Intra-backbone and Inter-backbone Peering Among Internet Service Providers*

Narine Badasyan† and Subhadip Chakrabarti‡

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Abstract

We consider a model with two backbones and a finite number of Internet Service Providers (ISPs) connected to the backbones. ISPs decide on private peering agreements, comparing the benefits of private peering to costs. Intra-backbone peering refers to peering between ISPs connected to the same backbone, whereas inter-backbone peering refers to peering between ISPs connected to different backbones. We formulate the model as a two-stage game. In the first stage, ISPs decide on peering agreements. In the second stage they compete in prices a la Bertrand. We examine the effects of peering on profits of ISPs. Peering affects profits through two channels - reduction of backbone congestion which we call the symmetric effect and ability to send traffic bypassing or circumventing congested backbones which we call the asymmetric effect. The first has a negative or ambiguous effect while the second has a generally positive effect on firm profits. The two often act against each other making the net effect ambiguous. We also conduct simulations to determine pairwise stable peering configurations in a six-provider model and find that there is a paucity of inter-backbone peering in asymmetric settings.

JEL codes: D21, D43

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†Department of Economics, Virginia Tech, Blacksburg, VA 24060, email: nabadsya@vt.edu

‡Department of Economics, Virginia Tech, Blacksburg, VA 24060, email: schakrab@vt.edu

1 Introduction

The Internet is comprised of many distinct networks, which are operated by different firms. Each firm has its own network, where the connected users can communicate with each other. The end-users of the Internet are consumers and websites. End-users generally want to have access to all other possible end-users, regardless of the network they are attached to. To provide such universal connectivity to their users, the firms must interconnect with each other and share their network infrastructure. Two main forms of interconnection emerged following privatization of the National Access Points - peering under which firms carry each other's traffic without any payments and transit under which the downstream firm pays the upstream firm a certain settlement payment for carrying its traffic.

Universal connectivity requires some structure on the connectivity agreements. The Internet has a loosely hierarchical structure (Ross and Kurose, 2000). At the top of the hierarchy are the backbones, also called Internet Access Providers (hereafter IAPs), that own national and/or international high speed networks. The four largest IAPs in the U.S. are UUNET/MCI (27.9%), AT&T (10%), SPRINT (6.5%) and GENUITY/LEVEL3 (6.3%) (Haynal, 2003). The second layer of the hierarchy includes so called retail Internet Service Providers (hereafter ISPs). At the bottom of the hierarchy are the end users, i.e. namely consumers who browse the web and websites.

In general,end-users connect to ISPs. ISPs connect to backbones. Backbones connect to each other at the National Access Points (hereafter NAPs) as illustrated in Figure 1. With regard to connectivity agreements, large IAPs mostly peer with each other at the NAPs and these are called public peerings. The growing congestion at the NAPs have increasingly necessitated private peering between the IAPs which refer direct connections between providers bypassing NAPs. ISPs generally have transit agreements with backbones but they also privately peer with other ISPs which are called retail peerings. We refer peering between two ISPs connected to the same backbone as intra-backbone peering and that between two ISPs connected to different backbones as inter-backbone peering. Inter and intra-backbone peerings among Internet service providers have two distinct (though not completely unrelated effects) on quality of service and ultimately profits of the firms. First, intra-backbone peering

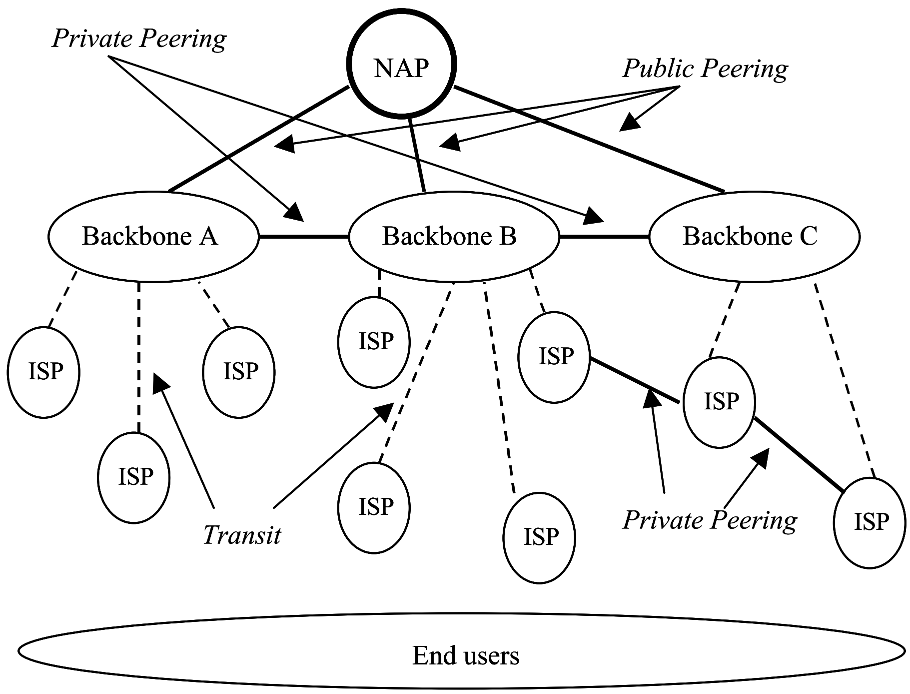

Figure 1:

reduces the traffic within the backbone and raises the quality of all ISPs connected to that backbone. Inter-backbone peering reduces the traffic between backbones and raises the quality of all ISPs connected to both backbones. We call this the symmetric effect or traffic diverting effect. Due to the symmetric effect, peering in general has strong positive externalities. The chief reason why ISPs may be interested in peering is not the symmetric effect but the asymmetric effect or the circumventing effect. Users (both web sites and consumers) who connect through privately peered lines avoid the congestion/delays associated with going through backbones and National Access Points. This raises the quality of the ISPs who peer and increases the demand of those ISPs. We call this the asymmetric or circumventing effect.

The first effect has been captured by DangNyugen and Penard (1999) using a model of club behavior and vertical differentiation. They consider a model with two asymmetric backbones and identical retail ISPs connected to those backbones. The retail ISPs engage in both intra-backbone peering and inter-backbone peering. They make the assumption that ISPs connected to the same backbone behave like a club and collude with regard to pricing and connectivity behavior. The authors extend the standard model of vertical product differentiation to show that ISPs connected to the high quality backbone will always peer with each other. But they may or may not peer with the ISPs connected to the low quality backbone. Also, in the latter case, the ISPs of the low quality backbone will peer with each other. The results are illustrated with some evidence from the French Internet market. The second effect has been captured among others, by Badasyan and Chakrabarti (2003) who show that threat of traffic diversion creates strong incentives for peering among backbones or IAPs.

Before starting the model, we briefly discuss the literature. Generally, one would expect investment in the Internet infrastructure to be sub-optimal under peering using the standard argument of the "tragedy of the commons", given that providers share a common backbone, especially in presence of large asymmetries. A number of authors have made this point in a variety of contexts (see Little and Wright (1999), Cremer et al. (2000)). Such problems can be avoided under transit or settlement payments. However, transit between large backbones may be impractical given prohibitive costs of monitoring the Internet traffic and may account for the paucity of transit agreements among large backbones. In the future, when monitoring technology improves bringing down monitoring costs, we expect to see peering agreements being replaced by transit agreements.

There is a related body of literature in Internet economics that deal with pricing that we mention in passing. Mackie-Mason and Varian (1995) have pointed out that the current practice of flat-rate pricing (charging a fixed fee to consumers unrelated to usage) encourages overusage and hence congestion. Their solution to the problem is setting up a smart market to price consumers according to usage. Of course, this involves a technological leap but may become feasible in near future.

Currently, Internet services offered by an ISP have little horizontal differentiation offering the same basic services of web-browsing, emails, real time conversation and some Internet telephony but there is some vertical differentiation. Some authors have examined price and product differentiation in services. Odlyzko (1997) suggests multiservice mechanism, where users can choose between first and second class service and pay accordingly, even though the quality is not necessarily different. Gibbens et al (2000) discusses the competition between two Internet service providers, when either or both of them choose to offer multiple service classes. Assuming a uniform distribution of user preferences towards congestion and a finite number of networks, they prove that, even when Internet service providers are free to set capacities as well as prices, multiproduct competition is not sustainable in a profit maximizing equilibrium.

In this paper, we examine incentives for peering among the retail Internet service providers taking into account both the symmetric and asymmetric effects, where ISPs can choose to peer both within and across backbones. A two stage game is considered. In the first stage, ISPs decide on peering and in the second stage, given the contractual configuration, they compete a la Bertrand. We find that symmetric and asymmetric effects of peering often have opposite effects on firms' profits making the net effect ambiguous. Hence, the decision to peer or not largely depends on underlying parameters such as network capacity, number of ISPs connected to each backbone and number of consumers and websites connected to each ISP. Under simulations with six ISPs and two backbones, we find that a variety of configurations emerge in equilibrium depending on the values of the aforesaid parameters.

The rest of the paper is organized as follows. In section 2, we discuss the assumptions underlying the model. In section 3, we analyze the second stage and we determine equilibrium prices. In section 4, we analyze the first stage and determine the equilibrium network. Finally, we conclude.

2 The Model

We begin with the assumptions underlying the model. We consider two backbones A and B and a finite set of N ISPs denoted by $\eta = \{1, 2, 3, \ldots, N\}$ where $N \in \mathbb{N}$. We assume that each ISP is connected to one of the backbones, but not both. Every ISP has a transit agreement with the backbone it is connected to. According to that agreement the ISP pays the backbone a certain settlement for interconnection which is given exogenously. Without loss of generality, we assume that ISPs $1, \ldots, m$ are connected to backbone A and ISPs $m+1, \ldots, N$ are connected to backbone B. Let $\eta_A = \{1, \ldots, m\}$ and $\eta_B = \{m+1, \ldots, N\}$. Generally we will index backbones by l,

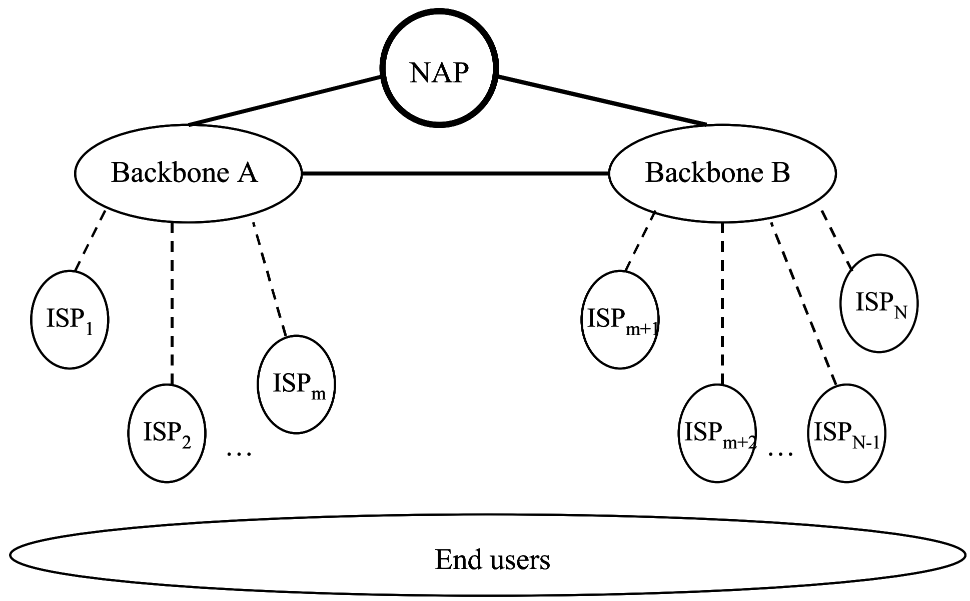

Figure 2:

$l \in \{A, B\}$ and ISPs by i, j, $i, j \in \eta$. We illustrate this in Figure 2. We assume that each ISP pays an access fee (transit) F_l if it is connected to backbone l.We distinguish between two types of end-users - consumers and websites. There is a continuum of consumers of mass 1 and a continuum of websites of mass 1 as well. We assume away any micro-payments from websites to consumers and vice versa. Let α_i denote the proportion of consumers connected to ISP i and d_i be the proportion of websites connected to ISP i. Obviously

$$\sum_{i=1}^{N} \alpha_i = 1 = \sum_{i=1}^{N} d_i$$

Also denote

$$\sum_{i=1}^{m} \alpha_i = \alpha$$

α_i's are given exogenously. di's will be determined endogenously by explicit modelling of website preferences. Thus, α is the proportion of consumers who have chosen the ISPs connected to backbone A. Similarly, $1 - \alpha$ is the proportion of consumers who have chosen the ISPs connected to backbone B.

In general it can be asserted that there is little traffic between websites. Traffic between consumers while not negligible are miniscule compared to traffic from websites to consumers. Thus, most of the traffic between the websites and the consumers is unidirectional, i.e., from websites to consumers. To capture this traffic pattern in its simplest form, we ignore all traffic between consumers, and from consumers to websites and focus exclusively on the traffic from websites to consumers. We assume following Laffont et. al (2001) that consumers are interested in all websites independently of their network choices. A consumer is as likely to request a page from a given website belonging to her network and another website belonging to a rival network. This is referred in the aforesaid paper as "balanced calling pattern". Hence, we assume each consumer requests one unit of traffic from each website. This gives precise measure to the proportion of traffic originating in ISP i and terminating in ISP j, namely,

$$t_{ij} = d_i \cdot \alpha_j$$

To be consistent with the standard terminology, we refer to traffic between ISPs belonging to the same backbone as on-net traffic and traffic going from one backbone to another as off-net traffic. With regard to traffic movement between backbones, backbone providers follow what is called "hot potato routing" - pass off net traffic as soon as possible. Given this pattern of traffic, it is not unrealistic to assume that all off-net traffic is borne by the receiver backbone. Hence, traffic requested by consumers in A from websites in B will be borne by backbone A. Similarly, traffic requested by consumers in B from websites in A will be borne by B.

Let U_i be the utility derived by a website connected to ISP i:

$$U_i = V - \delta_i - p_i$$

where p_i is the price charged by ISP i and δ_i is the delay associated with ISP i, and V is the value from the connectivity to the Internet. We assume V to be sufficiently large so that no consumer drops out of the market.

Delays occur due to excessive traffic in the backbone. Define s_l to be the network capacity of backbone l ($l \in \{A, B\}$). The network capacity is the maximum amount of traffic that the backbone can handle without experiencing delay. We assume that the network capacities are exogenously given. Thus, if t_l is the amount of traffic coming into backbone l, the delay in backbone l is $t_l - s_l$.

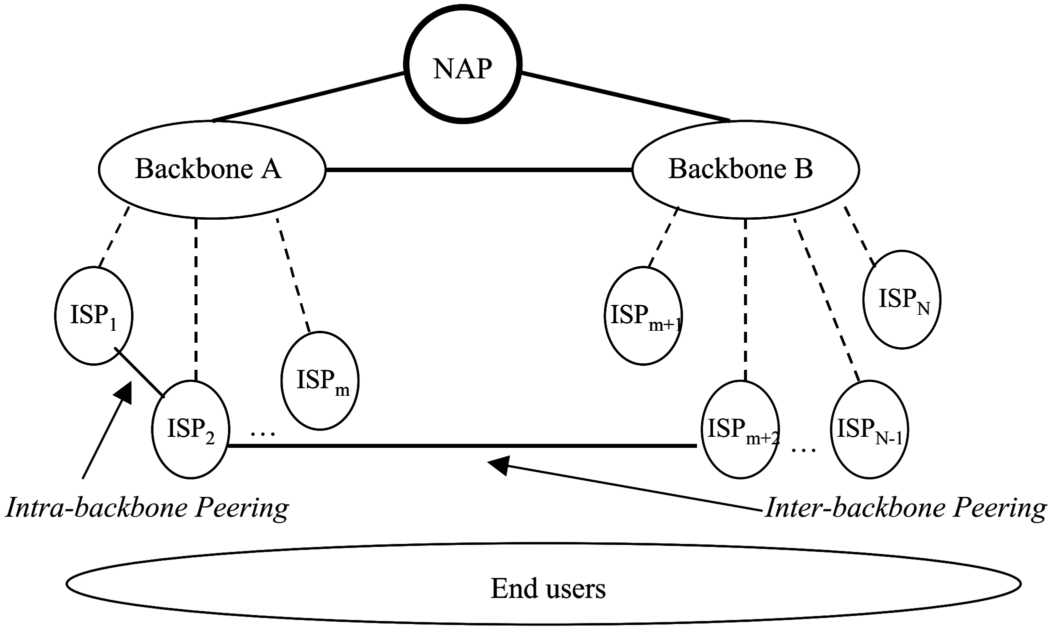

Figure 3:

Given the balanced calling pattern and the hot potato routing,

$$\begin{aligned} t_A &= \sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i \cdot d_j + \sum_{i=1}^{m}\sum_{j=m+1}^{N}\alpha_i \cdot d_j = \alpha \\ t_B &= \sum_{i=m+1}^{N}\sum_{j=1}^{m}\alpha_i \cdot d_j + \sum_{i=m+1}^{N}\sum_{j=m+1}^{N}\alpha_i \cdot d_j = 1-\alpha \end{aligned}$$

ISPs can enter into private peering agreements with each other. If two ISPs build a private peering link, then that peering is referred to as intra-backbone peering, if the ISPs are connected to the same backbone, and inter-backbone peering, if the ISPs are connected to different backbones. For example, in Figure 3, ISP 2 has an intra-backbone peering agreement with ISP 1 and an inter-backbone peering agreement with ISP $m+2$. In order to model private peering, we introduce a standard network notation borrowed from Goyal and Joshi (2000). For any two distinct ISPs i and j, define a binary variable $g_{ij} \in \{0,1\}$ where

$$g_{ij} = \begin{cases} 1 \text{ if a private peering arrangement exists between } i \text{ and } j \\ 0 \text{ if no such arrangement exists} \end{cases}$$

Obviously, $g_{ij} = g_{ji}$. If $g_{ij} = 1$, we say a link exists or a link is formed between i and j. The network $g = \{g_{ij}\}_{i,j\in\eta,i\neq j}$ is then a collection of links. Let $g - g_{ij}$ denote the network obtained by severing an existing link between i and j from the network g while $g + g_{ij}$ is the network obtained by adding a new link between i and j in the network g. The network g for which $g_{ij} = 1$ for all $i, j \in \eta, i \neq j$ is called the complete network. The network g for which $g_{ij} = 0$ for all $i, j \in \eta, i \neq j$ is called the empty network.

We assume each private peering link entails a transfer of traffic amounting to σ between i and j, that is capacity of each private peering link is equal to σ. Usage is shared equally and costs are borne equally. Assuming quadratic costs, each such link costs σ^2 and the cost borne by each of the two ISPs forming a link is $\sigma^2/2$. We assume that the private lines are completely uncongested and hence traffic traversing such links experience zero delays. Given that private links carry traffic that otherwise would have been carried by backbones, private peering links also reduce backbone congestion by reducing backbone traffic.

Now we can compute the average delay experienced by ISP i, namely, δ_i. Let n_{AA} denote the number of intra-backbone links for backbone A, n_{BB} be the number of intra-backbone links for backbone B, and n_{AB} be the number of inter-backbone links. Obviously,

$$\begin{aligned} n_{ll} &= \sum_{j\in\eta_l, i\in\eta_l, i<j} \sum g_{ij} \\ n_{AB} &= \sum_{j\in\eta_B} \sum_{i\in\eta_A} g_{ij} \end{aligned}$$

Further, let n_{iA} be the number of links ISP i has with members of η_A and n_{iB} be the number of links ISP i has with members of η_B. Let n_i be the total number of private links of ISP i. Hence,

$$\begin{aligned} n_{il} &= \sum_{j\in\eta_l, j\neq i} g_{ij} \\ n_i &= n_{iA} + n_{iB} \end{aligned}$$

Let ω_l be the reduction or leakage in the traffic of backbone l as a result of private peerings. Then,

$$\begin{aligned}\omega_A &= (n_{AA})\cdot\sigma + (n_{AB})\cdot(\sigma/2)\\ \omega_B &= (n_{BB})\cdot\sigma + (n_{AB})\cdot(\sigma/2)\end{aligned}$$

The traffic of backbone l is reduced by the amount of on-net traffic diverted through intra-backbone private peering links and by the amount of the outgoing off-net traffic diverted through inter-backbone peering links. Each intra-backbone link reduces on-net traffic by σ and each inter-backbone link reduces off-net outgoing traffic by $\sigma/2$ given that usage of peered links is shared equally.

Hence, if θ_l is the amount of congestion in backbone l, then

$$\theta_l = t_l - s_l - \omega_l$$

Traffic going through private peering links experience zero delays because we explicitly assume that ISPs keep their private links uncongested. All extra traffic is routed through the backbones. The traffic going through backbone A experience congestion of θ_A and the traffic going through backbone B experience a congestion of θ_B. If $\theta_A > \theta_B$, we refer to A as the low quality backbone and B as the high quality backbone and vice versa.

Consider an ISP connected to backbone A. Given that this ISP has n_{iA} intra-backbone links and each such link can carry $\sigma/2$ of traffic, on-net traffic circulating through privately peered lines is equal to $n_{iA}\cdot(\sigma/2)$ and this traffic experiences zero delay. Given that total volume of on-net traffic going out from this ISP is $\alpha\cdot d_i$, the traffic that traverses backbone A and experiences a delay of θ_A is $\alpha\cdot d_i - n_{iA}\cdot(\sigma/2)$. Next, the off-net traffic going out from this ISP is $(1-\alpha)\cdot d_i$ of which again $n_{iB}\cdot(\sigma/2)$ amount of traffic travels through privately peered lines and experiences zero delay. Hence, $(1-\alpha)\cdot d_i - n_{iB}\cdot(\sigma/2)$ amount of traffic traverses both backbones and is largely borne by the receiver backbone B and hence experiences delay θ_B. Therefore, if $i \in \eta_A$[1],

$$\delta_i = \theta_A\cdot(\alpha\cdot d_i - n_{iA}\cdot(\sigma/2)) + \theta_B\cdot((1-\alpha)\cdot d_i - n_{iB}\cdot(\sigma/2))$$

[1] Traffic going from websites to consumers connected to the same ISP do not have to traverse any backbone or private peering link. So strictly speaking, we have to deduct this leakage from traffic flows through backbones. We assume that this leakage is relatively small and can be ignored.

Similarly, if $j \in \eta_B$,

$$\delta_j = \theta_B \cdot ((1-\alpha) \cdot d_j - n_{jB} \cdot (\sigma/2)) + \theta_A \cdot (\alpha \cdot d_j - n_{jA} \cdot (\sigma/2))$$

Equilibrium demands are determined by equating utilities of all websites given that in equilibrium all websites must derive an identical utility from each ISP. As far as the ISPs are concerned, we will analyze a two-stage game. In the first stage, ISPs form links that determines the network structure g. In the second stage they compete in prices a la Bertrand. Consistent with the logic of backward induction, we start with the second stage.

3 Analysis of the Second Stage

Given a certain network g and the fact that firms compete as Bertrand oligopolists, we can solve for equilibrium prices in the second stage. We show the details in Appendix 1. Here we just present the results in the form of Proposition 1.

Proposition 1 *Let ,*

$$\begin{aligned}
\gamma_i &= \frac{\sigma\left(n_{iA} \cdot \theta_A + n_{iB} \cdot \theta_B\right)}{2} \text{ for all } i \in \eta \\
\widetilde{\theta} &= \alpha \cdot \theta_A + (1-\alpha) \cdot \theta_B \\
\theta_A &= \alpha - s_A - \sigma\left(n_{AA} + \frac{n_{AB}}{2}\right) \\
\theta_B &= (1-\alpha) - s_B - \sigma\left(n_{BB} + \frac{n_{AB}}{2}\right)
\end{aligned}$$

The equilibrium price, demand and profit, respectively, of $i \in \eta_l$ in stage 2 is given by

$$p_i^* = \left(\frac{\widetilde{\theta}}{N-1}\right) + \frac{1}{2N-1}\left(N \cdot \gamma_i - \sum_{j=1}^{N} \gamma_j\right) \tag{1}$$

$$d_i^* = \left(\frac{N-1}{N \cdot \widetilde{\theta}}\right)\left[\left(\frac{\widetilde{\theta}}{N-1}\right) + \frac{1}{2N-1}\left(N \cdot \gamma_i - \sum_{j=1}^{N} \gamma_j\right)\right] \tag{2}$$

$$\pi_i^* = \left(\frac{N-1}{N \cdot \widetilde{\theta}}\right)\left[\left(\frac{\widetilde{\theta}}{N-1}\right) + \frac{1}{2N-1}\left(N \cdot \gamma_i - \sum_{j=1}^{N} \gamma_j\right)\right]^2 - \left(\frac{\sigma^2}{2}\right) n_i - F_l \tag{3}$$

Of course, the above proposition represents the unique interior solution. For the interior solution to exist and be applicable, we need certain parameter constraints. Specifically, $\theta_A > 0$ and $\theta_B > 0$ requires

$$\begin{aligned} \sigma &< \frac{2(\alpha - s_A)}{(2 \cdot n_{AA} + n_{AB})} \\ \sigma &< \frac{2(1 - \alpha - s_B)}{(2 \cdot n_{BB} + n_{AB})} \\ s_A &< \alpha \\ s_B &< 1 - \alpha \end{aligned}$$

Given that

$$\begin{aligned} n_{AA} &\leqslant \frac{m(m-1)}{2} \\ n_{BB} &\leqslant \frac{(N-m)(N-m-1)}{2} \\ n_{AB} &\leqslant m(N-m) \end{aligned}$$

we have

$$\begin{aligned} \frac{2 \cdot n_{AA} + n_{AB}}{2} &\leqslant \frac{m(N-1)}{2} \\ \frac{2 \cdot n_{BB} + n_{AB}}{2} &\leqslant \frac{(N-m)(2 \cdot N - 2 \cdot m - 1)}{2} \end{aligned}$$

In order to ensure that the above constraints are satisfied for all possible networks, we assume the following parameter constraints:

$$0 < s_A < \alpha < 1 \tag{4}$$

$$0 < s_B < 1 - \alpha \tag{5}$$

$$0 < \sigma < Min\left\{\frac{2(\alpha - s_A)}{m(N-1)}, \frac{2(1 - \alpha - s_B)}{(N-m)(2 \cdot N - 2 \cdot m - 1)}\right\} \tag{6}$$

To allow the possibility of link formation within each backbone, we also assume that $m > 1, N > 3$.

We have stated before that θ_A is the measure of congestion in backbone A and θ_B is the measure of congestion in backbone B. The backbone with the lower level of congestion will be referred to as the high quality backbone and that with the higher level of congestion will be referred to as the low quality backbone.

Hence, $\widetilde{\theta} = \alpha \cdot \theta_A + (1-\alpha) \cdot \theta_B$ is the level of congestion in each backbone weighted by the traffic flowing through each backbone is an average measure of congestion in the whole network. $\widetilde{\theta}$ captures the symmetric effect of peering because changes in backbone congestion primarily affect this parameter.

As far as the asymmetric effect is concerned, γ_i will be referred to as the "link density factor" of ISP i. It is proportional to the number of links ISP i has in each backbone weighted by the level of congestion. The weights reflect the fact that forming links in congested backbones to divert traffic is more valuable than forming links in uncongested backbones. Next, define

$$\kappa_i = N\left(\gamma_i - \frac{1}{N}\sum_{j=1}^{N}\gamma_j\right)$$

κ_i which is proportional to the difference between the link density factor of ISP i and the average link density factor captures the asymmetric effect of peering because it is proportional to γ_i which reflects the impact of circumventing congested backbones on delay.

To see this clearly, we can express the delay or congestion of ISP i, namely δ_i in terms of $\widetilde{\theta}$ and γ_i.

$$\delta_i = d_i \cdot \widetilde{\theta} - \gamma_i$$

Higher the overall backbone congestion, greater is the delay, but delay can be reduced by increasing the link density factor because traffic can now circulate through uncongested lines. Increasing demand also increases delay because larger amount of the traffic passes through congested backbones.

We can express prices, demands and profits in terms of $\widetilde{\theta}$ and κ_i.

$$\begin{aligned}
p_i^* &= \left(\frac{\widetilde{\theta}}{N-1}\right) + \left(\frac{\kappa_i}{2N-1}\right) \\
d_i^* &= \left(\frac{N-1}{N \cdot \widetilde{\theta}}\right)\left[\left(\frac{\widetilde{\theta}}{N-1}\right) + \left(\frac{\kappa_i}{2N-1}\right)\right] \\
\pi_i^* &= \left(\frac{N-1}{N \cdot \widetilde{\theta}}\right)\left[\left(\frac{\widetilde{\theta}}{N-1}\right) + \left(\frac{\kappa_i}{2N-1}\right)\right]^2 - \left(\frac{\sigma^2}{2}\right)n_i - F_l
\end{aligned}$$

By gross profits we mean profits gross of link formation costs or profits not taking into account link formation costs. Hence, gross profit is simply the product of price and demand minus the transit fee paid to the backbones. We can examine the impact that symmetric and asymmetric effects of peering have on gross profits.

Next let us examine how peering affects gross profits. First note that

$$\begin{aligned}
\frac{\partial p_i}{\partial \widetilde{\theta}} &= \frac{1}{N-1} > 0 \\
\frac{\partial d_i}{\partial \widetilde{\theta}} &= \frac{-(N-1)\cdot \kappa_i}{(2\cdot N-1)\cdot N\cdot \left(\widetilde{\theta}\right)^2} < 0 \text{ if } \kappa_i > 0 \text{ and } > 0 \text{ if } \kappa_i < 0.
\end{aligned}$$

Hence, increasing the level of overall congestion increases prices.[2] For ISPs with above average link density, increasing the level of congestion reduces demand while reducing the link density increases demand. For ISPs with below average link density, the opposite is the case. Increasing the level of overall congestion increases demand and vice versa.

Now an intra-backbone link reduces $\widetilde{\theta}$ by $\alpha\sigma$ through its effect on the backbone the ISPs belong to. An inter-backbone link reduces $\widetilde{\theta}$ by $\sigma/2$ through its impact on both backbones. While those boost consumer utilities, they have either a negative or an ambiguous effect on firm profits. For ISPs with below average link density ($\kappa_i < 0$) the impact is definitely negative because given the signs of the derivatives above, both prices and demands fall and hence gross profits fall. For ISPs with above average link density ($\kappa_i < 0$), the impact is ambiguous. While prices fall, demands rise and hence the net effect depends on the relative magnitudes of the two effects.

It is somewhat counter-intuitive that peering should have a negative or ambiguous effect on gross profit. This is because the effect of reducing overall congestion on price is always negative. This is because,

$$\delta_i - \delta_j = (d_i - d_j)\cdot \widetilde{\theta} - (\gamma_i - \gamma_j)$$

Hence, any increase in $\widetilde{\theta}$ accentuates the difference in quality or congestion or the level of vertical differentiation between firms i and j. The increase in vertical differentiation softens price competition and enables both firms to charge higher prices. In fact, both prices rise by an exactly equal amount in equilibrium and hence final

[2] This is a standard result for congested goods (see De Palma and Leruth (1989)).

differences in prices as well as differences in congestion remain unchanged. Hence the initial increase in the difference in congestion is compensated by appropriate changes in demands.

Next, consider the asymmetric effects of peering. We have

$$\begin{aligned} \frac{\partial p_i}{\partial \kappa_i} &= \frac{1}{2 \cdot N - 1} > 0 \\ \frac{\partial d_i}{\partial \kappa_i} &= \frac{(N-1)}{(2 \cdot N - 1) \cdot N \cdot \widetilde{\theta}} > 0 \end{aligned}$$

Consider the impact of intra and inter-backbone links on κ_i. There is a direct effect owing to the fact the traffic can now travel through uncongested lines and there is an indirect effect owing to the impact of link formation on congestion in the backbones θ_A and θ_B. We ignore the indirect effect because it involves terms σ^2 which is negligible if σ is small. Thus an intra-backbone link for $i \in \eta_l$ boosts κ_i by $\frac{N \cdot \sigma \cdot \theta_l}{2}$ and hence has a positive impact on gross profits. An inter-backbone link increases κ_i by $\frac{\sigma[(N-1)\theta_{-l} - \theta_l]}{2}$ where $-l$ refers to the backbone other than l. If i belongs to the high quality backbone, namely, $\theta_{-l} > \theta_l$, inter-backbone links increase gross profits. If i belongs to the low quality backbone, $\theta_{-l} < \theta_l$, inter-backbone links increase gross profits provided the quality difference between the two backbones is not very high, i.e. $(N-1)\theta_{-l} > \theta_l$. However, if the quality difference between the two backbones is very high, $(N-1)\theta_{-l} < \theta_l$, then inter-backbone peering has indeed a negative effect on gross profits.

Hence we get Proposition 2.

Proposition 2 *(a) For ISPs with link density that is below the average link density, the symmetric effects of intra-backbone or inter-backbone peering on gross profits are always negative. For those with link density above the average, the effect is ambiguous.*

(b) For all ISPs, the asymmetric effect of intra-backbone peering on gross profits is always positive. For ISPs connected to high quality backbones, the effect of inter-backbone peering is also positive. However, for ISPs connected to low quality backbones, the effect is positive if the difference in quality between the two backbones is sufficiently small and negative otherwise.

We can summarize these facts in form of the following tables. Let HH denote the fact that the ISP in question belongs to the high quality backbone and has higher than average link density. Let HL denote the fact that the ISP in question belongs to the high quality backbone and has lower than average link density. Let LH denote the fact that the ISP in question belongs to the low quality backbone and has higher than average link density. Let LL denote the fact that the ISP in question belongs to the low quality backbone and has lower than average link density.

Table 1: Effect of peering on gross profit when $(N-1)\theta_{-l} > \theta_l$

	Intra-backbone Peering		Inter-backbone Peering	
	Symmetric Effect	Asymmetric Effect	Symmetric Effect	Asymmetric Effect
HH	Ambiguous	Positive	Ambiguous	Positive
HL	Negative	Positive	Negative	Positive
LH	Ambiguous	Positive	Ambiguous	Positive
LL	Negative	Positive	Negative	Positive

Table 2: Effect of peering on gross profit when $(N-1)\theta_{-l} < \theta_l$

	Intra-backbone Peering		Inter-backbone Peering	
	Symmetric Effect	Asymmetric Effect	Symmetric Effect	Asymmetric Effect
HH	Ambiguous	Positive	Ambiguous	Positive
HL	Negative	Positive	Negative	Positive
LH	Ambiguous	Positive	Ambiguous	Negative
LL	Negative	Positive	Negative	Negative

Hence, one finds that quite often the symmetric effect and asymmetric effects actually work against each other. This is because while peering improves quality and hence prices, demands and profits, its effect on backbone congestion has the unintended effect of reducing vertical differentiation, stiffening price competition and reducing prices and profits. The net effect us almost always ambiguous and stating anything further would require comparison of relative magnitudes which in turn would depend on relative values of parameters α, s_l and m. Hence one would expect to find a multitude of equilibrium networks depending on the values of these parameters, a fact that we illustrate in the next section with simulations.

However, we can state one thing with certainty. If the quality difference between the two backbones is substantial, namely $(N-1)\theta_A < \theta_B$ or vice versa, then ISPs connected to the quality backbone have no interest in interpeering if it also has lower than average link density. This is because gross profits reduce with inter-backbone peering given that both the symmetric and asymmetric effects are negative, and the added cost of link formation further reduces net profits. This actually forms the basis of Proposition 3.

4 Analysis of the First Stage

We will analyze the first stage using the notion of pairwise stability which was introduced by Jackson and Wolinsky (1996). A network is pairwise stable if given the network, there is no incentive to either form links or destroy links. Since links are formed unilaterally but can be broken bilaterally, we can formally define it as follows:

Definition: *Let $\pi_i(g)$ denoted the reduced profits of stage 1 for a network g. The network g is pairwise stable if for all $i, j \in \eta$:*

$$\begin{aligned}
\text{(a) If } g_{ij} &= 1, \text{then } \pi_i(g) \geqslant \pi_i(g - g_{ij}) \text{ and } \pi_j(g) \geqslant \pi_j(g - g_{ij}) \\
\text{(b) If } g_{ij} &= 0 \text{ and } \pi_i(g + g_{ij}) > \pi_i(g), \text{ then } \pi_j(g + g_{ij}) < \pi_j(g)
\end{aligned}$$

The intuition here is quite simple. Links are formed bilaterally but can be broken unilaterally. Hence, in a pairwise stable network, neither player should gain by breaking a link while at least one player must lose or remain indifferent through forming a new link.

Before we embark on simulations, let us formalize our analysis in the previous section in the form of Proposition 3.

Proposition 3 *Let $(N-1)\theta_{-l} < \theta_l$ where l represents the low quality backbone, namely the quality differential between the two backbones is sufficiently large. Then, in any pairwise stable network, there will be no inter-backbone peering between two ISPs with different link densities if the ISP belonging to the low quality backbone has lower than average link density.*

In all other cases, given that net effects of peering are ambiguous, we cannot state anything for certain. However, we can do some simulations. We will only study six-provider networks. The choice of six is not entirely arbitrary. Since the number of ISPs attached to each backbone has to be greater than or equal to two for any meaningful analysis, six is the smallest number that allows us to consider both symmetry as well asymmetry in the number of firms attached to individual backbones. We will consider only the following eight possible cases or eight possible networks.[3]

1. All ISPs peer with each other. The resulting network called a complete network is denoted by g_{AB1}.

2. There is no peering whatsoever. The resulting network called an empty network is denoted by g_{000}.

3. All ISPs belonging to both backbones engage in intra-backbone peering but there is no inter-backbone peering. We refer to the resulting network by g_{AB0}.

4. All ISPs belonging to backbone A engage in intra-backbone peering. However there is neither any inter-backbone peering nor any intra-backbone peering in backbone B. We refer to this network as g_{A00}.

5. All ISPs belonging to backbone B engage in intra-backbone peering. However there is neither any inter-backbone peering nor any intra-backbone peering in backbone A. We refer to this network as g_{0B0}.

6. All ISPs belonging to backbone A engage in intra-backbone peering. There is no intra-backbone peering in backbone B but there is inter-backbone peering. We refer to this network as g_{A01}.

7. All ISPs belonging to backbone B engage in intra-backbone peering. There is no intra-backbone peering in backbone A but there is inter-backbone peering. We refer to this network as g_{0B1}.

8. No intra-backbone peering whatsoever but there is inter-backbone peering. We refer to this network as g_{001}.

Case 1: We start with perfect symmetry, namely, $\alpha = 0.5$, $m = 3$, $s_A = s_B = s$ (say). Then the area where all parameter constraints (4)-(6) are satisfied is repre-

[3]Checking more complicated networks for pairwise stability requires some programming which is reserved as a future endeavour.

sented by figure 4 where we plot σ and s along the two axes. This figure and all subsequent figures are in Appendix 2. We will refer to the parameter range for which constraints (4)-(6) are satisfied as the feasible parameter range or the feasible range.[4]

We find that with complete symmetry, two networks are pairwise stable in the feasible parameter range, namely, the complete network and the empty network. Figure 5 represents the area where the complete network is pairwise stable. Figure 6 represents the area where the empty network is pairwise stable. The two areas represent mutually exclusive parameter ranges, hence, for a given set of parameter values, there is an unique pairwise stable network which is either the complete or the empty network. This is clear from Figure 7 where we represent the two areas in the same figure. Under complete symmetry, all firms face exactly the same benefits and costs. Hence, it is expected that we get symmetric outcomes, namely either all firms will peer or nobody will peer. Depending on the magnitudes of σ and s either outcome is feasible.

Case 2: Next we will introduce asymmetries. We find that introduction of asymmetries result in other pairwise stable networks. We begin with an asymmetry in the network capacity. Specifically, assume that $s_A > s_B = s$. We plot σ, s_A and $s_B = s$ along the three axes. We find that there are two additional pairwise stable network configurations within the feasible parameter range besides the complete and the empty network, namely, the network g_{AB0} and the network g_{0B0}. We summarize this in figures 8 to 12. The figures are three dimensional to account for the fact that s_A and s_B are represented in two different axes. Figure 8 represents the feasible parameter range. Figure 9 represents the area where the complete network is pairwise stable. Figure 10 represents the area where the empty network is pairwise stable. Figure 11 represents the area where the network g_{AB0} is pairwise stable. Figure 12 represents the area where the network g_{0B0} is pairwise stable.

Case 3: Next, starting from perfect symmetry, let us introduce an asymmetry in the number of firms connected to each backbone. Here our options are quite limited. We can consider only the case where $m = 4$ and $n = 6$. We plot σ and $s(= s_A = s_B)$ on the two axes. We find that again there are two possible pairwise stable networks

[4]The figures have by developed using Mathematica. Computations are available at www.filebox.vt.edu/users/schakrab/computations.htm.

besides the complete and the empty network in the feasible parameter range, namely, the network g_{AB0} and the network g_{0B0}. We represent this situation in figure 13 to 18. Figure 13 represents the feasible parameter range. Figure 14 represents the area where the complete network is pairwise stable. Figure 15 represents the area where the empty network is pairwise stable. Figure 16 represents the area where the network g_{AB0} is pairwise stable. Figure 17 represents the area where the network g_{0B0} is pairwise stable. The areas are mutually exclusive hence for a certain parameter value we get an unique stable network. This can be seen from figure 18 where we bring all the different areas in one figure.

Case 4: Finally, let us explore asymmetries in the consumer base. Assume, for instance, starting from a perfectly symmetric setup with $n = 6$, that $\alpha > 0.5$. We represent this situation in figure 19 to 23. The figures are three dimensional and we plot α, σ and $s(= s_A = s_B)$ along the three axes. This time we find that besides the complete and the empty network, two more networks namely g_{AB0} and g_{A00} are pairwise stable within the feasible parameter range. Figure 19 represents the feasible parameter range. Figure 20 represents the area where the complete network is pairwise stable. Figure 21 represents the area where the empty network is pairwise stable. Figure 22 represents the area where the network g_{AB0} is pairwise stable. Figure 23 represents the area where the network g_{A00} is pairwise stable.

We summarize these results of our simulations in the form of Proposition 4.

Proposition 4 *Let $n = 6$ and parameter values satisfy constraints (4)-(6). Then, (1) for complete symmetry, namely, $\alpha = 0.5, m = 3$ and $s_A = s_B$, there are two pairwise stable networks, namely the complete network and the empty network; (2) for an asymmetry in the network capacity, namely, $\alpha = 0.5, m = 3$ and $s_A > s_B$, there are four pairwise stable networks, namely the complete network, the empty network, g_{AB0} and g_{0B0}. (3) for an asymmetry in the number of firms connected to each backbone, namely, $\alpha = 0.5, m = 4$ and $s_A = s_B$, there are four pairwise stable networks, namely the complete network, the empty network, g_{AB0} and g_{0B0} (4) for an asymmetry in the consumer base, namely, $\alpha > 0.5, m = 3$ and $s_A = s_B$, there are four pairwise stable networks, namely the complete network, the empty network, g_{AB0} and g_{A00}.*

We summarize this in the following table.

Table 3

Nature of asymmetry	**Pairwise Stable Networks**
None	g_{AB1}, g_{000}
Asymmetry in network capacity $s_A > s_B$	$g_{AB1}, g_{000}, g_{AB0}, g_{0B0}$
Asymmetry in the the number of firms $m > n/2$	$g_{AB1}, g_{000}, g_{AB0}, g_{0B0}$
Asymmetry in the website base $\alpha > 1/2$	$g_{AB1}, g_{000}, g_{AB0}, g_{A00}$

We find there are two major trends in our simulations:

(a) First, when ISPs connected to backbones whose congestion without taking to account leakage due to peering $(\theta_l + \omega_l)$ is higher tend to intrapeer, the ISPs belonging to the other backbone do not intrapeer.

(b) There is no interpeering in presence of asymmetries in networks other than the complete network.

Now, (b) could be partly a due to the proposition 2 especially given the fact that N is small. But there is likely other effects that we are unable to capture in a formal manner.

We will briefly compare our results with that of DangNyugen and Penard (1999). In their model, ISPs connected to each backbone collude with each other and behave like a club while ours is a purely non-cooperative game. Further, in their model, the only asymmetry is with regard to an exogenously given quality, while in ours there are many sources of asymmetry and quality is determined endogenously by a complex interaction of several factors. Their model only takes the symmetric effect into account while ours take both effects into account. If we extend their results in context of pairwise stability, one would be likely to observe the network configuration g_{A00}, g_{AB0} and g_{0B0} depending on whether A or B is the high quality backbone. We also find that in our simulations, these three networks recur with some regularity. However, depending on parameter values, two other networks namely, the complete and empty network are also pairwise stable.

5 Conclusions

We find that there are two main avenues by which peering affects gross profits. The impact on the quality of service offered by the firm given that traffic can circumvent congested backbones which we term the asymmetric effect and the impact on backbone congestion which we term the symmetric effect. The asymmetric effect generally increases gross profits and the symmetric effect has a negative or ambiguous impact on gross profits. The two effects often work against each other making the net effect ambiguous as well.

One may object to this paper on the grounds that it does not have a punch line. Yet, it is precisely that absence of a punchline that we strive to show. Retail peering has quite complicated effects on firm profits and it is by no means certain that improving the quality of service by forming peering agreements would automatically increase gross profits even if we disregard the costs of peering. Peering among retail ISPs have both a positive and negative effect with regard to gross profits. On the positive side, peering improves quality, increases demand and enables firms to charge higher prices. On the negative side, it reduces differentiation and promotes stiffer price competition. Also, it may lead to overuse of one's network without adequate reciprocity. The relative magnitude of these factors help or hinder peering. In complex settings, such magnitudes also quite complex to analyze and give rise to a multitude of configurations depending on the various exogenously determined factors. This paper helps to illustrate this with the help of simulations.

Appendix 1: We will solve for the Nash equilibrium from the second stage. Given the network g, we know n_{il} for $i \in \eta$, $l \in \{A, B\}$. Further, we know n_{AA}, n_{AB} and n_{BB}.

Let ,

$$\begin{aligned}
\gamma_i &= \frac{\sigma\left(n_{iA} \cdot \theta_A + n_{iB} \cdot \theta_B\right)}{2} \text{ for all } i \in \eta \\
\widetilde{\theta} &= \alpha \cdot \theta_A + (1-\alpha) \cdot \theta_B \\
\theta_A &= \alpha - s_A - \sigma\left(n_{AA} + \frac{n_{AB}}{2}\right) \\
\theta_B &= (1-\alpha) - s_B - \sigma\left(n_{BB} + \frac{n_{AB}}{2}\right)
\end{aligned}$$

We know that if $i \in A$,

$$\delta_i = \theta_A \cdot (\alpha \cdot d_i - n_{iA} \cdot (\sigma/2)) + \theta_B \cdot ((1-\alpha) \cdot d_i - n_{iB} \cdot (\sigma/2))$$

If $i \in B$,

$$\delta_i = \theta_B \cdot ((1-\alpha) \cdot d_i - n_{iB} \cdot (\sigma/2)) + \theta_A \cdot (\alpha \cdot d_i - n_{iA} \cdot (\sigma/2))$$

Hence,

$$\delta_i = d_i \cdot \widetilde{\theta} - \gamma_i \text{ for all } i \in \eta$$

Equating utilities across websites, for all $i \neq j$

$$\begin{aligned}
U_i &= U_j \\
&\Rightarrow \delta_i + p_i = \delta_j + p_j \\
&\Rightarrow d_i \cdot \widetilde{\theta} - \gamma_i + p_i = d_j \cdot \widetilde{\theta} - \gamma_j + p_j = \lambda \text{ (say)} \\
&\Rightarrow d_i = \frac{\lambda + \gamma_i - p_i}{\widetilde{\theta}} \text{ for all } i \in \eta
\end{aligned}$$

Now,

$$\begin{aligned}
\sum_{i=1}^{N} d_i &= 1 \\
&\Rightarrow \frac{N \cdot \lambda - \sum_{i=1}^{N} p_i + \sum_{i=1}^{N} \gamma_i}{\widetilde{\theta}} = 1 \\
&\Rightarrow \lambda = \frac{\widetilde{\theta} + \sum_{i=1}^{N} p_i - \sum_{i=1}^{N} \gamma_i}{N}
\end{aligned}$$

Hence,

$$\begin{aligned}
d_i &= \left(\frac{1}{\widetilde{\theta}}\right) \cdot \left[\left(\frac{\widetilde{\theta} + \sum\limits_{j=1}^{N} p_i - \sum\limits_{j=1}^{N} \gamma_i}{N}\right) + \gamma_i - p_i\right] \\
&= \frac{1}{N} + \left(\frac{1}{N \cdot \widetilde{\theta}}\right)\left(\sum_{j=1}^{N}(p_j - \gamma_j) - N \cdot (p_i - \gamma_i)\right) \\
&= \frac{1}{N} + \left(\frac{1}{N \cdot \widetilde{\theta}}\right)\left(\sum_{j \neq i}(p_j - \gamma_j) - (N-1) \cdot (p_i - \gamma_i)\right) \qquad (7)
\end{aligned}$$

The above equation gives us equilibrium demands for all ISPs. Next we will solve for equilibrium prices. Profits for ISP i are given by

$$\pi_i = p_i \cdot d_i - C_i$$

where costs of ISP i connected to backbone l are given by

$$C_i = \left(\frac{\sigma^2}{2}\right) \cdot n_i + F_l$$

Since C_i does not depend on prices, it can be treated as a constant.

Hence,

$$\begin{aligned}
p_i \cdot d_i &= \frac{p_i}{N} + \left(\frac{p_i}{N \cdot \widetilde{\theta}}\right)\left(\sum_{j \neq i}(p_j - \gamma_j) - (N-1) \cdot (p_i - \gamma_i)\right) \\
&= \frac{p_i}{N} + \left(\frac{p_i}{N \cdot \widetilde{\theta}}\right)\left(\sum_{j \neq i}(p_j - \gamma_j)\right) - \left(\frac{N-1}{N \cdot \widetilde{\theta}}\right) \cdot (p_i^2 - \gamma_i \cdot p_i) \\
\frac{\partial \pi_i}{\partial p_i} &= \frac{1}{N} + \left(\frac{1}{N \cdot \widetilde{\theta}}\right)\left(\sum_{j \neq i}(p_j - \gamma_j)\right) - \left(\frac{N-1}{N \cdot \widetilde{\theta}}\right) \cdot (2 \cdot p_i - \gamma_i) = 0 \\
&\Rightarrow \; p_i = \left(\frac{1}{2 \cdot N - 1}\right) \cdot \left[\widetilde{\theta} + \sum_{j=1}^{N} p_j - \sum_{j=1}^{N} \gamma_j + N \cdot \gamma_i\right]
\end{aligned}$$

Let

$$\sum_{j=1}^{N} p_j = \widetilde{p} \text{ and } \sum_{j=1}^{N} \gamma_j = \widetilde{\gamma}$$

.

Then

$$p_i = \left(\frac{1}{2 \cdot N - 1}\right) \cdot \left[\widetilde{\theta} + \widetilde{p} - \widetilde{\gamma} + N \cdot \gamma_i\right]$$

Summing up for all i,

$$\begin{aligned}
\widetilde{p} &= \left(\frac{N}{2 \cdot N - 1}\right) \cdot \left[\widetilde{\theta} + \widetilde{p} - \widetilde{\gamma}] + \left(\frac{N}{2 \cdot N - 1}\right) \cdot \widetilde{\gamma}\right] \\
&= \left(\frac{N}{2 \cdot N - 1}\right) \cdot (\widetilde{\theta} + \widetilde{p}) \\
&\Rightarrow \ \widetilde{p} = \frac{\widetilde{\theta} \cdot N}{N - 1}
\end{aligned}$$

Hence,

$$\begin{aligned}
p_i &= \left(\frac{1}{2 \cdot N - 1}\right) \cdot \left[\widetilde{\theta} + \frac{\widetilde{\theta} \cdot N}{N - 1} - \sum_{j=1}^{N} \gamma_j + N \cdot \gamma_i\right] \\
&= \frac{\widetilde{\theta}}{N - 1} + \left(\frac{1}{2N - 1}\right) \left(N \cdot \gamma_i - \sum_{j=1}^{N} \gamma_j\right) \qquad (8)
\end{aligned}$$

The above equation gives us equilibrium prices. From the above two equations, after some manipulation, we get equilibrium demand of ISP i :

$$d_i = \left(\frac{N - 1}{N * \widetilde{\theta}}\right) \left[\frac{\widetilde{\theta}}{N - 1} + \left(\frac{1}{2N - 1}\right) \left(N \cdot \gamma_i - \sum_{j=1}^{N} \gamma_j\right)\right] \qquad (9)$$

Hence the profits are given by:

$$\pi_i = \left(\frac{N - 1}{N * \widetilde{\theta}}\right) \left[\frac{\widetilde{\theta}}{N - 1} + \left(\frac{1}{2N - 1}\right) \left(N \cdot \gamma_i - \sum_{j=1}^{N} \gamma_j\right)\right]^2 - \left(\frac{\sigma^2}{2}\right) \cdot n_i - F_l \qquad (10)$$

where ISP i is connected to backbone l.

Appendix 2:.

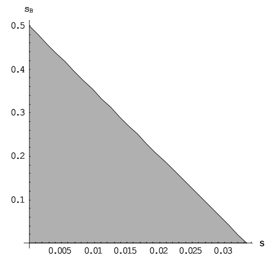

Figure 4

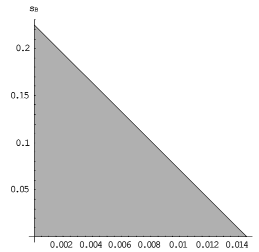

Figure 5

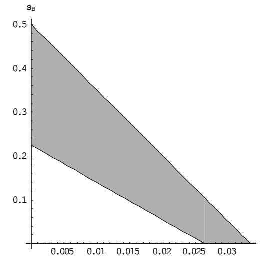

Figure 6

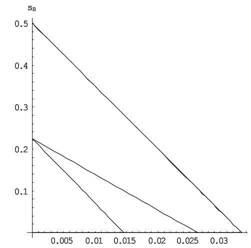

Figure 7

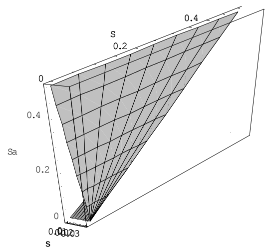

Figure 8

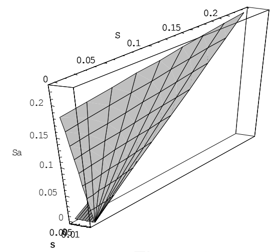

Figure 9

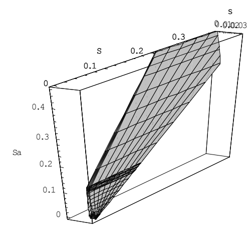

Figure 10

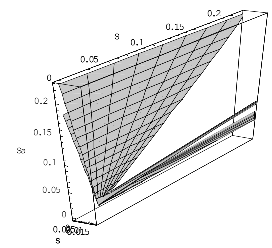

Figure 11

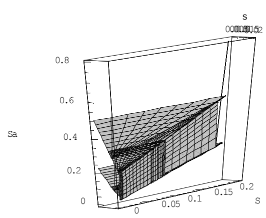

Figure 12

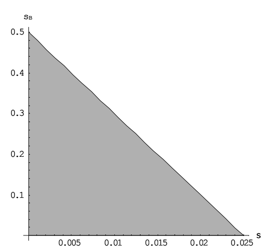

Figure 13

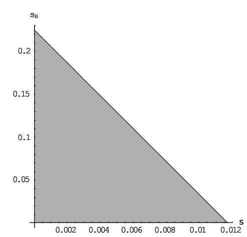

Figure 14

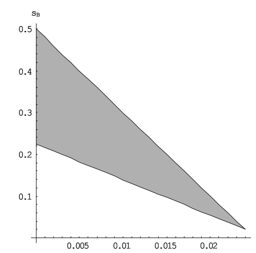

Figure 15

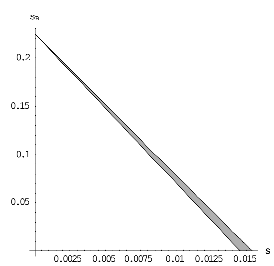

Figure 16

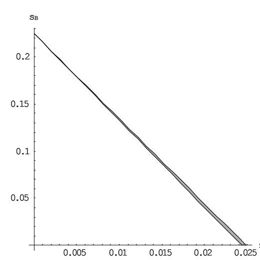

Figure 17

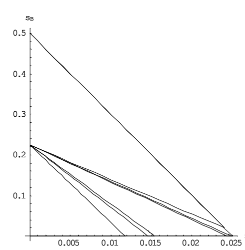

Figure 18

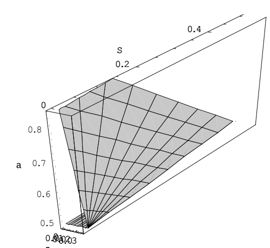

Figure 19

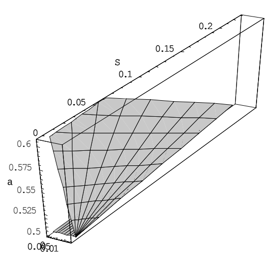

Figure 20

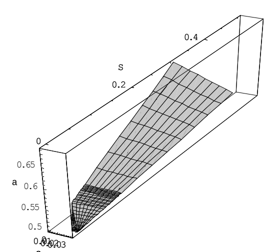

Figure 21

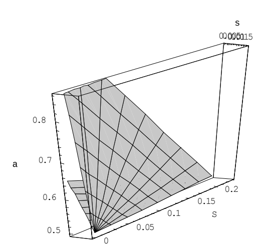

Figure 22

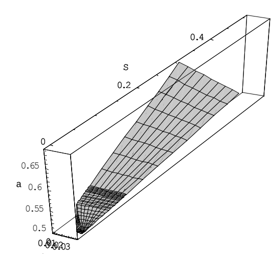

Figure 23

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