

Though Arrow Says It's Impossible, It Happens Everyday.

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Abstract: Arrow's Impossibility Theorem shows that transitive social preference is impossible. This note shows that in the general case of exchange, social preference need not be transitive. Indeed, it shows that social preference must be non-transitive to allow gainful exchange to maximize social welfare. Thus though Arrow says it is impossible, it actually happens everyday and everywhere.

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1. Background and Purpose

Arrow (1951) correctly argues that transitive social preference is impossible. But Gani (2003) shows that it is not a problem First, social preference need not be transitive. Secondly, social preference must be non-transitive to allow gainful trade and maximization of welfare. Since transitive social choice is neither necessary nor possible, it is irrelevant to economic analysis.

2. Arrow's Claim: Social Preference cannot be Transitive

To see Arrow's claim, suppose that three agents (A, B, C) have the following preferences over 3 goods of equal value. U denotes utility for the agent shown by the superscript over the good in the argument.

$$\begin{aligned}U^A(y) &< U^A(x) < U^A(z) \\U^B(z) &< U^B(y) < U^B(x) \\U^C(x) &< U^C(z) < U^C(y)\end{aligned}$$

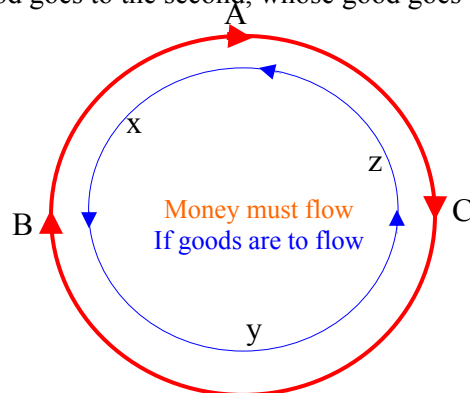
It is easy to see that there is no possible way to combine the preferences of all agents to derive a transitive social preference. It is possible to make coalitions of two people who share common preference over two goods, but not over all three. Thus coalition (A+B) prefers x to y , but they have opposite preferences with regard to z , as A prefers z to (x and y) while B prefers (x and y) to z . Similar problems happen with coalitions (B+C) and (A+C). Even when just two goods are taken by each coalition, the coalitions have preferences that are not resolved into transitive social preference. Thus coalition (A+B) prefers x to y , coalition (B+C) prefers y to z , but Coalition A+C prefers x to z . Transitivity is impossible.

3. Our First Claim: Social Preference Need Not be Transitive

Let us agree that social preference cannot be transitive. But is that a problem? Social preference need not be transitive. To see this, we take the same preference structure as above, and let A own x , B own y , and C own z . We first show that despite the non-transitive preferences, gainful exchange happens, and gives every agent the most preferred good. It maximizes social welfare. In the example, the good most preferred by A is z , that for B is x , and that for C is y . If the most desired good can be given to the individual, each stands to get the largest possible gain utility. Thus A will gain if he gets z in exchange for x (but lose utility if he gets y), because he prefers z to x . Next, B will gain if he can get x in exchange for y , because he prefers x to y . Lastly, C will gain utility if he can get y in exchange of z , as he prefers y to z . Society's problem is to make it possible for A to get z in exchange for x , and B to get x in exchange of y , and C to get y in exchange of z . Let A have z , B have x , and C have y . Everybody gets the most preferred good, and social welfare is surely maximized.

The general case of the exchange is indirect exchange, of which direct exchange is a special case. In the example, the possible exchange is indirect. In such an exchange, the first agent's good goes to the second, but the second agent's good does not go to the first. There is no barter between the first and the second. The second agent's good goes to the third, but the third agent's good does not go to the second. There is no barter between the second and the third agent. Lastly, in the specific example, the third agent's good goes to the first, but the first agent's good does not go to the third agent. There is no barter between the first and the third agents. In the general case with n goods, the first good goes to the second agent, whose good goes to the third, and so on, while the last agent's good goes to the first.

In the general case of exchange, the first agent is involved with two others. The second agent is his customer and the last agent is his supplier. There must be some tool to transfer the real good of the second agent (the customer) to the last agent (the supplier) directly or indirectly. That instrument is called money. In the following circuit showing indirect exchange, goods move in the counterclockwise direction while money moves in the clockwise direction. Thus x goes from A to B, y goes from B to C, and z goes from C to A. Money goes from B to A, then from A to C, and lastly from C to B. The direction of money shows the direction of preferences: A prefers C's good to the other goods and the money goes from A to C. Next, C prefers B's good to the other goods and the money goes from C to B. Lastly, B prefers A's good to the other goods and the money goes from B to A. The payment circuit may have any number of goods n . One special case is $n=2$, where it becomes barter. In that case, no money is possible or necessary. The first agent's good goes to the second, whose good goes to the first in the good's circuit.



4. Our Second Claim: Social Preference Must be Non-transitive

Let us conclude with an even more astonishing observation.. Trade must be gainful to all parties. This is possible if the buyer prefers the good he buys to the good he delivers in payment. Thus A gains if he prefers z (which he buys) to x (which he sells or delivers in payment). But owner of the other good also must gain. Thus had it been barter, C would have preferred x to z ; but in indirect trade, he does not. Society comes to the assistance in a peculiar manner by finding someone who prefers x to z . C prefers y to z , and the owner of y prefers x to y , so that the coalition (B+C) prefers x to z . Money makes the coalition possible so that C and B together take x from A, and then B keeps x while giving his good y to C in compensation. Though x is not really preferred to z by C, it is indeed used as commodity money which is artificially preferred to z . But it goes to someone (B) who really prefers x to z . It is as if C sells z to A for x , and then sells x to B for y , turning x into commodity money. Without non-transitivity, this is not possible. Hence, non-transitivity is necessary to permit gainful trade, whether directly with a real good in barter or indirectly with money. Choice and welfare maximization is possible, but transitivity is not.

References

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