Economic Analysis (3rd Edition)

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Abstract

A framework for economic analysis based on matrix algebra. In this edition, the mathematical model introduced in earlier works has been extended.

AC (March 2005)

PS: My name is inconsistent with the email address and URL given on the title page because I published previous versions of this work under the pseudonym "Mitchell Leonards". I decided to use my real name on the edition because *Economic Analysis* is now fairly mature. I will continue to use the Mitch Leonards email and website.

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Introduction

Section 1: Parameters

Let an element be defined as a phenomenon that is, or is treated as, irreducible.
Let a compound be defined as a set of elements.

Let composition be defined as the set of elements that constitute a compound.
Let structure be defined as the arrangement of elements in a compound.
Let state be defined as a compound's composition and structure.

Let a transformation be defined as an alteration in a compound's composition.
Let a translation be defined as an alteration in a compound's structure.
Let a transition be defined as a transformation or transition.

Let a dimension be defined as a set of elements that define a parameter.
Let a space be defined as an environment defined by a given set of dimensions.
Let a process be defined as the set of transitions undergone by a set of compounds in a given space.

Section 2: Analysis

Let inspection be defined as the identification of a compound's composition or structure.
Let a principle be defined as a concept that defines the general state of a compound.
Let a protocol be defined as the application of a principle in a particular compound.

Let data be defined as records pertaining to any given set of compounds.

Let tomography be defined as the derivation of a compound's composition or structure based on a given set of data.
Let induction be defined as the derivation of a principle based on a given set of data.
Let deduction be defined as the derivation of a protocol based on a given set of data.

<u>Processes</u>

Section 1: Composition

Let a reactant be defined as a compound that undergoes a set of transitions.

Let an accelerant be defined as a compound that accelerates a set of transitions or allows it to occur but is not a reactant.
Let a decelerant be defined as a compound that decelerates a set of transitions or prevents it from occurring but is not a reactant.
Let a regulator be defined as an accelerant or decelerant.

Let an input be defined as a reactant or regulator.

Let a product be defined as a compound that is formed by a set of transitions and can be an input in the same space.
Let a by-product be defined as a compound that is formed by a set of transitions and cannot be an input in the same space.
Let an output be defined as a product or by-product.

Let a resource be defined as an input or output.

Section 2: Structure

Let α_γ be a 1 x A matrix of inputs for the γ^{th} transition in a given process.
Let β_γ be a 1 x B matrix of outputs for the γ^{th} transition in a given process.

Let vec(γ) be the identifier of the γ^{th} transition in a given process.

Let co(vec(γ)) denote a progression to the γ^{th} transition in a given process.
Let au(vec(γ)) denote synchronization with the γ^{th} transition in a given process.
Let fb(vec(γ)) denote a regression to the γ^{th} transition in a given process.

From these definitions, a given process, *PS*, can be expressed as

$$PS = [\text{vec}(1);\ \text{co}(\text{vec}(\gamma))\ \ldots \quad \text{vec}(\Gamma);\ \text{fb}(\text{vec}(\gamma))] \qquad (2.1)$$

where *PS* contains Γ transitions.

The properties of a process can be expressed in a separate matrix or set of equations.

Depending on the set of outputs used to define a process, it may be convenient to let a given vec(γ) represent a set of transitions.

Economic Processes

Section 1: Composition

Let a good be defined as a resource that can be valued in terms of another set of resources.
Let an agent be defined as an autonomous regulator that can regulate multiple processes.
Let a vertex be defined as a fixed regulator that regulates a given set of agents.

Let search be defined as the process of locating a set of goods that satisfy a given set of criteria.
Let acquisition be defined as the process of obtaining the set of goods located in a search.
Let evaluation be defined as the process of expending the set of goods obtained in an acquisition.

Let notification be defined as the process of presenting a set of data about a set of goods.
Let distribution be defined as the process of transferring a set of goods to a set of agents.
Let engineering be defined as the process of constructing a set of goods.

Let production be defined as the set of processes that result in the distribution of a set of goods.
Let consumption be defined as the set of processes that result in the acquisition of a set of goods.

Let a lattice be defined as a given set of agents and vertices.

Section 2: Structure

Let a lane be defined as a vertex that connects a set of vertices.
Let a laboratory be defined as a vertex where the engineering of a set of goods occurs.
Let a directory be defined as a vertex where search and notification with respect to a set of goods converge.
Let a market be defined as a vertex where acquisition and distribution with respect to a set of goods converge.
Let a theater be defined as a vertex where the evaluation of a set of goods occurs.

Let – denote a lane.
Let L denote a laboratory.
Let D denote a directory.
Let M denote a market.
Let T denote a theater.

Let the identifier of a vertex be called a codon. Where more than one economic process occurs in a vertex, a codon with multiple letters can be used to identify it.
For dual process vertices, the codons are LD, LM, LT, DM, DT, and MT.
For triple process vertices, the codons are LDM, LDT, LMT, and DMT.
For quadruple process vertices, the codon is LDMT.

Where a subset of the economic processes that occur in a multi-process vertex do not interact with other vertices via lanes, the codon denoting that process can be contained in parentheses. The name of a vertex can be entered as a superscript to its codon.

Using process notation,
let vec(a) denote engineering,
let vec(b) denote notification, and
let vec(c) denote distribution.
From these definitions, production, *prod*, can be expressed as

prod = [vec(a); co(vec(b))
vec(b); co(vec(c)); fb(vec(a))
vec(c); (fb(vec(a), vec(b))] (2.1)

Similarly,

let vec(A) denote search,
let vec(B) denote acquisition, and
let vec(C) denote evaluation.
From these definitions, consumption, *cnsm*, can be expressed as

cnsm = [vec(A); co(vec(B))
vec(B); co(vec(C)); fb(vec(A))
vec(C); (fb(vec(A), vec(B))] (2.2)

Production and consumption of a given good interact at directories and markets. The production and consumption of different goods interact when they involve common agents or vertices.

Economic Networks

Section 1: Composition

Let an economic network be defined as a set of lattices.

Let A_t be the set of agents encompassed by an economic network at time t, such that

$$A_t = [A_1 \ldots A_m] \quad (1.1)$$

Let B_t be the set of agents in A_t that are producers at time t, such that

$$B_t = [B_1 \ldots B_i] \quad (1.2)$$

where $i \leq m$.

Let C_t be the set of agents in A_t that are consumers at time t, such that

$$C_t = [C_1 \ldots C_r] \quad (1.3)$$

where $r \leq m$. From these definitions, it is possible for a given agent to belong to subsets B_t and C_t at the same point in time.

Let N_t be an i x I matrix of the set of goods produced by the agents in B_t, such that

$$N_t = [N_1 \ldots N_I] \quad (1.4)$$

for I types of goods.

Let n_t be an r x R matrix of the set of goods consumed by the agents in C_t, such that

$$n_t = [n_1 \ldots n_R] \quad (1.5)$$

for R types of goods.

Let b_t be a 1 x I matrix of the set of goods produced by the agents in B_t, such that

$$b_t = [B_t][N_t] \quad (1.6)$$

where B_t is a 1 x i matrix.

Let c_t be a 1 x R matrix of the set of goods consumed by the agents in C_t, such that

$$c_t = [C_t][n_t] \quad (1.7)$$

where C_t is a 1 x r matrix.

From equation 1.6, the number of units of goods produced is given by

$$\sum_{g=1}^{i} \sum_{h=1}^{I} N_{gh} = \sum_{h=1}^{I} b_{1h} \quad (1.8)$$

and the number of units of goods of the Eth type produced is given by

$$\sum_{g=1}^{i} N_{gE} = b_{1E} \quad (1.9)$$

From equation 1.7, the number of units of goods consumed is given by

$$\sum_{j=1}^{r} \sum_{k=1}^{R} n_{jk} = \sum_{k=1}^{R} c_{1k} \quad (1.10)$$

and the number of units of goods of the Eth type consumed is given by

$$\sum_{j=1}^{r} n_{jE} = c_{1E} \quad (1.11)$$

Section 2: Structure

Given the structure of an economic network, a set of ratios can be constructed:

i_t / m_t (r - 2.1)

r_t / m_t (r - 2.2)

i_t / r_t (r - 2.3)

I_t / R_t (r - 2.4)

$$\sum_{h=1}^{i} b_{1h} / i \quad (r - 2.5)$$

$$b_{1E} / i \quad (r - 2.6)$$

$$\sum_{k=1}^{R} c_{1k} / r \quad (r - 2.7)$$

$$c_{1E} / r \quad (r - 2.8)$$

$$b_{1E} / \sum_{h=1}^{I} b_{1h} \quad (r - 2.9)$$

$$c_{1E} / \sum_{k=1}^{R} c_{1k} \quad (r - 2.10)$$

$$b_{1E} / c_{1E} \quad (r - 2.11)$$

$$\sum_{h=1}^{I} b_{1h} / \sum_{k=1}^{R} c_{1k} \quad (r - 2.12)$$

First-Order Dynamics

Let currency be defined as a good used to express the value of a set of goods.

The value of a process can be calculated by expressing the per unit value of each resource involved in the process in terms of a given currency and multiplying by the number of units of each resource.

For inputs,

$$C_t = \alpha_t \delta_t \quad (1.1)$$

where C is cost, α is a 1x R matrix of input quantities, and δ is the corresponding R x 1 matrix of per unit input prices.

For outputs,

$$R_t = \beta_t \varepsilon_t \quad (1.2)$$

where R is revenue, β is a 1 x I matrix of output quantities, and ε is the corresponding I x 1 matrix of per unit output prices.

Let π_t be profit at time t, such that

$$\begin{aligned} \pi_t &= R_t - C_t \\ &= \beta_t \varepsilon_t - \alpha_t \delta_t \quad (1.3) \end{aligned}$$

Accordingly, for an exchange network,

$$\pi_t = b_t Y_t - c_t Z_t \quad (1.4)$$

where Y_t is an I x 1 matrix of per unit output prices, and Z_t is an R x 1 matrix of per unit input prices.

Let

$$\Pi_t = \sum_{t=1}^{T} \pi_t \quad (1.5)$$

for t = 1...T periods.

Second-Order Dynamics

Section 1: Scale

Let scale be defined as the relationship between the price and quantity of a good, such that

$$P_{\theta t} = x_{\theta t} Q_{1t} + x_{1t} \tag{1.1}$$

where x_{1t} is a Q_{1t} independent component of $P_{\theta t}$.

Let increasing returns to scale (IRS) be defined as a scale where $x_{\theta t} < 0$. Let constant returns to scale (CRS) be defined as a scale where $x_{\theta t} = 0$. Let decreasing returns to scale (DRS) be defined as a scale where $x_{\theta t} > 0$.

Section 2: Elasticity

Let elasticity be defined as the relationship between the quantity and price of a good, such that

$$Q_{\iota t} = x_{\iota t} P_{\theta t} + x_{2t} \tag{2.1}$$

where x_{2t} is a $P_{\theta t}$ independent component of $Q_{\iota t}$.

Let inelastic elasticity (IE) be defined as an elasticity where $x_{\iota t} < -1$. Let neutral elasticity (NE) be defined as an elasticity where $x_{\iota t} = -1$. Let elastic elasticity (EE) be defined as an elasticity where $x_{\iota t} > -1$.

Section 3: Symmetry

Let symmetry be defined as the relationship between the profits of separate economic processes, such that

$$\pi_{\kappa t} = x_{\kappa t} \pi_{\iota t} + x_{3t} \tag{3.1}$$

where x_{3t} is a $\pi_{\iota t}$ independent component of $\pi_{\kappa t}$.

Let increasing symmetry (IS) be defined as a symmetry where $x_{\kappa t} > 0$. Let neutral symmetry (NS) be defined as a symmetry where $x_{\kappa t} = 0$. Let decreasing symmetry (DS) be defined as a symmetry where $x_{\kappa t} < 0$.

Section 4: Conclusion

Equation 3.1 can be expressed as

$$\begin{aligned} \pi_{\kappa t} &= x_{\kappa t}(q_{\zeta \iota t} p_{\eta \theta t} - q_{\delta \iota t} p_{\varepsilon \theta t}) + \overline{q}_{\zeta \iota t} \overline{p}_{\eta \theta t} - \overline{q}_{\delta \iota t} \overline{p}_{\varepsilon \theta t} \\ &= x_{\kappa t}(b_{\iota t} Y_{\theta t} - c_{\iota t} Z_{\theta t}) + \overline{b}_{\iota t} \overline{Y}_{\theta t} - \overline{c}_{\iota t} \overline{Z}_{\theta t} \end{aligned} \tag{4.1}$$

Let $\Pi_{\kappa t}$ be defined as

$$\Pi_{\kappa t} = \sum_{t=1}^{T} \pi_{\kappa t} \tag{4.2}$$

for t = 1...T periods.

Third-Order Dynamics

Constraints can be imposed by protocols or the structure of an economic network, or constructed from assumptions. Given second-order dynamics, there are nine types of constraints:

$x_{\kappa t} = \upsilon_t$ (1.1)

$b_{\iota t} = \phi_{bt}$ (1.2)

$Y_{\theta t} = \phi_{Yt}$ (1.3)

$b_{\iota t} = \chi_{ct}$ (1.4)

$Z_{\theta t} = \chi_{Zt}$ (1.5)

$\bar{b}_{\iota t} = \psi_{bt}$ (1.6)

$\bar{Y}_{\theta t} = \psi_{Yt}$ (1.7)

$\bar{c}_{\iota t} = \omega_{ct}$ (1.8)

$\bar{Z}_{\theta t} = \omega_{Zt}$ (1.9)

These constraints can be integrated into the equation for $\Pi_{\kappa t}$ using Lagrange multipliers, producing the equation

$$\Pi_{\kappa t} = \sum_{t=1}^{T} [\pi_{\kappa t} - \lambda_{1t}(x_{\kappa t} - \upsilon_t) - \lambda_{2t}(b_{\iota t} - \phi_{bt}) - \lambda_{3t}(Y_{\theta t} - \phi_{Yt}) - \lambda_{4t}(c_{\iota t} - \chi_{ct}) - \lambda_{5t}(Z_{\theta t} - \chi_{Zt})$$
$$- \lambda_{6t}(\bar{b}_{\iota t} - \psi_{bt}) - \lambda_{7t}(\bar{Y}_{\theta t} - \psi_{Yt}) - \lambda_{8t}(\bar{c}_{\iota t} - \omega_{ct}) - \lambda_{9t}(\bar{Z}_{\theta t} - \omega_{Zt})]$$

Fourth-Order Dynamics

Section 1: Type 1 Statistics

Let M be mass.

$$M_{xt} = (\sum_{h=1}^{I} b_{1h}) + (\sum_{k=1}^{R} c_{1k}) \qquad (1.1)$$

$$M_{yt} = b_t Y_t + c_t Z_t \qquad (1.2)$$

Let V be velocity.

$$V_{xt} = [(\sum_{h=1}^{I} b_{1h}) + (\sum_{k=1}^{R} c_{1k})] / t$$
$$= M_{xt} / t \qquad (1.3)$$

$$V_{yt} = (b_t Y_t + c_t Z_t) / t$$
$$= M_{yt} / t \qquad (1.4)$$

Section 2: Type 2 Statistics

Let g be growth.

$$g_{xt} = (M_{xt} - M_{xt-1}) / t \qquad (2.1)$$
$$g_{yt} = (M_{yt} - M_{yt-1}) / t \qquad (2.2)$$

Let a be acceleration.

$$a_{xt} = (V_{xt} - V_{xt-1}) / t$$
$$= (M_{xt} - M_{xt-1}) / t^2 \qquad (2.3)$$

$$a_{yt} = (V_{yt} - V_{yt-1}) / t$$
$$= (M_{yt} - M_{yt-1}) / t^2 \qquad (2.4)$$

Section 3: Type 3 Statistics

Let P be momentum.

$$P_{xt} = (M_{xt})(V_{xt})$$
$$= (M_{xt})^2 / t \qquad (3.1)$$

$$P_{yt} = (M_{yt})(V_{yt})$$
$$= (M_{yt})^2 / t \qquad (3.2)$$

Let F be force.

$$F_{xt} = (g_{xt})(a_{xt})$$
$$= (M_{xt} - M_{xt-1})^2 / t^3 \qquad (3.3)$$

$$F_{yt} = (g_{yt})(a_{yt})$$
$$= (M_{yt} - M_{yt-1})^2 / t^3 \qquad (3.4)$$

Section 4: Type 4 Statistics

A: Lower Thresholds for Type 1 Statistics

For a given A_t, let $C_{t(min)}$ be the minimum set of consumers at time t and let $n_{t(min)}$ be the minimum set of goods they consume, where there are α types of goods.

From these definitions,

$$c_{t(min)} = [C_{t(min)}][n_{t(min)}] \tag{4.1}$$

At minimum mass,

$$b_{t(min)} = [0] \tag{4.2}$$

Accordingly,

$$M_{xt(\min)} = \sum_{k=1}^{\alpha} c_{1k} = c_{1k(\min)} \tag{4.3}$$

$$M_{yt(min)} = c_{1k(min)}Z_{t(min)} \tag{4.4}$$

$$V_{xt(min)} = M_{xt(min)} / t \tag{4.5}$$

$$V_{yt(min)} = M_{yt(min)} / t \tag{4.6}$$

B: Upper Thresholds for Type 1 Statistics

For a given A_t, let $B_{t(max)}$ be the maximum set of producers and let $N_{t(max)}$ be the a maximum set of goods they produce, where there are Ω types of goods.

From these definitions,

$$b_{t(max)} = [B_{t(max)}][N_{t(max)}] \tag{4.7}$$

At maximum velocity c_t is identical to $b_{t(max)}$, so

$$b_{t(max)} = c_{t(max)} \tag{4.8}$$

Accordingly,

$$M_{xt(\max)} = (\sum_{h=1}^{\Omega} b_{1h}) + (\sum_{k=1}^{\Omega} c_{1k}) = 2\sum_{h=1}^{\Omega} b_{1h} \tag{4.9}$$

$$M_{yt(min)} = b_{t(max)}Y_t + c_{t(max)}Z_t = 2b_{t(max)}Y_t \tag{4.10}$$

$$V_{xt(max)} = M_{xt(max)} / t \tag{4.11}$$

$$V_{yt(max)} = M_{yt(max)} / t \tag{4.12}$$

<u>Section 5: Type 5 Statistics</u>

A: Lower Thresholds for Type 2 Statistics

$$g_{xt(min)} = (M_{xt(min)} - M_{xt-1}) / t \tag{5.1}$$

$$g_{yt(min)} = (M_{yt(min)} - M_{yt-1}) / t \tag{5.2}$$

$$a_{xt(min)} = (M_{xt(min)} - M_{xt-1}) / t^2 \tag{5.3}$$

$$a_{yt(min)} = (M_{yt(min)} - M_{yt-1}) / t^2 \tag{5.4}$$

B: Upper Thresholds for Type 2 Statistics

$$g_{xt(max)} = (M_{xt(max)} - M_{xt-1}) / t \quad (5.5)$$
$$g_{yt(max)} = (M_{yt(max)} - M_{yt-1}) / t \quad (5.6)$$

$$a_{xt(max)} = (M_{xt(max)} - M_{xt-1}) / t^2 \quad (5.7)$$
$$a_{yt(max)} = (M_{yt(max)} - M_{yt-1}) / t^2 \quad (5.8)$$

Section 6: Type 6 Statistics

A: Lower Thresholds for Type 3 Statistics

$$P_{xt(min)} = (M_{xt(min)})(V_{xt(min)})$$
$$= [M_{xt(min)}]^2 / t \quad (6.1)$$

$$P_{yt(min)} = (M_{yt(min)})(V_{yt(min)})$$
$$= [M_{yt(min)}]^2 / t \quad (6.2)$$

$$F_{xt(min)} = (g_{xt(min)})(a_{xt(min)})$$
$$= [M_{xt(min)} - M_{xt-1}]^2 / t^3 \quad (6.3)$$

$$F_{yt(min)} = (g_{yt(min)})(a_{yt(min)})$$
$$= [M_{yt(min)} - M_{yt-1}]^2 / t^3 \quad (6.4)$$

B: Upper Thresholds for Type 3 Statistics

$$P_{xt(max)} = (M_{xt(max)})(V_{xt(max)})$$
$$= [M_{xt(max)}]^2 / t \quad (6.5)$$

$$P_{yt(max)} = (M_{yt(max)})(V_{yt(max)})$$
$$= [M_{yt(max)}]^2 / t \quad (6.6)$$

$$F_{xt(max)} = (g_{xt(max)})(a_{xt(max)})$$
$$= [M_{xt(max)} - M_{xt-1}]^2 / t^3 \quad (6.7)$$

$$F_{yt(max)} = (g_{yt(max)})(a_{yt(max)})$$
$$= [M_{yt(max)} - M_{yt-1}]^2 / t^3 \quad (6.8)$$