A Search Model of Centralized and Decentralized Trade*

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Abstract

This paper presents a search model of centralized and decentralized trade. In a centralized market, trades are intermediated by market makers at publicly posted bid-ask prices. In a decentralized market, traders search counterparties. Prices are negotiated and transactions are conducted in private meetings among traders. Traders can choose which market to enter. The determinant of bid-ask spreads and liquidity is analyzed. The welfare consequence of the market fragmentation is also analyzed. Some limiting results and convergence to the Walrasian equilibrium as search fricitions or transaction costs vary are established.

Keywords: search, matching and bargaining, bid-ask spread, liquidity, welfare, Walrasian equilibrium

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1 Introduction

In the modern economy, some commodities and assets are traded in both centralized and decentralized markets. In centralized markets, trades are intermediated by market makers at publicly posted bid-ask prices. In decentralized markets, traders search counterparties. Prices are negotiated and transactions are conducted in private meetings among traders. For example, in certain securities and futures markets, there are both centralized trading and off-floor trading. In recent years, equity trading is becoming less centralized as third and fourth market activity has expanded greatly.

The above observation raises the following questions: What determines liquidity and the bid-ask spread? Why and under what conditions can the centralized and decentralized markets coexist? How does the market fragmentation influence liquidity and the bid-ask spread? What is the social consequence of the market fragmentation?

This paper provides a simple search model to shed light on these questions. The model is based on Rubinstein and Wolinsky (1985) and Gale (1987). At each date, there are potential inflows of new buyers and sellers. Buyers are heterogeneous in their valuation of an asset, while sellers are homogeneous. Buyers and sellers can choose to trade in the centralized market or in the decentralized market. Once a trader makes a transaction, he leaves the economy. Trading in the decentralized market is costly since search incurs time and contact costs. Trading in the centralized market is costly since there are transaction costs for market making.

I analyze and characterize stationary equilibrium. I find that a trader's choice between the two markets has important externalities on other traders.[1] First, a trader's participation decision depends on other traders' participation decisions. This may cause multiplicity of equilibria. If a trader anticipates that all other traders go to one of the markets, then he will also trade in that market. Thus, concentration of trade may occur. The conditions under which the market fragmentation or concentration

[1]Search externalities are emphasized in the labor market models, e.g., Diamond (1982), Mortensen (1982), and Hosios (1990).

can occur depend on the parameter values describing the relative efficiency of the two markets. Second, if a trader enters the centralized market, then he leaves the pool of searchers in the decentralized market. Thus, the bid-ask prices in the centralized market influences the market tightness in the decentralized market, and hence its equilibrium outcome, including negotiated prices and traders' payoffs. These two externality effects are important for understanding the working of the model.

The main results of the paper are as follows. First, I analyze the determinant of the bid ask spread.[2] I show that no matter how bid-ask prices are set in the centralized market, there is a positive bid-ask spread as long as the centralized and decentralized markets coexist in equilibrium. Moreover, the average negotiated price in the decentralized market is inside the bid-ask spread (see the supporting experimental evidence reported by Campbell et al (1991)). This result is related to the idea that market makers provide service of immediacy, as pointed out by Demsetz (1968). Thus, the quoted ask price includes a premium for immediate buying and the bid price reflects a concession required for immediate sale.

I also show that the bid-ask spread reflects the transaction cost in the centralized market and the search frictions in the decentralized market. In particular, under competitive marketmaking, the bid-ask spread is equal to the transaction cost. Under monopolistic marketmaking, the bid-ask spread exists even if there is no transaction cost. Further, it is positively related to the search frictions in the decentralized market reflected by the discount rate and contact rate. That is, the bid-ask spread is narrower if traders are more patient or can more easily find other traders. Another testable result is that under both competitive and monopolistic marketmaking, the bid-ask spread is positively related to the average negotiated price in the decentralized market.

Second, I show that liquidity in the centralized market measured by trading vol-

[2]The main traditional theories of bid-ask spreads are based on inventory risk (Garman (1976), Amihud and Mendelson, and Ho and Stoll (1981)) or asymmetric information (Bagehot (1971), Glosten and Milgrom (1985), and Kyle (1985)).

ume is negatively related to the bid-ask spread, and positively related to search frictions in the decentralized market. This result demonstrates that trading volume reflects the relative efficiency of the centralized and decentralized markets because of competition. It is supported by some experimental and empirical studies (e.g., Campbell et al (1991), Lamoureux and Schnitzlein (1997), Stoll (2000))

Third, I establish some limiting results. Specifically, consider perfectly competitive market makers. I show that starting from an equilibrium in which centralized and decentralized markets coexist, then the decentralized market is driven out of the economy if the search frictions in the decentralized market becomes large enough, or the transaction cost in the centralized market becomes small enough. A counterpart result is obtained for the centralized market. Furthermore, if either search frictions or transaction costs vanish, then the limiting equilibrium becomes Walrasian in the flow sense of Gale (1987).

Finally, I show that the fragmentation of a centralized market improves social welfare if the bid-ask spread after the fragmentation is small enough. In particular, under competitive market making, the fragmentation always improves social welfare. I also show that the opening of a centralized market in a decentralized market economy may not improve social welfare. These results seem surprising since trading in the centralized market provides immediacy and saves search costs, which should benefit traders. The reasons for my results are as follows: (i) Each transaction in the centralized market incurs a cost. Under the market fragmentation, traders have an additional marketplace to trade, which can save transaction costs. This effect may dominate so that fragmentation improves social welfare.[3] (ii) The opening of a centralized market in a decentralized market benefit high valuation buyers since low valuation buyers do not enter the centralized market. However, it also imposes negative externalities on the decentralized market since it makes the market tighter. Thus, buyers in the decentralized markets are worse off.

[3] See Campbell et al (1991) for experimental evidence.

I also study the case where market makers act as a social planner to select bid-ask prices so as to maximize social welfare, given search frictions. Compared to this constrained social optimum, both competitive and monopolistic marketmaking imply too much entry to the centralized market and too narrow bid-ask spread.

This paper is related to the literature on the search models of exchange. As mentioned earlier, the seminal papers are Rubinstein and Wolinsky (1985) and Gale (1987). Mortensen and Wright (2002) extend these papers by adding pecuniary search costs and considering a general matching technology and bargaining rule. They show that constrained efficiency can be obtained if third-party market makers set up a complete set of submarkets and traders can select into appropriate submarkets.

Rubinstein and Wolinsky (1987) introduce middlemen into a search model. Their model has been generalized by a number of papers (e.g., Li (1998) and Shevchenko (2004)). In my model, intermediation is conducted by market makers, instead of middlemen. As a result, all traders can trade at publicly posted bid-ask prices. This is different from the middlemen model in which traders can search for and bargain with middlemen or counterparties. My model is closer to Gehrig (1993), who studies a static search model with market makers. However, my dynamic model has many implications different from Gehrig (1993).[4] Spulber (1996) and Rust and Hall (2003) study dynamic search models of intermediation, but they do not consider traders searching each other in the decentralized market. Neeman and Vulkan (2003a) provide a different model of centralized and decentralized trade. They show that the entry of a market maker causes a complete unraveling of direct negotiations, and in perfect equilibrium almost all trade takes place in the centralized market.

The link between liquidity and search is pointed out by Lippman and McCall (1986). The issue of liquidity, concentration and fragmentation of trade across markets is studied by Pagano (1989) in a static model without intermediaries. Similar questions are analyzed in models based on asymmetric information or inventory risk

[4]For example, Gehrig's model implies that trading volume is independent of search costs or transaction costs.

(e.g. Mendelson (1987), Biais (1993), Madhavan (1995)).

My model is also related to several search models of asset markets initiated by Duffie et al (2003a). Duffie et al (2003b) generalize Rubinstein and Wolinsky's (1987) model. Although their paper differs from mine in terms of addressed questions and modelling details, they also show that the bid-ask spread is smaller if traders can find other traders more easily. The underlying mechanism is very different. In their model, increased search efficiency improves a trader's bargaining position relative to the middlemen, while in my model it provides competitive pressure on the centralized market. Vayanos and Wang (2003) generalize Duffie et al (2003a) and consider that traders can trade two identical assets in two decentralized markets. They study the welfare implication of the concentration of liquidity.

The paper is organized as follows. Section 2 presents a search model without intermediaries. Section 3 introduces a centralized market and analyzes equilibria with competitive market makers and with a monopolistic market maker. Section 4 conducts welfare analysis. Section 5 concludes.

2 Trade in the Decentralized Market

As a benchmark, I start with the case where there is no intermediary and all trades are conducted in the decentralized market. I model trades in the decentralized market as a process of search, matching and bargaining (see Rubinstein and Wolinsky (1985), Gale (1987), Mortensen and Wright (2002)).

Time is continuous and continues forever. At each date, there are f_B buyers and $f_S < f_B$ sellers potentially entering the market.[5] A trader enters the market if his expected payoff is positive and only if his expected payoff is non-negative. All traders are risk neutral. Each seller has one unit of an asset (or indivisible good) to sell and each buyer demands only one unit of the asset. Once a buyer buys or a seller sells, he exits the market. Buyers are heterogeneous in their valuations. Each

[5] This assumption will be clear from equation (7) below.

buyer's valuation v is drawn from a uniform distribution over $[0,1]$. All sellers are homogeneous and can hold the asset without cost.

I focus on the steady state, in which there are M_B buyers and M_S sellers in the market at each date. These numbers will be determined endogenously in equilibrium. Moreover, there is a stationary distribution of buyers F in the market. This distribution will be endogenously determined in equilibrium. It may be different from the exogenous uniform distribution of potential entrants since not everyone enters.

2.1 Matching and Bargaining

Search frictions are modeled in two dimensions: (i) There is an implicit time cost in that all traders discount future values by the discount rate $r > 0$. (ii) There is an explicit search cost in that a trader contacts another trader randomly. Assume that a trader contacts another trader according to a Poisson process with intensity $\rho > 0$. Since a trader is a buyer with probability $\gamma = \frac{M_B}{M_S + M_B}$, a seller meets a buyer with rate $\gamma\rho$ and a buyer meets a seller with rate $(1-\gamma)\rho$.

Once a buyer meets a seller, they negotiate a price to trade. For simplicity, suppose one of the two, chosen at random, announces a take-it-or-leave-it price offer. Let $\theta \in (0,1)$ be the probability the buyer makes the offer. If an offer is rejected, the traders part and continue searching; if the offer is accepted, exchange occurs and they leave the market.

Note that this bargaining protocol is equivalent to the generalized Nash solution. To see this, let $V_B(v)$ be the expect payoff of a buyer with valuation v and V_S be the expected payoff of a seller. Then the negotiated price

$$p(v) = \theta V_S + (1-\theta)(v - V_B(v)). \tag{1}$$

is the solution to the following problem.

$$\max_p (v - p - V_B(v))^{\theta} (p - V_S)^{1-\theta}$$

subject to

$$v - p \geq V_B(v), \; p \geq V_S.$$

The parameter θ can be interpreted as the relative bargaining power of the buyer.

2.2 Value Functions

It can be shown that the value function $V_B(v)$ satisfies the following Bellman equation:

$$rV_B(v) = \rho(1-\gamma)\max\{v - p(v) - V_B(v), 0\}. \tag{2}$$

The interpretation is as follows. At any date, the buyer with valuation v meets a seller with probability $\rho(1-\gamma)$. If he trades with the seller he obtains capital gains $v - p(v) - V_B(v)$. Otherwise, he has no capital gains.

Similarly, the seller's value function satisfies the following Bellman equation:

$$rV_S = \rho\gamma E_F[\max\{p(v) - V_S, 0\}], \tag{3}$$

where E_F denotes the expectation operator with respect to the distribution F.

Using (1), one can rewrite the above Bellman equations as

$$rV_B(v) = \rho(1-\gamma)\theta\max\{v - V_B(v) - V_S, 0\}, \tag{4}$$

$$rV_S = \rho\gamma(1-\theta)E_F[\max\{v - V_B(v) - V_S, 0\}]. \tag{5}$$

Since it can be verified that $V_B(v)$ is increasing in v, there exists a cutoff value R such that only buyers with valuation $v \geq R$ have non-negative gains from trade. The cutoff value R satisfies

$$R - V_B(R) - V_S = 0, \; V_B(R) = 0, \tag{6}$$

and $V_B(v) = 0$ for $v < R$. Thus, only buyers with valuation $v \geq R$ enter the market. Moreover, every meeting results in trade since $v - V_B(v) - V_S \geq 0$ for $v \geq R$.

In a steady state, the following condition must be satisfied.

$$f_B(1-R) = (1-\gamma)\rho M_B = \rho\gamma M_S = f_S. \tag{7}$$

The first and last equalities require that the inflow and outflow of traders balance. The second equality says that buyers and sellers exit the market in pairs. Note that R is the Walrasian equilibrium price in the flow sense of Gale (1987) since the flow demand $(1-R)f_B$ is equal to the flow supply f_S.

2.3 Search Equilibrium

I now define equilibrium. A steady state search equilibrium without intermediaries can be described by the value functions (V_B, V_S), the negotiated price $p(v)$, the marginal participating type R, the stocks of buyers and sellers in the market (M_B, M_S), and the distribution of buyers in the market F, such that (i) (V_B, V_S) satisfies (4) and (5), (ii) $p(v)$ is given by (1), (iii) R satisfies (6), and (iv) (M_B, M_S) satisfies the flow market-clearing condition (7).

To solve for an equilibrium, first observe that the stationary distribution of buyers in the market is uniform and its density is given by $dF(v) = \frac{dv}{1-R}$. Next, it follows from (7) that

$$R = 1 - \frac{f_S}{f_B}. \tag{8}$$

Thus, the cutoff value R is determined exclusively by the flows of entrants.

I now solve for value functions. It follows from (6) that

$$V_S = R. \tag{9}$$

Thus, a seller's expected payoff is equal to the marginal participating buyer's valuation. The intuition is simple. When a seller meets the marginal participating buyer, they trade at the price equal to the marginal valuation. Moreover, the gain from trade is exactly equal to zero. Thus, the seller's reservation value must be equal to the marginal valuation.

Substituting (9) into (4) and simplifying yield the buyer value function

$$V_B(v) = \frac{\rho\theta(1-\gamma)}{r+\rho\theta(1-\gamma)}(v-R) \text{ for } v \geq R. \tag{10}$$

Inserting (9)-(10) into (1) yields the negotiated price

$$p(v) = R + \frac{r(1-\theta)}{r+\rho(1-\gamma)\theta}(v-R). \tag{11}$$

Substituting (9) and (10) into (5) and simplifying yield

$$R = \frac{\rho\gamma(1-\theta)}{r+\rho(1-\gamma)\theta}\int_R^1 [v-R]\frac{dv}{1-R}. \tag{12}$$

From this equation, one can solve for the matching probability γ,

$$\gamma = \frac{2R(r+\rho\theta)}{\rho(2R\theta+(1-\theta)(1-R))}. \tag{13}$$

Given the value of γ, the number of buyers and sellers, M_B and M_S, can be solved from (7), and an equilibrium is constructed.

I now summarize the above discussion in the following proposition.

Proposition 1 *If the parameter values* $(r, \theta, \rho, f_B, f_S)$ *satisfy*

$$\frac{2R(r+\rho\theta)}{\rho(2R\theta+(1-\theta)(1-R))} \in (0,1),$$

where $R = 1 - f_S/f_B$*, then there exists a unique search equilibrium without intermediaries.*

It is intuitive that traders can negotiate lower prices, when they are more patient, or when they can contact each other more easily, or when buyers have more bargaining power. This intuition is formalized in the following proposition.

Proposition 2 *Let the assumption in Proposition 1 holds. Then the negotiated price* $p(v)$ *is increasing with* r*, and decreasing with* ρ *and* θ *for all* $v > R$*.*

Proof. Substituting the expression for γ in (13) into (11) yields

$$p(v) = R + \frac{r(3\theta R - R + 1 - \theta)}{(r + \rho\theta)(1 - R)}(v - R).$$

Simple algebra delivers $\frac{\partial p(v)}{\partial \rho} < 0$, and

$$\begin{aligned}
\frac{\partial p(v)}{\partial r} &= \frac{\theta\rho(3R\theta - R + 1 - \theta)}{(r + \rho\theta)^2(1 - R)}(v - R), \\
\frac{\partial p(v)}{\partial \theta} &= -r\frac{r + \rho - \rho R - 3Rr}{(r + \rho\theta)^2(1 - R)}(v - R).
\end{aligned}$$

By assumption, one can verify that $\frac{\partial p(v)}{\partial r} > 0$ and $\frac{\partial p(v)}{\partial \theta} < 0$. ■

Since the support of F, $[R, 1]$, does not depend on r, ρ, and θ, the above result also holds for the expected negotiated price $E_F[p(v)]$.

Finally, the limiting equilibrium becomes Walrasian as the cost of search vanishes. A similar limiting result for r is obtained by Gale (1987).

Proposition 3 *If $r \to 0$, or if $\rho \to \infty$, then the limiting search equilibrium is Walrasian.*

Proof. Substituting (13) into (11) and letting r converges to 0, or letting ρ converges to infinity, one obtains that the limiting price is given by R for all v. Thus, all traders in the market trade at the Walrasian price. Thus, the limiting equilibrium is Walrasian. ■

3 Search Equilibrium with Market Makers

I now introduce a centralized market, in which traders can observe bid and ask prices and trades are intermediated by market makers. Traders have the option to trade in the centralized or decentralized market. Market makers remain in the market forever and do not hold inventory. Their role is to channel trade between buyers and sellers. A transaction incurs a fixed cost k. For now, I assume that in each period there are constant bid and ask prices b and a, publicly posted in the centralized market, and do not study how market makers determine these prices. I will consider this issue by analyzing two cases in subsections 3.4-3.5. In the first case, there is a unit mass of identical perfectly competitive risk-neutral market makers. In the second case, there is a monopolistic risk-neutral market maker.

Before I turn to the formal model, I briefly describe the equilibrium when only the centralized market is available. It is clear that only buyers with valuation $v \geq a$ enter the market, and that sellers enter the market if their payoffs are nonnegative. In the steady state, the flow market-clearing condition must be satisfied $(1-a) f_B = f_S$. Thus, the ask price $a = 1 - f_S/f_B = R$. If market makers are competitive, then they make zero profit and the bid price $b = a - k$. If there is a monopolistic market maker, the bid price is set to $b = 0$. To ensure the existence of an equilibrium, $0 \leq k \leq 1 - f_S/f_B = R$.

3.1 Matching

As in the previous section, I focus on the steady state, in which there are N_S sellers and N_B buyers at each date. Moreover, there is a fraction λ_S of sellers and a fraction λ_B of buyers trading in the centralized market. Market makers set bid and ask prices so that demand is equal to supply; that is,

$$\lambda_S N_S = \lambda_B N_B. \tag{14}$$

The remaining traders trade in the decentralized market, where a trader contacts another trader according to a Poisson process with intensity $\rho > 0$. Since a trader in the decentralized market is a buyer with probability

$$\alpha = \frac{(1-\lambda_B) N_B}{(1-\lambda_B) N_B + (1-\lambda_S) N_S}, \tag{15}$$

a seller meets a buyer with rate $\rho\alpha$ and a buyer meets a seller with rate $\rho(1-\alpha)$.

3.2 Value Functions

At each date, a new entrant faces the following problem. He first chooses which market to enter. If he decides to enter the centralized market and if he is a buyer with valuation v, then he buys the asset from the market makers at the ask price a and obtains utility $v - a$. On the other hand, if he is a seller, then he sells the asset to the market makers at the bid price b and obtains utility b. After trade, the trader leaves the economy. If the trader chooses to enter the decentralized market, then he has to find a counterparty and negotiates a price. After trade, he leaves the economy. If the trader does not meet a counterparty, then he has to make the same decisions described above again.

Formally, if a buyer with valuation v meets a seller in the decentralized market, they negotiate a price $p(v)$, which is determined as in subsection 2.1. That is,

$$p(v) = \theta U_S + (1 - \theta)(v - U_B(v)). \tag{16}$$

where U_S and $U_B(v)$ are the expected payoffs of a seller and a buyer with valuation v.

The expected payoffs of a buyer and a seller satisfy the following Bellman equations if they trade in the decentralized market:

$$rU_B(v) = \rho(1 - \alpha)\max\{v - p(v) - U_B(v), 0\}, \tag{17}$$

$$rU_S = \rho\alpha E_G[\max\{p(v) - U_S, 0\}], \tag{18}$$

where G is the conditional distribution of buyers in the decentralized market and will be determined in equilibrium.

As in the previous section, there is a cutoff value R such that only buyers with valuation $v \geq R$ enter the markets. Moreover, every meeting results in trade. The cutoff value R satisfies

$$R - U_B(R) - U_S = 0, \ U_B(R) = 0, \tag{19}$$

and $U_B(v) = 0$ for $v < R$. Thus,

$$U_S = R. \tag{20}$$

It follows that in order to have both trade in centralized and decentralized markets, the market makers must set the bid price $b = R$ so that sellers are indifferent between the two markets.

Since it can be verified that $v - U_B(v)$ given below is increasing in v, there is a marginal buyer v_m such that buyers with valuation $v \geq v_m$ go to the centralized market and buyers with valuation $v \in [R, v_m]$ engage in direct trade (see Figure 1). Substituting (16) and (20) into (17), one can show that

$$U_B(v) = \frac{\rho\theta(1-\alpha)}{r + \rho\theta(1-\alpha)}(v - R) \text{ for } R \leq v \leq v_m \tag{21}$$

The cutoff value v_m satisfies

$$v_m - a = U_B(v_m). \tag{22}$$

That is, the marginal buyer v_m is indifferent between trading in the decentralized market and in the centralized market. Solving the above equation yields

$$v_m = \frac{1}{r}\left[(r + \rho\theta(1-\alpha))a - \rho\theta(1-\alpha)R\right]. \tag{23}$$

The value v_m must satisfy $R \leq v_m \leq 1$. It follows that the ask price a must satisfy

$$R \leq a \leq 1 - \frac{(1-\alpha)\rho\theta}{r + \rho\theta(1-\alpha)}(1 - R). \tag{24}$$

Substituting (20) and (21) into (16) yields the bargained price between buyer v and a seller:

$$p(v) = R + \frac{r(1-\theta)}{r + \rho(1-\alpha)\theta}(v - R) \tag{25}$$

I now turn to the seller's problem (18). Because of the cutoff nature of buyers' choice, the distribution of buyers in the decentralized market G is uniform over $[R, v_m]$

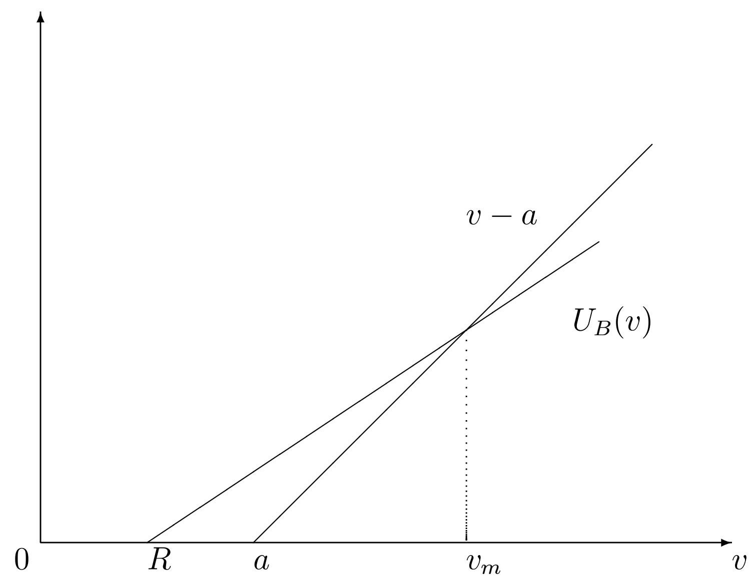

Figure 1: **The determination of the cutoff value** v_m**.**

and its density is $\frac{dv}{v_m-R}$. Thus, substituting (25) into (18) yields

$$rU_S = \rho\alpha\left(1-\theta\right)\int_R^{v_m}\left[v-U_S-U_B\left(v\right)\right]\frac{dv}{v_m-R}$$

Substituting U_S and U_B given in (20) and (21) into the above equation yields

$$R = \frac{\rho\alpha\left(1-\theta\right)}{r+\rho\left(1-\alpha\right)\theta}\int_R^{v_m}\left[v-R\right]\frac{dv}{v_m-R}. \tag{26}$$

Finally, after a trader makes a transaction in either the centralized or decentralized market, he leaves the economy. To maintain a steady state, the following flow market-clearing condition must hold

$$\left(1-R\right)f_B = \lambda_B N_B + \rho(1-\alpha)\left(1-\lambda_B\right)N_B = \lambda_S N_S + \rho\alpha\left(1-\lambda_S\right)N_S = f_S. \tag{27}$$

The interpretation is similar to that for (7).

3.3 Equilibrium

I first study equilibrium with given bid-ask prices. I defer the analysis of profit maximizing market making in later subsections. A (steady state) search equilibrium with market makers quoting bid-ask prices (a,b) is defined by value functions for buyers and sellers (U_B,U_S), the negotiated price $p\left(v\right)$ in the decentralized market, the marginal participating types (R,v_m), the stocks of buyers and sellers in the markets (N_B,N_S), and the fraction of buyers and sellers trading in the centralized market (λ_B,λ_S), such that (i) (U_B,U_S) satisfies (18) and (17), (ii) $p\left(v\right)$ satisfies (16), (iii) (R,v_m) satisfies (19) and (22), and (iv) $(N_B,N_S,\lambda_B,\lambda_S)$ satisfies the flow market-clearing conditions (14) and (27).

It can be seen that there may exist two degenerate equilibria where traders concentrate on only one market: If all buyers or sellers conjecture that there is no counterparty trading in one of the markets, then there is no trade on that market.[6] A similar multiplicity of equilibria issue is addressed in Pagano (1989) and Gehrig (1993). Here,

[6]Note that market makers do not hold any inventory.

I focus on the equilibrium where the two markets coexist, and analyze the degenerate case by studying the limits.

To derive the equilibrium, observe first that the bid price $b = R$ as discussed earlier. Next, I solve for the matching probability α and the cutoff value v_m. Specifically, use (26) to solve for v_m,

$$v_m = R\frac{2\left(r + \theta\rho\left(1-\alpha\right)\right) + \rho\alpha\left(1-\theta\right)}{\rho\alpha\left(1-\theta\right)}. \tag{28}$$

Equate the right-hand sides of equations (23) and (28) to deliver a quadratic equation for α. Solving this equation yields two roots. One root is given by $\frac{r+\rho\theta}{\rho\theta}$, which must be ruled out since it is bigger than 1. The other root is given by

$$\alpha = \frac{2Rr}{\rho\left(a-R\right)\left(1-\theta\right)}. \tag{29}$$

Substituting this root into (28) yields

$$v_m = \frac{a\left(1-\theta\right)\left(r+\rho\theta\right) - R\theta\left(2r + \rho\left(1-\theta\right)\right)}{\left(1-\theta\right)r}. \tag{30}$$

Once v_m is obtained, the proportion of buyers entering the centralized market can be derived,

$$\lambda_B = \frac{1-v_m}{1-R}. \tag{31}$$

Finally, it follows from (27) that

$$N_B = \frac{f_S}{\lambda_B + \rho\left(1-\alpha\right)\left(1-\lambda_B\right)},\ N_S = \frac{f_S}{\lambda_S + \rho\alpha(1-\lambda_S)}. \tag{32}$$

Substitute these expressions into (14) and solve for λ_S to obtain,

$$\lambda_S = \frac{\alpha\lambda_B}{\left(1-\alpha\right)\left(1-\lambda_B\right) + \alpha\lambda_B}.$$

Thus, one can solve for (N_B, N_S) and an equilibrium is constructed.

To have a nondegenerate equilibrium, the following conditions must be met

$$N_S, N_B > 0,\ \text{and}\ \alpha, \lambda_S, \lambda_B \in \left(0,1\right).$$

From the above expressions, it is clear that the requirement that

$$\alpha \in (0,1)\,, \text{ and } v_m \in (R,1)\,,$$

is sufficient.

The above discussion is summarized in the following proposition.

Proposition 4 *If the ask price a and parameter values (r,ρ,θ,f_B,f_S) satisfy the following conditions*

$$\frac{2Rr}{\rho\,(a-R)\,(1-\theta)} \in (0,1) \tag{33}$$

$$\frac{a\,(1-\theta)\,(r+\rho\theta)-R\theta\,(2r+\rho\,(1-\theta))}{(1-\theta)\,r} \in (R,1)\,, \tag{34}$$

where $R = 1 - f_S/f_B$, then there exists a unique nondegenerate search equilibrium with market makers quoting the bid price $b = R$ and the ask price a.

An important property of the equilibrium is described in the following proposition.

Proposition 5 *In any nondegenerate search equilibrium with market makers quoting bid-ask prices (a,b), the bid-ask prices and the expected price in the decentralized market satisfy*

$$b < E_G\,[p\,(v)] < a.$$

Proof. Since a buyer with valuation v in the support of G, $[R, v_m]$, trades in the centralized market, it follows from (17) that his payoff satisfies

$$\begin{aligned} rU_B\,(v) &= \rho\,(1-\alpha)\,(v - p\,(v) - U_B\,(v))\,, \\ U_B\,(v) &\geq v - a. \end{aligned}$$

Thus,

$$U_B\,(v) = \frac{\rho\,(1-\alpha)}{r+\rho\,(1-\alpha)}\,(v - p\,(v)) \geq v - a.$$

It follows from $v > p(v)$ that

$$a \geq v - \frac{\rho(1-\alpha)}{r+\rho(1-\alpha)}(v - p(v)) > p(v).$$

Taking expectation with respect to G yields $a > E_G[p(v)]$.

I now show that $b < E_G[p(v)]$. It follows from (18) that

$$rU_S = \rho\alpha E_G[p(v) - U_S].$$

Thus,

$$E_G[p(v)] = \frac{r+\alpha\rho}{\alpha\rho}U_S.$$

Since sellers are indifferent between trading in the two markets, $b = R = U_S$. Thus,

$$E_G[p(v)] > b,$$

which completes the proof. ■

Proposition 5 shows that no matter how bid-ask prices are determined and no matter whether there is transaction cost in the centralized market, a positive bid-ask spread exists as long as the centralized and decentralized markets coexist in equilibrium. Moreover, the expected price in the decentralized market lies between the bid and ask prices. The intuition is simple. Any trader faces the following trade-off: he may either wait to transact at a negotiated price in the decentralized market or choose immediate execution at the current bid or ask price in the centralized market. Trading in the decentralized market incurs search and delay costs. Thus, the quoted ask price must include a premium for immediate buying and the bid price must reflect a concession required for immediate sale.

Another important property of the equilibrium is that the centralized market imposes externality to the decentralized market. More specifically, it makes the decentralized market tighter as described in the following proposition.[7]

[7]Market tightness is defined as the buyer-seller ratio in the decentralized market.

Proposition 6 *In any nondegenerate search equilibrium with market makers quoting bid-ask prices* (a, b), $\alpha > \gamma$.

Proof. In a nondegenerate search equilibrium with market makers quoting bid-ask prices (a, b), the cutoff value satisfies $v_m \in (R, 1)$. From (12) and (26), it is straightforward to verify $\alpha > \gamma$. ■

In the next two subsections, I will analyze how market makers determine the bid-ask prices and characterize equilibria.

3.4 Competitive Market Makers

Competitive market makers make zero profits. Since a transaction yields profits $(a - b - k)$ and market markers are identical, they all quote the same prices, which are given by

$$b = R, \ a = b + k = R + k. \tag{35}$$

Thus, the bid-ask spread is equal to the transaction cost of market makers.

To solve for the equilibrium with competitive market makers, one only needs to substitute (35) into the equilibrium derived in the previous subsection. For example, the matching probability α is given by

$$\alpha = \frac{2Rr}{\rho k (1 - \theta)}. \tag{36}$$

The marginal participation type is given by v_m,

$$v_m = \frac{k (1 - \theta) (r + \rho\theta) + Rr (1 - 3\theta)}{(1 - \theta) r}. \tag{37}$$

The following result follows from Proposition 4.

Proposition 7 *If the values of parameters* $(k, r, \rho, \theta, f_B, f_S)$ *satisfy*

$$\frac{2Rr}{\rho k (1 - \theta)} \in (0, 1)$$

and

$$\frac{k(1-\theta)(r+\rho\theta)+Rr(1-3\theta)}{(1-\theta)r}\in(R,1),$$

where $R = 1 - f_S/f_B$*, then there is a unique search equilibrium with competitive market makers.*

To analyze the equilibrium, I first study some limiting results.

Proposition 8 *For any parameter values* $(k, r_0, \rho, \theta, f_B, f_S)$ *satisfying the assumption in Proposition 7, there exist values* $\overline{r} > 0$ *and* $\underline{r} > 0$ *such that if* r *converges up (below) to* $\overline{r}$ $(\underline{r})$ *from* r_0*,*[8] *then the decentralized (centralized) market is driven out of the economy. If* r *converges below further to 0, then the limiting equilibrium becomes Walrasian.*

Proof. Taking other parameter values as given, view v_m as a function of r. By assumption there is a value $r_0 > 0$ such that $v_m(r_0) \in (R, 1)$. One can also show that $v_m(r) \to \infty$ as $r \to 0$. Thus, there is a positive solution to the quadratic equation $v_m(r) = 1$. Take $\underline{r}$ as the maximum solution. When r converges to $\underline{r}$, v_m converges to 1 and hence no buyers go to the centralized market. The model then reduces to that in section 2. By Proposition 2, the economy becomes Walrasian if r converges below to 0.

Now consider increasing r from r_0. By (36) and (37), when r is sufficiently large, either α will exceed 1 or v_m will decrease below R. For both cases, all buyers prefer to go to the centralized market and the decentralized market disappears. ∎

A similar result for ρ can be established and its proof is omitted.

Proposition 9 *For any parameter values* $(k, r, \rho_0, \theta, f_B, f_S)$ *satisfying the assumption in Proposition 7, there exist values* $\overline{\rho} > 0$ *and* $\underline{\rho} > 0$ *such that if* ρ *converges*

[8] Note that $\overline{r}$ and $\underline{r}$ may depend on parameter values $(f_B, f_S, k, \theta, \rho)$. A similar remark holds for Propositions 9-10.

up (below) to $\overline{\rho}$ $(\underline{\rho})$ from ρ_0, then the centralized (decentralized) market is driven out of the economy. If ρ converges up further to infinity, then the limiting equilibrium becomes Walrasian.

Propositions 8-9 establishes that if either the time cost of search or the contact cost of search is small enough, then all traders prefer to trade in the decentralized market and the centralized market is driven out of the economy. When either type of search cost vanishes, the limiting equilibrium becomes Walrasian. On the other hand, if search cost is large enough, then there is no gain from trading in the decentralized market and all traders prefer to go to the centralized market.

Proposition 10 *For any parameter values $(k_0, r, \rho, \theta, f_B, f_S)$ satisfying the assumption in Proposition 7, there exist values $\underline{k} > 0$ and $\overline{k} > 0$ such that if k converges below (up) to $\underline{k}$ $(\overline{k})$ from k_0, then the decentralized (centralized) market is driven out of the economy. If k converges below further to zero, then the limiting equilibrium becomes Walrasian.*

Proof. By (36) and (37), when k is sufficiently small, either α tends to 1, or v_m tends to R. Thus, all buyers enter the centralized market and there is no trade in the decentralized market. When k converges below further to zero, then there is no bid-ask spread; that is, $a = b$. Moreover, only buyers with valuation $v \geq a$ enter the centralized market. Thus, to maintain a steady state, the following flow condition must be satisfied

$$f_B (1 - a) = N_B = N_S = f_S.$$

This implies that $a = R$ and the limiting equilibrium becomes Walrasian.

Finally, it follows from (37) that v_m increases with k. Thus, there exists a value $\overline{k} > 0$ such that v_m converges to 1 when k tends to $\overline{k}$. This implies that no traders go to the centralized market. ■

Proposition 10 establishes that if the transaction cost of market makers is small enough, or if the bid-ask spread is small enough, then all traders prefer to trade in the centralized market and the decentralized market is driven out of the economy. Moreover, when the bid ask spread converges to zero, the limiting equilibrium converges to the Walrasian equilibrium. On the other hand, when the bid-ask spread is large enough, no traders go to the centralized market and it is driven out of the economy.

I now study comparative statics with respect to parameters (k, r, ρ, θ). I will focus particularly on trading volume $\lambda_B N_B$ in the centralized market and the expected price in the decentralized market.[9] One can interpret trading volume as a measure of liquidity or a measure of the market participation.

Proposition 11 *Let the assumption in Proposition 7 hold. Then the trading volume in the centralized market, $\lambda_B N_B$, is decreasing with k, ρ, and increasing in r. It is increasing with θ if $k\rho(1-\theta)^2 < 2Rr$.*

Proof. It follows from (31)-(32) that

$$\begin{aligned}\lambda_B N_B &= \frac{\lambda_B f_S}{\lambda_B + \rho(1-\alpha)(1-\lambda_B)} = \frac{f_S}{1+\rho(1-\alpha)(1/\lambda_B - 1)} \\ &= \frac{f_S}{1+\rho(1-\alpha)((1-R)/(1-v_m)-1)}.\end{aligned}$$

From (36) and (37), it is straightforward to verify that α is decreasing with k and ρ, and that v_m is increasing with k and ρ. Thus, $\lambda_B N_B$ is decreasing with k and ρ.

From (36) and (37), simple algebra shows that

$$\frac{\partial v_m}{\partial r} < 0, \frac{\partial \alpha}{\partial r} > 0, \frac{\partial v_m}{\partial \theta} < 0, \frac{d\alpha}{d\theta} > 0$$

Thus, the desired result follows. ■

[9] It follows from (27) that the trading volume in the decentralized market is given by the difference between the inflows of traders and the trading volume in the centralized market.

Proposition 11 implies that trading volume in the centralized market is negatively correlated with the bid-ask spread (or transaction cost k), and positively correlated with the search cost reflected by the contact rate ρ in the decentralized market. This result is intuitive and simply says that trading volume should reflect the relative competitive position of the two markets.

One may expect that if the bargaining power of the buyers θ increases, then buyers can negotiate low prices as described in Proposition 2. As a result, they should find the decentralized market more attractive and there should be less trade in the centralized market. Surprisingly, Proposition 11 implies that in general there is no monotonic relation between trading volume in the centralized market and the bargaining power of the buyers θ. The intuition is as follows. In the short run, increasing the buyers' relative bargaining power θ raises a buyer's payoff and lowers a seller's payoff when they trade in the decentralized market. However, as shown earlier, a seller's steady state payoff must be equal to R, which does not depend on the bargaining power. To maintain this payoff, a seller must meet a buyer more often. That is, the proportion of buyers α must be higher as implied by equation (26). This imposes a negative externality to the buyers since it implies that the decentralized market becomes tighter. Thus, if this negative effect dominates the positive effect, then the buyer's payoff from trading in the decentralized market actually falls. This leads more buyers to entering the centralized market.

Proposition 12 *Let the assumption in Proposition 7 hold. Then the expected price in the decentralized market, $E_G[p(v)]$, is increasing with k and decreasing with θ.*

Proof. By (25), the expected price in the decentralized market is given by

$$E_G[p(v)] = R + \frac{r(1-\theta)}{r+\rho(1-\alpha)\theta}\frac{v_m - R}{2}. \tag{38}$$

Substituting the expressions for α and v_m given in (36)-(37) yields

$$E_G[p(v)] = R + \frac{k(1-\theta)}{2}.$$

Thus, $E_G[p(v)]$ is increasing with k and decreasing with θ ■

Proposition 12 shows that the average negotiated price in the decentralized market is positively correlated with the bid-ask spread. To understand this result, observe that there are two opposing effects of the bid-ask spread on the expected negotiated price: An increase in the bid-ask spread discourages some high valuation buyers from entering the centralized market. Hence it raises the average negotiated price. On the other hand, it also raises a seller's payoff since he is able to meet more high valuation buyers. But in the steady state, a seller's payoff is equal to R, which does not depend on the bid-ask spread. To maintain this value, the seller must meet a buyer less often. This implies a buyer can meet a seller more often, imposing a positive externality on the buyer's payoff. Thus, he can negotiate a lower price. The proposition shows that the former effect dominates.

As shown in Proposition 2, it is intuitive that the average negotiated price is negatively related to buyers' bargaining power. Different from Proposition 2, Proposition 12 implies that it does not depend on ρ and r. The intuition is similar to that described above. There are two opposing effects in force. An increase in the contact rate raises a buyer's payoff from trading in the decentralized market, and hence lowers the negotiated price. However, it also discourages high valuation buyers from entering the centralized market, leaving more high valuation buyers trading in the decentralized market. This raises the average negotiated price. Proposition 12 shows that these two effects offset each other. A similar analysis applies to r.

3.5 Monopolistic Market Maker

I now analyze the case with a monopolistic market maker. A search equilibrium with a monopolistic market maker is defined as the equilibrium described in subsection 3.3 for which the ask price is selected by the market maker so as to maximize profits. Formally, since I focus on steady states, the ask price is determined by the following problem

$$\max_a (a - R - k)\,\lambda_B N_B$$

subject to (33) and (34), where λ_B and N_B are given in subsection 3.3. Use (32) and (31) to rewrite this problem as

$$\max_a \frac{f_S\,(a - R - k)}{1 + \rho\,(1-\alpha)\,(v_m - R)\,/\,(1 - v_m)},$$

where α is given by (29) and v_m is given by (30).

The monopolistic market maker faces the following trade-off: Increasing the ask price raises the profits from a transaction. But it lowers the number of transactions since some buyers may find the decentralized market is more attractive. Because the objective function is a complicated function of the ask price a, no closed form solution is available. However, numerical analysis is straightforward. I select base case parameter values listed in Table 1.

Table 1. Base Case Parameter Values

f_B	f_S	r	k	ρ	θ
125	100	0.1	0.01	10	0.5

The equilibrium outcome is given in row 3 of Table 3. The equilibrium outcome under competitive marketmaking is listed in row 4 of Table 3. Thus, compared to the equilibrium with competitive market makers, the bid-ask spread is wider, the average negotiated price in the decentralized market is higher, and trading volume in the centralized market is lower. This result is intuitive since the monopolistic market maker makes positive profits, and hence sets a wider bid-ask spread. The high ask price discourages some high valuation buyers from entering the centralized market and lowers its trading volume. These high valuation buyers raise the average negotiated price in the decentralized market.

Based on the above benchmark parameter values, I conduct comparative statics analysis. Results are given in Table 2. Table 2 illustrates that the bid-ask spread is positively related to the transaction cost k of the market maker. The intuition is similar to that for the case with competitive market makers: A monopolistic market maker makes positive profits, and hence the bid-ask spread must cover the transaction cost. Importantly, Row 2 of Table 2 reveals that there exists a positive bid-ask spread even though there is no transaction cost in the centralized market. This result is in line with Proposition 5.

Table 2 also reveals that the bid-ask spread is positively related to the search frictions in the decentralized market represented by r and ρ. This is because when r decreases or ρ increased (search cost is lower), buyers get more payoffs from trading in the decentralized market. To attract buyers to trade in the centralized market, the market maker must lower the bid-ask spread.

Importantly, Table 2 reveals that there is no monotonic relation between the bid-ask spread and the bargaining power. The intuition is similar to that described after Proposition 9: An increase in θ raises a buyer's payoff from trading in the decentralized market, and hence encourages him to trade in that market. On the other hand, more searching buyers make it more difficult for a buyer to find a seller, and thus lowers the buyer's payoff. The overall effect on a buyer's payoff is ambiguous. The market maker adjusts the bid-ask spread so as to raise the profit from a single transaction and to attract more traders trading in the centralized market.

Table 2 also shows that trading volume in the centralized market is negatively related to k and ρ, and positively related to r and θ. This result is similar to that for competitive market makers shown in Proposition 9. The intuition is also similar: Trading volume or liquidity should reflect the relative efficiency of the markets.

The relation between the expected negotiated price in the decentralized market and the parameters k and θ is the same as that for competitive market makers described in Proposition 10. However, different from Proposition 10, the expected negotiated price is negatively related to ρ and positively related to r. This is because there are two opposing effects in force as discussed after Proposition 10. The overall effect depends on which effect dominates.

4 Welfare

An important question is what is the welfare implication of the market fragmentation. To answer this question, one has to adopt a welfare criterion. Since the Walrasian equilibrium of the benchmark frictionless economy is in the flow sense of Gale (1987), I also adopt the flow sense welfare criterion.

The social welfare W_w in the Walrasian equilibrium is given by the total buyer and seller surplus:

$$W_w = f_B (1 - R) \int_R^1 (v - R) \frac{dv}{1 - R} + f_S R = \frac{f_S (1 + R)}{2}. \tag{39}$$

This is the first-best value. The social welfare W_c in the pure centralized market equilibrium described at the beginning of section 3 is given by the total buyer and seller surplus plus market maker profits:

$$W_c = f_B (1 - a) \int_a^1 (v - a) \frac{dv}{1 - a} + f_S (a - b - k) + f_S b = f_S \left(\frac{1 - R}{2} + R - k \right), \tag{40}$$

where I use the fact that market maker sets the ask price $a = R$. Note that this is the social welfare under both competitive and monopolistic marketmaking. It is clearly less than the first best value due to the loss of transaction costs.

Consider next the social welfare in the search equilibrium without market makers described in section 2. It is given by

$$W_d = f_B \int_R^1 V_B (v) \, dv + f_S R = f_S \left[\int_R^1 V_B (v) \frac{dv}{1 - R} + R \right]. \tag{41}$$

Clearly, $W_d < W_w$ due to search frictions. From (40) and (41), one can see that search frictions reduce buyer surplus in the pure decentralized market economy, but transactions costs reduce social welfare in the pure centralized market economy. Thus, which one is bigger depends on the relative efficiency in the two markets described by the parameters k, ρ, and r.

Proposition 13 *If* $k < (>) \frac{1}{2} r \frac{3R\theta - R + 1 - \theta}{(1-\theta)(r+\rho\theta)}$, *then* $W_c > (<) W_d$

Proof. Substituting (10) and (13) and simplifying yield

$$\begin{aligned} W_c - W_d &= f_S \left(\frac{1-R}{2} + R - k \right) - f_S \left[\int_R^1 V_B(v) \frac{dv}{1-R} + R \right] \\ &= \frac{1}{2} r \frac{3R\theta - R + 1 - \theta}{(1-\theta)(r+\rho\theta)} - k. \end{aligned}$$

The result follows from the assumption. ■

Turn to the equilibrium where centralized and decentralized markets coexist. In this equilibrium, the social welfare W_f is equal to the buyer and seller surplus from trading in the centralized and decentralized markets plus market maker profits:

$$\begin{aligned} W_f &= (1-\lambda_B)(1-R) f_B \int_R^{v_m} U_B(v) \frac{dv}{v_m - R} + \lambda_B (1-R) f_B \int_{v_m}^1 (v-a) \frac{dv}{1-v_m} \\ &\quad + \lambda_B (1-R) f_B (a-b-k) + f_S U_S. \\ &= f_S \left[\int_R^{v_m} U_B(v) \frac{dv}{1-R} + \int_{v_m}^1 (v-R) \frac{dv}{1-R} + R - \lambda_B k \right], \qquad (42) \end{aligned}$$

where I have used (7), (31), and the fact that $U_S = R$ to derive the equality. Observe that the first term in the square bracket represents the surplus of buyers in the decentralized market, the second term represents the surplus of buyers in the centralized market, the third term represents the total surplus of sellers, the last term represents the loss of transaction costs.

It is ready to analyze the question whether market fragmentation improves social welfare. The following intuition is natural: The introduction of a centralized market to the decentralized market facilitates immediacy of trade and hence should improve social welfare. On the other hand, the fragmentation of a centralized market should lower social welfare since there are search frictions in the decentralized market. However, I will show that both claims are generally not true.

Consider first the question whether the opening of a centralized market improves the social welfare in the pure decentralized market economy. Rewrite (41) as

$$W_d = f_S \left[\int_R^{v_m} V_B(v) \frac{dv}{1-R} + \int_{v_m}^1 V_B(v) \frac{dv}{1-R} + R \right].$$

It is clear from (42) and the fact $V_B(v) < v - R$ for $v \geq v_m$ that high valuation buyers $v \geq v_m$ benefit from immediacy of trade. However, the centralized market imposes negative externality to the decentralized market in the sense that it makes the latter market tighter (see Proposition 6). Thus, buyers in the decentralized market are worse off, i.e., $V_B(v) > U_B(v)$ for $v \in [R, v_m]$. Moreover, market making incurs transaction costs. Consequently, the social welfare in the economy where the two markets coexist may not be higher than that in the pure decentralized market economy. For example, suppose market makers are competitive and denote by W_{cm} the social welfare in the two markets economy under competitive market making. For the base case parameter values in Table 1, I find that $W_d = 58.82 < W_{cm} = 59.07$. However, if $\rho = 20$, I find $W_d = 59.41 > W_{cm} = 59.38$.

Consider next the question whether the fragmentation of a centralized market improves social welfare in the pure centralized market economy. Rewrite W_c as

$$W_c = f_S \left[\int_R^{v_m} (v - R) \frac{dv}{1 - R} + \int_{v_m}^1 (v - R) \frac{dv}{1 - R} + R - k \right].$$

Compared with (42), it is clear that the centralized market provides immediacy of trade and hence improve the welfare of traders with valuation $v \in [R, v_m]$ as $v - R > U_B(v)$. However, since in a completely centralized market all traders trade with market makers, the total transaction costs $f_S k$ is bigger than that in the two markets economy $f_S \lambda_B k$. The following proposition shows that if the bid-ask spread after the market fragmentation is small enough, then the second effect dominates. In particular, under competitive market making, the fragmentation of centralized markets always improve social welfare.

Proposition 14 *If the bid-ask spread $(a - R)$ in the two markets economy satisfies $(a - R) < (>)\, 2k$, then $W_f > (<)\, W_c$. In particular, under competitive market making, $W_{cm} > W_c$.*

Proof. Substituting (29) and (30) into (42) and (40) and simplifying yield

$$\begin{aligned}
W_f - W_c &= f_S \left[\int_R^{v_m} U_B(v) \frac{dv}{1-R} + \int_{v_m}^1 (v-R) \frac{dv}{1-R} + R - \lambda_B k \right] \\
&\quad - f_S \left[\int_R^{v_m} (v-R) \frac{dv}{1-R} + \int_{v_m}^1 (v-R) \frac{dv}{1-R} + R - k \right] \\
&= f_S k (1-\lambda_B) - f_S \int_R^{v_m} (v - R - U_B(v)) \frac{dv}{1-R} \\
&= \frac{v_m - R}{1-R} f_S \left(k - \frac{r}{r + \theta\rho(1-\alpha)} \frac{v_m - R}{2} \right) \\
&= \frac{v_m - R}{1-R} f_S \left[k - (a-R)/2 \right].
\end{aligned}$$

Since for competitive market makers $a - R = k$, the result follows from the assumption.

■

I finally address the following questions. What is the constrained socially optimal bid-ask prices? How do the equilibria under competitive and monopolistic market-making compare with the constrained social optimum?

Again the bid price must be equal to R so that sellers are indifferent between the two markets. Thus, the social planner chooses the ask price a only so as to maximize W_m. This is equivalent to choose the cutoff value v_m.

Proposition 15 *The socially optimal cutoff value v_m^* satisfies*

$$v_m^* - R - k = U_B(v_m^*) + \int_R^{v_m^*} \frac{\partial U_B(v)}{\partial v_m} dv. \tag{43}$$

This value is bigger than the cutoff value under competitive market making. Furthermore, the socially optimal bid-ask spread is wider than that under competitive market making.

Proof. Taking first-order condition for (42) yields (43). Using (21), (29), and (30), one can easily show that $\frac{\partial U_B(v)}{\partial v_m} > 0$. Since under competitive market making, the ask price $a = R + k$ and the cutoff value v_m satisfies

$$v_m - R - k = U_B(v_m).$$

Thus, $v_m^* > v_m$. The last result follows from the fact that the ask price increases with v_m by (30). ■

This proposition implies that under competitive market making, the bid-ask spread is too tight and there is too much entry into the centralized market, compared with the constrained social optimum. The main reason is that the social planner internalizes the externality of the bid-ask prices on the decentralized market. This externality effect is captured by the second term on the right-hand side of (43).

Finally, for monopolistic market making, because there is no closed form solution, no clear-cut welfare comparison can be made. Simple numerical illustration may be useful. According to the base case parameter values in Table 1, the outcome of the constrained social optimum is described in Row 2 of Table 3. In particular, the constrained efficient social welfare is 59.118, and the bid ask spread is 0.0139. Rows 3-4 of Table 3 reveal that under monopolistic market making, the social welfare is closer to the constrained social optimum. Moreover, the bid ask spread is also closer to the social optimum.

5 Conclusion

This paper provides a search model of centralized and decentralized trade. The model has a number of testable implications. As mentioned in the introduction, some are consistent with empirical and experimental evidence.

The main results of the paper can be summarized as follows. First, a positive bid-ask spread exists if the centralized and decentralized markets coexist in equilibrium. Moreover, the average negotiated price in the decentralized market is inside the bid-ask spread. Second, under monopolistic marketmaking, the bid-ask spread is positively related to the transaction cost, search frictions, and average negotiated price. Third, liquidity in the centralized market measured by trading volume is negatively related to the bid-ask spread and positively related to search frictions. Fourth, the fragmentation of a centralized market improves social welfare if the bid-ask spread after the fragmentation is small enough. However, the opening of a centralized market in a decentralized market economy may not improve social welfare. Finally, several limiting results and convergence to the Walrasian equilibrium are established.

The model is highly stylized and may not describe perfectly any specific market in reality. It intends to capture in a simple manner some crucial elements of trades in the centralized and decentralized markets. As a result, the model can be extended in a number of dimensions.

First, since the model is stylized and it is difficult to gather decentralized trade data, an experimental study is helpful for testing the model. Related experimental studies have been carried out by Campbell et al (1991), Lamoureux and Schnitzlein (1997), and Neeman and Vulkan (2003b). Apparently, further work along this line is interesting.

Second, in the model, the only benefit of the centralized market is its publicity of prices and immediacy of trade. Centralized markets have other important advantages such as economies of scale and network externalities. A simple way to capture these advantages is to assume that the transaction cost k decreases with the volume of trade.

Finally, in order to keep the model tractable and to derive analytical results, I assume that sellers are homogenous, which seems reasonable when a single homogenous asset is traded. It would be interesting to consider the case where sellers are heterogeneous. For example, they may have different costs of holding the asset.

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Table 2. Comparative statics under the monopolistic market making. The base case parameter values are listed in Table 1. When conducting comparative static analysis for a parameter, the other parameter values are fixed as in Table 1.

		$b-a$	$E_G[p(v)]$	$\lambda_B N_B$	tightness
Base	Case	0.0137	0.2034	0.2844	1.393 5
k	0.0	0.0087	0.2022	0.9545	11.2676
	0.012	0.0157	0.2039	0.1671	1.033 8
	0.014	0.0175	0.2044	0.1029	.841 28
	0.016	0.0191	0.2048	0.0645	.723 84
r	0.07	0.0124	0.2031	0.0987	.821 16
	0.08	0.0130	0.2032	0.1482	.971 22
	0.09	0.0134	0.2033	0.2108	1.161 2
ρ	8	0.0151	0.2038	0.4943	1.956 8
	12	0.0131	0.2033	0.1471	1.041 2
	16	0.0114	0.2029	0.0550	.778 73
θ	0.4	0.0147	0.2044	0.2004	.826 82
	0.6	0.0134	0.2027	0.5137	2.9412
	0.8	0.0207	0.2021	0.9619	27.011

Table 3. Comparison of welfare under constrained social optimum and search equilibrium with competitive and a monopolistic market makers. The parameter values are listed in Table 1.

	$b-a$	$E_G[p(v)]$	$\lambda_B N_B$	tightness	Welfare
Social Optimum	0.0139	0.2035	0.2725	1.3559	59.118
Monopolistic MM	0.0137	0.2034	0.2844	1.3935	59.117
Competitive MM	0.0100	0.2025	0.7582	4.0000	58.82