DOES COURNOT COMPETITION NECESSARILY YIELD SPATIAL AGGLOMERATION ? *

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Abstract

Recent work suggests that Cournot oligopolists competing in a spatial model, with a uniform distribution of consumers, agglomerate in the center of the market. In this paper we analyze the robustness of this result with a general symmetric distribution of consumers and show that if the population density is "too thin" at the center of the market, then Cournot competition does not yield spatial agglomeration. Central agglomeration will result if the population is sufficiently dense at any point in the distribution. An example with two spatially separated markets further examines a case where this condition is not satisfied and we show that for any arbitrary number of firms, Cournot competition will never yield central agglomeration. Notably, the model explains spatial dispersion with perfectly overlapping markets.

Introduction

Most theoretical models of spatial competition have found that firms will never agglomerate in a location - price game. The reason is that coincident locations by firms offering essentially identical products will intensify price competition (a la Bertrand) and drive profits to zero.

Clearly firms want to differentiate to soften price competition. [1]

It was Hotelling who argued, in his seminal (1929) paper, that firms would locate back to back in the center of the market, in a location - mill price framework, a result referred to as the Principle of Minimum Differentiation. It was subsequently shown by d'Aspremont, Gabszewicz and Thisse (1979) that there does not exist a price equilibrium in pure strategies if firms locate close to the center of the market. Allowing mixed strategies over prices, Osborne and Pitchik (1987) derived first stage location equilibria, which were away from the center of the market. In fact, d'Aspremont, Gabszewicz and Thisse (1983) showed that back to back locations can never be sustained as an equilibrium precisely because each firm can earn positive profits by choosing a separate location.

Considering an alternative spatial price policy, first discussed in Hoover (1937), Lederer and Hurter (1986) analyzed the price discrimination analogue of the Hotelling model. They found that the socially efficient locations at the market quartiles can be sustained as a market equilibrium. Again, in order to soften price competition, firms do not choose coincident locations.

Another related body of literature deals with Cournot competition in spatial models. [2] Much of the early work in this area is due to Greenhut and Greenhut (1975), Greenhut and Ohta (1975), Norman (1981), Greenhut, Norman and Hung (1987) and Ohta (1988). These papers focus mainly on the pattern of equilibrium prices and do not consider locations as strategic choice variables. Recent work by Anderson and Neven (1991) and Hamilton, Thisse and Weskamp (1989) studies spatial discrimination under Cournot

[1]For an exception see DePalma et al (1985) where firms are differentiated by attributes other than location and hence there may exist an equilibrium in which both firms locate at the center.

[2]For a discussion of monopoly with spatial price discrimination, see for example Holahan (1975), Beckmann (1976) and Greenhut and Greenhut(1977).

oligopoly where locations are strategic choice variables.[3] In direct contrast to the nonagglomeration result with price competition, these papers establish that Cournot competition yields spatial agglomeration.

Without arguing the relative merits of Bertrand versus Cournot competition in spatial models, one may convincingly argue that both spatial dispersion and spatial agglomeration are observable phenomena. Typically, competing airlines choose spatially dispersed airports as their major hubs and firms differentiate their products for strategic purposes. On the other hand, manufacturers and retailers of a wide range of goods and services ranging from car dealerships to ice cream parlors, agglomerate at certain locations. In view of the existing literature, can we conclude that price competition leads to spatial dispersion whereas Cournot competition leads to spatial agglomeration? Although previous research does suggest a generalization in this direction, these results have been derived under the assumption of a uniform distribution of consumers. Further analysis of the robustness of these two distinct theoretical predictions is therefore desirable. In this paper, we are particularly interested in the effect that alternative distributions of consumers will have on the pattern of firm locations. With Bertrand competition, the nonagglomeration result is generally robust, irrespective of the pattern of consumer distribution. However, the same does not apply to a Cournot oligopoly. In particular, the result that Cournot oligopolists agglomerate at the center of the market, as derived in Anderson and Neven (1991) and Hamilton, Thisse and Weskamp (1989), rests on the assumption of a uniform distribution of consumers. The focus of this paper is to analyze the robustness of the central agglomeration result under alternative specifications of consumer distributions.

Our work addresses a further issue. The literature to date suggests that in most spatial models based on Bertrand competition, both for

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[3] Anderson and Neven (1991) allow convex transportation costs while Hamilton, Thisse and Weskamp (1989) compare the outcomes of Bertrand and Cournot competition in the context of a spatial duopoly.

Hotelling-type mill pricing and spatial price discrimination, there is no overlapping of market areas. As Anderson and Thisse (1988) note, under Bertrand competition each firm has an exclusive market space. In contrast, with Cournot competition there is overlapping of market areas; indeed as Anderson and Neven (1991) and Hamilton, Thisse and Weskamp (1989) show, firms' markets may overlap over the entire space. Do we then conclude that Bertrand competition leads to spatial dispersion and non-overlapping markets while Cournot competition leads to spatial agglomeration and overlapping markets? If so, how do we then reconcile this with the common observation of spatial dispersion and overlapping markets? This paper seeks to explain this observation and we show that a non-uniform distribution of consumers may generate an equilibrium outcome

where Cournot competitors display spatial dispersion but have perfectly overlapping markets.

We first study a model with a general symmetric distribution of consumers. Section 1 outlines the model. Section 2 states and discusses the main results. We find that for any symmetric distribution of consumers, if agglomeration occurs, it can occur only at the center of the market. Moreover, firms will not agglomerate if the population density at the market center is "too thin". We further show that for any symmetric distribution of consumers, agglomeration at the center of the market will be sustained as an equilibrium as long as the population density is not "too thin" at any point in the market.

In Section 3, we focus on a simple example with two spatially separated markets. This highlights the extreme case of zero density at all but the two endpoints of the market. While such models have been used in international trade to explain intra-industry trade [4] they could apply to regional markets inside a country, for example two large urban centers connected by an interstate highway, and to national markets, for example

[4] See Neven and Phlips (1985), Brander and Krugman (1983).

the U.S. and Canada, which are separated by some barriers.

We find that Cournot duopolists will never choose a location intermediate between the two markets, a result clearly consistent with those of the earlier section. In the context of a monopoly, this result is well-known (see Hakimi (1964) and Hwang and Mai (1990)). We further find that depending on the specific demand and cost parameters, the duopolists might agglomerate in the larger market, or choose dispersed locations, one in each market. This result illustrates an equilibrium outcome where Cournot duopolists choose dispersed locations but serve perfectly overlapping markets.

Finally, we show that for any arbitrary number of firms, Cournot oligopolists will never choose an intermediate location between the two markets.

<u>Section 1: Model</u>

Let two firms, 1 and 2, produce and sell a homogenous output to consumers located along a linear city of length L.

Let $\phi(x)$ denote the distribution of consumers, where $x \in [0, L]$ denotes the location of a consumer point and $\phi(.)$ is assumed to be symmetric around L/2 and twice continuously differentiable. The inverse demand at each point x is given by $p = a - b\,Q(x)$, where p is the price of the product.

Let $x_i \in [0, L]$, $i = 1, 2$, denote the location of firm i. Assume that $0 \leq x_1 \leq x_2 \leq L$. Firms have identical technology with constant marginal cost of production (normalized to zero). Moreover, firms pay the cost of transporting the product from its plant to the consumer, which equals $t|x - x_i|$. We assume there is no arbitrage between consumers. To ensure that both firms will always serve the whole market, we assume that $a > 2tL$.

We solve and study the subgame perfect equilibrium of a two-stage game. In the first stage, the firms simultaneously choose their locations. In the second stage, each firm observes its competitor's location and simultaneously makes its output choice. As usual, we first determine the second stage equilibrium for given locations.

Following the Cournot assumption, firms compete in quantity at each point. The second stage Cournot equilibrium can be characterized by a set of independent Cournot equilibria, one for each point $x \in [0, L]$, given the assumption of constant marginal cost of production.

Under these assumptions, firm i's profit at any point x is given by

$$\pi_i(x) = [a - bQ(x) - t|x - x_i|]q_i(x), \ i = 1, 2$$

where $Q(.) = q_1(.) + q_2(.)$ and $q_i(x)$ is firm i's output sold at point x.

Standard calculations yield the following unique Cournot - Nash equilibrium at all $x \in [0, L]$:

$$q_1(x) = (1/3b)(a + t|x_2 - x| - 2t|x_1 - x|)$$
$$q_2(x) = (1/3b)(a + t|x_1 - x| - 2t|x_2 - x|)$$
$$p(x) = (1/3)(a + t|x_1 - x| + t|x_2 - x|)$$
$$Q(x) = (1/3b)(2a - t|x_1 - x| - t|x_2 - x|)$$

As Anderson and Thisse (1988) have pointed out, when firms compete in prices and sell homogenous products, there is usually no overlap of their market areas. This is true with both mill pricing in the Hotelling tradition and spatial price discrimination as in Lederer and Hurter (1986).[5] In this paper, as in Anderson and Neven (1991), when firms are Cournot competitors, both firms' market areas overlap over the entire market.

[5]If firms sell differentiated products, as in Anderson and dePalma (1988) and dePalma et al (1985) (see footnote 1), we can observe overlapping of markets in equilibrium.

Section 2: Agglomeration as the location equilibrium ?

The second stage equilibrium quantities help derive the first stage location equilibrium. Firm 1's profit for the first stage location game is

$$\pi_1(x_1, x_2) = \int_0^L [a - bQ(x) - t|x - x_1|]q_1(x)\, \phi(x)\, dx$$

The first proposition specifies a necessary condition for agglomeration to be the equilibrium outcome and helps identify the robustness of earlier results in the literature.

Proposition 2.1

Agglomeration will occur at the center only if $\phi(L/2) \geq t/a$.

Proof: Suppose agglomeration occurs at L/2 but $\phi(L/2) < t/a$. Note that

$$\frac{\partial^2 \pi_1}{\partial x_1^2} = \frac{8t[t - \{a + t(x_2 - x_1)\}\phi(x_1)]}{9b}$$

At $x_1 = x_2 = L/2$,

$$\frac{\partial^2 \pi_1}{\partial x_1^2} = \frac{8t\{t - a\phi(L/2)\}}{9b}.$$

Now, if $\phi(L/2) < t/a$, then $\frac{\partial^2 \pi_1^2}{\partial x_1^2} > 0$, which implies that $\pi_1(x_1)$ is strictly convex at $x_1 = L/2$. However, $\frac{\partial \pi_1}{\partial x_1}$ (at $x_1 = x_2 = L/2$)

$$= - \int_0^{L/2} [\frac{4at}{9b} + \frac{4t^2}{9b}\{(L/2) - x\}]\, \phi(x)\, dx$$

$$+ \int_{L/2}^{L} [\frac{4at}{9b} + \frac{4t^2}{9b}\{(L/2) - x\}\}\, \phi(x)\, dx = 0$$

Therefore π_1 cannot be maximized at $x_1 = L/2$. ▲

Proposition 2.1 claims that Cournot competition will not lead to central agglomeration if the population density at this point is "too thin". As an illustration, consider the following example. Let

$$\phi(x) = \frac{12}{L^3}(x - ½L)^2$$

where $\phi(.)$ is convex and symmetric around ½L. However, note that $\phi(x = ½L) = 0$ and therefore the firms never agglomerate at the center.

The next lemma provides a sufficient condition under which, if firms agglomerate, they will do so only at the center of the market.

Lemma 2.1

If $\phi(x) \geq t/a \ \forall \ x \in [0, L]$, then agglomeration can occur only at the center of the market.

Proof: Let agglomeration occur at x^*. It can be checked that $x^* = 0$ or $x^* = L$ is not possible. That is, x^* must lie in between 0 and L. Therefore, if agglomeration occurs at x^* then

$$\frac{\partial \pi_1}{\partial x_1} = 0, \text{ (at } x_1 = x_2 = x^*).$$

$$=> \int_0^{x^*}[a + t(x^* - x) - 2t(x^* - x)]\phi(x)dx$$

$$= \int_{x^*}^{L}[a + t(x - x^*) - 2t(x - x^*)]\phi(x)dx$$

$$=> t(x^* - (½)L) = a[\int_0^{x^*}\phi(x)dx - \int_{x^*}^{L}\phi(x)dx]$$

Let $g(x^*) = t(x^* - (½)L)$ and $f(x^*) = a[\int_0^{x^*}\phi(x)dx - \int_{x^*}^{L}\phi(x)dx]$

Then $g'(x^*) = t > 0$ and $f'(x^*) = 2a\phi(x) > 0$. Also $g(x^*) = f(x^*)$ at $x^* = (½)L$. If $\phi(x) \geq t/a \ \forall \ x \in [0, L]$ then $g'(x^*) < f'(x^*) \ \forall \ x^* \ \forall \ [0, L]$.

Therefore $g(x^*)$ and $f(x^*)$ intersect only at x^*. This implies that agglomeration can occur only at $x^* = (½)L$. ▲

Finally, using Lemma 2.1, the first stage location equilibrium is characterized in the next proposition.

Proposition 2.2

If $\phi(x) \geq t/a$ for all $x \in [0, L]$, then the firms agglomerate at the center of the market.

Proof: At $x_1 = x_2 = ½L$, note that the first order condition is satisfied for both firms

$$\frac{\partial \pi_1}{\partial x_1} = \frac{\partial \pi_2}{\partial x_2} = 0.$$

The second derivative of firm 1's profit function is

$$\frac{\partial^2 \pi_1}{\partial x_1^2} = t - [a + t(x_2 - x_1)]\phi(x_1)$$

$$\leq t - [a + t(x_2 - x_1)](t/a) \quad [\text{since } \phi(x) \geq t/a]$$

$$= - (t^2/a)[x_2 - x_1] < 0.$$

It can be similarly checked that the second order condition for firm 2 is satisfied. ▲

From Proposition 2.2 we conclude that the central agglomeration result for Cournot duopoly is robust under any symmetric distribution of consumers, provided that their density is not "too thin" at any point. For example, even if the distribution of consumers is U shaped, we know that central agglomeration is the location equilibrium, subject to, of course, the sufficiency condition being satisfied. In another instance, even a multimodal distribution of consumers preserves the central agglomeration property.

Section 3: Spatially Separated Markets and Cournot Oligopoly

In this section we investigate firm location patterns when the population density is indeed "too thin" at some points along the market. We focus on a simple example with two spatially separated markets, in the extreme case of zero density at all but the two endpoints of the market. The point at issue remains whether, in equilibrium, Cournot duopolists agglomerate - either between the markets or in either market - or choose to soften competition by each locating in one market.

In this example firms 1 and 2 produce and sell a homogenous output in two spatially separated markets. Each firm must choose a location for its plant along a linear highway of length L, which connects the two markets. Firm i (i = 1, 2) sells q_i^j in market j (j = 1, 2). Therefore, the total output produced by firm i is $(q_i^1 + q_i^2)$. Let x_i be the distance of firm i from market 1, with $0 \leq x_1, x_2 \leq L$. Therefore firm i's total tranport cost is $t[x_i q_i^1 + (L - x_i)q_i^2]$. As in Section 1, firms have identical technology and constant marginal cost of production (normalized to zero). We assume there is no arbitrage between the consumers.

Again we consider a two-stage game. In Stage I, the firms simultaneously choose their locations. In Stage II, each firm observes its rival's location and then, the firms simultaneously decide how much to sell in each market. Demand in each market is assumed to be linear. The price in market j is $p_j = a_j - b_j(q_1^j + q_2^j)$. To ensure that both firms serve both markets, we assume that $a_j > 2tL$, j = 1, 2.

In the first proposition, we show that irrespective of possible differences in demand across the markets, the duopolists never choose an intermediate location.

Proposition 3.1

The duopolists never choose a location intermediate between the two markets.

Proof: Let x_i and x_k be the distance of firm i and k respectively, from market 1. Without loss of generality consider firm i. The second stage equilibrium outputs of firm i are

$$q_i^1 = (1/3b_1)[a_1 + tx_k - 2tx_i] \text{ and}$$

$$q_i^2 = (1/3b_2)[a_2 + t(L - x_k) - 2t(L - x_i)].$$

Substituting these expressions into firm i's profit function, it can be checked that firm i's second stage profit is

$$\pi_i = (1/9b_1)[a_1 + tx_k - 2tx_i]^2 + (1/9b_2)[a_2 + t(L - x_k) - 2t(L - x_i)]^2$$

However note that $\partial^2\pi_i / \partial x_i^2 > 0$. Hence π_i is strictly convex in x_i, which implies that firm i does not choose any location between the two markets.▲

Next, we derive the first stage SPNE locations. Depending on the difference in demand, the duopolists will either agglomerate in the "larger" market, or will follow the Principle of Maximal Differentiation, where one firm locates in market 1 and the other locates in market 2.

<u>Proposition 3.2</u>

Let $a_2/b_2 \geq a_1/b_1$. If $[(a_2/b_2) - (a_1/b_1)] \geq tL/b_2$, then the unique SPNE location pair is one where both firms locate in market 2.

If $[(a_2/b_2) - (a_1/b_1)] < tL/b_2$, then there are two SPNE location pairs. At each of these equilibria, one of the firms locates in market 1 and the other locates in market 2.

<u>Proof</u>: By Proposition 3.1, we exclude intermediate locations. Thus either both firms locate in the same market or they locate in different markets. Without loss of generality, let $a_2/b_2 \geq a_1/b_1$. Let firm k locate in market 2. If firm i locates in market 1, its profit is

$$\pi_i(x_i = 0) = (1/9b_1)(a_1 + tL)^2 + (1/9b_2)(a_2 - 2tL)^2.$$

If firm i locates in market 2, then its profit is

$$\pi_i(x_i = L) = (1/9b_1)(a_1 - tL)^2 + (1/9b_2)(a_2)^2.$$

Now, it can be checked that

$$\pi_i(x_i = 0) > \pi_i(x_i = L) \text{ if and only if } \{(a_2/b_2) - (a_1/b_1)\} < tL/b_2.$$

Similarly if firm i locates in market 1, firm k locates in market 2 iff

$$(a_1/b_1) - (a_2/b_2) < tL/b_1.$$

Since $a_2/b_2 \geq a_1/b_1$, this condition is always satisfied. Therefore

if $(a_2/b_2) - (a_1/b_1) \geq tL/b_2$, then both firms locate in market 2.

If $(a_2/b_2) - (a_1/b_1) < tL/b_2$, then there are two SPNE location pairs. At each of these equilibria, one of the firms locates in market 1 and the other locates in market 2. ▲

The intuition behind Proposition 3.2 is as follows. By locating in different markets, the firms soften competition and increase their local monopoly power. On the other hand, if there is a difference in demand between the two markets, there is a clear disadvantage from locating in the smaller market. With a positive transportation cost, the firm in the smaller market incurs a relatively higher delivered marginal cost to the larger market. The effect of these two countervailing incentives is that firms will locate in the same market only if that market is "much larger" than the other. Otherwise, they will locate in different markets. It is worth noting that the location equilibrium where each duopolist locates in one market but both serve both markets, explains the casual observation of dispersed locations but perfectly overlapping markets. It also follows that if both markets are of equal size, then agglomeration will never occur. Agglomeration occurs only if one market is significantly larger than the other.

It is also worthwhile to point out that if both firms agglomerate in the larger market, the price may be lower in the smaller market. To illustrate this point, let $b_1 = b_2 = 1$ for simplicity and let $a_1 > a_2 + tL$, so that both firms locate in market 1. This implies

$p_1 = (1/3)a_1$

$p_2 = (1/3)(a_2 + 2tL)$ and

$p_1 - p_2 = (1/3)(a_1 - a_2 - 2tL)$.

If $a_1 > a_2 + 2tL$, then $p_1 > p_2$.

So far, the analysis has concentrated on a duopoly. What are the implications of the location equilibrium we have just derived, if we extend this analysis to an arbitrary number of firms? With more than two firms, say three firms, is it possible that the third firm chooses a location intermediate between the two markets in order to have a cost advantage over at least one competitor in each market? In fact, Proposition 3.3 provides an answer to this question.

<u>Proposition 3.3</u>

For any number of firms, Cournot oligopolists never choose a location intermediate between the two markets.

Proof: Let the number of Cournot oligopolists be n. Let x_i, i = 1,...,n, be the distance of each firm from market 1. Let $p_i = a_i - b_i(q_1^i + q_2^i)$ be the inverse demand in market i, i = 1, 2. Without loss of generality consider firm 1. The second stage equilibrium outputs for firm 1 are

$$q_1^1 = \frac{a_1 - b_1 \sum_{i=2}^{n} q_i^1 - tx_1}{2b_1}$$

$$q_2^1 = \frac{a_2 - b_2 \sum_{i=2}^{n} q_i^2 - t(L - x_1)}{2b_2}$$

Substituting these expressions into firm 1's profit function, it can be checked that firm 1's second-stage profit is, π_1 =

$$[\, a_1 + t\sum_{i=1}^{n} x_i - (n + 1)tx_1][a_1 - ntx_1 + (n - 1)t\sum_{i=2}^{n} x_i \,] \frac{1}{b_1(n + 1)^2}$$

$$+ [\, a_2 + t\sum_{i=1}^{n} (L - x_i) - (n + 1)t(L - x_1)]$$

$$[a_2 - nt(L - x_1) + (n - 1)t\sum_{i=2}^{n} (\, L - x_i)] \frac{1}{b_2(n + 1)^2}$$

The second derivative of firm 1's profit is

$$\frac{\partial^2\pi_1}{\partial x_1^2} = 2[\frac{1}{b_1} + \frac{1}{b_2}] [\frac{nt}{n + 1}]^2 > 0.$$

Hence π_1 is strictly convex, which implies that firm 1 does not choose any location intermediate between the two markets. ▲

Concluding Comments

With Bertrand competition, firms selling homogenous products seek to differentiate their locations to soften price competition. In contrast to Cournot competition, this location pattern is robust to different consumer distributions. This paper focused on a common finding in the literature that Cournot oligopolists tend toward central agglomeration, and we show that the result is robust to any general symmetric distribution of consumers if the population density is not "too thin" at any point in the distribution. If the population density is "too thin" at the center, then Cournot competition does not yield central agglomeration. The case of two spatially separated markets was employed as an example of markets which are "too thin" at the center. For that case we find central agglomeration will never occur and firms may agglomerate in the larger market or seek maximal differentiation. This last result is the first which serves to explain the casual observation of dispersed locations but overlapping markets.

Although our model is quite simple, with a linear market and linear demand (designed to keep the analysis tractable), it highlights the contribution of a general consumer distribution on the pattern of firm locations. Alternative consumer distributions and their effects on location choice are a relatively neglected area in the context of Cournot oligopoly and this work is a first step to explore that area.

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