

# Optimal Auditing Under Intermediated Contracting

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## Abstract

This paper builds on Faure-Grimaud and Martimort's [Economics Letters 71 (2001) 75-82] analysis of intermediated contracting. I argue that intermediated contracting permits one form of auditing, in which the sub-contract offered to the firm is examined, contingent on the intermediary's report. Auditing reduces the intermediary's rent and increases allocative efficiency.

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## 1 Introduction

Faure-Grimaud and Martimort (2001)<sup>1</sup> in a three-tier model with principal, intermediary, and productive agent have filled an important lacuna in the literature of regulatory contracting under delegation and supervision by addressing an aspect that had been overlooked previously. In their setting, the principal delegates the *design* of the contract to the intermediary<sup>2</sup> (regulator) to induce production by the productive agent (firm) that can be of three types. Since the design of the sub-contract offered by intermediary is not contractible, a new form of agency cost arises: The intermediary can appropriate some of the information rent provided in the budget to permit production by one of the two most efficient firms, by playing a gamble and offering a contract that only the most efficient firm would accept. If the intermediary is lucky and the agent is of the most efficient type, she can pocket a surplus.

The FGM model's merit is to derive a form of agency costs directly from the limits in the contract design when an intermediary is needed to filter out an unwanted third type from producing. In turn, leaving the intermediary this freedom is costly: Should truth-telling be ensured and the intermediary be kept from gambling, the principal needs to increase her information rent.

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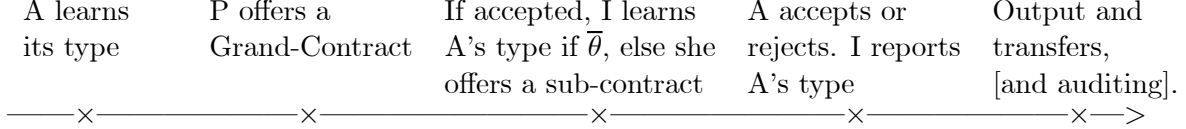
<sup>1</sup>henceforth FGM.

<sup>2</sup>As common in this literature, I use female pronouns for the intermediary.

Interesting as this viewpoint is as a next step toward a theory of bureaucracy, the FGM model still rests on a simplifying assumption, namely that the principal has no further means to reduce the intermediary's discretion through some form of bureaucratic control via internal or external auditing. It is not only useful from a theory perspective to clarify which auditing schemes are accessible to this form of contracting, and contingent on which report auditing is optimally performed. Under intermediated contracting, the principal cannot interact with the agent and may remain uninformed *ex-post* about the terms of the sub-contract, which intuitively calls for additional measures of control. Public administration as well as private organizations in virtually all countries make use of auditing policies that give the organizational design additional bite. Existing Government Auditing Standards<sup>3</sup> furthermore underpin the arguments presented in this paper.

## 2 The Grand-Contract in the FGM Model

This section provides a brief summary of the Grand Contract in the FGM model.<sup>4</sup> The agent's type space  $\Theta = \{\underline{\theta}, \hat{\theta}, \bar{\theta}\}$  includes the possible marginal costs of production. Type  $\bar{\theta}$  has excessive marginal costs such that it never should be offered a contract. To avoid type  $\bar{\theta}$  to produce, the principal hires the intermediary. I use the following notation for the distribution of prior beliefs for the three types, namely  $(1-p)\nu$ ,  $(1-p)(1-\nu)$ , and  $p$ . The timing of the contracting game including principal (P), intermediary (I) and agent (A) reads as follows:



The principal maximizes  $S(q) - s$ , with  $s$  denoting the budget, and with a marginal surplus of  $S' > 0$ , with  $S'' < 0$ . The intermediary has a VNM utility function of  $V(s-t) = \frac{1-e^{-r(s-t)}}{r}$ . The risk-neutral firm receives a transfer  $t$ .<sup>5</sup>

FGM (2001:79) have shown that principal can design a delegation-proof grand-contract with budget and production targets of  $(\underline{s} = s(\underline{q}), \underline{q}), (\hat{s} = s(\hat{q}), \hat{q}), (\bar{s} = s(\bar{q}), \bar{q} = q(\bar{\theta}) = 0)$ . Using  $\underline{v} = \underline{s} - \underline{\theta}\underline{q}$  and  $\hat{v} = \hat{s} - \hat{\theta}\hat{q}$  to denote the intermediary's information rents for the firm's respective types  $\underline{\theta}$  or  $\hat{\theta}$ , the downward incentive constraints can be expressed as

$$\underline{v} \geq \hat{v} + \Delta\theta\hat{q} \tag{1}$$

and

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<sup>3</sup>GAO (2003).

<sup>4</sup>Two *errata* in the original FGM paper somewhat harm the readability of their interesting model:

- The condition assuring the Principal to not induce the extremely inefficient type (page 77, line 5) should read in the original FGM notation:

$$\bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta + \frac{1}{1-\nu} \frac{\nu\Delta\theta}{e^{r\Delta\theta\bar{q}} - \nu} < S'(0) < \theta^\infty.$$

- The intermediary's interim participation constraint (9) on page 79 should read in the original notation:  $\nu V(\underline{v} - \Delta\theta\bar{q}) + (1-\nu)V(\bar{v}) \geq 0$ .

<sup>5</sup>As in the standard screening contract, the agent's participation constraint for the less efficient type  $\hat{u} \geq 0$ , and the incentive constraint for the efficient type  $\underline{u} \geq \hat{u} + \Delta\theta\hat{q}$  are binding.

$$\nu V(\underline{v} - \Delta\theta\hat{q}) + (1 - \nu)V(\hat{v}) \geq \nu V(\underline{v}) + (1 - \nu)V(s(\bar{q})). \quad (2)$$

The intermediary's two participation constraints are

$$V(\bar{s}) \geq 0 \quad (3)$$

and

$$\nu V(\underline{v} - \Delta\theta\hat{q}) + (1 - \nu)V(\hat{v}) \geq 0. \quad (4)$$

Stating the Grand-Contract as a problem over transfers and quantities leads to

$$\max_{\{(\underline{s}, \underline{q}), (\hat{s}, \hat{q}), (\bar{s})\}} (1 - p) [\nu (S(\underline{q}) - \underline{\theta}\underline{q} - \underline{v})] + p(1 - \nu) [S(\hat{q}) - \hat{\theta}\hat{q} - \hat{v}] - ps(\bar{q}) \quad (5)$$

s.t. (1), (2), (3), (4), with (1) binding and (2) rewritten as

$$V(\hat{v}) \geq \nu V(\hat{v} + \Delta\theta\hat{q}). \quad (6)$$

**Proposition 1** (FGM 2001) *The grand-contract does not entail any distortion on the efficient coalition's output, but optimally reduces the less efficient firm's production target by adding  $\frac{1}{1-\nu} \frac{\nu\Delta\theta}{e^{r\Delta\theta\hat{q}-\nu}}$  to the downward output distortion.<sup>6</sup>*

*The optimal contract leads the following output scheme:*

$$S'(\underline{q}^I) = \underline{\theta}$$

and

$$S'(\hat{q}^I) = \hat{\theta} + \frac{\nu}{1-\nu}\Delta\theta + \frac{1}{1-\nu} \frac{\nu\Delta\theta}{e^{r\Delta\theta\hat{q}-\nu}}.$$

An interior solution exists if the principal's objective function is strictly concave and  $|S''(\cdot)|$  is sufficiently large;  $\hat{\theta} + \frac{\nu}{1-\nu}\Delta\theta + \frac{1}{1-\nu} \frac{\nu\Delta\theta}{e^{r\Delta\theta\hat{q}-\nu}} < S'(0) < \bar{\theta}$  must hold to ensure that the principal does not want to have the  $\bar{\theta}$ -type in the regime.

Equation (6) is essential for an understanding of the limits of this design: The intermediary may risk to offer a shutdown contract to the firm. which will be only accepted by the  $\underline{\theta}$ -firm. If the firm turns out to be of type  $\underline{\theta}$ , this permits the intermediary to pocket the virtual costs  $\Delta\theta\hat{q}$  herself; if the agent is of type  $\hat{\theta}$ , she would misreport to the principal this type to be of  $\bar{\theta}$ . To ensure compliance, the intermediary receives a compensation:  $\hat{v}$  is increased to fulfill (6). The more risk averse the intermediary, the easier it is for the principal to do so; an  $r$  close to infinity creates an upper bound on the information rent.

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<sup>6</sup>See FGM (2001:79).

### 3 The Grand-Contract Under Auditing

It is unlikely to find real-world organizations that do not further restrict the intermediary's discretion by including some form of bureaucratic control into the contract design. What we observe is a variety of organizations that use different rules and control schemes to limit the freedom of intermediate agents. Typically, auditing standards are designed to curb bureaucratic discretion to an optimal degree, given that verifiability is limited and auditing is costly.

Intermediated contracting and the information structure of the gaming imposes some limits on the design of an auditing scheme.<sup>7</sup> The principal cannot interact with the firm but receives the intermediary's report from which he can infer the existence of a subcontract. This calls for a specific auditing scheme in which the principal does not audit the firm but examines whether sub-contract was carried out according to his intention. As explained below, such a treatment is not only intuitive, it follows directly from the principal's design options and the timing of the game.

#### 3.1 Incentive-Feasible Auditing Mechanisms

To extend the Grand Contract design to include auditing I start with the following assumptions on how the auditing process is carried out:

**Assumption 1** *The principal cannot interact with the firm but designs a Grand Contract that involves examination and potential punishment of the intermediary, not of the firm.*

**Assumption 2** *Auditing occurs contingent on the intermediary's report  $\tilde{\theta} \in \{\underline{\theta}, \hat{\theta}, \bar{\theta}\}$ .*

Assumption 1 follows from the FGM model and is appropriate since the intermediary is hired to design a screening contract in order to have both the  $\hat{\theta}$  and  $\underline{\theta}$  type in the regime. The intermediary is hired to find out the extremely inefficient type  $\bar{\theta}$  but not to reduce the virtual costs that are necessarily included in the budget to ensure truth-telling by the  $\underline{\theta}$ -firm in a screening contract. Inflicting a punishment both on the intermediary *and* the firm would contradict the principal's intent to hire an intermediary.

Assumption 2 is appropriate when the auditor cannot find evidence without an existing sub-contract and the firm as third party is not known. It is also appropriate in cases in which the intermediary would be required to keep track of her (sub-)contract offer but no auditor may be able to reconstruct whether this documentation represents the true offer at the time it was made. An auditor, being unable to verify the state of nature cannot produce pertinent information. Without loss of generality I therefore limit auditing to an investigation in the case in which a sub-contract exists between intermediary and firm.

**Proposition 2** *Since auditing is costly, the principal optimally designs a Grand-Contract in which the intermediary is audited if and only if she reports to the principal that the agent is of type  $\underline{\theta}$ .*

**Proof. Case 1:** The intermediary reports  $\tilde{\theta} = \bar{\theta}$ . No sub-contract exists, and the report can either be true or false; in the case of a lie the true type is  $\theta = \hat{\theta}$ . This reveals that the

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<sup>7</sup>I exclude the unrealistic case of full verifiability, with marginal costs of auditing low enough to detect the true state with certainty. This occurs without loss of generality and can be seen as a borderline case. Under full verifiability the FGM effect would simply disappear and the model reduces to a two-tier principal-agent setting without further distortion.

intermediary lost her gamble when offering the shutdown contract. The principal does not lose money and the intermediary was unable to reap any rent.<sup>8</sup>

**Case 2:** The intermediary reports  $\tilde{\theta} = \hat{\theta}$ . This can only be a true report, and  $\theta = \hat{\theta}$ . A sub-contract was signed, and the report  $\tilde{\theta} = \hat{\theta}$  implies that the sub-contract was a (screening) contract designed for both types of firms, and the firm was of type  $\hat{\theta}$ . This all occurred according to the principal's intention, and we have no need for auditing.

**Case 3:** The intermediary reports  $\tilde{\theta} = \underline{\theta}$ . Only this case will be audited. A contract was signed but only costly examination can reveal if the intermediary has offered a shut-down contract, accepted by the  $\underline{\theta}$ -firm. This sub-contract did not include the virtual costs  $\Delta\theta\hat{q}$  in the budget and is captured by the misreporting intermediary. ■

Costs and probabilities of auditing relate as follows. The principal has now access to an (internal or external) audit technology that permits to detect the true state of the sub-contract with probability  $\varphi$ , while incurring strictly convex costs of auditing  $c(\varphi)$ , and  $c(0) = 0$ ,  $c' > 0$ , and  $c'' > 0$ . I assume that the Inada conditions  $c'(0) = 0$ ,  $c'(1) = +\infty$  are fulfilled.

The Grand-Contract, in addition to output targets  $q$  and budgets  $s$  now also includes the probability of audit  $\varphi(\tilde{\theta})$  and a punishment  $P(\theta, \tilde{\theta})$ . Since auditing is costly, I furthermore limit this probability to  $\underline{\varphi}$  since auditing can only reveal additional information when the announced type was  $\tilde{\theta} = \underline{\theta}$ . The punishment  $P^s$  is inflicted on the intermediary in the case a shutdown-contract was signed. I assume endogenous punishment, which implies that the intermediary is punished up to her information rent in case of a detected lie:

$$P^s \leq \Delta\theta\hat{q}. \quad (7)$$

Auditing under intermediated contracting relaxes the intermediate's incentive constraint (6) into

$$V(\hat{v}) \geq \nu[\underline{\varphi}V(\hat{v} + \Delta\theta\hat{q} - P^s) + (1 - \underline{\varphi})V(\hat{v} + \Delta\theta\hat{q})], \quad (8)$$

which reveals that the intermediary, with probability  $\underline{\varphi}$  is inflicted a punishment  $P^s$  in the case auditing detects a lie, while with probability  $1 - \underline{\varphi}$  she still keeps her information rent as before. The information rent that the intermediary can claim is therefore reduced. With (7) binding, (8) reduces to

$$V(\hat{v}) \geq \nu[\underline{\varphi}V(\hat{v}) + (1 - \underline{\varphi})V(\hat{v} + \Delta\theta\hat{q})]. \quad (9)$$

Solving (9) for  $\hat{v}$  shows that this rent is now  $\hat{v} = \frac{1}{r} \ln \frac{1 - \nu e^{-r(1 - \underline{\varphi})\Delta\theta\hat{q}}}{1 - \nu}$ . This permits to derive the closed-form expression of the Grand-Contract:

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<sup>8</sup>Hiring an auditor in this case would furthermore contradict the basic assumptions of intermediated contracting: since the intermediary was exactly hired because of his ability to discover the type  $\tilde{\theta}$  without incurring any cost, it is not intuitive to hire a next supervisor or auditor to perform the same task.

$$\begin{aligned} & \max_{\{(\underline{s}, \underline{q}), (\hat{s}, \hat{q}), (\underline{\varrho}), (\bar{s})\}} (1-p) \left[ \nu(S(\underline{q}) - \underline{\theta}\underline{q} - \frac{1}{r} \ln \frac{1 - \nu e^{-r(1-\underline{\varrho})\Delta\theta\hat{q}}}{1-\nu} - (1-\underline{\varrho})\Delta\theta\hat{q} \right. \\ & \left. - c(\underline{\varrho}) + (1-\nu)(S(\hat{q}) - \hat{\theta}\hat{q} - \frac{1}{r} \ln \frac{1 - \nu e^{-r(1-\underline{\varrho})\Delta\theta\hat{q}}}{1-\nu}) \right] + p\bar{s}, \end{aligned} \quad (10)$$

and to sum up the findings under the following propositions:

**Proposition 3** *Under auditing, the Grand-Contract entails no distortion on the efficient coalition's output:*

$$S(\underline{q}^A) = \underline{\theta}.$$

The information rent that the intermediary can obtain is reduced under auditing. The second-best solution for the regime with a  $\hat{\theta}$ -firm and endogenous punishment now leads to a lower information rent for the intermediary

$$S(\hat{q}^A) = \hat{\theta} + \frac{\nu}{1-\nu}(1-\underline{\varrho}^A)\Delta\theta + \frac{1}{1-\nu} \frac{\nu\Delta\theta(1-\underline{\varrho}^A)}{e^{r(1-\underline{\varrho}^A)\Delta\theta\hat{q}-\nu}} < \hat{\theta} + \frac{\nu}{1-\nu}\Delta\theta + \frac{1}{1-\nu} \frac{\nu\Delta\theta}{e^{r\Delta\theta\hat{q}-\nu}}.$$

**Proposition 4** *The principal optimally chooses a strictly positive probability of auditing until the marginal costs of auditing are equal to the marginal utility of the agent plus the virtual costs of production.*

**Proof of Proposition 3.** Deriving (10) w.r. to  $\underline{q}$  and  $\hat{q}$  yields the above stated results. ■

**Proof of Proposition 4.** Deriving the Grand-Contract w.r. to  $\underline{\varrho}$  yields:

$$\nu \frac{d}{d\underline{\varrho}} \left( \frac{1}{r} \ln \frac{1 - \nu e^{-r(1-\underline{\varrho})\Delta\theta\hat{q}}}{1-\nu} \right) + (1-\nu) \frac{d}{d\underline{\varrho}} \left( \frac{1}{r} \ln \frac{1 - \nu e^{-r(1-\underline{\varrho})\Delta\theta\hat{q}}}{1-\nu} \right) - c'(\underline{\varrho}) + \Delta\theta\hat{q} \doteq 0,$$

from which the requested result  $c'(\underline{\varrho}) = \frac{\nu\Delta\theta}{e^{r\Delta\theta\hat{q}-\nu}} + \Delta\theta\hat{q}$  is obtained. ■

## 4 Comparative Statics and Discussion

Firstly, it is easy to see that  $q^I(\hat{\theta}) < \hat{q}^A < q^{sb}(\hat{\theta})$  and that the Grand-Contract under auditing reaches a more efficient allocation since the principal relies less often on output distortions.

Second, a second-order Taylor expansion of the intermediary's rent under auditing,  $\frac{1}{r} \ln \frac{1 - \nu e^{-r(1-\underline{\varrho})\Delta\theta\hat{q}}}{1-\nu}$ , further reveals how auditing improves the contract design. For small levels of the efficient firm's information rent  $\underline{u} = \Delta\theta\hat{q}$  we can approximate the intermediary's rent by

$$\hat{v} \approx \frac{\nu}{1-\nu}(1-\underline{\varrho})\underline{u} - \frac{r\nu}{2(1-\nu)^2}(1-\underline{\varrho})\underline{u}^2,$$

which shows in its first term on the R.H.S. that the stake  $\underline{u}$  that the intermediary can obtain is lowered since multiplied with  $(1-\underline{\varrho})$ . The higher the probability of auditing, the lower the rent that the principal needs to include to the intermediary to satisfy (9). The second term indicates that also the rate at which the intermediary can appropriate this stake is

affected. The probability of auditing  $\underline{\varrho}$  enters multiplicatively via the term  $(1 - \underline{\varrho})$ , both affecting the stake of the intermediary and its rate.

Auditing makes sense particularly when the intermediary's risk aversion  $r$  is not too high. The less risk averse the intermediary, the more the principal can benefit from auditing, which permits to increase allocative efficiency. High values of  $r$  instead impose an upper bound on the intermediary's rent<sup>9</sup> and thus on the agency costs of intermediated contracting. The higher  $r$ , the higher  $\underline{\varrho}$  needs to be to show any effect. If  $r$  reaches values close to infinity, auditing would lose its bite.

## 5 Conclusion

This paper sets out to make a simple point. There exists an optimal auditing scheme applicable to intermediated contracting that follows straightforward from the timing and the information structure in the FGM model. The principal has the sub-contract examined contingent on the intermediary's report. This scheme is not only intuitive since it follows general auditing practices to bundle the efforts toward contractual states that are worthwhile to be examined. The particular property is that auditing curbs the bureaucrat's (intermediary's) discretion. The assumptions on intermediated contracting in the base FGM model render this approach particularly intuitive.

An infinitely risk-averse intermediary does not need auditing; the result here is the same as in the original model of FGM. But whenever the costs of the audit technology permit partial revelation and the Inada conditions are fulfilled, auditing reduces the rent  $\hat{v}$  that the intermediary can require for executing the contract according to the principal's will. Auditing gives the principal additional leeway to improve the organizational design, and to increase allocative efficiency.

## 6 Acknowledgements

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## 7 References

- Faure-Grimaud, A., Martimort, D., 2001. On some agency costs of intermediated contracting. *Economics Letters* 71, 75-82.
- GAO, 2003. *Government Auditing Standards, 2003 Revision*, United States General Accounting Office, Washington, D.C. .
- Laffont, J.-J., Martimort, D., 2002. *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton.
- Tirole, J., 1986. Hierarchies and Bureaucracies: On the Role of Collusion in Organizations. *Journal of Law, Economics and Organization* 2, 181-224.

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<sup>9</sup>See FGM (2001).