General Equilibrium with Endogenous Securities and Moral Hazard

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February 3, 2004

Forthcoming in Economic Theory (Expected 2005)

Abstract

This paper studies a class of general equilibrium economies in which the individuals' endowments depend on privately observed effort choices and the financial markets are endogenous. The environment is modeled as a two-stage game. Individuals first make strategic financial-innovation decisions. They then act in a Radner-type economy with the previously designed securities. Consumption goods, portfolios, and effort levels are chosen competitively (i.e., taking prices as given). An equilibrium concept is adapted for these moral hazard economies and its existence is proven. It is shown through an example how incentive motives might lead to the endogenous emergence of financial incompleteness.

Keywords: general equilibrium, moral hazard, endogenous incomplete markets, non-exclusive securities. JEL Classification: D52, D82, G10, G22.

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*This paper is based on an essay from my Ph.D. dissertation at the University of Chicago. I am thankful for comments from Fernando Alvarez, Rodrigo Cerda, Pierre-André Chiappori, Rubens P. Cysne, Carlos E. da Costa, Milton Harris, Lars Stole, Juan P. Torres-Martínez, Paulo K. Monteiro, Philip Reny, Iván Werning, an anonymous referee, and seminar participants at the 6th SAET Conference and the 22th SBE Meeting. Financial support from CNPq is gratefully acknowledged.

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1 Introduction

The research agenda on financial innovation seeks to understand the forces that determine the design of financial markets. Based on the trade-off between innovation revenues and intermediation costs, this literature analyzes how different market frictions and innovation technologies affect the equilibrium asset structure—see, for example, Allen and Gale [1] and [2], Chen [13], Pesendorfer [27], and Bisin [8]. This paper extends the innovation analysis to general equilibrium economies with moral hazard. Moral hazard adds a new trade-off to the financial-innovation decision. On the one hand, individuals have an interest in issuing new securities in order to hedge risks posed by uncertainty. On the other hand, as is well-known from partial equilibrium models, higher insurance possibilities reduce effort incentives. The strategic balance of incentives and risk-sharing possibilities is an important aspect of financial innovation which is still unexplored in a general equilibrium context.

This paper models a class of general equilibrium economies with moral hazard and endogenous financial markets. Each individual is endowed with a productive project whose output depends on one's privately observed effort. The support of the output distribution is finite, so that the economy has a finite number of states of nature. In this world, a security is characterized by a vector of contingent payments and a list of transaction constraints. The setup consists of a two-stage game. First, individuals strategically make their financial-innovation decisions. Any individual is allowed to open a clearinghouse and issue securities in zero net supply. Once the financial market is designed, individuals act in a Radner-type economy in which consumption goods, portfolios, and effort levels are chosen competitively (i.e., as a best response to other effort levels, but taking prices as given).

The equilibrium concept combines strategic and competitive elements.[1] Individuals are atomistic, but they understand that the financial-innovation decision directly affects the economy's equilibrium. Therefore, when making such a decision, they act strategically and anticipate relevant elements of the second stage, such as equilibrium prices and quantities. Once the financial market has been designed, individuals choose consumption bundles, portfolios, and effort levels taking prices as given. In this second stage, individuals bear in mind the fact that their effort choices affect probabilities, but act as if all their choices had no effect on prices. Price taking is a fundamental behavioral assumption in the competitive tradition. Price-taking behavior associated with strategic effort choice is also assumed in Arnott and Stiglitz

[1] Equilibrium concepts with a strategic first stage and a competitive second stage are standard in the literature of financial innovation (e.g., Bisin [8] and Pesendorfer [27]).

[5] and Citanna and Villanacci [14] to study multi-good moral hazard economies.

The modeling strategy here allows one to use standard techniques to prove the existence of an equilibrium. In the first stage of the game, intermediaries are allowed to use mixed strategies. They have expectations about future allocations and prices. These expectations are modeled through endogenous sharing rules, as in Simon and Zame [31]. In this way, one can prove the existence of a Nash equilibrium for the financial-innovation subgame. (This same technique is used in Bisin [8].) Once the financial assets have been designed, individuals act in a competitive economy with moral hazard. The non-existence problem raised by Helpman and Laffont [23] and Bisin and Gottardi [9] and [10] is avoided by means of a convexity assumption that allows the application of Kakutani's fixed-point techniques.

The model incorporates many important features of real-world financial markets, such as the possibility of credit rationing, redundant securities, bilateral arrangements, financial clubs, anonymous competitive assets, and latent contracts (those that are not transacted in equilibrium, but are issued to inhibit other innovations). The equilibrium financial structure is typically incomplete due to incentive reasons. This point is illustrated through a simple example. Furthermore, the equilibrium tends not to implement the second-best allocation. An important source of inefficiency is the fact that securities are non-exclusive (i.e., individuals can trade multiple assets with different intermediaries).

The remainder of this paper is organized as follows. Section 2 discusses the related literature; Section 3 describes the basic model; Section 4 presents the equilibrium concept, proves its existence, and discusses some features and possible extensions of the model; Section 5 illustrates how moral hazard can generate financial incompleteness; and Section 6 concludes.

2 Related Literature

Related papers from two different fields are discussed here. Initially, a few selected topics on financial innovation are mentioned. (More detailed reviews can be found in Allen and Gale [3], Duffie [16], and Duffie and Rahi [17].) The literature on moral hazard is then briefly summarized.

It is well-known that any equilibrium allocation is Pareto optimal in competitive economies with complete markets. This is an important benchmark for the financial-innovation analysis. It is also known that issuing an arbitrarily new security need not be Pareto improving when financial markets are incomplete. In fact, Cass and Citanna [12] and Elul [18] and [19] show that, generically, there exists a Pareto improving financial innovation when markets are incomplete. However, as stressed in Cass and Citanna [12] and Elul [18], introducing a new asset into an economy with multiple goods and incomplete financial markets can move the vector of equilibrium utilities in any direction (so that, in particular, innovation could also make everyone worse-off). Thus, in the presence of exogenous frictions, the choice of which assets should exist has important welfare effects. This motivates the research agenda on optimal security design.

Many papers on financial innovation focus on the trade-off between profitable innovation and costly financial intermediation. Allen and Gale [1] and [2] study the optimal use of debt and equity to finance an endogenously chosen production plan in an incomplete markets context. Chen [13] shows that arbitrage valuation does not hold in frictional economies and that short-selling restrictions create value for securitization. Pesendorfer [27] models intermediaries designing new securities collateralized by standard assets and shows that the equilibrium financial structure may exhibit redundancies even when marketing a new security is costly. Bisin [8] shows that financial incompleteness endogenously arise in general equilibrium economies with profit-maximizing intermediaries facing intermediation costs.

Another group of papers studies single-good economies with normally distributed endowments, CARA utility functions, and asymmetric information (e.g., Rahi [30] and Demange and Laroque [15]). In rational expectations equilibria, securities provide hedge and transmit private information; this approach allows one to obtain explicit solutions for the optimal security design. Duffie and Rahi [17] present a detailed survey on these papers.

Moral hazard is another important element in this paper. The standard literature on this issue represents the equilibrium by a contract that solves the principal-agent problem (e.g., Grossman and Hart [22]). In the multi-agent setup, a benevolent planner (principal) allocates the economy's resources among individuals in order to solve the trade-off between incentives and risk sharing (e.g., Prescott and Townsend [28]).

An implicit assumption of these models is that individuals cannot obtain insurance from any other party besides the principal (exclusivity assumption). Exclusivity, however, might be difficult to implement in general equilibrium economies with competitive financial markets. Helpman and Laffont [23] present a general equilibrium analysis of moral hazard economies with exogenously complete asset markets where transactions are non-exclusive. Endogenizing the financial design is an important extension to be done, since complete markets need not be optimal in economies with moral hazard.

Non-exclusivity is also studied by other authors in different contexts. Jaynes [24], Arnott and Stiglitz [6], and Bisin and Guaitoli [11] model insurance companies offering non-exclusive contracts contingent on two possible events (accident and no accident). Similarly, Kahn and Mookherjee [25] model an economy where individuals make sequential offers to intermediaries, who cannot contract upon the transactions made previously with other intermediaries. These papers focus on economies with a single consumption good and insurance contracts offered on a take-it-or-leave-it basis, so that there is no price to be determined in equilibrium. Their results suggest that exclusivity is necessary for second-best efficiency of the equilibrium in moral hazard economies.

Finally, this paper is also related to the literature on common agency, which features multiple principals designing independent arrangements to a single agent (e.g., Bernheim and Whinston [7]).

3 An Economy with Moral Hazard

Consider an economy with $L \geq 1$ consumption goods and $I > 1$ individuals. Each individual, $i \in \mathbb{I} = \{1, ..., I\}$, is endowed with a productive project, a financial-innovation technology, and a unit of time to be allocated for leisure, $\ell^i \in [0, 1]$, productive effort, $e^i \in [0, 1]$, and financial activities. The allocation of time is privately observed and non-contractible.

The economy lasts for two periods. The individuals' productive projects deliver a fixed amount of goods in period zero, $y_0^i \in \mathbb{R}_{++}^L$, and a random outcome, $\tilde{y}^i \in \mathbb{R}_{++}^L$, in period one. The amount of goods received in period one depends stochastically on the time expended on productive effort, e^i, according to a conditional probability measure, $\alpha^i(\tilde{y}^i \mid e^i)$.[2] For simplicity, it is assumed that the support of $\alpha^i(\cdot)$ is finite, $\forall i \in \mathbb{I}$.

States of Nature

A state of nature, s, is understood as a complete and sufficiently fine description of all possible outcomes of uncertainty. Since α^i has finite support, say $\mathbb{Y}^i \subset \mathbb{R}_{++}^L$, there exists a finite set of states $\mathbb{S}$ (with cardinality $S > 1$) and a one-to-one mapping between $\mathbb{S}$ and $\mathbb{Y}^1 \times ... \times \mathbb{Y}^I$. The inverse mapping allows one to represent individual i's project as a function of the states, $y_s^i \in \mathbb{R}_{++}^L$, $\forall s \in \mathbb{S}$. Each state $s \in \mathbb{S}$ occurs

[2] The productive technology allows externalities, i.e., α^i need not be independent across i. In that case, the individuals' efforts would affect each other directly. Nevertheless, technological externalities are not necessary, since the individuals' choices of effort always affect each other via prices—a pecuniary effect defined by Greenwald and Stiglitz [21] as *moral hazard pecuniary externality*.

with probability $\pi_s(e)$ derived from the joint probability of outputs, $\alpha(\tilde{y}^1, ..., \tilde{y}^I \mid e)$, where $e = (e^1, ..., e^I) = (e^i, e^{-i}) \in [0,1]^I$.

In order to ensure that the output realization is non-informative about effort choices, assume that every state occurs with positive probability regardless of the effort vector, namely, $\inf_{e \in [0,1]^I} \pi_s(e) > 0$, for all $s \in \mathbb{S}$.

Preferences

Individuals care about their contingent consumption, $x^i \in \mathbb{R}_+^{L(S+1)}$, and leisure, $\ell^i \in [0,1]$. They also care about the vector of effort levels, $e \in [0,1]^I$, since it affects the probability of each state of nature. Assume that individual i's preference relation is represented by a continuous and quasiconcave utility function, $U^i : \mathbb{R}_+^{L(S+1)} \times [0,1]^{1+I} \to \mathbb{R} \cup \{-\infty\}$, such that: (i) $U^i\left(x^i, \ell^i, e\right) > -\infty$, $\forall \left(x^i, \ell^i, e\right) \in \mathbb{R}_{++}^{L(S+1)} \times [0,1]^{1+I}$; (ii) $\lim_{x_n^i \to 0} U^i\left(x^i, \cdot, \cdot\right) = -\infty$, $\forall n = 1, ..., L(S+1)$; and (iii) there exist continuous and strictly positive partial derivatives $\frac{\partial U^i}{\partial x_n^i} : \mathbb{R}_{++}^{L(S+1)} \times [0,1]^{1+I} \to \mathbb{R}_{++}$, $\forall n = 1, ..., L(S+1)$.

Timing

In this economy, there is no exogenous financial market. Before the beginning of period zero, all individuals are allowed to open clearinghouses and design securities in order to hedge the risk of unfavorable states of nature. This environment is represented by a two-stage game. Initially, individuals in $\mathbb{I}$ simultaneously choose a financial-innovation strategy. Next, they act in a competitive economy where: (i) before the realization of uncertainty, they choose a period-zero consumption bundle, $x_0^i \in \mathbb{R}_+^L$, a portfolio, $z^i \in \mathbb{R}^J$, and a level of productive effort, $e^i \in [0,1]$; and (ii) after the realization of s, they buy consumption goods, $x_s^i \in \mathbb{R}_+^L$, with the income from their productive project and portfolio.

3.1 Financial Innovation

In the first stage of the game, each individual is allowed to design a financial structure with J^i securities. Securities are in zero net supply and individuals (including the issuer) take long and short positions in each asset designed. The financial structure issued by individual i, $\digamma^i$, consists of payoffs and transaction constraints.

• *Payoffs:* Security j's payoff is represented by a vector, $a_j \in \mathbb{R}^S$, determining the amount of good 1 to be transferred in each state of nature. Each individual i must design exactly J^i securities, but the choice $a_j = \mathbf{0}$ is interpreted as the non-issuing decision.[3] Hence, there is a total of $J = \sum_{i \in \mathbb{I}} J^i$ securities in the economy, and those designed by individual i are indexed by $j \in \mathbb{J}^i = \left\{ \sum_{\hat{\imath}<i} J^{\hat{\imath}} + 1, ..., \sum_{\hat{\imath}<i} J^{\hat{\imath}} + J^i \right\}$.

• *Transaction Constraints:* Due to the presence of moral hazard, issuers might have an interest in restricting the participation of some individuals in some markets. They are allowed to do so by imposing personal constraints on short and long transactions. Namely, the issuer of security j chooses $\left(\lambda^{\hat{\imath}}_{buy,j}, \lambda^{\hat{\imath}}_{sell,j}\right)_{\hat{\imath}\in\mathbb{I}} \in \mathbb{R}^{2I}_{+}$, where $\lambda^{\hat{\imath}}_{buy,j}$ and $\lambda^{\hat{\imath}}_{sell,j}$ represent the maximum amount of security j that individual $\hat{\imath}$ is allowed to buy and sell, respectively.[4]

Remark 1. *Transaction constraints are allowed to depend on the individuals' names, $i \in \mathbb{I}$. This assumption is appropriate because the individuals' efforts have stochastic impact over specific states of nature. Personal transaction constraints expand the set of possible contracts, since they allow trading exclusion of individuals whose effort choices would compromise the implementation of a certain security. In particular, these constraints allow the existence of bilateral arrangements and financial clubs (which could be implemented by setting $\lambda^{\hat{\imath}}_{buy,j} = \lambda^{\hat{\imath}}_{sell,j} = 0$ for non-members).*

Remark 2. *It is important to stress that each intermediary chooses transaction constraints for the securities personally issued, but not for the securities issued by others. Therefore, these constraints do not implement trading exclusivity.*

Intermediation Cost

In order to rule out equilibria with weakly dominated strategies, let us assume that financial innovation is not free. Whenever $\mathcal{F}^i \neq \mathbf{0}$, individual i must expend an infinitesimal amount of time, $\bar{\tau}^i \in (0,1)$, to enforce transactions. This assumption is important to avoid innovations made by those who would have nothing to gain or lose with such a decision. More complex cost structures could be easily incorporated in this model. However, it is important to stress that intermediation costs are not driving the results here.

In short, the parameters $(J^i, \bar{\tau}^i)$ define individual i's financial technology. Security j is represented by $\mathcal{F}_j = \left(a_j, \left(\lambda^{\hat{\imath}}_{buy,j}, \lambda^{\hat{\imath}}_{sell,j}\right)_{\hat{\imath}\in\mathbb{I}}\right) \in \mathbb{R}^S \times \mathbb{R}^{2I}_{+}$. The financial structure designed by individual i is $\mathcal{F}^i = (\mathcal{F}_j)_{j\in\mathbb{J}^i}$. Individuals who choose to issue

[3] Throughout the paper, the bold character $\mathbf{0}$ is used to indicate the null vector.

[4] Note that when designing securities, intermediaries impose transaction constraints on all individuals $\hat{\imath} \in \mathbb{I}$ (including themselves). They can always impose flexible constraints on themselves, but they may choose not to do so whenever commitment is important.

at least one security are called intermediaries; the set of financial intermediaries is endogenously characterized by $\mathbb{H} = \{i \in \mathbb{I} : \digamma^i \neq \mathbf{0}\}$.

3.2 Competitive Markets

After observing the financial structure issued in the initial node, $\digamma = (\digamma^1, ..., \digamma^I)$, the individuals in $\mathbb{I}$ act in a Radner-type economy. A few definitions are necessary here. First, let $p_0 \in \mathbb{R}^L_+$ be the vector of commodity prices in period zero, and $p_s \in \mathbb{R}^L_+$ represent the commodity prices in state s of period one. Define $z^i \in \mathbb{R}^J$ as the vector with the number of shares of each security that is held by individual i and $q \in \mathbb{R}^J$ as the prices of those securities. Finally, define $\tau^i(\digamma^i)$ as the time spent on intermediation activities, where $\tau^i(\mathbf{0}) = 0$ and $\tau^i(\digamma^i) = \bar{\tau}^i$ for any $\digamma^i \neq \mathbf{0}$.

Before the realization of the uncertainty, individuals trade consumption goods and the existing securities. They also split their time endowment among leisure, ℓ^i, productive effort, e^i, and intermediation activities, $\tau^i(\digamma^i)$. Thus, the period-zero constraints are given by:

$$p_0 \cdot (x^i_0 - y^i_0) + q \cdot z^i \leq 0; \tag{1}$$

$$-\lambda^i_{sell,j} \leq z^i_j \leq \lambda^i_{buy,j}, \ \forall j \in \mathbb{J}; \tag{2}$$

$$\ell^i + e^i + \tau^i(\digamma^i) = 1. \tag{3}$$

Equation (1) states that individuals can use y^i_0 to buy consumption goods and securities; (2) represents the transaction constraints; and (3) is the time-allocation constraint.

After the realization of s, individuals can use the outcome of their productive project, $y^i_s \in \mathbb{R}^L_{++}$, and portfolio value to buy consumption goods. Therefore, the individuals' choices must also satisfy:

$$p_s \cdot (x^i_s - y^i_s) \leq p_{1,s} \textstyle\sum_{j \in \mathbb{J}} z^i_j a_{j,s}, \ \forall s \in \mathbb{S}. \tag{4}$$

Definition 1. *Individual i's budget set associated with the financial design* $\digamma$ *is* $B^i_\digamma(q,p) = \{(x^i, \ell^i, e^i) \in \mathbb{R}^{L(S+1)}_+ \times [0,1]^2 : \exists\ z^i \in \mathbb{R}^J$ *such that* (1)-(4) *hold*$\}$.

4 Equilibrium

Before defining the equilibrium concept, it is worth summing up the entire setup in a single assumption.

Assumption 1. *The economy ξ is defined as follows:*
Players: a finite set of individuals $(\mathbb{I})$;
States: a period zero and a finite set of period-one states of nature $(\mathbb{S})$;
Probabilities: $\pi : [0,1]^I \to [0,1]^S$ *such that:* (i) $\sum_{s\in\mathbb{S}} \pi_s(e) = 1$, $\forall e \in [0,1]^I$; *and* (ii) $\inf_{e\in[0,1]^I} \pi_s(e) > 0$, $\forall s \in \mathbb{S}$;
Preferences: for each $i \in \mathbb{I}$, *there is a continuous and quasiconcave function*, $U^i : \mathbb{R}_{+}^{L(S+1)} \times [0,1]^{1+I} \to \mathbb{R} \cup \{-\infty\}$, *such that:* (i) $U^i\left(x^i, \ell^i, e\right) > -\infty$, $\forall \left(x^i, \ell^i, e\right) \in \mathbb{R}_{++}^{L(S+1)} \times [0,1]^{1+I}$; (ii) $\lim_{x_n^i \to 0} U^i\left(x^i, \cdot, \cdot\right) = -\infty$, $\forall n = 1, ..., L(S+1)$; *and* (iii) *there exist continuous and strictly positive partial derivatives* $\frac{\partial U^i}{\partial x_n^i} : \mathbb{R}_{++}^{L(S+1)} \times [0,1]^{1+I} \to \mathbb{R}_{++}$, $\forall n = 1, ..., L(S+1)$;
Productive Technology: $y^i \in \mathbb{R}_{++}^{L(S+1)}$, $\forall i \in \mathbb{I}$;
Financial Technology: $J^i \geq 0$ *and* $\bar{\tau}^i \in (0,1)$, $\forall i \in \mathbb{I}$.

The equilibrium concept is presented in Definition 7. Since the economy ξ is modeled as a finite-horizon sequential game, this concept is characterized backwards.

4.1 Last Subgame

In each possible node of the last subgame $(\digamma)$, individuals choose consumption, portfolio, and allocation of time in a general equilibrium economy with exogenous financial markets.

Definition 2. *A competitive equilibrium associated with* $\digamma$ *is a vector* $(x^*, \ell^*, e^*, z^*, q^*, p^*) \in \mathbb{R}_{+}^{L(S+1)I} \times [0,1]^{2I} \times \mathbb{R}^{JI+J+L(S+1)}$ *such that:*
(i) $(x^{*i}, \ell^{*i}, e^{*i}, z^{*i})$ *maximizes* $U^i(x^i, \ell^i, e^i, e^{*-i})$ *in* $B^i_{\digamma}(q^*, p^*)$, $\forall i \in \mathbb{I}$;
(ii) *all markets clear, i.e.,* $\sum_{i\in\mathbb{I}}(x^{*i} - y^i, z^{*i}) = \mathbf{0}$.

The equilibrium concept for the last subgame combines the competitive and Nash concepts. It is a competitive equilibrium concept in the sense that individuals take prices as given, and it is a Nash equilibrium concept in the sense that each individual reacts to the other individuals' effort choices (e^{*-i}). Note that individuals do not consider the effect of their effort decision on prices (prices are always taken as given). As mentioned before, price taking is a fundamental behavioral assumption in the competitive tradition. Also, moral hazard models with multiple goods usually associate price-taking behavior with strategic effort choice (e.g., Arnott and Stiglitz

[5] and Citanna and Villanacci [14]). Lemma 1 in the appendix proves the existence of such an equilibrium for a general set of financial structures.

4.2 First Subgame

In the initial node, individuals face a financial innovation decision. Anticipating the competitive equilibria associated with each financial design, individuals in $\mathbb{I}$ move simultaneously and design a financial structure, $\digamma^i \in \Gamma^i$, to maximize their *expected utility payoff*, v^i. The reduced game is then given by $\left\{\mathbb{I}, \left(\Gamma^i, v^i\right)_{i\in\mathbb{I}}\right\}$.

The Strategy Space (Γ^i)

The strategy space Γ^i consists of issued and non-issued securities. Non-issued securities are represented by $\digamma_j = \left(a_j, \left(\lambda^{\hat{\imath}}_{buy,j}, \lambda^{\hat{\imath}}_{sell,j}\right)_{\hat{\imath}\in\mathbb{I}}\right) = \mathbf{0}$. The payoffs of issued securities are normalized to satisfy $\| a_j \| = \max\left(| a_{1,j} |, ..., | a_{S,j} |\right) = 1$. Moreover, following Radner [29], it is assumed that any promise to deliver more than the aggregate supply of the commodity is not credible. This imposes $\bar{y}_1 = \max_s \sum_{i\in\mathbb{I}} y^i_{1,s}$ as a natural upper bound for the sales of any issued security. Since each individual sells at most $\bar{y}_1$ units of each security issued, no one will be able to buy more than $(I-1)\bar{y}_1$ units of those securities. These restrictions make the strategy space compact.

Definition 3. *For any $i \in \mathbb{I}$, individual i's strategy space in the initial node is $\Gamma^i = \{\digamma^i \in \mathbb{R}^{SJ^i} \times \mathbb{R}^{2IJ^i}_+$: either $\| a_j \| = 1$ and $\left\| \left(\lambda^{\hat{\imath}}_{buy,j}, \lambda^{\hat{\imath}}_{sell,j}\right)_{\hat{\imath}\in\mathbb{I}} \right\| \leq I\bar{y}_1$ or $\digamma_j = \mathbf{0}, \forall j \in J^i\}$.*

The Expected Utility Payoffs (v^i)

The individuals' *expected utility payoffs* in the first subgame are real numbers associated with each asset structure, i.e., $v(\digamma) = \left(v^1(\digamma), ..., v^I(\digamma)\right), \forall \digamma \in \Gamma = \Gamma^1 \times ... \times \Gamma^I$. If the economy had a unique competitive equilibrium associated with each $\digamma$, say $(x^*, \ell^*, e^*, z^*, q^*, p^*)$, then $v^i(\digamma)$ would be naturally defined by $U^i(x^{*i}, \ell^{*i}, e^*)$. However, the possibility of multiple equilibria cannot be ruled out. In this case, $v(\digamma)$ must reflect the individuals' beliefs about what competitive equilibrium would be played in the second subgame. These beliefs are modeled as endogenous sharing rules, in the spirit of Simon and Zame [31].

Start by defining $\Xi(\digamma)$ as the set of all competitive equilibria associated with the financial structure $\digamma \in \Gamma$.

Definition 4. Ξ *is a correspondence mapping financial designs into competitive equilibria. Formally,* $\Xi : \Gamma \rightarrow \mathbb{R}_+^{L(S+1)I} \times [0,1]^{2I} \times \mathbb{R}^{JI+J+L(S+1)}$ *is such that* $(x^*, \ell^*, e^*, z^*, q^*, p^*) \in \Xi(F)$ *if and only if* $(x^*, \ell^*, e^*, z^*, q^*, p^*)$ *is a competitive equilibrium associated with* F *(see Definition 2).*

Next, let $V(F) = \left(V^1(F), ..., V^I(F)\right)$ be the set of all utility possibilities associated with the financial design F. Formally, $V(F) = \{\mathbf{v} \in \mathbb{R}^I : \exists\, (x^*, \ell^*, e^*, z^*, q^*, p^*) \in \Xi(F)$ *such that* $\mathbf{v}^i = U^i(x^{*i}, \ell^{*i}, e^*),\ \forall i \in \mathbb{I}\}$. The elements of $V(F)$ are the utility payoffs that can be achieved in each of the competitive equilibria in $\Xi(F)$. Note that $\mathbf{v} \in \mathbb{R}^I$ (rather than $\bar{\mathbb{R}}^I$) because $U^i(\cdot) = -\infty$ never occurs in equilibrium (see Lemma 1 in the appendix).

Now, define $\nabla(F)$ as the convex hull of $V(F)$.[5] From the definition of convex hull, any element in $\nabla(F)$ can be written as a convex combination of the elements in $V(F)$. Moreover, any convex combination of *utility payoffs* in $V(F)$ must belong to $\nabla(F)$. Therefore, since V is the correspondence of all possible *utility payoffs*, ∇ is the set with all possible *expected utility payoffs*.

Definition 5. $\nabla(F)$ *is the convex hull of* $V(F) = \{\mathbf{v} \in \mathbb{R}^I : \exists\, (x^*, \ell^*, e^*, z^*, q^*, p^*) \in \Xi(F)$ *such that* $\mathbf{v}^i = U^i(x^{*i}, \ell^{*i}, e^*),\ \forall i \in \mathbb{I}\},\ \forall F \in \Gamma$.

Then, define $v = (v^1, ..., v^I)$ as a measurable function selected from ∇. Since $\nabla(F)$ defines the universe of all possible *expected utility payoffs* associated with F, $v(F)$ is a particular choice that reflects the individuals' beliefs regarding which competitive equilibria would be played when the asset structure is F.

Definition 6. $v : \Gamma \rightarrow \mathbb{R}^I$ *is a Borel measurable function selected from* $\nabla : \Gamma \rightarrow \mathbb{R}^I$.

The Solution

Each individual chooses a strategy (potentially mixed) to maximize the *expected utility payoff*, $v^i(\cdot)$. Let $\mathcal{B}^i$ be the Borel sigma-algebra for Γ^i, and Λ_{Γ^i} be the set of probability measures on $(\Gamma^i, \mathcal{B}^i)$. Thus, individual i's financial-innovation strategy is a probability measure $\sigma^i \in \Lambda_{\mathbf{\Gamma}^i}$ that is chosen as a best response to the other individuals' strategies (σ^{-i}). Formally, the financial-innovation problem faced by each individual $i \in \mathbb{I}$ is:

$$\max_{\sigma^i \in \Lambda_{\Gamma^i}} \int v^i(F) d\sigma^i \times d\sigma^{-i}. \quad (5)$$

When making the issuing decision, individuals must balance aggregate effort incentives and personal risk sharing. By issuing a new security, one improves one's

[5] The convex hull of a given set Ω is the smallest convex set containing Ω.

personal risk sharing, but also changes the probability distribution over the states of nature (since insurance affects the individuals' effort choices).

Definition 7. *An Equilibrium with Endogenous Financial Markets consists of a Borel measurable function selected from* ∇ *and financial-innovation strategies, namely* $v = \left(v^1, ..., v^I\right)$ *and* $\sigma = (\sigma^1, ..., \sigma^I)$*, such that* σ^i *solves* (5)*, given* σ^{-i}*, for all* $i \in \mathbb{I}$.

Theorem 1. *There exists an Equilibrium with Endogenous Financial Markets for economy* ξ.

Proof. See appendix. ■

4.3 Further Comments

Theorem 1 guarantees the existence of a solution for the two-stage game defined in this paper. The equilibrium financial design may present many real-world features, such as anonymous competitive securities (those whose transaction constraints were non-binding for all individuals); bilateral arrangements and financial clubs (i.e., markets in which participation is restricted); redundant securities; and latent contracts (those not traded in equilibrium, but issued to affect other intermediaries' strategies).

Financial markets also tend to be incomplete. As is well-known from partial equilibrium models, imperfect risk sharing is an important generator of effort incentives. Section 5 elaborates on this topic through a simple example.

Furthermore, the *Equilibrium with Endogenous Financial Markets* tends not to implement the second-best allocation. There are many sources of inefficiency. First, the financial technology and the strategic nature of financial innovation are, per se, causes of inefficiency. For instance, as in Pesendorfer [27] and Bisin [8], the asset structure may present redundant securities even when there are fixed costs associated with the financial-innovation technology. The shape of $U^i(x^i, \ell^i, e)$ can also be important in driving the equilibrium to be constrained inefficient. As noted by Arnott and Stiglitz [5], in economies with multiple consumption goods and utility functions which are non-separable in leisure, competitive commodity prices will almost always fail to implement the second-best allocation.[6] However, the main source of inefficiency is related to the fact that contracts are non-exclusive. It is known that

[6]Another potential source of inefficiency, which is unrelated to moral hazard, comes from the fact that individuals choose competitive securities in a multi-good economy with potentially incomplete markets. As remarked by Geanakoplos and Polemarchakis [20], individuals generically do not incorporate the pecuniary externalities of their portfolio decisions when markets are incomplete.

second-best allocations tend to underinsure individuals (due to the trade-off between insurance and incentives), so that there would be groups interested in deviating from these allocations. Here, these individuals would be able to deviate by introducing new securities.[7] Inefficiency of moral hazard economies with non-exclusive contracts was first discussed by Richard Arnott and Joseph Stiglitz in some unpublished papers in the early eighties (as mentioned in Bisin and Guaitoli [11]). Since then, an extensive literature, which includes Helpman and Laffont [23], Jaynes [24], Arnott and Stiglitz [6], Bisin and Guaitoli [11], and Kahn and Mookherjee [25], has addressed this problem. These analyses suggest that exclusivity would be necessary for second-best efficiency.

Possible Extensions

The model presented here could be extended in many directions. For instance, it would be more realistic to allow individuals to charge for the intermediation service through a non-linear price schedule. The existence of an equilibrium would then obviously depend on the type of price schedule that is allowed. It is worth mentioning that the equilibrium would still exist if intermediaries were allowed to choose bid-ask spreads to be charged from traders, in the spirit of Bisin [8]. Although interesting, allowing for intermediation profits would introduce a new dimension to the financial-innovation decision without changing the main trade-off between incentives and risk-sharing possibilities. Moreover, profits would not be an important element in the analysis insofar as all individuals were able to issue securities at an infinitesimal cost.

Other possible extensions could explore different forms of relaxing the non-exclusivity problem. For instance, infinite repetition of the financial-innovation game could implement cooperation among financial intermediaries, alleviating this problem. However, introducing dynamics into the model complicates the analysis. Time raises the possibility of using the history to provide intertemporal incentives. As a consequence, long-lived contracts and time-dependent strategies would have to be considered. These issues are out of the scope of this paper.

However, it must be noted that markets are exogenously incomplete in that paper. Lisboa [26] argues that generic arguments may not be valid for economies with endogenous financial markets. Endowments and utilities in a set of measure zero may be exactly those associated with a particular financial structure when such a structure is endogenous.

[7] Intermediaries are allowed to set constraints on transactions made in their clearinghouse, but these constraints do not guarantee exclusivity. Intermediaries have no control over trades in the securities issued by the others.

5 Illustration

In order to illustrate how moral hazard can generate financial incompleteness, consider a simple economy with two individuals, no consumption in period zero, and a single consumption good in period one. Individual 1 is risk neutral and produces a constant level of output, $\bar{y}^1$. Individual 2 is risk averse and faces a random production that delivers $y^2_{high} > 0$ units of the good with probability $\delta + \beta e^2$ and $y^2_{low} \in (0, y^2_{high})$ units with probability $(1 - \delta - \beta e^2)$, where δ and β are strictly positive and $\delta + \beta < 1$. The set of states of nature is then: $\mathbb{S} = \{s_1 = (\bar{y}^1, y^2_{high}), s_2 = (\bar{y}^1, y^2_{low})\}$.

Preferences are represented by $U^1\left(x^1, \ell^1, e^2\right) = \sum_{s \in \mathbb{S}} x^1_s \pi_s(e^2) + \ell^1$ and $U^2(x^2, \ell^2, e^2) = \sum_{s \in \mathbb{S}} u(x^2_s)\pi_s(e^2) + g\left(\ell^2\right)$, where $u(0) = -\infty$, and $u(\cdot)$ and $g(\cdot)$ are strictly increasing, strictly concave, and C^2. Furthermore, $g''(\cdot)$ is sufficiently negative to make $U^2(\cdot)$ concave, and $\bar{y}^1$ is sufficiently large to ensure that individual 1 is able to insure individual 2.

Individual 1 has no effort choice to make ($e^1 = 0$), and individual 2's optimal effort must satisfy:

$$\beta\left(u(x^2_{s_1}) - u(x^2_{s_2})\right) - g'\left(1 - \tau^2\left(F^2\right) - e^2\right) + \theta_0 - \theta_1 = 0, \qquad (6)$$

where θ_0 and θ_1 are the Kuhn-Tucker multipliers of the restrictions $e^2 \geq 0$ and $\ell^2 = 1 - \tau^2\left(F^2\right) - e^2 \geq 0$, respectively.

This expression provides a simple rationale for market incompleteness. Since $g''(\cdot) < 0$, there is a positive relation between effort and the difference between the consumption level in the two states of nature. Thus, the higher the risk-sharing possibilities, the lower the effort level. In fact, it can be shown that $e^2 = 0$ in the equilibrium with complete markets.

Complete Markets

Suppose that individuals can trade two Arrow-Debreu securities at prices p_{s_1} and p_{s_2}. For $\bar{y}^1$ sufficiently large, one must have $\frac{p_{s_1}}{p_{s_2}} = \frac{\pi_{s_1}(e^2)}{\pi_{s_2}(e^2)}$ in any equilibrium—otherwise individual 1 would be willing to buy more than the aggregate endowment of the good in one of the states.

From individual 2's necessary and sufficient first-order conditions, one has:

$$\frac{\pi(e^2)u'(x^2_{s_1})}{\left(1 - \pi(e^2)\right)u'(x^2_{s_2})} = \frac{p_{s_1}}{p_{s_2}}. \qquad (7)$$

Note from (6)-(7) that $\frac{p_{s_1}}{p_{s_2}} = \frac{\pi(e^2)}{1-\pi(e^2)}$ implies $x^2_{s_1} = x^2_{s_2}$ and $e^2 = 0$. Furthermore, $x^2_{s_1} = x^2_{s_2} = \delta y^2_{high} + (1 - \delta)\, y^2_{low}$ and $e^2 = 0$ solve individual 2's optimization problem when $\frac{p_{s_1}}{p_{s_2}} = \frac{\delta}{1-\delta}$. Thus, these allocations and prices, together with $x^1_{s_1} = \bar{y}^1 + (1-\delta)\left(y^2_{high} - y^2_{low}\right)$, $x^1_{s_2} = \bar{y}^1 - \delta\left(y^2_{high} - y^2_{low}\right)$, and $\ell^i = 1 - \tau^i\left(F^i\right)$ for $i \in \{1,2\}$, constitute the competitive equilibrium when markets are complete.

Incomplete Markets

By taking prices as given, individual 2 does not incorporate the effect of effort on the equilibrium prices. As a consequence, one ends up trapped in an equilibrium where insurance is costly and the bad state occurs with high probability. In this economy, social welfare could be improved if individual 2 faced fewer insurance possibilities (either through missing assets or through trading constraints). For instance, consider a second-best allocation $\left(\hat{x}^1_{s_1}, \hat{x}^1_{s_2}, \hat{x}^2_{s_1}, \hat{x}^2_{s_2}, \hat{e}^2\right)$ which solves the following problem:

$$\max\ U^2(x^2, 1-\tau^2\left(F^2\right) - e^2, e^2) \tag{8}$$

$$s.t.\ \left(x^1, x^2\right) \in \mathbb{R}^4_+; \tag{9}$$

$$\textstyle\sum_{i\in\mathbb{I}}(x^i - y^i) = \mathbf{0}; \tag{10}$$

$$\pi\left(e^2\right)x^1_{s_1} + \left(1-\pi\left(e^2\right)\right)x^1_{s_2} = \bar{y}^1; \tag{11}$$

$$e \in \underset{\tilde{e}^2\in[0,1-\tau^2(F^2)]}{\operatorname{argmax}}\ U^2(x^2, 1-\tau^2\left(F^2\right) - \tilde{e}^2, \tilde{e}^2). \tag{12}$$

For appropriate parameters, one has $y^2_{high} > \hat{x}^2_{s_1} > \hat{x}^2_{s_2} > y^2_{low}$ and $\hat{e}^2 > 0$. Consider now an asset structure with two Arrow-Debreu securities and trading constraints (λ) such that individual 2 is not able to get more insurance than the second-best level (i.e., $x^2_{s_1} \geq \hat{x}^2_{s_1}$ and $x^2_{s_2} \leq \hat{x}^2_{s_2}$). Given this financial design, the allocation $\left(\hat{x}^1_{s_1}, \hat{x}^1_{s_2}, \hat{x}^2_{s_1}, \hat{x}^2_{s_2}, \hat{e}^2\right)$ together with $\hat{e}^1 = 0$, $\hat{\ell}^i = 1 - \tau^i\left(F^i\right) - \hat{e}^i$ for $i \in \{1,2\}$, and $\frac{p_{s_1}}{p_{s_2}} = \frac{\pi(\hat{e}^2)}{1-\pi(\hat{e}^2)}$ constitute an equilibrium. This incomplete financial structure Pareto dominates complete markets and would naturally arise as an *Equilibrium with Endogenous Financial Markets* in different circumstances.[8] It is worth noting that this economy does not present the elements discussed in Section 4.3 that typically yield equilibrium (second-best) inefficiency.

6 Conclusion

Incompleteness is an intriguing feature of modern financial markets. The literature on financial innovation uses the trade-off between innovation revenues and

[8] Consider, for example, the case where only individual 2 were allowed to issue securities (i.e., $J^1 = 0$).

intermediation costs to explain incompleteness as an equilibrium outcome. This paper extends the analysis of optimal security design to economies with moral hazard. The motivation for this extension is the existence of incompleteness explained by incentive problems associated with hidden actions. For instance, unemployment insurance is typically incomplete, in the sense that individuals are not able to equalize earnings across employed and unemployed states. Also, in agricultural markets, future and forward contracts provide insurance against the risk of price variations, but no contract covers the risk inherent to production. These types of incompleteness are clearly motivated by the fact that, once hedged, workers and farmers would have no interest in making efforts to avoid the undesirable states.

The paper presents a decentralized general equilibrium approach to the moral hazard problem where, instead of having a principal determining the allocation, individuals endogenously choose the financial span. Incomplete financial markets emerge as part of the equilibrium incentive structure. The equilibrium tends not to be optimal (in the second-best sense).

A Appendix: Proof of Theorem 1

The proof requires the following lemmas.

Lemma 1. *There exists a competitive equilibrium associated with each $\digamma \in \Gamma$.*

Proof. Given the assumptions on preferences and endowments (see Assumption 1), there exists $c > 0$ such that $U^i\left(x^i, \ell^i, e\right) \geq U^i\left(y^i, \ell^i, e\right)$ implies $\min\left(x^i\right) > c$, $\forall \left(\ell^i, e\right) \in [0,1]^{1+I}$. Define then the truncated demand correspondence to be $d^i_\digamma(q,p,e^{-i}) = \{(x^i, \ell^i, e^i, z^i) \in \arg\max U^i(x^i, \ell^i, e)$ *s.t.* $(x^i, \ell^i, e^i) \in B^i_\digamma(q,p)$ *and* $\left(x^i, z^i\right) \in K\}$, where $B^i_\digamma(q,p)$ is given in Definition 1 and $K = K_1 \times K_2 = \{x^i \in \mathbb{R}^{L(S+1)}_+ : \min\left(x^i\right) \geq c \text{ and } \|x^i\| \leq 2\|\sum_{i\in\mathbb{I}} y^i\|\} \times \{z^i \in \mathbb{R}^J : \|z^i\| \leq 2I\|\sum_{i\in\mathbb{I}} y^i\|\}$.[9] Normalize the commodity prices to lie in the simplex and note that the assets' payoffs are bounded for all $\digamma \in \Gamma$. By the Kuhn-Tucker necessary conditions, there exists $\bar{q} = \max_{\mathbb{I}\times K_1\times[0,1]^{1+I}} \frac{\sum_{s\in\mathbb{S}} \partial U^i/\partial x^i_{1,s}}{\sum_{l=1}^{L} \partial U^i/\partial x^i_{l,0}} > 0$ such that, for all $i \in \mathbb{I}$ and $j \in \mathbb{J}$, one has: (i) if $q_j \geq \bar{q}$ and $(x^i, \ell^i, e^i, z^i) \in d^i_\digamma(q,p,e^{-i})$, then $z^i_j = -\lambda^i_{sell,j}$; and analogously (ii) if $q_j \leq -\bar{q}$ and $(x^i, \ell^i, e^i, z^i) \in d^i_\digamma(q,p,e^{-i})$, then $z^i_j = \lambda^i_{buy,j}$. Therefore, one can focus on prices in $\Delta = \Delta_0 \times \Delta_1 \times ... \times \Delta_S$, where $\Delta_0 = \{(q,p_0) \in \mathbb{R}^J \times \mathbb{R}^L_+ : \|q\| \leq \bar{q}$ *and* $\sum_{l=1}^{L} p_{l,s} = 1\}$ and $\Delta_s = \{p_s \in \mathbb{R}^L_+ : \sum_{l=1}^{L} p_{l,s} = 1\}$, $\forall s \in \mathbb{S}$.

[9] For a given vector $\omega \in \mathbb{R}^n$, $\|\omega\| = \max(| \omega_1 |, ..., | \omega_n |)$.

Next, let $\phi_F : K_1^I \times [0,1]^{2I} \times K_2^I \times \Delta \to K_1^I \times [0,1]^{2I} \times K_2^I \times \Delta$ be such that $\phi_F(x,\ell,e,z,q,p) = (\varphi_F, \psi_F)$, where $\varphi_F(q,p,e) = \left(d_F^1, ..., d_F^I\right)$ and $\psi_F(x,z) = \{(q',p') \in \Delta : (q',p_0') \in \underset{(\tilde{q},\tilde{p}_0)\in\Delta_0}{\text{argmax}}\, \tilde{q} \cdot \sum_{i\in\mathbb{I}} z^i + \tilde{p}_0 \cdot \sum_{i\in\mathbb{I}}(x_0^i - y_0^i)$ *and* $p_s' \in \underset{\tilde{p}_s\in\Delta_s}{\text{argmax}}\, \tilde{p}_s \cdot \sum_{i\in\mathbb{I}}(x_s^i - y_s^i), \forall s \in \mathbb{S}\}$. For each fixed F, the budget set, $B_F^i : \Delta \to \mathbb{R}_+^{L(S+1)} \times [0,1]^2$, is nonempty and convex valued, and continuous in (q,p). Moreover, (x^i, ℓ^i, e^i, z^i) must lie in a compact set. Thus, by the Theorem of the Maximum, $d_F^i(q,p,e^{-i})$ is upper hemicontinuous, nonempty, and compact valued. Moreover, $d_F^i(\cdot)$ is convex valued, since U^i is quasiconcave and $B_F^i(q,p)$ is convex. From these results, it is straightforward to show that ϕ_F is an upper-hemicontinuous correspondence with nonempty and convex values that maps a compact set into itself. Therefore, by Kakutani's fixed point theorem, $\exists\ (x^*, \ell^*, e^*, z^*, q^*, p^*) \in \phi_F\ (x^*, \ell^*, e^*, z^*, q^*, p^*)$.

By summing the budget constraint (1) over i, one gets $q^* \cdot \sum_{i\in\mathbb{I}} z^{*i} + p_0^* \cdot \sum_{i\in\mathbb{I}}(x_0^{*i} - y_0^i) \leq 0$. Since q^* and p_0^* satisfy the maximization in ψ_F, one has $\sum_{i\in\mathbb{I}}(x_0^{*i} - y_0^i) \leq \mathbf{0}$ and, then, $x_0^{*i} \in int\,(K_1)$. Interior consumption and the fact that U^i is strictly increasing in $x^i \in \mathbb{R}_{++}^{L(S+1)}$ imply that (1) is binding for all i and, thus, $q^* \cdot \sum_{i\in\mathbb{I}} z^{*i} + p_0^* \cdot \sum_{i\in\mathbb{I}}(x_0^{*i} - y_0^i) = 0$. Using once again the definition of ψ_F, one gets $\sum_{i\in\mathbb{I}} z^{*i} = \mathbf{0}$ and, then, $\sum_{i\in\mathbb{I}}(x_0^{*i} - y_0^i) = \mathbf{0}$.[10] Furthermore, summing (4) over i and using the fact that $\sum_{i\in\mathbb{I}} z^{*i} = \mathbf{0}$, one gets $p_s^* \cdot \sum_{i\in\mathbb{I}}(x_s^{*i} - y_s^i) \leq 0$. The same argument implies $\sum_{i\in\mathbb{I}}(x_s^{*i} - y_s^i) = \mathbf{0}$, $\forall s \in \mathbb{S}$.

From the definition of φ_F, the allocation (x^*, ℓ^*, e^*, z^*) solves the individuals' truncated problems at prices (q^*, p^*). By continuity, convexity of the budget set, concavity of U^i, and the fact that $(x^{*i}, z^{*i}) \in int(K)$, $\forall i \in \mathbb{I}$, one has that (x^*, ℓ^*, e^*, z^*) solves the individuals' untruncated problems at prices (q^*, p^*). Therefore, $(x^*, \ell^*, e^*, z^*, q^*, p^*)$ is an equilibrium for the untruncated economy. ■

Lemma 2. *The correspondence* $\nabla : \Gamma \to \mathbb{R}^I$ *is bounded and upper hemicontinuous with nonempty, convex, and compact values.*

Proof. First, note that $V : \Gamma \to \mathbb{R}^I$ is bounded, since $U^i(\cdot)$ is continuous for all i and equilibrium allocations lie in the same compact set, $K_1^I \times [0,1]^{2I}$, for all $F \in \Gamma$. Moreover, V has a closed graph and is therefore upper hemicontinuous and compact valued. To see the closed-graph property, take any two sequences $\{F_n\} \to F^*$ and $\{\mathbf{v}_n\} \to \mathbf{v}^*$ s.t. $F_n \in \Gamma$ and $\mathbf{v}_n \in V(F_n)$, $\forall n \in \mathbb{N}$. From Lemma 1 and the definition of $V(\cdot)$, there exists a competitive equilibrium associated with F_n, say

[10] In any fixed point of ϕ_F, one must have $\sum_{i\in\mathbb{I}} z^{*i} = 0$ since: (i) $\sum_{i\in\mathbb{I}} z^{*i} > 0 \Rightarrow q_j^* = \bar{q} \Rightarrow \sum_{i\in\mathbb{I}} z^{*i} = -\sum_{i\in\mathbb{I}} \lambda_{sell,j} < 0$; and (ii) $\sum_{i\in\mathbb{I}} z^{*i} < 0 \Rightarrow q_j^* = -\bar{q} \Rightarrow \sum_{i\in\mathbb{I}} z^{*i} = \sum_{i\in\mathbb{I}} \lambda_{buy,j} > 0$ (contradiction).

$(x_n, \ell_n, e_n, z_n, p_n, q_n) \in K_1^I \times [0,1]^{2I} \times K_2^I \times \Delta$, such that $\mathbf{v}_n = \left(U^i(x_n^i, \ell_n^i, e_n)\right)_{i\in\mathbb{I}}$, $\forall n \in \mathbb{N}$. The sequence $\{x_n, \ell_n, e_n, z_n, p_n, q_n\}_{n=1}^{\infty}$ is bounded, so that there exists a subsequence converging to $(x^*, \ell^*, e^*, z^*, q^*, p^*)$. Since U^i is continuous, one must have $\mathbf{v}^{*i} = \lim_{n\to\infty} U^i(x_n^i, \ell_n^i, e_n) = U^i(x^{*i}, \ell^{*i}, e^*)$, for all $i \in \mathbb{I}$. Note that Γ^i was defined in such a way that $\tau^i : \Gamma^i \to \left\{0, \bar{\tau}^i\right\}$ is continuous, so that $B^i_{\digamma}(q,p)$ is continuous in $(\digamma, q, p) \in \Gamma \times \Delta$. The theorem of the maximum implies that (x^*, ℓ^*, e^*, z^*) solves the individuals' maximization problems at (q^*, p^*). Furthermore, since market clearing conditions are continuous, $(x^*, \ell^*, e^*, z^*, q^*, p^*)$ is a competitive equilibrium associated with $\digamma^*$ and, thus, $\mathbf{v}^* = U^i(x^{*i}, \ell^{*i}, e^*) \in V(\digamma^*)$.

Therefore, ∇ is bounded and compact valued since V is bounded and compact valued; and it is upper hemicontinuous since V is compact valued and upper hemicontinuous (see Aliprantis and Border, 1999, Theorem 16.35, pp. 542). Finally, ∇ is nonempty from Lemma 1 and convex from the definition of convex hull. ■

Lemma 1 proves the existence of a solution for the last subgame. In order to conclude the proof of Theorem 1, one needs only to show that there exists a Nash Equilibrium for the first subgame. Since Γ^i is compact, the result follows from Lemma 2 together with the main theorem in Simon and Zame [31], pp. 865. *Q.E.D.*

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