

Weber-Fechner's Law and Demand Function

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Abstract

We apply the Weber-Fechner's law, which represents the relation between the magnitude of physical stimulus and the magnitude of psychological sense in human being, to the utility function. We conclude that the utility function of n -types of goods is of separable type $u(x_1, x_2, \dots, x_n) = u_1(x_1) + u_2(x_2) + \dots + u_n(x_n)$, which gives the relation of the demand function in the form $p_i = du_i/dx_i$. The explicit quantitative form of each utility function, which is suggested by the Weber-Fechner's law, becomes $u_i(x_i) = A_i \log(x_i/x_i^{(0)})$. Then we obtain each demand function in the familiar form $p_i = A_i/x_i$.

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§1. Introduction

Nowadays it is widely said that the 20-th century *was* the century of physics but the 21st century *is* the century of biology. Here and there, the new trend of physics gradually appears, and pioneers in such new trend physics consider that the physics should treat not only the science of the nature but also the science of human being[1, 2]. As a result of the advance of the study of the complex system, we may realize the dream to understand the origin of life in near future. Today, the progress in biophysics, biochemistry, gene technology, brain physics *etc.* is surprising.

In near future, phenomena connected with human being such as behavioral psychological phenomena will be understood from physics. The important part of economics depends on the knowledge of behavioral psychology, especially behavioral psychology concerning about money. In this context, we consider economics as the application of behavioral psychology. In micro economics, the utility function is one of the most important things, and the functional form of this utility function must be determined from the knowledge of behavioral psychology. In macro economics, functional dependence of various macro functions such as the consumption function, the saving function and the investment function *etc.* must be determined from the knowledge of behavioral psychology. For example, the consumption function C is often assumed as $C = C(Y - T) = C_0 + c(Y - T)$ where Y : national income; T : tax; c : marginal propensity of consume [3]. This is considered as the law of behavioral psychology. This law is quite simple but universal and powerful, which governs the behavior of human being. While, the functional form of the utility function is still in qualitative level as people use this or that functional form without any scientific reason.

The most important law in behavioral psychology is Weber-Fechner's law[4, 5], which represents the relation between the magnitude of physical stimulus and the magnitude of psychological sense in human being. In previous paper, we apply this Weber-Fechner's law to the utility function, and give the explicit quantitative functional form for the utility function[6]. In this paper, we give the explicit quantitative functional form for the demand function by using the utility function proposed in the previous paper. We also give the interpretation of the relation of this demand function from the view point of consumer's surplus.

§2. Demand function from the Weber-Fechner's law

The well-known and quite important law in behavioral psychology is the Weber-Fechner's law[4, 5]. Modification of the Weber-Fechner's law is known as the Stevens' law, which is successful in the phenomena of sound[7]. This Weber-Fechner's law is expressed as "

the magnitude of psychological sense is proportional to the logarithm of the magnitude of physical stimulus.” More precisely, we denote the magnitude of psychological sense as u and the magnitude of physical stimulus as x , then we can express the Weber-Fechner’s law in the following form

$$u(x) = \begin{cases} A \log(x/x^{(0)}) & \text{if } x \geq x^{(0)} \\ 0 & \text{if } x < x^{(0)} \end{cases}, \quad (1)$$

where A is constant and $x^{(0)}$ is the threshold of the magnitude of the physical stimulus. In the following, we consider only in the region where the magnitude of the physical stimulus is above the threshold value.

We apply this Weber-Fechner’s formula to the utility function[6]. First, we assume that there is only one type of goods, then the Weber-Fechner’s law suggests that the utility function u_1 is given as the function of the quantity x_1 of this goods in the form

$$u_1(x_1) = A_1 \log(x_1/x_1^{(0)}). \quad (2)$$

Next we assume that there are two types of goods, then each utility function u_1 and u_2 is given as $u_1(x_1) = A_1 \log(x_1/x_1^{(0)})$ and $u_2(x_2) = A_2 \log(x_2/x_2^{(0)})$ as the function of each quantity x_1 and x_2 . It is the natural assumption that the total utility function is just the sum of each utility function, as each utility obtained by getting each goods is independent. Then the total utility function in this case is given by

$$u(x_1, x_2) = A_1 \log(x_1/x_1^{(0)}) + A_2 \log(x_2/x_2^{(0)}) \quad (3)$$

$$= \log\left((x_1/x_1^{(0)})^{A_1} \times (x_2/x_2^{(0)})^{A_2}\right). \quad (4)$$

The second expression in the above means that the total utility function is the logarithm of the Cobb-Douglas type function. We can generalize this analysis to n -types of goods, and the total utility function is given by

$$u(x_1, x_2, \dots, x_n) = u_1(x_1) + u_2(x_2) + \dots + u_n(x_n), \quad (5)$$

where

$$u_i(x_i) = A_i \log(x_i/x_i^{(0)}). \quad (6)$$

Next, we derive the demand function by using this utility function.

We assume that each price p_i of i -th goods is given. According to the standard analysis to maximize the utility function under budget constraints, we have

$$\frac{\partial u/\partial x_1}{p_1} = \frac{\partial u/\partial x_2}{p_2} = \dots = \frac{\partial u/\partial x_n}{p_n} = k(x_1, x_2, \dots, x_n). \quad (7)$$

Using the utility function of separable type Eq(5), we have

$$\frac{du_1(x_1)/dx_1}{p_1} = \frac{du_2(x_2)/dx_2}{p_2} = \dots = \frac{du_n(x_n)/dx_n}{p_n} = k(x_1, x_2, \dots, x_n). \quad (8)$$

The above relation is satisfied for any number n of types of goods, which means that $k(x_1, x_2, \dots, x_n) = \text{const.}$, because $k(x_1, x_2, \dots, x_n)$ is independent of the type of goods. Normalizing the utility function, we can choose $k = 1$ and obtain the relation of the demand function in the form

$$p_i = \frac{du_i(x_i)}{dx_i} \quad (i = 1, 2, \dots, n), \quad (9)$$

which is correct for the general utility function of the separable type. The above argument is interpreted as follows. Suppose we consider the $(n + 1)$ -th goods as money itself, then the $(n + 1)$ -th utility function $u_{n+1}(x_{n+1})$, which utility is measured by the unit of money, is given by $u_{n+1}(x_{n+1}) = p_{n+1}x_{n+1}$. Then $\frac{du_{n+1}(x_{n+1})/dx_{n+1}}{p_{n+1}} = 1$, which gives $k = 1$.

Further, we use the explicit form Eq.(6) of the utility function, which is suggested by the Weber-Fechner's law, and we have the explicit quantitative demand function in the familiar form

$$p_i = \frac{A_i}{x_i} \quad (i = 1, 2, \dots, n). \quad (10)$$

In this way, applying the Weber-Fechner's law to each utility function and assuming that total utility is just the sum of the utility of each goods, we obtain the quantitative demand function in the familiar form.

§3. Demand function from the consumer's surplus

We derive the relation of the demand function Eq.(9) by using the utility function from the view point of the consumer's surplus. Consumer's surplus is defined in the following way

$$(\text{Consumer's surplus}) = \int_0^x p(x')dx' - px, \quad (11)$$

where p and x is the price and the quantity of goods. We define the concept of "consumer's profit" $\rho(x)$ as $\rho(x) = u(x) - px$, that is, the "consumer's profit" means the profit of money in mind minus the loss of money paid to obtain goods. Then the concept of the consumer's surplus is equal to the "consumer's profit", that is, (consumer's surplus)=("consumer's profit") = $\rho(x) = u(x) - px$. This gives the relation

$$\int_0^x p(x')dx' = u(x), \quad (12)$$

where we assume $u(0) = 0$ as the utility function should have this property. Then we again obtain the relation of the demand function in the same form as Eq.(9)

$$p = \frac{du(x)}{dx}, \quad (13)$$

from the view point of the consumer's surplus.

We define the "producer's profit" $\pi(x)$ as $\pi(x) = px - c(x)$ where $c(x)$ is the cost function. As is well-known, the supply function is derived by maximizing this "producer's profit". Then the relation of the supply function is given by

$$\underline{\text{Supply function}} : \frac{d\pi(x)}{dx} = p - \frac{dc(x)}{dx} = 0. \quad (14)$$

Similarly, we can derive the demand function by maximizing the "consumer's profit", $\rho(x) = u(x) - px$. Then the relation of the demand function is given in the same form as Eq.(9)

$$\underline{\text{Demand function}} : \frac{d\rho(x)}{dx} = \frac{du(x)}{dx} - p = 0. \quad (15)$$

In this way, the relation of the demand function is derived just in the same way as the the relation of the supply function from the view point of the consumer's surplus and we again obtain the same relation Eq.(9) for the demand function.

If we assume $u(0) \neq 0$, the "consumer's profit" differs from the consumer's surplus by only the constant value $u(0)$. Then we can say that the demand function is determined by maximizing the "consumer's profit" or by maximizing the consumer's surplus. Similarly, the "producer's profit" differs from the producer's surplus by only the constant value $c(0)$. Then we can say that the supply function is determined by maximizing the "producer's profit" or by maximizing the producer's surplus.

§4. Summary and discussion

We try to reconstruct economics as the application of behavioral psychology. The most important law in behavioral psychology is the Weber-Fechner's law, which represents the relation between the magnitude of physical stimulus and the magnitude of psychological sense in human being. While the utility function is one of the most important thing in micro economics. In this paper, we apply the Weber-Fechner's law to determine the explicit quantitative utility function. Then utility function becomes in the form of the logarithm of the Cobb-Douglas type function. We conclude that *i*) the utility function of n -types of goods are separable $u(x_1, x_2, \dots, x_n) = u_1(x_1) + u_2(x_2) + \dots + u_n(x_n)$, *ii*) each utility function becomes $u_i(x_i) = A_i \log(x_i/x_i^{(0)})$. From the property *i*), we have the relation of the demand function in the form $p_i = du_i/dx_i$. Using also the property *ii*), which is suggested by the Weber-Fechner's law, we have the explicit quantitative demand function in the familiar form $p_i = A_i/x_i$. We also derive the relation of the demand function from the view point of the consumer's surplus and obtain the same relation $p_i = du_i/dx_i$ for the demand function.

In this way, as the application of the Weber-Fechner's law to the utility function, we obtain the quantitative demand function in the familiar form. Our analysis is the zero-th order approximation for the demand function. If we modify the Weber-Fechner's law into Stevens' law, we obtain the more complicated demand function.

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