SEQUENTIAL ASYMMETRIC AUCTIONS WITH ENDOGENOUS PARTICIPATION[1]

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1. Introduction

Milgrom and Weber's (1982) seminal paper was a major step towards building an economic theory of auctions, on the foundations laid by the pioneering work of Vickrey (1961). In this process of building such theory economists have concentrated on the sale of a single object. One of the major concerns of theorists has been to examine the conditions under which different auction forms generate the same revenue.[2]

This theory has typically neglected the sale of more than one object through sequential or simultaneous auctions. The last type of auction has attracted some attention since government bonds are usually sold that way. Under some circumstances, it is possible to obtain revenue equivalence for multi-unit auctions.[3]

When objects are sold sequentially, Weber (1983) shows that expected revenue is the same under first or second price sealed bid auctions of identical objects with private or common values. In particular, Weber

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[2] See, for example, the surveys by McAfee and McMillan (1987) and Milgrom (1989).

[3] See, for example, Weber (1983) and Maskin and Riley (1989).

shows that expected prices follow a martingale:bidders expect that on average prices remain constant throughout the sequence of auctions.

In Weber's model, winners have to drop from the auction since each bidder can acquire at most one object. Thus two opposite effects cancel out: the decreased competition that forces prices to go down; and the fact that the number of objects remaining decreases from one period to the next, which makes bidders more aggressive and consequently forces prices to increase.

Ashenfelter (1989) and McAfee and Vincent (1993) have shown empirical evidence that prices are decreasing throughout a sequence of auctions of identical wine. McAfee and Vincent explain this phenomenon, referred to as the "price decline anomaly", assuming that bidders are risk averse so that they care more for the effects of the decreasing number of opportunities than they care for the reduced competition.

The price decline anomaly is not limited to sequential auctions of identical objects.Ashenfelter and Genesove (1992) and Engelbrecht-Wiggans and Kahn (1992) also report this phenomenon in auctions of,respectively, real estate and dairy cattle. While houses are different from each other,they have many common characteristics.The same is true for dairy cattle.

Thus Engelbrecht-Wiggans (1991) considers an independent private value model where bidders are symmetric and their values for the objects are independent draws from a fixed distribution. For these statistically identical objects, expected prices are decreasing if the sequence of auctions is sufficiently large and the distribution of values is bounded.

A common characteristic of these models is that bidders are allowed to buy only one object.Further it is assumed that losing bidders always stay for the next round and bid even if he or she has a very low probability of winning the next object.In fact,Engelbrecht-Wiggans and Menezes(1992) provide a necessary condition for all losing bidders to stay when they are allowed to drop out at any round. They may want to do so because bidders face delay costs if they decide to continue.This necessary condition requires that we start with the "right" number of objects and bidders.Hence requiring bidders to stay is a strong restriction.

Therefore,in this paper we consider a sale of stochastically independent objects:bidder's values for the objects are independent draws from potentially distinct distributions.In our model players face participation costs,bidders may buy as many objects they wish and are allowed to drop out at any round.

As in Menezes(1993), bidders learn their values prior to deciding whether or not to enter for the next sale.Bidders can drop out at any time,but they cannot come back to the auction.As a result, bidders stay for the remaining auctions only if their values are greater than a certain cut-off point. In particular we can determine the number of participants and expected prices in equilibrium.

Although the assumption of stochastically independent objects may limit the analysis,we claim that our model,despite its specialized nature,captures the flavor of certain actual auctions.For example,there are a number of similar objects that are usually sold sequentially:farm machinery,used restaurant equipment,dairy cattle,used cars,...

In our framework,bidder's decision set consists of two elements: a decision whether or not to stay(and consequently whether or not to incur participation costs) and a decision on how to bid.Since winners do not need to drop out,these auctions are independent in a very natural manner.The only link among auctions is the number of participants.

The contribution of this paper goes beyond the intent to explain the price decline anomaly.As we have mentioned, a number of auctions involve the sequential sale and it is useful to have a characterization of equilibrium in such auctions.Moreover it is our opinion that this model may shed some light on the construction of an economic theory that would explain why bidders participate in auctions.

This paper is organized as follows. We describe the model in section 2, while section 3 contains the main results. In section 4 we present some examples. Finally, the conclusions are summarized in section 5.

2. Model and Notation

We examine the case of two goods being sold sequentially through second price sealed bid auctions.[4] There is an arbitrary number of potential participants represented by the set I. Agents are risk neutral.

Let $\Omega = S \times T$ be the sample space representing the values of the two goods for each potential participant, that is, $X^i : S \to R_+$ denotes the value of the first object for participant $i \in I$ and $Y^i : T \to R_+$ the value for agent i of the second object. We assume that $\{X^i\}_{i \in I}$ and $\{Y^j\}_{j \in I}$ are independent and integrable.

Our analysis is concentrated in the independent private value model. However, it differ from the previous literature in at least two respects: bidders may be asymmetric – that is, they may draw their values from distinct distributions, and the number of participants at each stage is endogenous. Furthermore, agent $i \in I$ incurs a participation cost $c_i^j, j = 1, 2$ if he decides to stay for auction j. He makes this decision after seen his value for object j.Objects are stochastically independent in the sense that agent $i \in I$ learns his value for the first object prior to the first round but he still does not know his value for the next object. He knows only its distribution. At auction j, bidder i knows only his value and the distribution of his opponents' values. Participation costs are common knowledge.

We model sequential auctions where a bidder may bid on as many objects he wishes but if he decides to drop out from the auction he

[4] Observe that there is nothing special about the two-object assumption, and we expect our results to hold for any number of objects.

cannot return. In our model bidders may want to drop out because there is an explicit cost to continue.

We show in the next section that there is an equilibrium that survives iterated elimination of dominated strategies consisting of two dimensions: the decision whether or not to stay and a bidding strategy. Bidder i decides to stay if his value is greater than a certain cut-off point, denoted by q_i^j, for $j = 1, 2$, which depends on the participation costs in both periods and on his expected profits for the following round. Telling the truth in both periods is an optimal bidding strategy under this scenario.

For $q^1 \in R^I$ (R^I is the set of real valued functions with domain I) and $s \in S$, we define $H(s) = H(q^1, s) = \{i \in I \colon X^i(s) \geq q_i^1\}$. For $N \subset I, q^2 \in R^N$, and $t \in T$, the following set is defined $H_N(t) = H_N(q^2, t) = \{i \in N \colon Y^i(t) \geq q_i^2\}$. In equilibrium, these sets indicate the optimal number of participants at each round.

In the next section we show that there exist such cut-off points and we construct an equilibrium where agents follow this rule in their decision whether or not to stay and they bid their true values at each round. Intuitively,one can understand why this happens. In Engelbrecht-Wiggans (1991), participants (exogenously determined) submit bids that are equal to the difference between their values and their future expected profits.This last amount represents how much one loses when winning an object and leaving the auction. Hence, our result may not sound surprising. However, in our model bidders pay a participation cost, and in general this would result in participants reducing their bids by the same amount. Since in our model bidders decision to stay is endogenous, if they decide to stay then the costs incurred by them become sunk costs.

3. Main Theorem

Our objective in this section is two-fold. First, we want to show that there are $(q_i^1)_{i\in I}$ and $(q_i^2)_{i\in N, N\subset I}$ satisfying the following two conditions:

1) Agent $i, i \in I$, participates in the first auction if and only if $X^i(s) \geq q_i^1$;
2) If $N = H(s), i \in N$ participates in the second auction if and only if $Y^i(t) \geq q_i^2$.

Second, we want to compute an optimal bidding strategy. This result is summarized in the next theorem.

Theorem. *If the distributions of X^i and Y^i are continuous and integrable then there exist $(q_i^1)_{i\in I}$ and $(q_i^2)_{i\in N, N\subset I}$ satisfying (1) and (2). Further, there is an equilibrium that survives the iterated elimination of dominated strategies such that each remaining participant submits a bid equal to his value and he decides to stay or not according to the rule above.*

We need the following lemmas.

Lemma 1. *If $\emptyset \neq N \subset I$ then there exist $q^N \in \mathbf{R}_+^N$ such that:*

$$E_t\left\{\left(q_i^N - \max_{j\in N\setminus\{i\}} Y^j \chi_{Y^j \geq q_j^N}\right)^+ - c_i^2\right\} = 0, \quad \forall\ i \in N$$

Proof. For $p \in \mathbf{R}_+^N$ and $i \in N$, let

$$\varphi_i(p) = E_t\left\{\left(p_i - \max_{j\in N\setminus\{i\}} Y^j \chi_{Y^j \geq p_j}\right)^+ - c_i^2\right\}$$

We define $\varphi(p) = (\varphi_i(p))_{i\in N}$ and $H(p) = -\varphi(p) + p$.

Observe that H is continuous because the distribution of the Ys is continuous. Let $\tilde{c} = (c_i^2)_{i\in N}$

From $\varphi_i(p) \leq p_i - c_i^2$, we have that

$$H(p) \geq -p + \tilde{c} + p = \tilde{c}$$

But

$$\varphi_i(p) \geq E_t\Big[\Big(p_i - \max_{j\neq i} Y^j\Big)^+ - c_i^2\Big]$$

$$= \int\limits_{p_i \geq \tilde{Y}_i} (p_i - \tilde{Y}_i) dF_i - c_i^2, \quad \text{where}$$

$$\tilde{Y}_i = \max_{j\neq i} Y^j$$

Hence,

$$H_i(p) \leq c_i^2 - \int\limits_{p_i \geq \tilde{Y}_i} (p_i - \tilde{Y}_i) dF_i + p_i$$

$$= c_i^2 + \int\limits_{p_i \geq \tilde{Y}_i} \tilde{Y}_i dF_i + \int\limits_{p_i < \tilde{Y}_i} p_i dF_i \leq c_i^2 + E\tilde{Y}_i =: \alpha.$$

Thus, if $X = \prod_{i\in N} [c_i^2, \alpha_i]$, $H|_X : X \to X$,Brouwer's Fixed Point Theorem guarantees that H has a fixed point, and consequently φ has a zero. $\square$

Lemma 2. *There exist $(q_i^1)_{i\in I} \in \mathbf{R}_+^I$ and $(q_i^2)_{i\in N, N\subset I}$ satisfying the following two conditions:*

i)

$$E\left\{\Big(q_i^1 - \max_{j\in H(s)\setminus\{i\}} X^j\Big)^+ - c_i^1 + \Big(E\Big[\Big(Y^i - \max_{j\in H_{H(s)}(t)\setminus\{i\}} Y^j\Big)^+ - c_i^2 | Y^i\Big]\Big)^+\right\} \geq 0$$

with equality if $q_i^1 > 0$; and for $N = H(s)$:

ii)

$$E_t\left\{\Big(q_i^2 - \max_{j\in H_N(t)\setminus\{i\}} Y^j\Big)^+ - c_i^2\right\} = 0, \quad \text{if} \quad i \in N$$

Proof. First, we obtain (ii). Define $(q_i^2)_{i\in N} = q^N$ as given by lemma 1.

Next, we obtain (i). For $q^1 \in \mathbf{R}_+^I$, and for $i \in I$, we define $\psi^i(q^1)$ as follows(please keep in mind that q_j^2 below depends on $H(s)$)

$$\psi^i(q^1) = E\left\{ \left(q_i^1 - \max_{j \neq i} X^j \chi_{X^j \geq q_j^1}\right)^+ - c_i^1 + \right.$$
$$\left. + \left(E\left[\left(Y^i - \max_{j \in H(s)\setminus\{i\}} Y^j \chi_{Y^j \geq q_j^2}\right)^+ - c_i^2 | Y^i\right]\right)^+ \right\}$$

Note that

$$\max_{j \neq i} X^j \chi_{X^j \geq q_j^1} = \max_{j \in H(s)\setminus\{i\}} X^j$$

and

$$\max_{j \in H(s)\setminus\{i\}} Y^j \chi_{Y^j \geq q_j^2} = \max_{j \in H_{H(s)}(t)\setminus\{i\}} Y^j$$

we then set $\theta(q^1) = (\psi^i(q^1) \wedge q_i^1)_{i \in I}$ and $H(q^1) = -\theta(q^1) + q^1$.

We need to show that H has a fixed point. We first show that H is continuous. For that, it suffices to show that ψ^i is continuous:

$$\psi^i(q^1) = \sum_{N \subset 2^I} E\chi_{H(s)=N} \left\{ \left(q_i^1 - \max_{j \in N\setminus\{i\}} X^j\right)^+ - c_i^1 + \left(E\left[Y^i - \max_{j \in N\setminus\{i\}} Y^j \chi_{Y^j \geq q_j^2}\right]\right)^+ \right\}$$

But $\chi_{H(s)=N} = \prod_{j \in N} \chi_{X^j \geq q_j^1} \prod_{j \notin N} \chi_{X^j < q_j^1}$ and the joint distribution of $(X^1, \cdots, X^I)$ is continuous. Therefore, ψ^i is continuous.

From $\psi^i(q^1) \wedge q_i^1 \leq q_i^1$, we conclude that $-\theta^i(q^1) \geq -q_i^1$ and, thus, $H^i(q^1) \geq 0$.

Further, $\psi^i(q^1) \wedge q_i^1 \geq \psi^i(q^1)$ implies that

$$-\theta^i(q^1) \leq -\psi^i(q^1), \quad \text{and}$$

$$H^i(q^1) \leq q_i^1 - \psi^i(q^1)$$

We are now able to compute $q_i^1 - \psi^i(q^1)$.

$$\psi^i(q^1) \geq E\left(q_i^1 - \max_{j \neq i} X^j \chi_{X^j \geq q_j^1}\right)^+ - c_i^1 \geq E(q_i^1 - \tilde{X}^i)^+ - c_i^1,$$

where $\tilde{X}^i = \max_{j \neq i} X^j$.

We conclude that $\psi^i(q^1) \geq \int_{q_i^1 \geq \tilde{X}_i} (q_i^1 - \tilde{X}^i) dF_i - c_i^1$; and consequently

$$q_i^1 - \psi^i(q^1) \leq q_i^1 + c_i^1 - \int_{q_i^1 \geq \tilde{X}_i} (q_i^1 - \tilde{X}_i) dF_i =$$

$$\int_{q_i^1 < \tilde{X}_i} q_i^1 dF_i + \int_{q_i^1 \geq \tilde{X}_i} \tilde{X}_i dF_i + c_i^1 \leq E\tilde{X}_i + c_i^1 =: \beta^i$$

Hence, if $q \in X = \prod_{i \in I} [0, \beta^i]$, $\quad 0 \leq H_i(q^1) \leq \beta^i$, that is, $H|_X : X \to X$. Let q^1 be a fixed point of H. Then, $\theta^i(q^1) = 0 = \psi^i(q^1) \wedge q_i^1 \leq \psi^i(q^1)$. Finally if $q_i^1 > 0$ then $\psi^i(q^1) = 0$, which ends the proof of this lemma. $\square$

We now prove our main theorem

Proof. If $q_i^1 = 0$, then i's expected profit is non-negative (Lemma 2(i)). Hence, he participates in the first auction if $q_i^1 = 0$. Lemma 2(i) also guarantees that if $q_i^1 > 0$ then his expected profits when $X^i(s) = q_i^1$ are equal to zero. Whenever $X^i(s) > q_i^1$, his expected profits are positive, and they are negative if $X^i(s) < q_i^1$. Consequently, i participates in the first auction if $X^i(s) \geq q_i^1$. Lemma 2(ii) may be used to show that i's participation constraint also holds in the second auction.

It remains to be shown that "i always submits a bid equal to his value" is an equilibrium bidding strategy for agent i. In fact, an equilibrium in our model is described by a pair $(b_i^*, q_i^*)_{i \in I}$ representing, respectively i's bidding strategy and his cut-off point,$q_i^* = (q_i^1, q_i^2)$ for each round.

Notice that the last auction is a single-object second price real bid auction with $H_N(t)$ agents. In this case, it is immediate that bidding his true value is an optimal bidding strategy.

Participation costs are sunk costs. Given that i has paid his participation costs the best he can do is to bid his value. But the same reasoning applies to the first round, since the only difference is the number of players at each round. □

4. Examples

In this section we examine various simple examples where the application of our equilibrium analysis is straightforward.

4.1 $I = \{i\}$, two stochastically independent objects

The solution in this case is trivial, namely:

$$q_i^1 = \Big(c_i^1 - E\Big(E\big[(Y^i - \max_{j \in H_N(t)\setminus\{i\}} Y^j)^+ - c_i^2 | Y^i\big]\Big)^+\Big)^+$$

$$q_i^2 = c_i^2$$

that is, bidder i participates in the first auction if his value is greater than $c_i^1 - E[Y^i] - c_i^2$ where $E[Y^i]$ denotes this expected value for the second object.

4.2 $I = \{i, j\}$, two stochastically independent objects.

Let X^i, Y^j be random variables independent and uniformly distributed on $[0, 1]$. We assume that $c_i^2 \geq c_j^2$ and $\gamma^i = c_i^2 + \sqrt{(c_i^2) - (c_j^2)^2} \leq 1$; and $c_j^1 = 0$. We have that $q_{\{i\}}^2 = c_i^2, \quad q_{\{j\}}^2 = c_j^2$, and

$$(q_{\{i,j\}}^i, q_{\{i,j\}}^j) = \left(\sqrt{\gamma^i}, \frac{c_j}{\sqrt{\gamma^i}}\right)$$

4.3 Two stochastically identical objects,I symmetric participants:

$$X^a \sim X, Y^a \sim Y \sim X, \;\; c_a^1 = c_a^2 = c, \;\; \forall\, a, a \in I.$$

This example generalizes the work of Menezes (1993) by allowing bidders to buy more than one object.

Let $q_i^1 = q^1, \ \forall\, i \in I$ and $q_i^2 = q_N^2, \ \forall\, i \in N$. We want a solution for the following equations:

(i)

$$E\left\{ \left(q_i^1 - \max_{j \in H(s)\setminus\{i\}} X^j\right)^+ - c + \left(E\left[\left(Y^i - \max_{j \in H_N(t)\setminus\{i\}} Y^j\right)^+ - c|Y^i\right)^+ \right\} = 0$$

and

$$\text{(ii)} \qquad E\left\{ \left(q_i^2 - \max_{j \in H_N(t)\setminus\{i\}} Y^j\right)^+ - c\right\} = 0 \quad \forall\, i \in H(s)$$

Therefore we need that

$$q_N^2 P(Y^j < q_N^2, \quad \forall\, j \in N \setminus \{i\}) = c$$

or

$$q_N^2 F_X(q_N^2)^{\#N-1} = c$$

where $\#N-1$ denotes the number of elements in $N-1$. Let $\overline{q}_n$ be the solution of $qF_Y(q)^{n-1} = c, \quad 1 \leq n \leq \#I$. Thus $q_N^2 = \overline{q}_{\#N}$. Further $c = \overline{q}_1 \leq \overline{q}_2 \leq \cdots \leq \overline{q}_{\#I}$ because

$$\overline{q}_n F_Y(\overline{q}_n)^{n-1} = c = \overline{q}_{n+1} F_Y(\overline{q}_{n+1})^n \leq$$
$$\leq \overline{q}_{n+1} F_Y(\overline{q}_{n+1})^{n-1} \quad \text{implies that} \quad \overline{q}_{n+1} \geq \overline{q}_n$$

since $F_Y(q)$ is non-decreasing in q. Therefore, the solution to the decision whether to stay for the second stage is equivalent to the solution indicated by Menezes (1993). Remember that i's optimal bidding strategy is still to submit a bid equal to his value.

We now compute the solution for the first stage, which differs from Menezes (1993), because we now assume that participants may bid for both objects.

For $N \subset 2^I$, define

$$A_N := E \prod_{i \in N} \chi_{X^i \geq q^i} \prod_{i \notin N} \chi_{X^i < q^i} \left\{ \left(q^1 - \max_{j \in N \setminus \{i\}} X^j \right)^+ - c + \right.$$

$$\left. + \left(E\left[\left(Y^i - \max_{j \in N \setminus \{i\}} Y^j \right)^+ - c | Y^i \right] \right)^+ \right\}.$$

But $\chi_{H(s)=N} = \prod_{i \in N} \chi_{X^i \geq q^1} \prod_{i \notin N} \chi_{X^i < q^1}$, and consequently $\Sigma_{N \subset I} A_N = 0$.

Let $\alpha = F_X(q^1)$. By definition:

$$A\phi = \alpha^{\#I} \{ q^1 - c + E(Y^i - c)^+ \}$$

$$A_{\{i\}} = (1-\alpha)\alpha^{\#I-1} \{ q^1 - c + E(Y^i - c)^+ \}$$

Notice that if $s \in N$, $N \neq \{i\}$, then

$$\prod_{i \in N} \chi_{X^i \geq q^1} \cdot (q^1 - \max_{j \in N \setminus \{1\}} X^j)^+ = 0.$$

Hence:

$$A_N = (1-\alpha)^{\#N} \alpha^{\#I-\#N} \left\{ -c + E\left(E\left[\left(Y^1 - \max_{\substack{j \in N \setminus \{1\} \\ Y^j \geq q_N^2}} Y^j \right)^+ - c | Y^1 \right] \right)^+ \right\}$$

If $s \notin N$, $N \neq \phi$ then

$$A_N = (1-\alpha)^{\#N} \alpha^{\#I-\#N} \left\{ -c + E\left(E\left[\left(Y^1 - \max_{j \in N} Y^j \right)^+ - c | Y^1 \right] \right)^+ \right\}.$$

Let

$$\hat{Y}^n = \max_{2 \leq j \leq n+1} Y^j \chi_{Y^j \geq \overline{q}_n},$$

$$\tilde{Y}^n = \max_{2 \leq j \leq n} Y^j \cdot \chi_{Y^j \geq \overline{q}_n} \quad \text{and} \quad \tilde{Y}^i := 0.$$

We define

$$U^n = E\Big(E(Y^1 - \tilde{Y}^n)^+ - c|Y^1]\Big)^+ \quad \text{and}$$

$$V^n = E\Big(E(Y^1 - \hat{Y}^n)^+ - c|Y^1]\Big)^+ .$$

Therefore, we can write

$$A\phi + \sum_{n=1}^{\#I} \sum_{1\in N} A_N + \sum_{n=1}^{\#I-1} \sum_{\substack{\#N=n \\ 1\notin N}} A_n = 0$$

which implies that

$$\alpha^{\#I}\{q^1 - c + E(Y^1 - c)^+\} + \sum_{n=1}^{\#I} C_{I-I}^{n-1}(1-\alpha)^n \alpha^{\#I-n}\{-c + U^n\} +$$

$$+ \sum_{n=1}^{\#I-1} C_{I-1}^{n}(1-\alpha)^n \alpha^{\#I-n}\{-c + V^n\} = 0.$$

Using the known identity $C_{I-1}^{n-1} + C_{I-1}^{n} = C_I^n$ we have that:

$$\alpha^{\#I}\{q^1 + E(Y^1 - c)^+\} + \sum_{n=1}^{I-1}(C_{I-1}^{n-1}U^n + C_{I-1}^{n}V^n)(1-\alpha)^n\alpha^{\#I-n} + (1-\alpha)^{\#I}U^{\#I} = c\,.$$

This equation represents the solution q^1 for each bidder's decision to stay for the first auction. It differs from the second round's decision since bidders take into consideration the possibility that they have a value lower than their entry cost in the first round, but a value for the second object that more than compensates his first period losses.

We now compute expected prices when F_X is equal to F_Y. The expected price in the first round is the second order statistics of $(X^j \chi_{X^j \geq q^1})_{j\in N}$, if $H(s) = N$ and $N \neq \phi$. Notice that this amounts to the second order statistics of $(X^j)_{j\in I}$. Thus, the expected price in the first round equals the second order statistics of $(X^j)\ j \in I$ for all $s \in S$. Similarly, the expected price in the second round equals $(Y^j \chi_{Y^j \geq q_N^2})_{j \in N}$ for $H(s) = N$. But this is equal to the second order statistics of $(Y^j)_{j\in N}$, which has the same distribution of $(X^j)_{j\in N}$. Notice that $(X^j)_{j\in N}$ is not greater than the second order statistics of $(X^j)_{j\in I}$. As a result, expected prices are non-increasing.

The next example suggests what may happen when objects are identical.

4.4. Two identical objects, $I = \{i, j\}$ symmetric participants.

For simplicity let us consider that valuations are uniformly distributed on $[0, 1]$ and $c_i = c_j = c \in [0, 1]$. First we suggest that the following equilibrium survives the iterated elimination of weakly dominated strategies: "l do not participate in the auction if $v_l \leq c$ and l submits a bid of $2v_l - 2c, \quad l = i, j$, in the first round otherwise. If agent j wins the first object then he bids his value for the second object and agent i drops out. If i wins the first object he bids his value in the second round and player j drops out".

Note that both players obtain non-negative expected profits in the whole game. Furthermore this equilibrium is robust to a possible tremble of the losing player, who may bid a positive amount in the next round. This profile remains if such tremble is lower than the difference between the two values.

The intuition for this equilibrium is the notion of signalling. Each player wants to show that he can accept losses in the first auction given that by doing this he guarantees being the only player left in the second round.Signalling results from the fact that bidders may invert the bidding strategies to discover their opponnents' values.

For the same reason, the equilibrium bidding strategy described in the previous section is not even a Nash equilibrium for this game, that is, if both bidders submit their values in the two rounds (conditional on having a value greater than q), it is not optimal for the losing player to continue.

Finally, the equilibrium bidding strategy described by Weber (1983), which is equivalent in this example to have players bidding their values (net of participation costs) in the second round and 1/2 of their values (net of costs) in the first round, is not a Nash equilibrium for this game either. The losing player in the first round would drop out again.

5. Conclusion

In this paper we examine sequential asymmetric auctions with an endogenous number of participants. In our model asymmetric players may buy as many objects they want and are allowed to drop out from the auction at any time.

For the case of stochastically independent objects, we show that every bidder who decides to continue submits a bid that is equal to his value at each round. This happens because only those players with a value greater than a certain cut-off value show up and pay their participation costs.

Thus in equilibrium bidders choose both an optimal bidding strategy and a cut-off point such that if they have values greater than this,they decide to stay. Otherwise,they drop out.

When players are symmetric and objects are stochastically identical we are able to show that expected prices are decreasing. In this case,why should a seller choose a sequential auction instead of a simultaneous auction? However,when players are asymmetric, as we expect in real auctions,expected prices can go in either direction.

Finally,example 4.4 suggest that when players may buy more than one object,in auctions of identical goods,Weber's result fails to hold.Expected prices may decrease as a consequence of signalling effects.Nevertheless,we still have to provide a more general theory encompassing the case of identical objects.

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