Stochastic demand correspondences and their aggregation properties

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Abstract. Bandyopadhyay, Dasgupta and Pattanaik [3] have presented a construction that aggregates *standard* demand functions by a stochastic demand function, in such way that the latter satisfies the weak axiom of stochastic revealed preference when the former fulfil Samuelson's weak axiom of revealed preference. We introduce a concept of stochastic demand correspondences and generalize WASRP to this new setting. Also, we propose a procedure to aggregate *stochastic* demand correspondences that (a) includes the previous one by those authors, (b) aggregates the weak axiom of stochastic revealed preference (and thus stochastic substitutability when we started with tight stochastic demand functions), and (c) produces a *stochastically rational* stochastic demand correspondence when the primitive ones are stochastically rational. Interpretations of the model are provided too. An analogous to the result that rationalizability of stochastic demand functions by stochastic orderings implies WASRP holds for the correspondence setting. We also discuss the convenience of adopting a formal variation of the notion of being stochastically rational that was proposed in the literature.

1 Introduction

This work originates with the disappointing fact that a representative consumer endowed with the wealth of the group whose aggregate demand represents may fail to satisfy Samuelson's weak axiom of revealed preference (WARP) even though all the individual consumers fulfil it (Mas-Colell et al. [9, Example 4.C.1]). This results in procedural difficulties when consistent predictions on aggregate behavior are to be performed, as long as the standard analysis presumes WARP in the aggregate, being derived from the preference maximization hypothesis.

A recent proposal to overcome such technical handicap appeals to stochastic demand functions for representing aggregate behavior instead (cf. Bandyopadhyay, Dasgupta and Pattanaik [3]). Such stochastic functions are then requested to satisfy a certain weak axiom of stochastic revealed preference (WASRP). This requirement had been introduced in Bandyopadhyay et al. [2]. Then, [2, Proposition 3.5] proves that it is strictly weaker than rationalizability by stochastic orderings. This fact accounts, at individual level, for its justification as a sensible rationality property. Further, it addresses the reader to a rationalizability problem to which Falmagne [8] and, afterwards, Barberà and Pattanaik [5], gave answers. Also, Bandyopadhyay et al. [4] was devoted to show that such weak axiom of stochastic revealed preference is equivalent to another restriction on stochastic demand behavior that they called Stochastic Substitutability, for *tight* stochastic demand functions. This latter achievement extends some fundamental results in the classical demand theory.

In order to make that new aggregation proposal fully useful one needs to guarantee that in suitable circumstances, the representative consumer has a certain stochastic demand function that satisfies WASRP. In Bandyopadhyay et al. [3] that is accomplished by associating a significant stochastic demand function with a group of competitive consumers displaying standard demand functions, which, provided that WARP holds for all the individual consumers, satisfies WASRP in turn. Explicitly, what is postulated is that a fictitious representative consumer captures the groups' behavior in the following sense: for any price-wealth situation, her stochastic demand function measures the proportion of agents whose demanded bundle lies in each part of the corresponding budget set.

Our main aim is to complete the study considering these and other aggregability issues with regard to aggregation of stochastic demand functions or *correspondences*. The latter notion, that is intrinsically interesting because it permits to associate a (stochastic) representative agent with any society whose members have multivalued demands, will be motivated and introduced as a generalization of the concept of stochastic demand function.

Particularly relevant is the analysis of the rationalizability problem. Let us recall briefly some key antecedents in the standard approach. What must be highlighted in the first place is that the standard aggregation procedure fails to keep stronger rationality assumptions such as the Strong Axiom or representability (by preferences or utility functions) as well. Consequently, some classical contributions had to narrow their focus to individual demand functions that have rather restrictive structures in order to be able to assure that the appropriate rationality postulate carries over to the aggregate. As for the representability issues, Chipman [6] and Eisenberg [7] come to mind. In a much related line of inquiry, Shafer [10] is concerned with the Strong Axiom. After his work we know that if some individual demand functions are homogeneous of degree one in income and each of them satisfies the budget equality and the strong axiom of revealed preference, then their aggregate demand satisfies (a strengthened version of) the strong axiom too. Given these antecedents, it is not only noteworthy but also probably surprising that the stochastic procedure of aggregation does perform well when it comes to keeping rationality properties. To illustrate this characteristic before introducing the necessary technicalities, we anticipate a particular statement of this achievement: the stochastic representative agent of a society where everyone derived his/her demand function from an ordering (complete or connected plus transitive) can be also explained as the result of optimizing a stochastically rational behavior, although its standard aggregate demand could be non-rationalizable by an ordering.

We have organized this work as follows. In Section 2 we introduce the concept of stochastic demand correspondence. For these correspondences, we specify the appropriate version of WASRP, that coincides with the previous formulation in the restricted instance of stochastic demand functions. Afterwards, in Section 3 we justify that if a number of (individual or representative) consumers display stochastic demand correspondences that satisfy WASRP, then their aggregation by a certain sensible stochastic demand correspondence satisfies it too. Other aggregability issues of our construction will be put forward. In particular, certain structural form of the *stochastic* aggregate of classical demand functions that satisfy Walras' law plus the compensated law of demand will be derived easily, which adds to the mentioned fact that their standard aggregate fulfils the compensated law of demand too. Our proposal for aggregating stochastic demand has different readings. On the one hand, it permits to deal with individual consumers whose choices were stochastic in the beginning, e.g. because they derive them from a probability distribution over utility scales or just linear preference orderings (see Falmagne [8] and Barberà and Pattanaik [5] for a more thorough exposition of this latter model). The motivating instance that Bandyopadhyay et al. analyzed, where all the initial agents have a deterministic behavior, is encompassed in this case. On the other hand, it may permit capturing in a single representative the choice behavior of a group of different collectives, each of whose demand behaviors were determined (or estimated, e.g. through samplings or polls) by a representative stochastic consumer. Whatever the reading, the relevant issue is that, under our construction, the corresponding (stochastic or not) weak axiom translates into WARSP of the stochastic aggregate. In Section 4 we shall remark that under the appropriate rationalizability restriction, stochastic demand correspondences satisfy WASRP, which highlights its role as a plausible rationality restriction on stochastic demand behavior. A short discussion of the concepts of rationality in the stochastic setting, together with possible variations on it, are explicited too. We also correct an inaccuracy in a proof contained in a previous reference. The rationalizability properties that are carried over to the aggregate stochastic demand correspondence are put forward in Section 5. A concluding Section 6 will put an end to this contribution.

2 Stochastic demand correspondences and WASRP

The settings that we are going to consider will be modeled through the following common axiomatics, as will be explained in Section 4. We assume henceforth that prices are positive and incomes non-negative. Each consumer's consumption set is $\mathbb{R}^m_+$. The next definitions are known.

Firstly, we focus on d (individual or not) stochastic demand function (SDF henceforth). That is, d is a rule assigning, to every price-income situation (p, w), one finitely additive probability measure over the class of all subsets of the budget set $B(p, w)$: cf. Bandyopadhyay et al. [3, Definition 2.1, Remark 2.2]. Thus, whenever $A \subseteq B(p, w)$, $d(p, w)(A)$ represents the probability that the agent's demanded bundle will lie in A under prices p and income w. Any standard deterministic demand function d can be identified with a (degenerate) SDF $\widetilde{d}$ through the expression: $\widetilde{d}(p, w)(A) = 1$ when $d(p, w) \in A$, $\widetilde{d}(p, w)(A) = 0$ otherwise, for each possible (p, w) -see also [4, Definition 2.2]. We shall use the shorthand DDF as well.

We say that the stochastic demand function d satisfies the *weak axiom of stochastic revealed preference* (WASRP) if, for all price-income situations (p, w), (p', w'), and for every $A \subseteq B(p, w) \cap B(p', w')$: $d(p, w)(B(p, w) \setminus B(p', w')) \geqslant d(p', w')(A) - d(p, w)(A)$. For the sake of the comparison, Remark 3.7 in [2] shows that a certain stochastic version of WARP holds under WASRP. Namely: for all price-income situations (p, w), (p', w') such that $d(p, w)(B(p, w) \cap B(p', w')) = 1$, and for every $A \subseteq B(p, w) \cap B(p', w')$:

$$d(p', w')(A) \leqslant d(p, w)(A) \leqslant d(p', w')(A) + d(p', w')(B(p', w') \setminus B(p, w)) \qquad (1)$$

Thus far, we have been concerned with the stochastic counterpart of demand *functions* alone. However, demand correspondences are also relevant in economic theory and abstract choice theory, and therefore must appeal to an appropriate stochastic version. We propose the following:

DEFINITION 1 *A stochastic demand correspondence d is a rule assigning, to every price-income situation (p, w), one subadditive and monotone function $d(p, w) : \mathcal{P} \to [0, 1]$ -where $\mathcal{P}$ denotes the class of all subsets of the budget set $B(p, w)$-, with $d(p, w)(\varnothing) = 0$ and $d(p, w)(B(p, w)) = 1$. We shall also use the abbreviated term SDC.*

We recall that subadditivity means $d(p, w)(A_1 \cup ... \cup A_p) \leqslant \sum_{i=1}^{p} d(p, w)(A_i)$ for any $A_1, ..., A_p \subseteq B(p, w)$, and monotonicity means $d(p, w)(A) \leqslant d(p, w)(B)$ if $A \subseteq B \subseteq B(p, w)$.

Our motivation for such concept, that is illustrated in Example 1 below, goes as follows. A common argument for the use of demand (or choice) correspondences is that they serve to model the first step in a multistep procedure aiming at selecting one single bundle (see e.g. Aizerman and Aleskerov [1]). That is: if c denotes our classical demand correspondence, then $c(p, w)$ would produce a subset of $B(p, w)$ -say, any number of available bundles with which we would be most satisfied- and subsequent steps would end in a final decision among them. This suggests the following digression. Any subset of the budget set can be assigned a binary "satisfaction value" under c: that is, we write $c(p,w)(A)=1$ if A meets $c(p,w)$, and $c(p,w)(A)=0$ otherwise. The interpretation is plain: A offers "full" satisfaction if it contains *anything* optimal (so that the final decision is to be made "at full satisfaction"), and "void" satisfaction if no optimal element is available among A. Observe that, with this interpretation in mind, subadditivity and monotonicity are quite natural though additivity is not any more. Correspondingly, after that abstraction process, for any d stochastic demand correspondence we are led to interpret that $d(p,w)(A)$ reflects the probability that A contains *at least one* bundle that could potentially be demanded by the consumer (recall the possible multiplicity of the optimal bundles as presented above). Any such assignement must be naturally monotone and subadditive though not necessarily additive, as Example 1 will illustrate.

It is clear that deterministic demand correspondences -DDC henceforth- are *degenerate*[1] SDC through the standard identification explained above. Due to the definition of a measure, any SDF is SDC. Besides, we must admit that the term "stochastic" as is applied to SDC's may seem misleading since there are not probabilities in Definition 1. Nonetheless, we shall keep it for homogeneity.

We say that the stochastic demand correspondence d satisfies the *weak axiom of stochastic revealed preference* (WASRP) if, for all price-income situations (p,w), (p',w'), and for every $A\subseteq B(p,w)\cap B(p',w')$: $d(p,w)(A\cup(B(p,w)\setminus B(p',w')))\geqslant d(p',w')(A)$.

OBSERVATION 1 *We observe that if d was in fact SDF, then the definition coincides with that given by Bandyopadhyay et al., since $A\cap(B(p,w)\setminus B(p',w'))=\varnothing$ and thus we would have*

$$d(p,w)(A\cup(B(p,w)\setminus B(p',w')))=d(p,w)(A)+d(p,w)(B(p,w)\setminus B(p',w')))$$

In [2, pp. 100-101] they offer an explanation of the intuition underlying WASRP in the SDF realm. Also, the manuscript "'Regular' choice and the weak axiom of stochastic revealed preference" by Dasgupta and Pattanaik explores the relation between this postulate for rational choice behavior and regularity. The latter is kind of a stochastic version of Chernoff's condition -as they explain in their Remark 2.6- and thus it is a very weak rationality property of stochastic choice.

3 Aggregating stochastic demand correspondences

Suppose now that we have $d_1,...,d_m$ SDC's associated with each of a finite number n of agents, and that there are known, prefixed $\delta_1,...,\delta_n$ strictly positive and summing up to 1. We shall explain below possible interpretations of these key amounts, that depend on the original context. Then, we introduce the following:

[1]For this we mean that, for each (p,w), there is $A\subseteq B(p,w)$ such that $d(p,w)(B)=1$ whenever $A\cap B\neq\varnothing$, and $d(p,w)(B)=0$ whenever $A\cap B=\varnothing$

DEFINITION 2 *The stochastic demand correspondences $d_1, ..., d_n$ induce the stochastic demand correspondence d associated with $\delta_1, ..., \delta_n$, according to:*

$$d(p, w)(A) = \delta_1 d_1(p, w)(A) + ... + \delta_n d_n(p, w)(A)$$

for all $A \subseteq B(p, w)$.

The fact that d is SDC is immediate. It is also plain that when in particular the d_i's are stochastic demand functions then d is SDF as well. In this Section we analyse some properties of the stochastic demand correspondences (or functions) that carry over to their aggregate stochastic demand correspondence defined as above. The main property of this construction is Proposition 1 below. Before stating it, we explicit a simple example that illustrates Definitions 1 and 2:

EXAMPLE 1 *In a competitive situation with 2 agents consuming 2 goods, their respective demand behaviors are given –for any positive price and non-negative income– by the following demand correspondences:*

1. $d_1((p_1, p_2), w) = \{(\frac{w}{p_1}, 0), (0, \frac{w}{p_2})\}$
2. $d_2((p_1, p_2), w) = \{(\frac{w}{2p_1}, \frac{w}{2p_2})\}$

The SDC associated with d_1 is given by the expression: for any $A \subseteq B(p, w)$,

$$d_1((p_1, p_2), w)(A) = \begin{cases} 1 & \text{if } (\frac{w}{p_1}, 0) \in A \text{ or } (0, \frac{w}{p_2}) \in A \\ 0 & \text{otherwise} \end{cases}$$

It is easy to check that it is subadditive but not additive by computing at $\{(\frac{w}{p_1}, 0), (0, \frac{w}{p_2})\} = \{(\frac{w}{p_1}, 0)\} \cup \{(0, \frac{w}{p_2})\}$. With respect to the SDF associated with d_2, it is defined by: for any $A \subseteq B(p, w)$,

$$d_2((p_1, p_2), w)(A) = \begin{cases} 1 & \text{if } (\frac{w}{2p_1}, \frac{w}{2p_2}) \in A \\ 0 & \text{otherwise} \end{cases}$$

The stochastic representative consumer associated with the society ($\delta_1 = \delta_2 = \frac{1}{2}$) will be represented by the SDC given by:

$$d((p_1, p_2), w)(A) = \begin{cases} \frac{1}{2} & \text{if [either } (\frac{w}{p_1}, 0) \in A \text{ or } (0, \frac{w}{p_2}) \in A] \text{ and } (\frac{w}{2p_1}, \frac{w}{2p_2}) \notin A \\ \frac{1}{2} & \text{if } (\frac{w}{2p_1}, \frac{w}{2p_2}) \in A \text{ and } (\frac{w}{p_1}, 0) \notin A \text{ and } (0, \frac{w}{p_2}) \notin A \\ 1 & \text{if } (\frac{w}{2p_1}, \frac{w}{2p_2}) \in A \text{ and [either } (\frac{w}{p_1}, 0) \in A \text{ or } (0, \frac{w}{p_2}) \in A] \\ 0 & \text{otherwise} \end{cases}$$

for any $A \subseteq B(p, w)$, irrespective of p and w.

PROPOSITION 1 *For any list of stochastic demand correspondences (respectively: functions) $d_1, ..., d_n$ that satisfy WASRP and any choice of $\delta_1, ..., \delta_n$ strictly positive and summing up to 1, the stochastic demand correspondence (respectively: function) d associated with them satisfies WASRP.*

Proof. We consider the SDC case. Fix price-income situations (p, w), (p', w'), and $A \subseteq B(p, w) \cap B(p', w')$. We need to check $d(p, w)(A \cup (B(p, w) \setminus B(p', w'))) \geqslant d(p', w')(A)$. That is,

$$\sum_{i=1}^{n} \delta_i d_i(p, w)(A \cup (B(p, w) \setminus B(p', w'))) \geqslant \sum_{i=1}^{n} \delta_i d_i(p', w')(A)$$

Now, because each d_i satisfies WASRP,

$$d_i(p, w)\left(A \cup (B(p, w) \setminus B(p', w'))\right) \geqslant d_i(p', w')(A)$$

which ends the proof for the case of demand correspondences. The SDF case is alike. ■

Interpretations and consequences of the property exposed in Proposition 1 vary with the setting that originates the construction. In general, the functional form is easier to explain and we focus on that case for expository convenience.

Bandyopadhyay et al. [3, Proposition 3.2 (i)] is a valuable result stating that the *stochastic* aggregate of a finite number of deterministic demand funcions satisfying WARP fulfils WASRP even though this does not preclude that the classical (deterministic) aggregate demand fails to satisfy WARP. In their model, *the primitive demand functions are classical* (i.e., each one associates, with each price-income situation, a bundle: the basket of commodities demanded by the corresponding agent). This is a degenerate case of stochastic demand functions, after a certain identification process is performed (cf. [2, Remark 2.3] and [3, Remark 2.2]). Also, through that standard identification, asking the demand functions to satisfy WARP means exactly that their stochastic counterparts fulfil WASRP (cf. [3, Remark 2.4]). Thus [3, Proposition 3.2 (i)] is encompassed in Proposition 1, where $\delta_i = \frac{1}{n}$ for all i and the d_i's are standard demand functions upon the aforementioned identification. Likewise, we can deduce a deterministic statement for classical demand correspondences.

REMARK 1 *For those who intend to link this model to the rationalizability achievements in Falmagne [8] and Barberà and Pattanaik [5], the next alternative construction may clarify some points while capturing the same conceptual meaning.*

As in [3], suppose $d_1(p, w), ..., d_n(p, w)$ are standard demand functions. Let us define $c(p, w) = \{d_1(p, w), ..., d_n(p, w)\}$, whose cardinality is at most n . We construct the following probability assignment $\widetilde{d}(p, w)$ over $c(p, w)$ -cf. [5, Definition 2.1]-, given by:

$$\widetilde{d}(p, w)(a) = \frac{|\{i : a = d_i(p, w)\}|}{n}$$

The concept represented by d as in [3, Definition 3.1], and therefore through Definition 2, can be identified with that captured by this probability assignment over $c(p, w)$. Notice that $d(p, w)(A) = \sum_{a \in A \cap c(p,w)} \widetilde{d}(p, w)(a)$ for all $A \subseteq B(p, w)$, because the d_i's are univalued.

REMARK 2 *Incidentally: WASRP translates into a property of $\widetilde{d}(p,w)$ too. Namely,*

$$\sum_{b \in B(p,w) \cap B(p',w') \cap c(p,w)} \widetilde{d}(p,w)(b) \geqslant \sum_{a \in A \cap c(p,w)} (\widetilde{d}(p',w')(a) - \widetilde{d}(p,w)(a))$$

for all $A \subseteq B(p,w) \cap B(p',w')$.

The case where we aggregate demand correspondences through Definition 2 permits to answer the next question: *What kind of representative consumer could we associate with a society whose members have multivalued demands?* In our proposal, any stochastic demand correspondence d representing such collective captures in $d(p,w)(A)$ the proportion of members in the society that have at least one satisfactory bundle of $B(p,w)$ in A.

We are aware that on occasions the construction in Definition 2 could be explained by splitting each d_i in an adequate number of fictitious deterministic members of a collective the d_i represents, and then defining d through the procedure laid down in Bandyopadhyay et al. [3]. However, the restrictions to proceed in this alternative way are by no means trivial. A very significant one arises from [3, Footnote 5]: any stochastic demand function defined by aggregation of a finite number of standard demand functions must take rational values only. In other words: if, for example, we have as primitive components of a demand model some stochastic demand functions that permit the probability of demanding some bundle in a fixed A to vary continously with prices and/or wealth, then that decomposition procedure can not be conducted succesfully.

In order to end this Section we shall produce yet another statement in the line of Proposition 1. In Corollary 1 below we analyze the rendering of our aggregation proposal with regard to some other normative properties. We need some previous notation.

Following Bandyopadhyay et al. [4, Definition 2.4 (ii)], we say that the SDF d is *tight* if $d(p,w)(A) = 1$, where $A = \{x \in B(p,w) : px = w\}$. Thus, a DDF satisfies Walras' law exactly when the degenerate SDF that is identified with it is tight. Besides, aggregating tight SDF's produces a tight SDF. The stochastic role of the Compensated Law of Demand (CLD henceforth, or Samuelson's inequality in the terminology of [4]) is played by the following concept: the tight SDF d satisfies *Stochastic Substitutability* if for all price-income situations (p,w), (p',w') and for every $A \subseteq \{x \in \mathbb{R}^m_+ : px = w,\, p'x = w'\}$:

$$d(p',w')(\{x \in \mathbb{R}^m_+ : px > w,\, p'x = w'\}) + d(p',w')(A) \geqslant$$

$$d(p,w)(\{x \in \mathbb{R}^m_+ : px = w,\, p'x < w'\}) + d(p,w)(A)$$

Then, we recall that [4, Claim 3.3] establishes that, for tight DDF's, CLD/Samuelson's inequality is equivalent not only to WARP (Mas-Colell et al. [9, Proposition 2.F.1]) but also to Stochastic Substitutability (under the usual identification).

COROLLARY 1 *For any list of tight stochastic demand functions $d_1, ..., d_n$ that satisfy Stochastic Substitutability and any choice of $\delta_1, ..., \delta_n$ strictly positive and summing up to 1, the stochastic demand function d associated with them satisfies Stochastic Substitutability as well. In particular: the stochastic aggregate of a finite number of deterministic demand functions that satisfy Walras' law plus CLD satisfies Stochastic Substitutability.*

Proof. The first statement admits an immediate direct proof. Or, alternatively: it holds because Stochastic Substitutability is equivalent to WASRP for any tight SDF ([4, Proposition 3.4]), and Proposition 1 ensures that WASRP carries over to the stochastic aggregate. Then, because the stochastic aggregate is forcefully tight, we use the same equivalence to prove that it satisfies Stochastic Substitutability.

As for the deterministic instance [2], we just recall that, for tight DDF's, CLD is equivalent to Stochastic Substitutability. ■

4 WASRP and the representability by stochastic orderings

A focal result in Bandyopadhyay et al. [2] is their Proposition 3.5, where it is ensured that rationalizability by stochastic orderings -i.e., the possibility of explaining the primitive demand behavior through the optimality procedure that would have used a stochastically rational agent to derive her demand- implies WASRP. As we recalled in the Introduction, its importance lies on the role that such fact assigns to WASRP: if a certain stochastic demand behavior is to be considered rational, that axiom is mandatory. In this Section we address ourselves to such issue and extend it to the SDC setting conveniently, which constitutes a direct argument for establishing WASRP as a significant rationality restriction on stocahstic demand behavior in the SDC setting too.

Following [2, Definition 3.2], we define T as the set of all orderings (reflexive, connected, transitive) on $\mathbb{R}^m_+$. Also, $T(G, H)$ will denote the set of all orderings in T that possess a unique greatest element in G and it belongs to H. Then, the stochastic demand function d is *rationalizable by stochastic orderings* (RSO) if there is a probability measure r on the set of all subsets of T such that

$$d(p, w)(A) = r(T(B(p, w), A))$$

for each possible (p, w) and $A \subseteq B(p, w)$.

For reasons that are related to the general structure of our SDC concept, it seems unavoidable to introduce the following extended version of the RSO concept, whose interpretation is similar to that of RSO.

DEFINITION 3 *Let $T'(G, H)$ denote the set of all orderings in T whose greatest elements in G exist and intersect H. Then, a stochastic demand correspondence d is weakly rationalizable by stochastic orderings (WRSO) if there is a probability measure r on the set of all subsets of T such that $d(p, w)(A) = r(T'(B(p, w), A))$ for each possible (p, w) and $A \subseteq B(p, w)$.*

[2]The particular case cited in Corollary 1 is all but a particularization of [3, Proposition 3.2 (i)]. Observe that both the primitive demand functions exhaust the income and their stochastic aggregate is tight, and that, under such restrictions, WARP is equivalent to CLD -primitive level- while WASRP is equivalent to Stochastic Substitutability -aggregate level.

It is clear that any RSO stochastic demand function is a WRSO stochastic demand correspondence. Moreover, notice that r in Definition 3 satisfies $1 = r(T'(B(p,w), B(p,w)))$ for all possible (p,w). We shall return to this relevant fact after proving our next result.

In order to fully grasp the reach of Definition 3, it pays to explicit what to be stochastically rational means in our general setting. Our rational agent is presumed to have a collection of orderings on $\mathbb{R}^m_+$ at his disposal. When he has to decide what he finds satisfactory for prices p and available income w he selects one of them according to a probability distribution r. Such ordering forcefully has at least one maximal element in $B(p,w)$, and possibly more. If he assigns to each collection of feasible options $A \subseteq B(p,w)$ the probability that it contains one such "demanded" bundles, what he thus derives is a stochastic demand correspondence. Conversely then, a given SDC is considered (weakly) rational if it is possible to explain it through that precise procedure.

Thus, and following this discussion, now we are in a position to prove a fact that establishes the WRSO concept as a sensible rationality property of stochastic demand behavior:

PROPOSITION 2 *Any WRSO stochastic demand correspondence satisfies WASRP.*

Proof. Let d denote the SDC under inspection. By assumption, there is a certain probability measure r that justifies its WRSO condition.

The following derivation will be mostly useful, also regarding intuitive interpretations of the proof. For any $B = B(p,w)$, denote $N_B = T \setminus T'(B,B)$. Such subset is formed by the collection of all orderings on $\mathbb{R}^m_+$ without maximal elements in B. The fact that $1 = r(T'(B(p,w), B(p,w)))$ yields $r(N_B) = 0$.

In order to check that WASRP holds under such circumstances, let us fix price-income situations (p,w), (p',w'), and $A \subseteq B(p,w) \cap B(p',w')$. We need to prove $d(p,w)(A \cup (B(p,w) \setminus B(p',w'))) \geqslant d(p',w')(A)$, or, equivalently,

$$r(T'(B(p',w'), A)) \leqslant r(T'(B(p,w), A \cup (B(p,w) \setminus B(p',w')))$$

For convenience, we use the shorthands $B = B(p,w)$ and $B' = B(p',w')$

Pick $\succcurlyeq \in T'(B', A)$. By definition, there is $a \in A$ such that $a \succcurlyeq x$ for all $x \in B'$. Two cases arise: either $\succcurlyeq \in N_B$, or $\succcurlyeq \in T'(B,B)$. In this second case, where there is a maximal element b in B, forcefully $\succcurlyeq \in T'(B, A \cup (B \setminus B'))$. Indeed, if $b \in B \setminus B'$ this is obvious. Otherwise, $b \in B \cap B'$, but then $a \succcurlyeq b$ and therefore $\succcurlyeq$ has a maximal element on B, namely a, that belongs to A.

The digression above shows that $T'(B', A) \subseteq N_B \cup T'(B, A \cup (B \setminus B'))$. This yields the desired inequality. ■

The introduction of the N_B subsets has a plausible motivation and it is backed by a clear intuition. If the agent would only consider orderings that have maximal elements *on every budget set*, a determinant inclusion $T'(B', A) \subseteq T'(B, A \cup (B \setminus B'))$ would be reached in a straightforward manner. However, taking T as it was proposed in the literature forces us to deal with two different types of orderings: those with some maximal element on B, which are selected almost surely, and those without any maximal bundle, which happen with null probability. This technical handicap is probably unnecessary because it could be overcome through the use of an adapted WRSO/RSO concept (i.e. taking T as the set of all orderings on $\mathbb{R}^m_+$ with a greatest element on every possible budget set instead) that is normatively equivalent to the original one. We would obtain a result (only superfluosly) weaker than the Proposition above but as we have argued, it would lose none of its essence while producing a gain in intuition and clarity of exposition.

In order to close this Section, we deduce from Proposition 2:

COROLLARY 2 (Bandyopadhyay, Dasgupta and Pattanaik) *Any RSO stochastic demand function satisfies WASRP.*

Proof. Immediate. The stochastic demand function fulfils WRSO and therefore WASRP. Recall that despite the different formal statement, WASRP means the property of stochastic demand functions that Bandyopadhyay, Dasgupta and Pattanaik had defined (cf. Observation 1). ■

OBSERVATION 2 *There seems to be an inaccurate statement in the proof of Bandyopadhyay et al. [2, Proposition 3.5], which does not affect the truth of the enunciate as proven above. These authors give an argument aiming at showing that:*

$$T(B(p,w), E \setminus A) \subseteq T(B(p',w'), B(p',w' \setminus A))$$

(E denotes $B(p,w) \cap B(p',w')$). Such statement is false, and examples are easy to provide. The reason is that there is the implicit assumption that every ordering in $T(B(p,w), E \setminus A)$ has a greatest element in $B(p',w')$, which is incorrect under RSO. To put it shortly: nothing prevents an ordering in $T(B(p,w), E \setminus A)$ from having no greatest element in $B(p',w')$ at all. This difficulty is overcome through the use of the adapted RSO concept we mentioned before. Anyway, not only their Proposition 3.5 derives from Proposition 2 above but in fact it is possible to modify the original proof given by Bandyopadhyay, Dasgupta and Pattanaik suitably and keep the property that is really needed, namely:

$$r(T(B(p,w), E \setminus A)) \leqslant r(T(B(p',w'), B(p',w') \setminus A))$$

for each possible (p,w) and $A \subseteq B(p,w) \cap B(p',w')$. We just need to check that

$$r(T(B(p,w), E \setminus A)) \leqslant 1 - r(T(B(p',w'), A) \leqslant r(T(B(p',w'), B(p',w') \setminus A))$$

which holds because r is finitely additive and both $T(B(p,w), E \setminus A), T(B(p',w'), A)$ and $T(B(p',w'), A), T(B(p',w'), B(p',w') \setminus A)$ are pairs of disjoint subsets. Indeed, a short argument proves the first disjointness, and the second one is plain. Actually, that second inequality holds with equality: the definition of RSO forces $r(T(B(p',w'), B(p',w'))) = 1$.

5 The rationalizability of the aggregate of rational stochastic demands

In this Section we shall be concerned with a topic that seems to have been understudied. We focus on a group of agents whose demands are "stochastically rational" -the appropriate notion is related to the exact form of their respective demand correspondences. Then, moving up to studying their "total demand", we shall infer that the same property can be expected of their stochastic representative agent. As we have argued before, such behavior can be labelled as no less than unexpected due to the discouraging antecedents in the classical literature.

PROPOSITION 3 *For any list of stochastic demand correspondences (respectively: functions) $d_1, ..., d_n$ that are WRSO (respectively: RSO) and any choice of $\delta_1, ..., \delta_n$ strictly positive and summing up to 1, the stochastic demand correspondence (respectively: function) d associated with them is WRSO (respectively: RSO) too.*

Proof. We consider the SDC case firstly. There exists a list of probability measures r_i on the set of all subsets of T such that $d_i(p, w)(A) = r_i(T'(B(p, w), A))$ for each possible (p, w) and $A \subseteq B(p, w)$. Let r be the probability measure on the set of all subsets of T given by $r(S) = \sum_{i=1}^{n} \delta_i r_i(S)$ for all $S \subseteq T$. It is then a simple routine to check that $d(p, w)(A) = r(T'(B(p, w), A))$ for each possible (p, w) and $A \subseteq B(p, w)$, since that equality boils down to $\sum_{i=1}^{n} \delta_i d_i(p, w)(A) = \sum_{i=1}^{n} \delta_i r_i(T'(B(p, w), A))$.

The SDF case is analogous. ■

The next Corollary is highly relevant by comparison with the outcome that produces the deterministic setting.

COROLLARY 3 *The stochastic representative agent of a society where everyone derived his/her demand function from an ordering is RSO.*

Its importance lies in the fact that the classical aggregate consumer associated with this society might be non-derivable from an ordering, an issue that poses hard difficulties to the analysis of aggregate behavior. Nonetheless, even in such case we can guarantee that its stochastic representative agent is rationalizable by a stochastic ordering with a precise form.

6 Conclusions

We have shown that a sensible concept of stochastic demand correspondence can be used to generalize the concept of stochastic demand function. We also proved that if a number of (individual or representative) consumers display stochastic demand correspondences that satisfy (the extended variation of) WASRP, then their aggregation by a certain sensible stochastic demand correspondence satisfies it too. Moreover, it those agents are "stochastically rational" then so is their aggregation by such stochastic demand correspondence. As a consequence, other aggregability properties of the construction are obtained: particularly, we deduced a Corollary that extends to the stochastic realm the classical deterministic result that the Compensated Law of Demand is an aggregable restriction when the agents always exhaust their income. Thus, gathering classical and original statements, one has: given any finite number of standard demand functions that satisfy Walras' law and CLD, their deterministic aggregate always satisfies CLD and their stochastic aggregate always satisfies Stochastic Substitutability. Under a weaker rationalizability restriction than the RSO used by Bandyopadhyay, Dasgupta and Pattanaik, stochastic demand correspondences fulfil WASRP. This reinforces the relevance of the extended version of WASRP that we have proposed as an unavoidable requirement for rational stochastic demand behavior. We also argued in favour of a normatively equivalent notion of what to be stochastically rational means. It is our view that adopting such variation helps to educate one's intuition as far as proofs are concerned.

We can brief a good part of our results in one sentence: as long as a full analysis of stochastic demand is available, its application to aggregate behavior seems to overturn many of the handicaps that the deterministic case poses.

Besides, although it is out of the scope of this work, we point out that the interested reader may find it challenging to check that many of these achievements can be exported to an abstract setting, i.e. to a choice framework that is not necessarily linked to the restrictions of demand theory.

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