal-dependent opportunity set?

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