Endogenous leadership in teams*

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Abstract

In this paper we study the mechanics of "leading by example" in teams. Leadership is beneficial for the entire team when agents are conformists, i.e., dislike effort differentials. We also show how leadership can arise endogenously and discuss what type of leader benefits a team most.

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1 Introduction

In a recent experimental study Gächter and Renner (2004) illustrate the mechanics of "leading by example". In a team of agents one team member acts as leader by choosing his effort prior to all others. Gächter and Renner observe that the leader's effort influences the effort choice of all team members. The higher the leader's effort, the higher the effort of the other team members. Strikingly, this holds even though there are no monetary incentives that would induce such complementarities. Nevertheless, team members moving at the second stage follow the example set by their leader—which, in fact, reduces their monetary payoff.[1] Consequently, "bold" leadership, i.e. exerting high efforts as a first mover, can be beneficial, both for the leader and the team as a whole.

In this paper we suggest a way of modeling such leadership mechanics and show how leadership can arise endogenously. Our model is driven by the assumption that some agents might dislike effort differentials. For obvious reasons we shall call such agents, who have a tendency to be influenced by their peers, "conformists". A tendency of agents to match efforts of their peers has been documented in various recent empirical studies. For example, Falk and Ichino (2003) document peer effects in a controlled field experiment and Bandiera, Barankay, and Rasul (2004) observe strong peer effects among fruit pickers.

In team production that we study here conformism turns out to be a two-edged sword. While it tends to reduce efforts of highly productive agents it tends to increase the efforts of less productive agents. Nevertheless, we can show that teams always benefit (weakly) from exogenously imposed or

[1]Remarkably, Gächter and Renner make this observation even for one-shot games.

endogenously arising leadership. Material output and payoffs are higher in the presence of a leader.

Furthermore we show that endogenous leadership arises if and only if there is at least one team member who is a conformist and we analyze whose leadership is most desirable. Interestingly, it turns out that, everything else being equal, team output is maximized if the least productive agent takes on the role of team leader. Moreover, if agents vary in their degree of conformism, team output is maximized if a comparative non-conformist is leader.

Previous theoretical attempts to model leadership have invoked asymmetric information. In Hermalin's (1998) model leaders have private information about the team's productivity and, thus, can signal the team's productivity by their effort choice. While this is an extremely plausible model, it cannot explain Gächter and Renner's data where information is symmetric and, indeed, complete.

This note is organized as follows. In Section 2 we introduce two simple static models with two agents where the timing of agents' effort choices is exogenous. We show that sequential moves, i.e., having a leader always increases outputs as long as there is at least one conformist in the team. Furthermore, we demonstrate the main comparative statics results in this section. In Section 3 we allow for endogenous timing, following the modelling approach of Hamilton and Slutsky (1990). We show that, whenever at least one agent is a conformist, agents will indeed choose effort sequentially, increasing team output. Section 4 concludes.

2 Exogenous timing

Consider two agents $i = 1, 2$ who produce some joint output that they share equally. Each agent chooses some effort $x_i \geq 0$. For simplicity, let the output, y, be linear in efforts, i.e.,

$$y = 2(k_1 x_1 + k_2 x_2) \tag{1}$$

where $k_i \geq 0$ is agent i's constant productivity.[2] [3]Also for simplicity, we assume that the physical cost of exerting effort is quadratic such that agent i's *material* payoff is given by

$$\pi_i = \frac{y}{2} - \frac{1}{2} x_i^2. \tag{2}$$

Materially efficient production is therefore reached if agents choose $x_i^{EFF} = 2k_i$ which, as we know from Holmström (1982) and will see in some detail below, they will not do with standard preferences. The efficient total output is $y^{EFF} = 4(k_1^2 + k_2^2)$.

In our model, an agent's utility depends on his material payoff and may depend on the difference between the agent's effort and the effort of his peer. More specifically, let

$$u_i = \pi_i - \frac{b_i}{2}(x_i - x_j)^2 \tag{3}$$

where $(x_i - x_j)^2$ measures effort differences and $b_i \geq 0$ measures agent i's degree of conformism.[4] Standard preferences are obtained as a special case of (3) for $b_i = 0$.

[2]One might argue that team production is more likely to occur when efforts are complementary. However this complicates the algebra while our main qualitative results remain robust.

[3]Notice that in Gächter and Renner (2004) $k_1 = k_2$.

[4]In this environment, similar utility functions can be justified with other social preferences, for example, a variant of Fehr and Schmidt's (1999) inequality aversion. In their

2.1 No leadership: Simultaneous moves

Now suppose that efforts are chosen simultaneously at some given point in time. Taking first-order conditions we can derive agent i's best-reply function as

$$x_i(x_j) = \frac{k_i + b_i x_j}{1 + b_i}. \tag{4}$$

It is easy to see that efforts are strategically independent only for $b_i = 0$, the standard case. However, with conformism efforts become *strategic complements*. Solving the two simultaneous equations we compute equilibrium efforts as

$$x_i^{SIM} = \frac{k_i(1 + b_j) + k_j b_i}{1 + b_i + b_j} \text{ for } i = 1, 2 \text{ and } i \neq j. \tag{5}$$

Analyzing the comparative statics we find that

$$\text{sign } \frac{dx_i^{SIM}}{db_i} = -\text{sign } \frac{dx_i^{SIM}}{db_j} = \text{sign } (k_j - k_i) \tag{6}$$

In words, the more productive agent's effort is decreasing in his own conformism and increasing in the other agent's conformism and vice versa for the less productive agent. The intuition for this result is simple. In order to reduce differences in efforts, agents adjust their effort choice towards the efforts of others. Thus, the more productive agent lowers his effort. And the

model agents receive a utility penalty that depends linearly on the difference between agents' material payoffs. Since $\pi_i - \pi_j = \frac{1}{2}(x_j^2 - x_i^2)$ and $(x_j - x_i)^2 = \left(\frac{x_j^2 - x_i^2}{x_i + x_j}\right)^2$ our "non-conformism penalty" can be obtained from their inequality penalty by normalizing with respect to total effort and taking the square. However, in more complex environments conformism and this form of inequality aversion do not necessarily coincide. Notice also that conformism, as we model it here, does not depend on symmetry. Agents care about choosing similar actions despite poetentially different productivities. In a richer model, the degree of conformism could also depend on how similar or different agents are. In the context of a principal-agent problem such an approach is taken, for example, by Hehenkamp and Kaarboe (2004).

more conformist he is the more he will lower it. On the other hand, the less productive agent increases his effort. Again the size of this adjustment is increasing in the degree of his conformism.

Total equilibrium output is easily calculated as

$$y^{SIM} = 2\frac{k_1^2(1+b_2)+k_2^2(1+b_1)+(b_1+b_2)k_1k_2}{1+b_1+b_2}. \tag{7}$$

Again we can take first derivatives in order to analyze the effect of conformism on output. It is easy to see (and, in fact, follows immediately already from (6)) that

$$\text{sign } \frac{dy^{SIM}}{db_i} = \text{sign } (k_j - k_i). \tag{8}$$

In words, total output is increasing in the less productive agent's conformism and decreasing in the more productive agent's conformism. Notice that any (moderate) increase in output implies increased material efficiency. If agents are equally productive, conformism has no effect on production in the simultaneous-move equilibrium.

2.2 Exogenous leadership: Sequential moves

Let us now assume that agents decide about their efforts sequentially, the second mover knowing the first mover's choice.[5] Notationwise, let agent 1 be the first mover and agent 2 the second mover. Solving by backwards

[5] This does not necessarily require that agents work at totally separated times. Rather it might be that the first agent starts a little earlier than the second and that there is some inertia when efforts are exerted over time. In fact, when efforts are exerted over time there will always be an element of sequentiality as long as agents can observe what others are doing. Assuming two periods and a simple leader-follower structure is just a convenient way of capturing this.

induction it is obvious that agent 2 has to choose his effort according to (4), i.e., $x_2(x_1) = \frac{k_2+b_2x_1}{1+b_2}$. Anticipating this, the first agent maximizes

$$u_1 = k_1x_1 + k_2\frac{k_2 + b_2x_1}{1 + b_2} - \frac{1}{2}x_1^2 - \frac{b_1}{2}(x_1 - \frac{k_2 + b_2x_1}{1 + b_2})^2. \tag{9}$$

Solving the first-order condition we obtain

$$x_1^{SEQ} = \frac{k_1(1 + b_2)^2 + k_2(b_1 + b_2 + b_2^2)}{(1 + b_2)^2 + b_1} \tag{10}$$

and, accordingly, along the equilibrium path

$$x_2^{SEQ} = \frac{k_2 + b_2x_1^{SEQ}}{1 + b_2}, \tag{11}$$

and

$$y^{SEQ} = 2(k_1x_1^{SEQ} + k_2x_2^{SEQ}). \tag{12}$$

We have seen above that under simultaneous moves with conformism, the more productive agent has an incentive to reduce his effort while the less productive agent has an incentive to increase his effort. Let us refer to this as the *pure conformity effect*. It still applies here. But with sequential play there is a second effect to which we refer as the *commitment effect*. Since efforts are strategic complements, the first mover knows that by increasing his effort he will also increase the effort of the second mover. This implies that his return on effort is greater than under simultaneous moves. This commitment effect is always positive. However, if the more productive agent moves first, his level of conformism must not be too great because otherwise the negative conformity effect can outweigh the positive commitment effect.

The relative sizes of the pure conformity and commitment effects are crucial for a comparison of output under simultaneous play and output under sequential play. The intuition we gained from above tells us that sequential play will be particularly good if the two effects are aligned, i.e., when the less productive agent moves first (because he will increase his effort due to both the conformity and the commitment effect!). However, the actual comparison of outputs,

$$y^{SEQ} - y^{SIM} = b_2 \frac{(k_1 + k_1 b_2 + k_2 b_2)(b_1 k_1 + k_2 b_2 + k_2)}{(1 + 2b_2 + b_2^2 + b_1)(1 + b_1 + b_2)} \tag{13}$$

shows, since all parameters are positive, that the commitment effect *always* exceeds the conformity effect as long as the second mover shows a minimal tendency toward conformism.[6]

Examining (13) also reveals that, everything else being equal, it is always better for the team if the less productive agent moves first. For agents with equal (or similar productivity) it is, furthermore, better when the one who is more independent (that is, less conformist) moves first.[7]

Result 1 *Output with leadership always (weakly) exceeds output under simultaneous play. If the second mover is prone to conformism this holds strictly. Moreover, for agents equally prone to conformism, the less productive agent is preferable as leader. Finally, for equally productive agents, the agent who is less prone to conformism is the leader who maximizes output.*

[6] For equal productivities, $k_1 = k_2 = k$ (the case of Gächter and Renner 2004) this becomes $\frac{k^2 b_2 + 2k^2 b_2^2}{b_1 + 2b_2 + b_2^2 + 1}$. Hence, while there is no effect of conformism with equal productivities and simultaneous moves, our model does predict positive effects of leadership in a sequential-move game even if productivities are identical.

[7] The first claim can be easily established by substituting k_2 in (13) by $k_1 + \delta$ and then taking the first derivative w.r.t. δ. For the second claim one can simply normalize productivities to 1 and then evaluate (13) as $b_2 \frac{2b_2 + 1}{b_1 + 2b_2 + b_2^2 + 1}$.

3 Endogenous timing

In the absence of a firm owner and principal who implements a leadership structure, it seems unclear how the agents themselves should decide about the order of moves and the issue arises whether agents are able to achieve the benefits of sequential play. Of course, they might be able to engage in some bargaining prior to choosing their efforts. But if they are able to reach binding agreements, the free rider problem should disappear in any case. Thus, we here outline what will happen in the probably more realistic and more interesting case when they cannot reach binding agreements.

A natural way to find an answer to this question is to model the agents' problem as a game with endogenous timing. Here we adapt Hamilton and Slutsky's (1990) framework which studies the emergence of Stackelberg leadership in (market) games. Let there be two periods, $t = 1, 2$. In the first period, agents either exert some effort or decide to wait. This happens simultaneously. In the second period agents who have decided to wait, learn what happened in $t = 1$ and then choose their effort. Applying backward induction, we find, similar to Hamilton and Slutsky, that there are three subgame perfect equilibria, one symmetric and two asymmetric ones. In the symmetric SPE, both agents choose x_i^{SIM} in $t = 1$. In the two asymmetric SPE, one of the agents chooses x_1^{SEQ} in $t = 1$ while the other waits and chooses x_2^{SEQ} in $t = 2$.[8] [9]

As Hamilton and Slutsky, we can deselect the first symmetric equilibrium because it is in weakly dominated strategies. Simply notice that if the other

[8] Off the equilibrium path, agents simply play best replies.

[9] To see that the latter are indeed equilibria notice that the agent who moves first picks his best point on the other agent's response function. Thus, x_1^{SEQ} is a best reply to the other's waiting strategy.

player chooses an effort in $t = 1$, an agent is always weakly better off by waiting since he can then play the best reply against this effort in $t = 2$. Moreover, if the other waits, waiting too is equally good as playing x_i^{SIM} in $t = 1$ as in both cases, both agents eventually choose x_i^{SIM}. Hence, waiting can never be worse than playing x_i^{SIM} in $t = 1$ and is sometimes better. Thus, we should expect one of the two asymmetric equilibria where agents move indeed sequentially.

We refrain from selecting a unique solution.[10] Instead we observe that, with endogenous timing, agents will always achieve (weakly) higher output than when forced to play simultaneously. (This follows immediately from the first part of Result 1 above.)

Result 2 *If the timing of effort choices is endogenous and at least one agent is a conformist, agents will choose their efforts sequentially which strictly increases material efficiency.*

Notice that we assume that the leader-follower structure emerges because agents maximize utility and not their material payoff. However, as we see in Result 2 this will also increase their material payoffs. Thus, we see that, when timing is endogenous, teams with at least agent who is a conformist have a substantial advantage over teams where agents have standard preferences. In contrast to standard agents, agents with positive b's will benefit from the endogenously emerging leader-follower structure.

[10] For Hamilton and Slutsky's game, van Damme and Hurkens (1999) provide a unique solution applying Harsanyi-Selten style equilibrium selection arguments.

4 Incomplete information

So far, we have always assumed that both, agents' productivities as well as agents' degrees of conformism, are commonly known. This is obviously a heroic assumption and the question arises whether or not the results are robust if there is some incomplete information. Since, arguably, productivities are much easier to observe in the setting that we have in mind, the more pressing question is what would happen if agents have to face some uncertainty about their partners' degree of conformism. While a full-fledged analysis of this problem would go far beyond the scope of this paper we did analyze the robustness of our results for the special case of equal productivities k (which is, in fact, the setup of Gächter and Renner 2004) and two possible values of b, zero and a strictly positive $\overline{b}$. The common prior attaches probability p to the latter type, and $1-p$ to the former (the standard type of economic theory).

With equal productivities we know that conformism has only bite if agents move sequentially and this, of course, remains true in the presence of incomplete information: both agents simply choose $x_i = k$ when they move simultaneously. With sequential moves the analysis becomes slightly more tedious but remains essentially straightforward. The analysis is greatly simplified through the observation that the type of the first mover is completely irrelevant for the second mover who only cares about the first mover's action. Hence, signalling is not an issue and there is a unique sequential equilibrium. (Some algebraic results are contained in the appendix.) Also, for $p \to 1$ this equilibrium converges to the equilibrium of the game with complete information where $b_1 = b_2 = \overline{b}$. What is more, output under sequential moves again exceeds output under simultaneous moves, for all parameters.

The analysis of the game with incomplete information and endogenous timing is a little more elaborate but it turns out that the results we obtained above do again carry over. (See also the appendix.) In particular, there are sequentially rational equilibria with endogenous leadership. However, now there are two possible types of such equilibria—equilibria where, say, agent 1 moves first regardless of his b and agent 2 waits regardless of his b (or vice versa) and equilibria where the decision when to move is a function of b. It turns out that both types of equilibria coexist. First of all, the equilibria of the complete information case where leadership depends on the identity of agents are robust. In the game with incomplete information there are always asymmetric equilibria where, say, agent 1 moves first and agent 2 waits. In addition, there is also a symmetric equilibrium where high types with $b = \overline{b}$ move first and low types with $b = 0$ wait. Of course, in this equilibrium where the conformists become leaders and complete non-conformists followers, production is just as under simultaneous moves. Due to the insensitivity of the (endogenous) follower, there is neither a conformity nor a commitment effect. And the more desirable symmetric outcome where the non-conformist becomes the endogenous leader and the conformist the follower is, as it turns out, not an equilibrium. The reason for this is that a conformist has an incentive to deviate and move first since there is a chance that the other agent is a conformist, too, who can be stipulated to work harder *if* the deviating agent decides to lead by example.

The bottom line is that, in this simple model of incomplete information, endogenous leadership is predicted to arise but will only be beneficial for the team if agents coordinate on one of the asymmetric equilibria where the first agent leads regardless of his type.

5 Conclusion

In this paper we have illustrated a model that captures the mechanics of "leading by example" documented in recent experiments (Gächter and Renner 2004). The model takes as its central assumptions one of the key results of Gächter and Renner's study, namely that agents exhibit a substantial degree of conformism, i.e., a tendency to reduce effort differentials even if this is costly for them. We show that with such conformism leadership is always beneficial for the team. Moreover, we show that leadership need not be imposed exogenously. When at least one agent is prone to conformism, leadership will, in fact, arise endogenously. Moreover, we show that, somewhat counterintuitive, teams should select the least productive agent as leader. This is because then the incentives induced through a pure conformity effect and a commitment effect are aligned. Finally, for equally productive agents, it is better for the team to have a "free spirit", i.e. somebody who is less prone to conformism, as leader.

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Appendix

In this appendix we sketch the equilibrium solutions of the game with exogenous sequential moves and incomplete information and the game with endogenous timing and incomplete information.

The game with exogenous sequential moves can be solved by simple backward induction (since the first mover only influences the second mover's payoff via his action and not via his type). The low type ($b = 0$) will simply choose

$$x_2^L = k$$

in the second period regardless of what the first mover did. The high type ($b = \overline{b}$) would instead react to the leader's decision and choose his effort according to

$$x_2^H(x_1) = \frac{k + \overline{b}x_1}{1 + \overline{b}}.$$

As first movers both types would utilize the commitment effect. Specifically, the low type would choose

$$x_1^L = k\frac{1 + \overline{b}(1 + p)}{1 + \overline{b}}$$

and the high type

$$x_1^H = k\frac{3\overline{b} + \overline{b}p + 3\overline{b}^2 + \overline{b}^3 - \overline{b}^2 p - \overline{b}^3 p + 1}{1 - 2\overline{b}^2 p + 3\overline{b}^2 + \overline{b}^3 + 3\overline{b} - \overline{b}^3 p}.$$

This also describes what would happen in one of the asymmetric equilibria of the game with endogenous timing, where one of the agents becomes leader because of his "name".

We conclude this appendix by mentioning the first-period effort that a high type would choose deviating from a proposed symmetric outcome where low types move first and high types second. The optimal deviation would then be to choose

$$x_1^+ = k\frac{\overline{b}^2 p^2 - 3\overline{b} - \overline{b}p - 3\overline{b}^2 - \overline{b}^3 - 1 + \overline{b}^3 p^2}{2\overline{b}^2 p - 3\overline{b}^2 - \overline{b}^3 - 3\overline{b} + \overline{b}^3 p - 1}.$$