"Giffen's Paradox and Falsifiability," **Weltwirtschaftliches Archiv**, Bd. 107, Heft 1 (1971), pp. 139-146.

by Gordon Welty
Wright State University
Dayton, OH 45435 USA

[//139] The problems of falsifiability of economic theory are well-known./1/ Especially problematic is the so-called Giffen paradox. In the face of a suitable income effect, the slope of the demand curve is indeterminate. Since the greater part of the testing of economic propositions involves considerations of signs and sign changes, the possibility of the Giffen good renders the theory of consumer behavior irrefutable.

Professor Louis De Alessi has recently given a methodological appraisal of Giffen's paradox./2/ We will consider the methodological problem which the Giffen paradox poses for the economist intent on establishing the empirical status of microeconomic theory. We will then turn to more general considerations of the falsifiability of economic theory based on some recent discussions of Professor Adolf Grünbaum. These considerations will lead us to reject De Alessi's proposal for rectifying the problem of the paradox. Finally, we will turn to the problem of ceteris paribus as it bears on De Alessi's argument, and observe that he has not resolved this problem.

I.

It is well-known in elementary logic that there are two tautological implications, modus ponens and modus tollens. In the first case, from P and $P \rightarrow Q$, we infer Q. In modus tollens, from not Q and $P \rightarrow Q$, we infer not P. For scientific explanations we can interpret the antecedent as a [139/140] theory and the consequent as a deduced observation statement. By the very meaning of the word "theory," we cannot directly establish the truth of the antecedent. Thus we are restricted to modus tollens, and seek continually to express theories so that they are falsifiable. That is to say, we require that the implicates of any theory possibly be false./3/

A given theory is falsifiable if some of its sentences are possibly false. If sentences deducible from, say, traditional consumer's demand theory could not be known to be false, then the theory is tautological, hence not empirical. From the theory of demand we can derive, for instance, a sentence to the effect that not all the commodities in a system can be complements. If conditions can then be specified, whereby under a compensated price change, the signs of all the variations in demand X_{rs} are negative, then we would say that consumer's demand theory was falsifiable.

Of course the latter would be true if the theory was falsifiable but not actually false. In other words, we seek to specify truth-conditions of the sentences of the theory's implicates, while excluding certain truth-values for those sentences. It might be remarked that we can weaken or strengthen the test of a theory by appropriate modification of the criterion of truth, weakening falsifiability to disconfirmability and vice versa. The logic or structure of explanation remains the same, however.

It is to that structure of explanation that we now turn. In traditional consumer's demand theory we find the "Fundamental Equation of Value Theory."/4/ We deduce for a commodity of quantity x and price p, under budget M, the uncompensated price effect:

$$\partial x/\partial p = [\partial x/\partial p_{M=Mo} - \partial x/\partial M_{p=po}]$$

By Cramer's Rule, and the appropriate second order conditions, the first term on the right-hand side is known to be negative. This is the substitution effect, which guarantees that for a compensated price change, as price goes up, the quantity demanded goes down.

Of the partial derivative of quantity demanded with respect to income, we know less. $\partial x/\partial M$ can be positive or negative. If this term is merely [140/141] negative, the commodity is called an inferior good. This is the income effect and if it is large enough in concert with x, it can dominate the substitution effect. In this case, the commodity is called a Giffen good. The composite is then indeterminate in sign.

If $\partial x/\partial p$ is indeterminate in sign, the demand curve can be upward sloping. This is the Giffen paradox: it is supposed that at a low level of income, as the price of a Giffen good rises, a greater quantity of the good is consumed. How does this indeterminacy bear on the falsifiability of consumer's demand theory?

In terms of our discussion of modus tollens, let us call traditional consumer's demand theory H. Then from the theory we deduce an equivocally signed demand curve:

$$H \rightarrow [\partial x/\partial p \geq 0] \text{ v } [\partial x/\partial p < 0]$$

It is readily apparent that this conditional is a tautology. Hence, under these conditions, demand theory is not falsifiable.

To rectify this patently undesirable state of affairs, De Alessi suggests imposing as a condition on the form of individual utility functions that the absolute value of the deduced income effect is less than the absolute value of the deduced substitution effect in the case of inferior goods./5/

This condition, which we will call A, can be expressed formally as:

$$A \equiv [\partial x/\partial M < 0] \rightarrow [|\partial x/\partial M| < |\partial x/\partial p|] .$$

The conjunction of A and H gives the following conditional:

$$H \,\&\, A \rightarrow \partial x/\partial p < 0.$$

If we examine just the consequent of this conditional, we notice that the consequent is no longer tautologically true. Thus the consequent can be empirically tested and possibly false. We can consider that we have derived, in Samuelson's words, the "so-called law of diminishing demand." We then ask whether the introduction of condition A renders the

theory of consumer's demand falsifiable. We should note that Samuelson calls such an assumption as A "demonstrably arbitrary"./6/ [141/142]

II.

Professor De Alessi inquires as to criteria for determining where an assumption, such as his condition A, is arbitrary. As we shall see, there are methodological grounds sufficient for this judgment. These methodological grounds require the introduction of more general considerations.

The problem of the falsifiability of a component of a theoretical system in the presence of auxiliary conditions such as A has recently received great attention in the literature of philosophy of science. The problem has been labelled the "D-thesis" and has been signally studied by Professor Adolf Grünbaum. We can summarize the problem and his argument as follows./7/ Consider a theory H and an auxiliary condition A. The theory is to be tested by virtue of an observation O. As we have seen, by modus tollens, from H $\rightarrow$ O and not O, we can infer not H. Thus, the theory is falsifiable. If, however, we conjoin to H the auxiliary condition A, then we find H & A $\rightarrow$ O, and from not O, we have not H or not A. The D-thesis asserts that there is no logical ground for the rejection of not A as an implicate, hence the theory is not falsifiable.

Further, in the presence of a disconfirming instance O, the investigator can always preserve the theory by rejecting the auxiliary hypothesis. As Andreas Papandreou has noted, the economist typically conjoins to his theory a set of auxiliary assumptions because he refuses to take his chances with a theory that is prima facie refutable./8/ Thus the D-thesis would seem to be quite germane to economics.

Professor Grünbaum has proposed the following criterion in such cases: The rejection of A is legitimate precisely to the extent that its verification independent of the assumption of H suffers from inductive uncertainty./9/ Thus, we must provide answers to the questions:

(a) Was A verified independently of H?

(b) Was this verification inductively certain?

Evidently the answer to (b) awaits an affirmative answer to question (a). This concurs with De Alessi's argument that the acceptability of A is "determined by the empirical validity of its implications relative to [142/143] the empirical validity of the implications of competing 'assumptions'."/10/ We will merely note here that not A is certainly a "competing assumption," and has the virtue of simplicity vis a vis A, were the two equivalent in other respects./11/

To verify A independently of the assumption of H requires that the following relationship be observable:

$$\partial x/\partial p < \partial x/\partial M < 0$$

There are two considerations here. One is that to insure sufficient degrees of freedom to

estimate these two effects, it is necessary that a nested design be employed, whereby all income effects are dichotomized into those positively and those negatively signed. Then within the latter set, an examination of all substitution effects must insure that the above relationship exists.

Secondly, it is further necessary, both to insure the independence of H and A, as well as to prevent the confounding in effect of ΔM and Δp, that the income effects temporally as well as logically precede the substitution effects. As Professor Lloyd has pointed out, "we need at least two different observations of consumption changes *at different points in time*"/12/.

But such intertemporal comparisons require the specification of ceteris paribus. For instance, consumer tastes, as well as a myriad of other factors should not change during the time period under study. Hence a necessary condition for the possibility of independent verification of A for DeAlessi is whether one can fulfil ceteris paribus here. Notice this is only necessary and not sufficient for the verification of A. We will see that it is not possible in his terms for De Alessi to fulfil ceteris paribus, hence impossible to establish A.

III.

It is a curious argument indeed which De Alessi forwards regarding ceteris paribus. He presents two points, the first, that

> The choice of variables that may be impounded in ceteris paribus depends upon the problem being considered [143/144]

and the second, that this choice is

> determined by the usefulness of the additional accuracy, if any, that may be obtained by explicitly considering additional variables relative to the additional cost of including such variables./13/

Let us trace briefly the history of this argument.

De Alessi cites James Buchanan on his first point. Indeed, Buchanan, in his analysis of the relationship between ceteris paribus and partial equilibrium analysis, indicates that if the purposes of analysis are purely formal, all of the variables except two can be placed literally in ceteris paribus./14/ This is clearly a special case of De Alessi's "types of problems under consideration." For the formal activity of partial equilibrium analysis, it is unexceptional that after conceptually isolating the market under study, the economist neglects the remainder of the variables./15/

However, where the activity is instead that of empirical validation of the implications of a theoretical assumption, which is De Alessi's activity,/16/ Professor Buchanan provides little support, other than noting that "the predictive error involved in neglecting all offsetting or compensating variations... need not be large."/17/ Further, in the case of empirical validation as contrasted with partial equilibrium analysis, De Alessi's first point is the logical error of petitio principii. It is invalid to suppose that one knows which of the set of possible factors do have a significant effect on the dependent variable. Regarding

De Alessi's first point, then, we must conclude that unless he can show a criterion for the impounding of variables,/18/ we must reject his first point: there is otherwise no "choice of variables."

We turn then to point two. Here De Alessi proposes, as a criterion for the consideration of additional variables, to ascertain the utility and cost of each if it was explicitly treated. Implicit is the supposition that an algorithm can be specified, whereby the investigator considers first this variable, then the next variable, etc. In each instance he compares cost and utility. Two comments are in order here.[144/145]

First, such a supposition requires that the number of variables to be considered is at most denumerable. The contrary, i. e. that the number of possibly relevant variables is non-denumerably many, has been maintained by a number of investigators, including Fisher, Georgescu-Roegen, and Hildreth./19/ If the number of variables which might be impounded is not denumerable, then by Cantor's Theorem, De Alessi's proposal is contradicted, since the implicit algorithm cannot exist.

A number of investigators have not only supposed that the number of variables confronting the investigator was denumerable, but also that it was finite. Both Arthur Marget/20/ and Richard Brumberg/21/ assumed that there was a finite number of variables under consideration when the extension of ceteris paribus was in question. These investigators, as well as Professor Friedman,/22/ were perhaps mislead by Alfred Marshall's tautological treatment of ceteris paribus, in his *Principles,* for analytical purposes.

Marshall, it will be recalled, in his general law of demand, stated that a functional relation existed between the quantity demanded and the price of a commodity. This commodity was constrained to a market "during a given time and under given conditions," which is an implicit ceteris paribus condition. He continues: "If the conditions vary in any respect the prices will probably require to be changed."/23/ Since all conditions [145/146] but quantity and price are constant, we have the purely formal analysis which Buchanan noted.

So much for the formalism of partial equilibrium analysis, where the conceptual restriction of the analysis to two variables, with several others impounded in ceteris paribus is legitimate. When we turn, on the other hand, to problems of empirical validation, we find clear evidence that Marshall carefully qualifies himself by asserting that in the real world of statistical estimation, "other things seldom are equal...."/24/

Certainly the proposal to compare the utility and cost of impounding additional variables has little support. It appears that De Alessi's proposed criterion for ceteris paribus may rest on a misreading of Marshall. Be that as it may, in light of the distinction between partial equilibrium analysis and the empirical validation of a set of assumptions the problem of fulfilling ceteris paribus does not appear possible of solution in this fashion. On this ground, we find ceteris paribus unfulfilled, hence De Alessi's suggestion of a condition A on the individual utility functions to be arbitrary.

IV.

In conclusion then, we have examined the problem the Giffen paradox presents to the refutability of micro-economic theory. To resolve the problem, De Alessi suggested a restriction on the admissible magnitude of income effects. While tracing out the implication of this suggestion, we found that a necessary condition was the fulfilling of ceteris paribus. As this is not accomplished in De Alessi's terms, and does not even appear possible in his terms, we must conclude his restriction to be demonstrably arbitrary.

Two points remain. We should not be so understood that ceteris paribus is considered impossible of fulfillment. As we have discussed this problem elsewhere,/25/ we will not pursue it further here. Secondly, to establish the refutability of traditional consumer's demand theory, and to resolve the Giffen paradox, seems to require an explicit institutional embedding of the socio-economic model under study./26/ There is certainly a problem here of parametric stability,/27/ but this problem is perhaps part of the economists' burden.[146//]

Notes

Remark: I would like to express my appreciation to Professor Oskar Morgenstern and Mr. Nick Hope for their comments and criticism. They are not responsible for any errors remaining in this paper, of course.

1. Cf. Gordon A. Welty, "Lloyd on the Falsifiability of Economic Theory," *Metroeconomica*, Vol. XXI, Bologna, 1969, pp. 81 sqq., and references there. Also, Emile Grunberg, "Notes on the Verifiability of Economic Laws," in: *Theorie und Realität, Ausgewählte Aufsätze zur Wissenschaftslehre der Sozialwissenschaften*, Die Einheit der Gesellschaftswissenschaften, Studien in den Grenzbereichen der Wirtschafts- und Sozialwissenschaften, Bd. 2, Tübingen, 1964, pp. 137 sqq.

2. Louis De Alessi, "A Methodological Appraisal of Giffen's Paradox," *Weltwirtschaftliches Archiv*, Bd. CI, 1968 II, pp. 287 sqq.

3. Early proponents of this, the hypothetico-deductive method, were Wm. Stanley Jevons, *Principles of Science*, New York, 1874, pp. 13 sqq., 262. Frank H. Knight, *Risk, Uncertainty, and Profit*, Hart, Schaffner & Marx Prize Essays, XXXI, Boston and New York, 1921, p. 7, note.

4. J. R. Hicks, *Value and Capital, An Inquiry Into Some Fundamental Principles of Economic Theory*, 2nd Ed., Oxford, 1946, p. 309.

5. De Alessi, op. cit., p. 293.

6. Paul Anthony Samuelson, *Foundations of Economic Analysis*, 5th Print, Harvard Economic Studies, Vol. LXXX, Cambridge, 1958, p. 115, note 17.

7. A. Grünbaum, "Can we Ascertain the Falsity of a Scientific Hypothesis ?" *Studium Generale*, XXII, Berlin, Heidelberg, New York, 1969, pp. 1061 sqq.

8. A. G. Papandreou, "Theory Construction and Empirical Meaning in Economics," *The American Economic Review*, Vol. LIII, Menasha, Wisc., 1963, p. 206. Consider also Poincare's reaction to Michelson-Morley's finding of a zero ether drift. Cf. Oskar Morgenstern, "L'attitude de la nature et le comportement rationnel," in: *Les Fondements philosophiques des systemes economiques*, Ed. Emil M. Classen, Paris, 1967, p. 136.

9. Grünbaum, op. cit., p. 1092. Cf. also Peter B. Medawar, "Induction and Intuition in Scientific Thought," *Memoirs of the American Philosophical Society*, Vol. LXXV, Philadelphia, Pa., 1969, pp. 53 sq.

10. De Alessi, op. cit., p. 293, note 9.

11. If A is found to be the equivalent of not A in terms of empirical implications, then not A's simplicity warrants Samuelson's judgment that A is an "additional, and demonstrably arbitrary, assumption" (cf. Samuelson, op. cit., p. 115, note 17).

12. C. Lloyd, "On the Falsifiability of Traditional Demand Theory," *Metroeconomica*, Vol. XVII, 1965, p. 21 (emphasis his).

13. De Alessi, op. cit., pp. 290 sq.

14. James M. Buchanan, "Ceteris Paribus: Some Notes on Methodology," *The Southern Economic Journal*, Vol. XXIV, Chapel Hill, N.C., 1957-1958, p. 260.

15. Cf. ibid.

16. Cf. De Alessi, op. cit., p. 293.

17. Buchanan, op. cit., p. 262. To paraphrase De Alessi here, while the error need not be large, it need not be small. Cf. De Alessi, op. cit., p. 293, note 8.

18. Cf. Oskar Morgenstern, *Wirtschaltsprognose, Eine Untersuchung ihrer Voraussetzungen und Moglichkeiten*, Wien, 1928, p. 112, note 1, on the necessity of resolving ceteris paribus.

19. On this point, see Irving Fisher, *The Theory of Interest*, New York, 1930, pp. 318 sqq. Nicholas Georgescu-Roegen, *Analytical Economics, Issues and Problems*, Cambridge, Mass., 1966, p. 39. Clifford Hildreth, Review of Georgescu-Roegen, op. cit., *The American Economic Review*, Vol. LVII, 1967, pp. 594 sq.

20. A. W. Marget, "Morgenstern on the Methodology of Economic Forecasting," *The Journal of Political Economy*, Vol. XXXVII, Chicago, Ill., 1929, pp. 338 sq., note 69. What Marget means by "practical purposes" in this context is unclear. It is not, in general,

possible to add a new variable to a theory in the face of a disconfirming instance, since the question is whether the theory was wrong or whether ceteris paribus did not hold. To add a variable, as Marget proposes, is another petitio principii, invalid for any purpose, practical or otherwise. We are reminded of Macrobius: sive quo alio nomine te appelari volueris. *Saturnalia* II, ix. Cf. also Welty, op. cit., pp.84sq. Emile Grunberg, "Some Methodological Observations on Macro Economics," *Konjunkturpolitik*, Jg. XIII, Berlin, 1967, pp. 25 sqq.

21. Richard Bromberg, "Ceteris Paribus for Supply Curves," *The Economic Journal*, Vol. LXIII, London, 1953, pp.462 sqq.

22. Milton Friedman, "The Marshallian Demand Curve," *The Journal of Political Economy*, Vol. LVII, 1949, pp. 463 sqq.

23. Alfred Marshall, *Principles of Economics, An Introductory Volume*, 8th Ed., London, 1922, Book III, Chapter III, § 6, p. 100. Note that Marshall earlier referred to ceteris paribus as "an implicit condition," which although implicit, nonetheless "should be made clear." Ibid., § 1, p. 94. This is perfectly straightforward, and to try to "infer the contents of ceteris paribus" appears to miss the spirit of Marshall's discussion. But cf. Friedman, op. cit., passim.

24. Marshall, op. cit., Book III, Chapter IV, § 5, p. 109.

25. Cf. G. A. Welty, "Methodological Reply to David Gold," *American Sociologist*, Vol. IV, 1969, p. 338.

26. Papandreou, op. cit., p. 206.

27. De Alessi, op. cit., p. 289, note 1.