The Pareto-Efficient Relativity of Relative Risk Aversion

Abstract

In a pure-exchange economy involving one perishable consumption good and risk-averse consumers, the elasticity of a consumer's Pareto-efficient consumption with respect to aggregate output equals the reciprocal of the ratio of the consumer's coefficient of relative risk aversion to average relative risk aversion. Therefore, this elasticity is unity for someone with average relative risk aversion, whereas consumers with above average relative risk aversion transfer some of their aggregate-output risk to consumers with below average relative risk aversion. This result has important implications on the financial securities needed to complete markets, inflation indexing, and central bank goals and targeting objectives.

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The Pareto-Efficient Relativity of Relative Risk Aversion

Consider a consumer's coefficient of relative risk aversion as a ratio to average relative risk aversion for the whole economy. That ratio, as this paper shows, is the sole determinant of the elasticity of the consumer's Pareto-efficient consumption with respect to aggregate output. In fact this elasticity just equals the reciprocal of the ratio. We label this relationship between this elasticity and this ratio as "the Pareto-efficient relativity of relative risk aversion."

This relationship implies that consumers with average relative risk aversion will have unity elasticities, which is what must happen on average since the sum of consumption must equal aggregate output. Consumers with above average relative risk aversion will have elasticities less than one, whereas consumers with below average relative risk aversion will have elasticities greater than one. In essence, in a competitive equilibrium with complete markets, consumers with above average relative risk aversion will transfer some of their aggregate-output risk to consumers with below-average-relative risk aversion.

In a pure exchange economy without storage, aggregate-output risk is the only source of undiversifiable risk that the market can handle.[1] Even in a more elaborate economy with production, capital, and storage; aggregate-output risk will remain an important risk. Thus, GDP futures as proposed by Shiller (1993) will be important not only to help diversify aggregate-output across countries as Shiller argues, but also to enable consumers with more than average relative risk aversion to transfer some aggregate-output risk to consumers with less than average relative risk aversion.

[1] Eagle (2005a) separates utility shocks into individual utility shocks that can be diversified and aggregate utility shocks that cannot be diversified. However, since Pareto-efficient consumption is unaffected by the aggregate utility shocks by Eagle's definition, the markets do not handle these aggregate utility shocks. Hence the only undiversifiable risk in a pure-exchange economy is aggregate-output risk.

The Pareto-efficient relativity of relative risk aversion has other "real-world" implications including central-bank goals and objectives and an advantage that nominal contracts have over inflation-indexed contracts. Section 4 of this paper discusses these implications.

Other economists have derived related results. Wilson (1968, see his theorem 5) derives the relationship between the derivative of a consumer's Pareto-efficient consumption with respect to aggregate output and how the consumer's absolute risk aversion compares to an aggregation of absolute risk aversion. Viard (1993) and Bodie, Kane, and McDonald (1985) use Merton's (1971) portfolio analysis to assess consumer welfare and relate choices involving inflation-indexed bonds to the harmonic mean of consumers' coefficients of relative risk aversion.

Since complete markets are required in order for a competitive equilibrium to achieve Pareto-efficiency, the next section (section 2) describes a very general Arrow-Debreu pure-exchange economy with state-contingent securities; this section then presents and proves the Consumption-Aggregate-Supply-Invariance Property. Section 3 then uses this property to derive relationship we call the Pareto-efficient relativity of relative risk aversion. Readers wishing to skip the model details, proofs, and derivations in sections 2 and 3 may skip directly to Section 4, which interprets the Pareto-efficient-relativity of relative risk aversion and discusses its real-world implications.

2. The Consumption-Aggregate-Supply Invariance Property

This section reviews a standard Arrow-Debreu[2] pure exchange economy with a finite horizon T and one nonstorable consumption good and then presents and proves the

[2] See Arrow (1953 and 1964) and Debreu (1959).

Consumption-Aggregate-Supply-Invariance Property. Other than assuming time-additively separable utility functions, we otherwise make the economy as general as we can by including individual utility shocks and individual utility functions.

Assume each consumer j's time-separable utility function is:

$$\xi_{j0}U_{j0}(c_{j0})+\sum_{t=1}^{T}\beta^t\sum_{s=1}^{n_t}\pi_{st}\xi_{jst}U_{jt}(c_{jst}) \qquad (1)$$

where c_{j0} is j's consumption at time 0, c_{jst} is j's consumption in state s at time t, β is the time discount factor,[3] and π_{st} is the probability of state s occurring at time t. The random variables ξ_{j0} and ξ_{jst} for state s and time t are the utility shocks for consumer j. The functions $U_{j0}(c_{j0})$ and $U_{jt}(c_{jst})$ are continuous, twice differentiable, strictly concave, strictly increasing, and satisfy the Inada conditions that $U'(0)\equiv\lim_{x\to 0}U'(x)=\infty$ and $U'(\infty)\equiv\lim_{x\to\infty}U'(x)=0$. The time frame for the s subscript is determined by the t subscript next to the s subscript. For example, the s in c_{js1} refers to one of the possible states that can occur at time 1.

At time 0, consumers can buy or sell state-contingent securities. These state-contingent securities are prepaid securities where the buyer pays the seller the price of the security at time 0. Let x_{jst} represent individual j's demand at time 0 for the state-contingent security that delivers one consumption good at time t if and only if state s occurs at time t. Define Ω_{st} so that the price of this security equals $P_0\pi_{st}\Omega_{st}$. With it so defined, Ω_{st} represents the real pricing kernel.

Each consumer j chooses x_{jst} for all s and t to maximize (1) subject to:

[3] Because we allow different consumers to have different utility functions, the assumption of a common time-discount factor is not restrictive. Suppose that instead, consumer j's utility is $\beta_{jit}^t\xi_{jst}\tilde{U}_{jt}(c_{jit})$ where $\tilde{U}$ denotes the true utility function. If we set our beta equal to one and defined $U_{jt}(c_{jit})\equiv\beta_{jit}^t\xi_{jst}\tilde{U}_{jt}(c_{jit})$, our formulation would take this situation into account.

$$P_0 c_{j0} + P_0 \sum_{t=1}^{T} \sum_{s=1}^{S_t} \pi_{st} \Omega_{st} x_{jst} = P_0 y_{j0} \tag{2}$$

$$c_{jst} = y_{jst} + x_{jst} \tag{3}$$

where (3) applies for all s=1,2,…,S_t for all t=1,2,…,T where S_t is the finite number of states of nature at time t.

The market clearing conditions are that $\sum_{j=1}^{n} c_{j0} = Y_0$, $\sum_{j=1}^{n} c_{jst} = Y_{st}$, and $\sum_{j=1}^{n} x_{jst} = 0$ for all states s at time t and for t=1,2,…T, where the aggregate supply of the consumption good is represented by Y_0 at time 0 and Y_{st} in state s at time t respectively. Consumer j's optimization problem is satisfied when $\frac{\xi_{j0} U'_{j0}(c_{j0})}{P_0} = \frac{\beta^t \pi_{st} \xi_{jst} U'_{jt}(c_{jst})}{P_0 \pi_{st} \Omega_{st}}$ for all s=1,2,…,S_t and for all t=1,2,…,T, which implies that

$$\Omega_{st} = \frac{\beta^t \xi_{jst} U'_{jt}(c_{jst})}{\xi_{j0} U'_{j0}(c_{j0})} \tag{4}$$

The left side of (4) is the real pricing kernel and the right side is the intertemporal marginal rate of substitution. Some literature mistakenly defines the pricing kernel as the intertemporal marginal rate of substitution (See, for example, Campbell, Lo, and MacKinlay, 1997, p. 294). The equality between the real pricing kernel and the intertemporal marginal rate of substitution shown in (4) is an equilibrium condition not a definition.

Since this is a standard one-good Arrow-Debreu pure-exchange economy with well behaved utility functions, a unique competitive equilibrium exists and that competitive equilibrium is Pareto efficient. Also, the following property holds:

Consumption-Aggregate-Supply Invariance Property:[4] In a pure exchange economy with one nonstorable consumption good and risk-averse consumers, each individual j's Pareto-efficient consumption at any time t will vary across states of nature only when one of two factors changes. These two factors are (i) aggregate output and (ii) utility shocks, either j's utility shocks or other consumers' utility shocks.

Proof: We need to show that for any two states of nature where aggregate output and utility shocks are the same, then each consumer j's Pareto-efficient consumption must be the same for those two states of nature. For some time t, let 1 and 2 represent any two different states of nature where $Y_{1t} = Y_{2t}$ and for all consumers j, $\xi_{j1t} = \xi_{j2t}$. So we can prove this property by contradiction, assume there are two individuals j and k such that $c_{j1t} < c_{j2t}$ and $c_{k1t} > c_{k2t}$. Since this is an Arrow-Debreu competitive equilibrium, the consumption allocation must be Pareto efficient. Define $\tilde{c}_{j1t} \equiv \frac{1}{2}\left(c_{j1t} + c_{j2t}\right)$ and $\tilde{c}_{k1t} \equiv \frac{1}{2}\left(c_{k1t} + c_{k2t}\right)$. Define a new consumption allocation where for all consumers, for all states of nature, and for all time periods, the new consumption equals the old consumption except that j's consumption in states 1 and 2 are both $\tilde{c}_{j1t}$ and k's consumption in states 1 and 2 are both $\tilde{c}_{k1t}$. The new consumption allocation is obviously feasible since the original allocation was feasible. Because both j and k are strictly risk averse, they are both better off with this new consumption allocation. However, that contradicts the statement that the original consumption allocation is Pareto

[4] In macroeconomics "aggregate supply" is used to represent an aggregation of aggregate outputs across different goods. However, much of macroeconomic theory assumes one consumption good. While when only one good exists, the term "aggregate supply" is synonymous with the term "aggregate output", the term "aggregate output" is less confusing. However, because the author is presenting consumption-aggregate-supply invariance property in macroeconomic literature as well, he has decided to keep the same label on this property in both literatures he writes in. Also, when he discusses central bank targeting later in the paper, he switches to using the term "aggregate supply" in order to be consistent with the macroeconomic literature.

efficient. We, therefore, conclude that the Pareto-efficient consumption allocation must be the same as long as aggregate output and the utility shocks are the same. Q.E.D.

To avoid confusion, we need to be careful what we mean by "the Pareto-efficient consumption allocation." Usually, there are many Pareto-efficient consumption allocations, perhaps an infinite number of these Pareto-efficient consumption allocations. It is true that usually a unique Pareto-efficient consumption allocation corresponds to the competitive Arrow-Debreu equilibrium for a given endowment. We could think about "the Pareto-efficient consumption allocation" as the allocation that corresponds to the existing endowment allocation. However, we need not do so as the following two paragraphs explain:

A consumption allocation is where c_{jst} is defined for all j, s, and t. In other words, a consumption allocation is a mapping from j, s, and t to consumption. The set of all Pareto-efficient consumption allocations is a subset of all possible consumption allocations. What the Consumption-Aggregate-Supply-Invariance Property states is that for a particular j and t; when s varies, any Pareto-efficient consumption allocation will vary when s varies only if either aggregate output at time t changes or some utility shock at time t changes. It is true that there could be a different Pareto-efficient consumption allocation that is a different mapping from j, s, and t to consumption. However, that Pareto-efficient consumption allocation would also have to meet the Consumption-Aggregate-Supply-Invariance Property in that if s varies, the Pareto-efficient consumption will change only if aggregate output changes or some utility shock changes.

By the Consumption-Aggregate-Supply-Invariance Property, we can write any Pareto-efficient consumption allocation at time t as a function solely of aggregate output and utility shocks. In other words, we can write this function as $\tilde{c}_{jt}(Y_{st},\vec{\xi}_t)$ where the vector

$\vec{\xi}_t \equiv (\xi_{1st}, \xi_{2st}, ..., \xi_{mst})$. Note that another Pareto-efficient consumption allocation can also be written as a different function of aggregate output and utility shocks. Both functions, however, will only vary with aggregate output and utility shocks.

The importance of the Consumption-Aggregate-Supply-Invariance Property is that it enables us to write any Pareto-efficient consumption allocation as a function solely of aggregate output and utility shocks. It is important to note that this function is a reduced form relationship; it is **not** the structural consumption function. To help us avoid this confusion, we refer to Y_t as aggregate output at time t, not income.

The next section uses such a function to derive the relationship between a consumer j's relative risk aversion and the elasticity of j's Pareto-efficient consumption with respect to aggregate output.

3. Derivation of the Pareto-Efficient Relativity of Relative-Risk-Aversion

The previous section showed that we can write Pareto-efficient consumption solely as a function of aggregate output and utility shocks. For notational simplicity, we will suppress the s subscripts in this section. Define $\tilde{a}_{jt}(Y_t, \vec{\xi}_t) \equiv -\frac{U''_{jt}\left(\tilde{c}_{jt}\left(Y_t, \vec{\xi}_t\right)\right)}{U'_{jt}\left(\tilde{c}_{jt}\left(Y_t, \vec{\xi}_t\right)\right)}$, which is the function of how the coefficient of absolute risk aversion varies with real aggregate supply and utility shocks. Define $\tilde{\rho}_{jt}\left(Y_t, \vec{\xi}_t\right) \equiv \tilde{c}_{jt}\left(Y_t, \vec{\xi}_t\right) \cdot \tilde{a}_{jt}\left(Y_t, \vec{\xi}_t\right)$, which is the function of how the relative risk coefficient varies with real aggregate supply and utility shocks. Also, define $\overline{\rho}_t\left(Y_t, \vec{\xi}_t\right) \equiv \sum_{j=1}^{m}\left(\tilde{\rho}_{jt}\left(Y_t, \vec{\xi}_t\right) \cdot \tilde{c}'_{jt}(Y_t, \vec{\xi}_t)\right)$, which is the average of the relative risk coefficients weighted by the partial derivatives of $\tilde{c}_{jt}$

with respect to aggregate output. Finally, define $\tilde{\alpha}_j\left(Y_t,\vec{\xi}_t\right)\equiv\frac{\tilde{\rho}_{jt}\left(Y_t,\vec{\xi}_t\right)}{\overline{\rho}_t\left(Y_t,\vec{\xi}_t\right)}$, which is how j's relative risk coefficient compares to the average relative risk coefficient. In each definition, the "wiggle" symbol "~" means the function applies to a consumption allocation that is Pareto-efficient.

Since equation (4) is true for all consumers, $\frac{\beta^t\xi_{jt}U'_{jt}(\tilde{c}_{jt})}{\xi_{j0}U'_{j0}(c_{j0})}=\frac{\beta^t\xi_{1t}U'_{1t}(\tilde{c}_{1t})}{\xi_{10}U'_{1,0}(c_{1,0})}$, which implies:

$$\frac{\xi_{jt}U'_{jt}(\tilde{c}_{jt})}{\xi_{j0}U'_{j0}(c_{j0})}=\frac{\xi_{1t}U'_{1t}(\tilde{c}_{1t})}{\xi_{10}U'_{1,0}(c_{1,0})} \tag{5}$$

for j=2..m. Totally differentiating (5) with respect to Y_t gives:

$$\frac{\xi_{jt}U''_{jt}(\tilde{c}_{jt})}{\xi_{j0}U'_{j0}(c_{j0})}\frac{\partial\tilde{c}_{jt}}{\partial Y_t}=\frac{\xi_{1t}U''_{1t}(\tilde{c}_{1t})}{\xi_{10}U'_{1,0}(c_{1,0})}\frac{\partial\tilde{c}_{1t}}{\partial Y_t}$$

Dividing the left and right sides of this by the left and rights sides of (5) respectively gives:

$$\frac{U''_{jt}(\tilde{c}_{jt})}{U'_{jt}(\tilde{c}_{jt})}\frac{\partial\tilde{c}_{jt}}{\partial Y_t}=\frac{U''_{1t}(\tilde{c}_{1t})}{U'_{1t}(\tilde{c}_{1t})}\frac{\partial\tilde{c}_{1t}}{\partial Y_t} \tag{6}$$

Substituting $\tilde{a}_{jt}\equiv-\frac{U''_{jt}\left(\tilde{c}_{jt}\right)}{U'_{jt}\left(\tilde{c}_{jt}\right)}$ into (6) and multiplying both sides by a minus sign and rearranging slightly gives:

$$\frac{\frac{\partial\tilde{c}_{jt}}{\partial Y_t}}{\frac{\partial\tilde{c}_{1t}}{\partial Y_t}}=\frac{\tilde{a}_{1t}}{\tilde{a}_{jt}} \tag{7}$$

By summing both sides of (7) over all consumers, we get:

$$\frac{\sum_{j=1}^{m}\frac{\partial\tilde{c}_{jt}}{\partial Y_t}}{\frac{\partial\tilde{c}_{1t}}{\partial Y_t}}=\tilde{a}_{1t}\sum_{j=1}^{m}\frac{1}{\tilde{a}_{jt}} \tag{8}$$

By equilibrium in the market for the consumption good at time t, $\sum_{j=1}^{m} \tilde{c}_{jt}(Y_t, \vec{\xi}_t) = Y_t$, which also implies that $\sum_{j=1}^{m} \frac{\partial \tilde{c}_{jt}}{\partial Y_t} = 1$. Therefore, solving (8) for $\frac{\partial \tilde{c}_{1t}}{\partial Y_t}$ gives $\frac{\partial \tilde{c}_{1t}}{\partial Y_t} = \frac{1/\tilde{a}_{1t}}{\sum_{j=1}^{m} \frac{1}{\tilde{a}_{jt}}}$. This and (7) imply that the following is true for all j.

$$\frac{\partial \tilde{c}_{jt}}{\partial Y_t} = \frac{1/\tilde{a}_{jt}}{\sum_{s=1}^{m} \frac{1}{\tilde{a}_{st}}} \tag{9}$$

This result was first derived by Wilson (1968, see his theorem 5).

Next, we need to determine the value of $\overline{\rho}_t(Y_t, \vec{\xi}_t)$. The following starts out with the definition of $\overline{\rho}_t(Y_t, \vec{\xi}_t)$, then substitutes in the definition of $\tilde{\rho}_{jt}(Y_t, \vec{\xi}_t)$ and the result of (9):

$$\overline{\rho}_t(Y_t, \vec{\xi}_t) \equiv \sum_{j=1}^{m} \left(\tilde{\rho}_{jt}(Y_t, \vec{\xi}_t) \cdot \frac{\partial \tilde{c}_{jt}}{\partial Y_t} \right) = \sum_{j=1}^{m} \left(\tilde{c}_{jt} \tilde{a}_{jt} \cdot \frac{\frac{1}{\tilde{a}_{jt}}}{\sum_{k=1}^{m} \frac{1}{\tilde{a}_{kt}}} \right) = \frac{\sum_{j=1}^{m} \tilde{c}_{jt}}{\sum_{k=1}^{m} \frac{1}{\tilde{a}_{kt}}}$$

However, the sum of consumption across all consumers in this pure exchange economy equals aggregate supply for that period. Therefore,

$$\overline{\rho}_t = \frac{Y_t}{\sum_{j=1}^{m} \frac{1}{\tilde{a}_{jt}}} \tag{10}$$

From the definition of $\tilde{\alpha}_j \equiv \frac{\tilde{\rho}_{jt}}{\overline{\rho}_t}$, we can write $\tilde{\rho}_{jt} = \tilde{\alpha}_j \overline{\rho}_t$ and then replace $\tilde{\rho}_{jt}$ with $\tilde{c}_{jt} \tilde{a}_{jt}$ and $\overline{\rho}_t$ with (10) to get $\tilde{c}_{jt} \tilde{a}_{jt} = \tilde{\alpha}_{jt} \frac{Y_t}{\sum_{j=1}^{m} \frac{1}{\tilde{a}_{jt}}}$. Dividing both sides by $\tilde{a}_{jt} Y_t$ gives

$\frac{\tilde{c}_{jt}}{Y_t} = \tilde{\alpha}_{jt} \frac{\frac{1}{\tilde{a}_{jt}}}{\sum_{j=1}^{m} \frac{1}{\tilde{a}_{jt}}}$. Using (9), we can rewrite this as: $\frac{\tilde{c}_{jt}}{Y_t} = \tilde{\alpha}_{jt} \frac{\partial \tilde{c}_{jt}}{\partial Y_t}$. Dividing both sides by $\tilde{\alpha}_{jt}$ gives us:

$$\frac{\partial \tilde{c}_{jt}}{\partial Y_t} = \frac{1}{\tilde{\alpha}_{jt}} \frac{\tilde{c}_{jt}}{Y_t} \tag{11}$$

We can rewrite (11) slightly differently by multiplying both sides by $\frac{Y_t}{\tilde{c}_{jt}}$ to get:

$$\frac{\partial \tilde{c}_{jt}}{\partial Y_t} \frac{Y_t}{\tilde{c}_{jt}} = \frac{1}{\tilde{\alpha}_{jt}} \tag{12}$$

The left side of (12) is the reduced-form elasticity of j's Pareto-efficient consumption with respect to aggregate output. Therefore, (12) states that this elasticity equals the reciprocal of the ratio of j's relative risk aversion to average relative risk aversion.

4. Interpretation of Result and Implications

Section 2 defined a general Arrow-Debreu pure-exchange economy of one nonstorable consumption good and risk-averse investors and showed that any Pareto-efficient consumption allocation is solely a function of aggregate output and utility shocks. To avoid confusing this with a structural consumption function, it is important to remember that this is a reduced-form relationship. Section 3 then derived the relationship we call "the Pareto-efficient relativity of relative risk aversion."

The elasticity of this function with respect to aggregate output equals $\frac{\partial \tilde{c}_{jt}}{\partial Y_t} \frac{Y_t}{\tilde{c}_{jt}}$. Again, it is important to remember that this is a reduced-form elasticity; not an elasticity of a structural consumption function. In section 3, we defined the ratio $\tilde{\alpha}_j \equiv \frac{\tilde{\rho}_{jt}}{\overline{\rho}_t}$, where $\tilde{\rho}_{jt}$ represents j's relative risk aversion and $\overline{\rho}_t$, is the average relative risk aversion for the whole economy. As defined in section 3, $\overline{\rho}_t \equiv \sum_{j=1}^{m}\left(\tilde{\rho}_{jt} \cdot \frac{\partial \tilde{c}'_{jt}}{\partial Y_t}\right)$, which means $\overline{\rho}_t$ is the average of all consumers relative risk aversion coefficients weighted by the partial derivative of each individual's consumption with respect to aggregate output. Therefore, $\tilde{\alpha}_j$ is the ratio of j's relative risk aversion to average relative risk aversion.

Section 3's purpose was to derive equation (12), which is reproduced below:

$$\frac{\partial \tilde{c}_{jt}}{\partial Y_t}\frac{Y_{it}}{\tilde{c}_{jt}} = \frac{1}{\tilde{\alpha}_{jt}} \qquad (12)$$

This states that elasticity of j's Pareto-efficient consumption with respect to aggregate output equals the reciprocal of the ratio of j's relative risk aversion to average relative risk aversion. This section will now explain what this means in terms of individuals with average relative risk aversion, above average relative risk aversion and below average relative risk aversion. We will then discuss some important implications of this relationship with respect to the financial securities needed to complete markets, inflation indexing, and central bank goals and targeting objectives.

Because in equilibrium, $\sum_{j=1}^{m}\tilde{c}_{jt} = Y_t$, the average elasticity of consumption with respect to aggregate output must equal 1. In other words when aggregate output decreases by 1%, on average each individual must decrease their consumption by 1%. In order for some consumers to decrease their consumption by less than 1%, others must decrease their consumption by more than 1%. Equation (12) shows that how one's relative-risk aversion compares to average relative-risk aversion is the sole determinant of the elasticity of an individual's Pareto-efficient consumption with respect to aggregate output. An individual with average relative-risk aversion will decrease his or her Pareto-Efficient consumption by 1% when aggregate output falls by 1%. On the other hand, a 1% decrease in aggregate output will cause an individual with twice the average relative-risk aversion to decrease their Pareto-Efficient consumption by ½%, whereas it will cause an individual with half the average relative-risk aversion to decrease their Pareto-Efficient consumption by 2%. In essence, a competitive equilibrium with complete markets will allow consumers with above average relative-risk aversion to transfer some of their aggregate output risk to individuals with below average relative-risk aversion.

A financial security that could enable consumers to transfer aggregate-output risk is Shiller's (1993) proposed GDP futures. While Shiller argues in favor of these GDP futures as a way to diversify GDP risk internationally, there would remain a component of this risk that will be undiversifiable. The risk that would-wide aggregate output will change cannot be diversified. Nevertheless, consumers with above average relative-risk aversion will want to transfer some of their aggregate output risk to those with below average relative-risk aversion.

Equation (12) also has some implications concerning nominal contracts vs. inflation-indexed contracts. Because the literature on inflation indexing is normally in the macroeconomics field and macroeconomists usually use the term "aggregate supply" instead of "aggregate output," the rest of this paper will use "aggregate supply" instead of aggregate output. Please note that from the perspective of this paper, the two terms are synonymous.

The equation of exchange states that

$$M_t V_t = N_t = P_t Y_t \tag{13}$$

where M_t is the money supply at time t, V_t is the income velocity[5] of money at time t, N_t is the nominal aggregate demand at time t, P_t is the price level at time t, and Y_t is the real aggregate supply at time t. While economists often call the equation of exchange an identity, only the left side, $M_tV_t = N_t$, is an identity. The right side, $N_t = P_tY_t$, is an equilibrium condition. If we solve the right side of the equation for the price level, we get $P_t = {}^{N_t}\!/_{Y_t}$.

Let X_t be the nominal payment at time t on a nominal contract, and let x_t be the real value of that payment, which equals $x_t \equiv {}^{X_t}\!/_{P_t}$. Substituting $P_t = {}^{N_t}\!/_{Y_t}$ for P_t gives $x_t = \frac{X_t}{N_t} Y_t$.

Therefore, when N_t is kept constant, the elasticity of this real payment with respect to real aggregate supply is $\frac{\partial x_t}{\partial Y_t}\frac{Y_t}{x_t} = \frac{X_t}{N_t}\frac{Y_t}{x_t} = \frac{X_t}{N_t}\frac{Y_t}{\left(\frac{X_t}{N_t}Y\right)} = 1$. This unitary elasticity is exactly what is needed by consumers with average relative risk aversion.

Now consider an inflation-indexed obligation. By definition or by design, the real payment on an inflation-indexed obligation is constant, which implies that its elasticticy with respect to aggregate supply is zero. By (12) such a contractual payment would be needed by a consumer with infinite relative risk aversion.

Because the average consumer may not be or need to be very shophisticated about the means of transferring aggregate-supply risk, such a consumer should not have to use GDP futures to transfer aggregate-supply risk. Only those with above average or below average relative-risk aversion should need to use GDP futures. When the central bank successfully

[5] Since the income velocity of money is defined as nominal aggregate demand divided by the money supply, calling it "income" velocity is not really appropriate since income is usually associated with aggregate output not aggregate demand. However, the precedent for this terminology is so strongly entrenched in the literature, that I need to go along with that terminology.

targets nominal income or nominal aggregate demand (i.e., $N_t=E_{t-1}[N_t]$), consumers with average relative-risk aversion would be able to use nominal contracts without having to transfer aggregate-supply risk to achieve their Pareto-Efficient consumption; they would have no need to use GDP futures. However, if such a consumer with average relative-risk aversion only had inflation indexed contractual payments, that consumer would have to use GDP futures to undo the inflation indexing.

Now suppose the central bank targets the price level instead of nominal income.[6] If the central bank is successful, the price level will remain the same if aggregate supply changes. Such a central-bank policy will have the effect of making all nominal contracts behave as inflation-indexed contracts, meaning the real payments on all nominal contracts will be constant even when aggregate supply changes. Again, receiving constant real contractual payments is appropriate for people with infinite relative risk aversion. Those with average relative risk aversion will need to use GDP futures to undo the effect of the price-level targeting, so that their net payments will have unitary elasticity with respect to changes in aggregate supply. The financial lives of consumers with average relative risk aversion would have been simpler with nominal contracts and the central bank targeting nominal income or nominal aggregate demand.

So far we have talked about consumers with average relative-risk aversion. Those with above average relative-risk aversion may find it useful to be on the receiving end of contracts with constant real payments. Also, those with below average relative-risk aversion may find it useful being on the paying side of these constant-real-payment contracts. However, there is no reason to expect that those on the receiving end of the contracts will have above average relative-risk aversion and those on the paying end will have below average relative-risk aversion. Also,

[6] A central bank targeting inflation will have an impact on nominal contracts similar to if they targeted the price level.

since few if any consumers will have infinite relative-risk aversion, the transfer of aggregate-supply risk on the contracts exceeds what people need. It is likely that even consumers with above average relative-risk aversion will need to use GDP futures to offset some of the inflation indexing or price-level targeting.

The discussion in favor of nominal contracts for consumers with average relative-risk aversion assumed that the central bank was successfully targeting nominal aggregate demand. If that assumption is violated, then holders of nominal contracts would be exposed to nominal-aggregate-demand risk, which is the risk that nominal aggregate demand will differ from its expected value. Under such circumstances, Eagle and Domian's (1995, 2003) quasi-real indexing may be needed to filter out the aggregate-demand-caused inflation while leaving the aggregate-supply-caused inflation intact.

Even when the central bank successfully targets nominal income or nominal aggregate demand, nominal contracts will not satisfy all the needs of consumers with average relative-risk aversion. In particular, these consumers could still incur endowment risk, where the ratio of their endowment to aggregate output would vary across different states of nature. Also, they have the risk of experiencing utility shocks (e.g., large medical expenses). Insurance contracts such as unemployment insurance or medical insurance would be needed by almost all consumers, including those with average relative-risk aversion, to completely manage their risks. Eagle (2005) discusses how four different types of contracts could approximately complete markets without the use of state-contingent securities. These include (i) endowment-sharing contracts, (ii) spending-sharing contracts, (iii) normal contracts, and (iv) Real-Aggregate-Supply-Risk-Transfer (RASRT) contracts. The first two contracts are insurance-like contracts to handle the endowment risk and utility-shock risk. The RASRT contracts are basically Shiller's GDP futures. Eagle defines "normal contracts" as contracts whose real payments have unitary elasticity with respect to aggregate-supply. Normal contracts would include nominal contracts when the central bank targets nominal income and would include quasi-real contracts regardless.

5. Summary

For a pure-exchange economy with one nonstorable consumption good and risk-averse consumers, this paper has derived the sole determinant of the elasticity of any individual j's Pareto-efficient consumption with respect to aggregate output. That elasticity equals the reciprocal of the ratio of j's relative-risk aversion to average relative-risk aversion. This means that the Pareto-efficient consumption of someone with average relative risk aversion should decrease (or increase) by 1% when aggregate output decreases (or increases) by 1%. For that 1% decrease (or increase) in aggregate output, the Pareto-Efficient consumption for someone with above average relative risk aversion should decrease (or increase) by more than 1% decrease, whereas it should decrease (or increase) by less than 1% for someone with below average relative-risk aversion. In essence the consumers with above average relative-risk aversion are transferring some of the aggregate-output risk from to consumers with below average relative-risk aversion.

To achieve this transfer of risk, Shiller's (1993) GDP futures are needed. Such futures would be needed for any closed economy, including the world-wide economy as the risk of changes in world-wide aggregate output cannot be diversified away.

Nominal contracts under nominal-income targeting would enable consumers with average relative-risk aversion to achieve their Pareto-Efficient consumption without using GDP futures, although they still may need insurance contracts to diversify away their endowment risks and the diversifiable part of their utility shocks. Inflation-indexed contracts would not be useful to consumers with average relative-risk aversion.

Under price-level targeting or inflation targeting, even nominal contracts would not enable consumers with average relative-risk aversion to avoid the need to use GDP futures. They would need to use the GDP futures to undo the effects of price-level targeting or inflation targeting to make the contracts behave as though they were nominal contracts under nominal income targeting. Thus, the lives of consumers with average relative-risk aversion would be simpler when the central bank targets nominal income rather than the price level or inflation.

REFERENCES

Arrow, Kenneth J., (1953), "The Role of Securities in the Optimal Allocation of Risk Bearing," *Colloques Intternationaux du Centre National de la Recherche Scientifique*, XL: 41-48; translated in the 1964 *Review of Economic Studies*, 31(#2):91-96.

___________, (1965), *Aspects of the Theory of Risk-Bearing* (Helsinki: Yrjo Jahnssonin).

Bodie, Zvi, Alex Kane, and Robert McDonald, (1983) "Inflation and the Role of Bonds in Investor Portfolios," NBER Wroking Paper Series #1091.

Campbell, Lo, and MacKinlay, (1997), *The Econometrics of Financial Markets* (Princeton University Press – Princeton).

Debreu, Gerard, (1959), *Theory of Value* (Wiley – New York).

Eagle, D., (2005), "Completing Markets in a One-Good, Pure Exchange Economy Without State-Contingent Securities," http://ideas.repec.org/p/wpa/wuwpfi/0501009.html, working paper accessed on January 22, 2005.

Eagle, D. and D. Domian, (1995), "Quasi-Real Bonds - A Financial Innovation to Eliminate a Danger Inherent in Inflation-Indexed Bonds," *Applied Economic Letters*, December 1995, 487-490.

__________, (2003), "Quasi-Real Indexing – The Pareto-Efficient Solution to Inflation Indexing," http://ideas.repec.org/p/wpa/wuwpma/0312010.html, accessed on November 14, 2004.

Shiller, Robert (1993), *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks*, (Oxford University Press – Oxford)

Viard, Alan D. (1993), "The Welfare Gain from the Introduction of Indexed Bonds," Journal of Money, Credit, and Banking, 25(#3):469-708.

Wilson, Robert (1968), "The Theory of Syndicates," *Econometrica*, 36(#1):119-132.