Short-term planning and the life-cycle consumption puzzle[1]

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Abstract

This paper provides a new explanation for the hump-shaped age-consumption profile observed in household data. Standard life-cycle models are based on an optimization problem that spans the entire life expectancy. Alternatively, we examine the consumption profile of an individual with a shorter planning horizon. The actual consumption profile is the envelope of a continuum of control problems because the agent's short-term planning horizon continually slides along the time-scale, and the agent is therefore continually re-optimizing. We derive analytical solutions to a deterministic, continuous-time control model with this characteristic and we show that hump-shaped consumption is a feature of the model.

Keywords: Short-term planning; Hump-shaped consumption

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[1] The authors have benefited from helpful conversations with Annamaria Lusardi and participants of a seminar at Northern Arizona University.

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1 Introduction

Standard life-cycle models predict that consumption will grow smoothly over the lifecycle if the interest rate exceeds the rate of time preference (or decay smoothly if the reverse is true). However, household data indicate that life-cycle consumption is hump-shaped, with a peak around 45 to 50 years of age (e.g., see Gourinchas and Parker 2002, Attanasio et al. 1999, Browning and Crossley 2001, and many others). This inconsistency is well-known and is a prominent puzzle in consumption theory.[4]

In order to solve this puzzle and to ensure the life-cycle model better fits additional aspects of household data, economists have modified the basic model to include precautionary and bequest motives for saving, uncertain lifetime, family size dynamics, endogenous labor supply, and borrowing constraints to name a few. It is important to note that these modifications are still "located well within the lifecycle tradition" (Browning and Crossley 2001), which typically assumes that individuals solve some type of optimization problem that spans the *entire* life expectancy.

However, casual observation and recent research (Lusardi 1999 and numerous others) suggest that for some individuals, the planning horizon may be much shorter than the entire life expectancy.[5] For these individuals, the retirement phase may not be considered during the early stages of the lifecycle. As the individual grows older, however, the retirement phase of the lifecycle will "come into view" and the individual will begin to consider the need to finance retirement consumption through current saving.

An individual with a planning horizon that is shorter than the entire lifespan is distinctly different from the traditional concept of impatience (a positive rate of time preference). The typical control problem of life-cycle consumption with impatience assumes the agent plans her consumption profile right up to the date of death, but simply attaches less importance to distant consumption. In this paper we are interested in a distinct notion, that is, some individuals may have short planning horizons that, for example, span 5, 10 or 20 years.

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[4] For a detailed literature review, see Browning and Crossley (2001). Also see Bütler (2001).

[5] By way of casual observation, Lusardi (2002) notes that the (non-academic) financial planning literature (e.g., Glink, 1999; Ernst and Young's Retirement Planning Guide, 1997) is replete with the notion that the 'most common mistake' made by savers is not getting started early enough. Furthermore, Miron (2001) notes that "there are no doubt some myopic persons…," that is, persons who "are consuming 'excessively' during their earnings years and then finding themselves with insufficient savings in retirement."

Our goal is to incorporate this type of behavior into a life-cycle model of optimal control and investigate the impact on the age-consumption profile. The actual consumption profile of an agent with short-term planning is the envelope of initial values from a continuum of control problems because the agent's short-term planning horizon continually slides along the time-scale. We develop a deterministic, continuous-time control model with this feature and show that hump-shaped consumption is a feature of the model. To our knowledge, this is the first attempt at such an exercise. Moreover, because we are able to derive analytical solutions to this complex system of control problems, this model can be used as a benchmark or textbook model for future work in this area.[6]

Apart from its contribution to the life-cycle puzzle, this paper makes a general contribution to optimal control theory by illustrating how to derive analytical solutions to a dynamically inconsistent problem involving short and moving planning horizons.

The remainder of the paper is as follows. In Section 2, we summarize the evidence in support of short-term planning horizons. In Section 3, we present a control model with short-term planning. Because the model is significantly different from traditional life-cycle models, and because this paper is the first to derive analytical solutions to a life-cycle model with a short and moving planning horizon, we devote a substantial amount of space to the derivation of the solution. Section 4 presents simulated profiles based on various short-term planning horizons. Section 5 concludes.

2 Evidence of Short Planning Horizons

Drawing from the Health and Retirement Study, Lusardi (1999) notes that roughly 9 percent of households in the sample report a planning horizon of about one year, 35 percent have a planning horizon of a few years, 32 percent have planning horizon of 5 to 10 years, and about 8 percent have a planning horizon greater than 10 years. Moreover, "respondents who have not thought about retirement are more often the younger ones." Lusardi (2002) notes that older workers "are more likely to report a retirement saving

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[6] We stress that our explanation for hump-shaped consumption is not necessarily inconsistent with the existing explanations. Instead, we view short-term planning as an *additional* and provocative explanation that should be emphasized.

goal." Even so, about one-third of people between 51 – 61 years of age have not yet begun to think about retirement, a stark contrast to the long-term planner that conventionally inhabits life-cycle models.

Similarly, in the Loewenstein et al. (1999) sample of individuals near retirement, respondents frequently report that they "should have started saving earlier." Moreover, these authors show that "financial regret" among the elderly is strongly tried to the age at which they started saving (which varied from "during their twenties" to "never").

As expected, wealth at retirement is correlated with how much thought households have given to the retirement period of the lifecycle (Lusardi 1999). This result is confirmed by Ameriks et al. (2003), who also verify that the causality runs from planning to wealth accumulation, not the other way around. Indeed, Venti and Wise (2001) estimate that "the primary determinant of the dispersion of wealth at retirement [after controlling for lifetime earnings] is evidently the choice to save or spend while young."

We view this seemingly widespread lack of planning for retirement, among those even relatively close to retirement, as undermining the general validity of the standard assumption that agents are perfectly forward looking and plan ahead for retirement. Likewise, Rabin (1999) says that "if there is a single realm where economists ought to be wary of the rationality assumption it is retirement planning."

The relevant question, of course, is *why* some individuals appear to insufficiently plan for retirement. The 1999 Retirement Confidence Survey shows that a substantial fraction of workers (one-third) are currently saving nothing for retirement, with the common explanation from respondents that "lots of time remains until retirement." The academic literature offers a number of additional reasons for why individuals, young or middle aged, may take no thought for the retirement period of the lifecycle (i.e., why individuals may have short planning horizons).

For example, Akerlof and Dickens (1982, p. 317) note that there may be persons who "would simply prefer not to contemplate a time when their earning power is diminished, and…the very fact of saving for old age [may] force persons into such contemplations…persons may find it uncomfortable to contemplate their old age. For that reason they may make the wrong trade-off, given their own preferences, between current consumption and saving for retirement." Lusardi (2002) agrees with this possibility and states that the Health and Retirement Study offers supporting evidence.

Lusardi (2003) offers additional reasons for why some individuals initially ignore the retirement portion of the lifecycle, which relate to the time-intensive planning costs such as properly understanding pension rules, social security distributions, etc. She also offers some evidence from the Health and Retirement Study which suggests that the life experiences of older siblings and of old parents affect the planning horizon of the individual. In addition, she stresses the role of information, financial literacy, and financial education as important determinants of the degree to which individuals plan (save) for retirement.

A different strand of research argues that individuals may lack the necessary willpower to follow the optimal life-cycle consumption program (Thaler 1994 and Thaler and Shefrin 1981). In fact, Thaler and Benartzi (forthcoming) report that a substantial fraction of low-saving individuals willingly enrolled in a prescriptive savings plan (called "Save More Tomorrow") for which people commit in advance to allocate a portion of their future salary increases toward retirement savings.[7]

In sum, both causal observation and empirical evidence suggest that the degree of planning for retirement is an important characteristic of the lifecycle experience that partially accounts for the vast differences in household wealth accumulation. We believe the evidence is enough to justify an exploratory study of lifecycle optimal control models with short planning horizons for comparison with the standard forward-looking model. This is the purpose of our paper. We turn now to the control model.

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[7] Another interesting explanation for the lack of saving by young individuals is given by McKenzie (1997). He argues that not all individuals perceive a unit of time in the same manner (e.g., individuals almost invariably report that time appears to speed up as they grow older). He states: "it may be that people save more as they age simply because the age at which they expect to retire…comes within the person's relevant time horizon….[This] inducement to save "is compounded by the actual or perceived shrinking units of time extending toward retirement, which means that the rewards from saving become ever more immediate."

3 A control model with short-term planning

The agent's work life is [0, *T*], but we assume the length of the individual's planning horizon, *x*, is less than *T* so that initially the agent does not plan for retirement. The date of retirement is exogenous and certain.[8] Also, *x* is constant over time, which implies dynamic inconsistency, i.e., the planning window slides along the time scale, and therefore, the agent is continually solving a fresh control problem at each and every instant in time. This formulation is consist with the academic and financial-planning literature cited above, as well as casual observation, that during the younger years of the lifecycle the individual may not consider the retirement phase, but with the passage of time the individual begins to plan for retirement.

For expositional convenience, the agent's work life is separated into two phases. Phase 1 is $[0, T-x]$, which is the interval of time for which retirement is not in the planning window, and phase 2 is the interval of time for which retirement is in view $[T-x, T]$. We start by analyzing phase 1, which is more straightforward than phase 2.

3.1 Phase 1

At any time $t_0 \in [0, T-x]$, the agent solves a standard, fixed endpoint problem

$$\text{(1)} \qquad \max: \int_{t_0}^{t_0+x} \exp(-\rho(t-t_0)) \frac{c(t)^{1-\phi}}{1-\phi} dt$$

subject to:

$$\text{(2)} \qquad \begin{aligned} & dk(t)/dt = w + rk(t) - c(t) \\ & k(t_0) \ \ given, \ \ k(t_0+x) = 0 \end{aligned}$$

where *k*(*t*) is the savings account balance, *w* is the wage rate and is constant for simplicity, *r* is the rate of interest, and ρ is the rate of time preference.[9] As is standard,

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[8] Note that *T* is autonomous. It does not depend on where the individual currently stands. The 1999 Retirement Confidence Survey, however, indicates that a substantial number of individuals end up retiring at a different age than initially planned; implying that the expected date of retirement is likely to evolve over the work life. To keep the model simple and the focus on short-term planning, we leave this extension for future research.

[9] Our formulation includes both the standard notion of impatience and a short planning horizon. Ameriks et al. (2003) estimate that a household's propensity to plan is unrelated to the subjective discount factor. Also, Lusardi (2003) estimates that the degree of planning is a significant predictor of wealth accumulation, even after controlling for the rate of time preference.

income is fungible.[10] The model is intentionally simple to ensure that short-term planning is the *only* possible explanation for hump-shaped consumption.

The assumption $k(t_0+x)=0$ does not imply that the capital stock runs dry before the end of the lifecycle, because the agent's planning horizon continually slides along the time scale. That is, although the agent *plans* to exhaust the savings account within x years, the agent never really exhausts the account until the end of the life-cycle because the horizon keeps moving.

In this problem, $c(t)$ and $k(t)$ are the control and state variables, respectively. The current-value costate variable is $\lambda(t)$, and the necessary conditions are

$$c(t)^{-\phi}=\lambda(t) \tag{3}$$

$$d\lambda(t)/dt=(\rho-r)\lambda(t) \tag{4}$$

which imply

$$c(t)=A^{-1/\phi}\exp(gt) \tag{5}$$

where A is an arbitrary constant and $g\equiv-(\rho-r)/\phi$.

Insert (5) into (2) and find the general solution

$$k(t)=\left\{D+\int^{t}\left[w-A^{-1/\phi}\exp(gs)\right]\exp(-rs)ds\right\}\exp(rt) \tag{6}$$

where D is an arbitrary constant. The fixed endpoints $k(t_0)$, $k(t_0+x)$ are used to identify constants D and A. Evaluate (6) at t_0, solve for D, substitute back into (6) and integrate

$$\begin{aligned}k(t)&=k(t_0)\exp(r(t-t_0))-w(1-\exp(r(t-t_0)))/r\\&\qquad-A^{-1/\phi}(\exp(gt)-\exp(gt_0+r(t-t_0)))/(g-r)\end{aligned} \tag{7}$$

Evaluate (7) at $t=t_0+x$, solve for A and insert into (7) and (5) to obtain the agent's *planned* time paths of $k(t)$ and $c(t)$ on $[t_0,t_0+x]$ for all $t_0\in[0,T-x]$

$$\begin{aligned}k(t)&=k(t_0)\exp(r(t-t_0))-w(1-\exp(r(t-t_0)))/r\\&\qquad-\left\{\frac{\left[k(t_0)\exp(rx)-w(1-\exp(rx))/r\right]}{\exp(g(t_0+x))-\exp(gt_0+rx)}\right\}(\exp(gt)-\exp(gt_0+r(t-t_0)))\end{aligned} \tag{8}$$

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[10] See Thaler (1990) for the reverse case.

$$(9) \qquad c(t) = \left\{ \frac{\left[k(t_0)\exp(rx) - w(1-\exp(rx))/r\right](g-r)}{\exp(g(t_0+x)) - \exp(gt_0+rx)} \right\} \exp(gt)$$

But since the planning window continually slides along the time-scale, the problem is dynamically inconsistent and (9) gives the actual level of consumption *only* at time t_0 since all $c(t)$ beyond t_0 end up being different than planned. Thus, consumption at time t_0 is given by replacing all t with t_0

$$(10) \qquad c(t_0) = \frac{\left[k(t_0)\exp(rx) - w(1-\exp(rx))/r\right](g-r)}{\exp(gx) - \exp(rx)}$$

However, t_0 was initially defined to represent *any* point in time on $[0, T-x]$. Thus, actual consumption during phase 1 is found by replacing all t_0 in (10) with t

$$(10') \qquad c(t) = \frac{\left[k(t)\exp(rx) - w(1-\exp(rx))/r\right](g-r)}{\exp(gx) - \exp(rx)}$$

Note that $c(t)$ is a function of $k(t)$; thus, it remains to find the actual time path of $k(t)$, which is (see the appendix for derivation)

$$(11) \qquad k(t) = k(0)\exp((r-z_1)t) + (w+z_2)(1-\exp(rt-z_1t))/(z_1-r)$$

where

$$z_1 \equiv \frac{\exp(rx)(g-r)}{\exp(gx)-\exp(rx)} \qquad z_2 \equiv \frac{\left[w(1-\exp(rx))/r\right](g-r)}{\exp(gx)-\exp(rx)}$$

Thus, (11) and (10'), along with $k(0) = 0$, give the actual program of capital and consumption on $[0, T-x]$ in analytical form. This program is the envelope of infinitely many initial values from the continuum of planned time paths.

3.2 Phase 2

Phase 2 is the interval of time during the work life, $[T-x, T]$, for which the retirement period is in view. Suppose the agent is currently standing at time $t_0 \in [T-x, T]$ and solves the following control problem

$$(12) \qquad \max : \int_{t_0}^{T} \exp(-\rho(t-t_0)) \frac{c(t)^{1-\phi}}{1-\phi} dt + \exp(-\rho(T-t_0))B(k(T))$$

where $B(k(T))$ is a scrap-value function that measures the value the agent derives from leaving $k(T)$ dollars in the savings account at retirement. (We revisit this concept below). The agent solves (12) subject to

$$\begin{aligned} &dk(t)/dt = w + rk(t) - c(t) \\ &k(t_0)\ given \end{aligned} \tag{13}$$

The first-order necessary conditions in current-value form are (3), (4) and the transversality condition $B'(k(T)) = \lambda(T)$. Thus,

$$\lambda(t) = \lambda(T)\exp((\rho - r)(t - T)) \tag{14}$$

and

$$dk(t)/dt = w + rk(t) - \lambda(T)^{-1/\phi}\exp(g(t - T)) \tag{15}$$

We can evaluate the general solution of (15) at time t_0 to obtain the planned (as opposed to actual) time path for the savings account on $[t_0, T]$ for each $t_0 \in [T - x, T]$

$$\begin{aligned} k(t) = {} & k(t_0)\exp(r(t - t_0)) - w(1 - \exp(r(t - t_0)))/r \\ & - \lambda(T)^{-1/\phi}(\exp(g(t - T)) - \exp(g(t_0 - T) + r(t - t_0)))/(g - r) \end{aligned} \tag{16}$$

It remains to find $\lambda(T)$. Since the agent is currently standing at some point in phase 2 of the work life $t_0 \in [T - x, T]$, she anticipates a retirement period $[T,\ t_0 + x]$ during which the control problem will solely involve the optimal withdrawal of retirement funds. This creates a multiple-period control problem with a switch in the state equation. As is standard in these types of problems (Kamien and Schwartz, 1991), we equate the initial marginal value of capital from the retirement problem to $\lambda(T)$.

The retirement control problem anticipated by an agent at time $t_0 \in [T - x, T]$, with planning horizon *x*, is

$$\max: \int_T^{t_0 + x} \exp(-\rho(t - T))\frac{c(t)^{1-\phi}}{1 - \phi}\,dt \tag{17}$$

subject to:

$$\begin{aligned} &dk(t)/dt = rk(t) - c(t) \\ &k(T)\ given,\ \ k(t_0 + x) = 0 \end{aligned} \tag{18}$$

The necessary conditions are, again, (3) and (4). Thus, we have

$$dk(t)/dt = rk(t) - G^{-1/\phi}\exp(gt) \tag{19}$$

where *G* is an arbitrary constant. The solution to (19) is

(20) $$k(t) = k(T)\exp(r(t-T)) - G^{-1/\phi}(\exp(gt) - \exp(gT + r(t-T)))/(g-r)$$

Evaluate (20) at $t = t_0 + x$, use $k(t_0 + x) = 0$, solve for *G* and insert into $\lambda(T) = G\exp((\rho - r)T)$ to obtain

(21) $$\lambda(T) = \left\{\frac{k(T)\exp(r(t_0 + x - T))(g-r)}{\exp(g(t_0 + x)) - \exp(gT + r(t_0 + x - T))}\right\}^{-\phi} \exp((\rho - r)T)$$

Next we solve for $k(T)$. As discussed earlier, we can insert (21) into (16). After doing so, evaluate at $t = T$ and solve for $k(T)$

(22) $$k(T) = y_1(t_0)/(1 + y_2(t_0))$$

which is the planned account balance at retirement from the perspective of time $t_0 \in [T - x, T]$, where

$$y_1(t_0) \equiv k(t_0)\exp(r(T - t_0)) - w(1 - \exp(r(T - t_0)))/r$$

$$y_2(t_0) \equiv \left\{\frac{\exp(r(t_0 + x - T))\exp(gT)}{\exp(g(t_0 + x)) - \exp(gT + r(t_0 + x - T))}\right\}(1 - \exp(g(t_0 - T) + r(T - t_0)))$$

Finally, planned consumption on $[t_0, T]$ from the perspective of $t_0 \in [T - x, T]$ is

(23) $$c(t) = \lambda(T)^{-1/\phi}\exp(g(t - T))$$

To find actual consumption on $[T - x, T]$, insert (22) into (21) and then insert into (23), and, as explained in the previous section, let $t_0 = t$

(24) $$c(t) = \frac{y_1(t)}{1 + y_2(t)}\left\{\frac{\exp(r(x + t - T))(g - r)}{\exp(g(x + t)) - \exp(gT + r(x + t - T))}\right\}\exp(gt)$$

Note that $y_1(t)$ is a function of *k*(*t*), thus we must find the actual time path of *k*(*t*) on $[T - x, T]$, which is found by inserting (24) into (13) and solving the differential equation. Doing this gives (see appendix for derivation)

(25) $$\begin{aligned} k(t) = {} & k(T - x)\exp((r - z_1)(t - T + x)) + w/(z_1 - r)(1 - \exp((r - z_1)(t - T + x))) \\ & - z_1 w/(rz_1 - r^2)(1 - \exp((r - z_1)(t - T + x))) \\ & + w(\exp(r(t - T)) - \exp(r(t - T) - z_1(t - T + x)))/r \end{aligned}$$

Thus, (24) and (25), along with $k(T-x)$, which is the solution endpoint from phase 1, represent the analytical solution program for phase 2. As with phase 1, this is the envelope of infinitely many initial values from the continuum of planned time paths.

4 Simulations

Parameter values are $r = 6\%$, $\rho = 3\%$, $w = 40{,}000$, $\phi = 1.5$, $T = 40$, and $k(0) = 0$. Figures 1, 2 and 3 illustrate hump-shape consumption profiles for various possible lengths of short-term planning windows. (We assume that time zero corresponds to age 25 and that time T corresponds to age 65).

Phase 1 of the work life coincides with the increasing side of the hump, since during this interval of time the agent's planning horizon does not bracket any portion of the retirement phase. However, as the planning horizon slides along the time-scale, eventually the date of retirement will come into view. The agent responds by scaling back consumption, and since the short-term horizon continually advances, the agent continues to scale back consumption commensurate with projected retirement needs.

This explanation has intuitive appeal and seems consistent with hundreds of years of folk wisdom. Lusardi (1999) states the matter plainly: "people without a [savings] plan for retirement…may be surprised as they approach retirement at how little they have accumulated and suddenly realize they must accept a sharp drop in living standards." In other words, they must scale back their consumption in order to provide for retirement. Our model is also consistent with Lusardi's (1999) empirical work, which suggests that the length of the planning horizon is a strong predictor of wealth accumulation. For planning windows of length 10, 15, and 20 years, accumulated wealth at retirement in our simulations is \$190,000, \$315,000, and \$445,000, respectively.

Another descriptive measure of the hump-shape is the ratio of peak consumption to initial consumption. This ratio is often used as a summary statistic (Bullard and Feigenbaum 2003) to assess whether simulated consumption paths are consistent with observation. Gourinchas and Parker (2002) estimate a ratio of 1.1, and Fernández-Villaverde and Krueger (2002) estimate 1.34.

In our short-term planning model, this ratio is $c(T-x)/c(0)$. For the planning windows of length 10, 15, and 20 years for example, we have ratios of 1.16, 1.22, and 1.23, respectively. These calculations fall within the above range (1.1 - 1.34). We take this as further evidence that the short-term planning model is consistent with observed features of life-cycle consumption.

Of course, the abstract theoretical profiles do not perfectly match the empirical profiles. For example, the individual theoretical profiles are not as smooth at the peaks as the empirical profiles. Yet, at an aggregate level, it is straightforward to smooth-out the inverted "V" by averaging over a set of individuals with various planning horizons. For example, Figure 4 represents the average age-consumption profile for an economy populated with an equal mix of individuals with planning horizons of length 10, 15, and 20 years. Using Lusardi's (1999) rough estimates of the planning composition of the population, we also obtain a pronounced hump, but the peak occurs a few years later than in figure 4.

5 Conclusion

We have offered an additional explanation for the hump-shaped age-consumption profile observed in household data. We develop a model of short-term planning with a moving horizon that continually slides along the time-scale. We solve the model analytically and show that the actual consumption program is the envelope of infinitely many initial values from a continuum of planned time paths. Hump-shape consumption is a feature of the model. This is taken as evidence that short-term planning is another plausible solution to the hump-shaped consumption puzzle.

An interesting area for future research would be the integration of this short-term planning model with other life-cycle modifications such as borrowing constraints, precautionary saving, uncertain lifetime, family size and endogenous labor supply.

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Appendix

***Derivation of equation* (11)**

Using z_1 and z_2 rewrite (10')

(A1) $c(t) = k(t)z_1 - z_2$

Substitute (A1) into (2) and solve for the general solution

(A2) $k(t) = \left\{ C + \int^t (w + z_2) \exp(-rs + z_1 s) ds \right\} \exp(rt - z_1 t)$

where *C* is an arbitrary constant. Evaluate (A2) at time zero, solve for *C*, and insert into (A2). Integrate to obtain (11).

***Derivation of equation* (25)**

Using $y_1(t)$, $y_2(t)$, and z_1 we simplify (24) (tedious algebraic manipulation to get here)

(A3) $c(t) = k(t)z_1 + z_1 w(1 - \exp(r(t - T)))/r$

Substitute (A3) into (13) and solve for the general solution

(A4) $k(t) = \left\{ C + \int^t \left[w - z_1 w(1 - \exp(r(s - T)))/r \right] \exp(-rs + z_1 s) ds \right\} \exp(rt - z_1 t)$

Evaluate (A4) at time $T - x$, solve for *C*, and insert into (A4). Integrate to obtain (25).

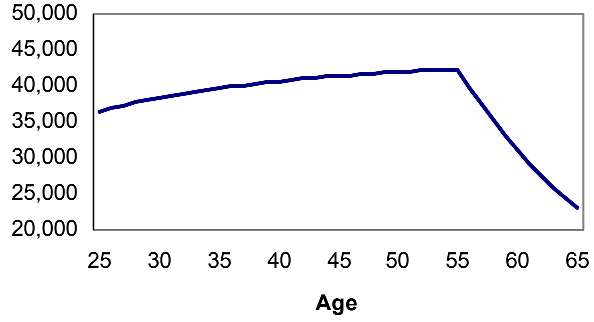

Figure 1. Work life consumption (10 year planning window)

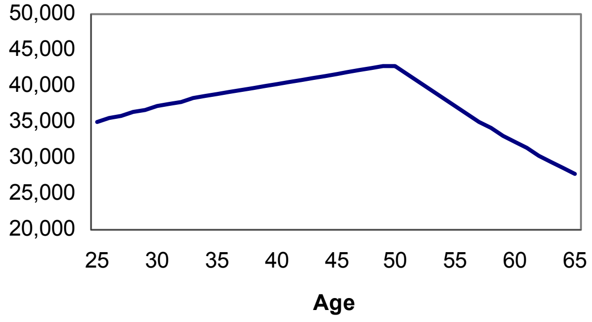

Figure 2. Work life consumption (15 year planning window)

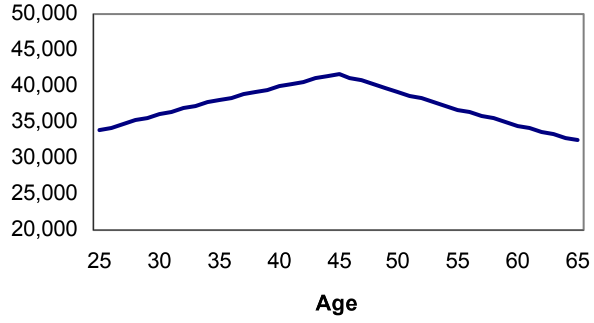

Figure 3. Work life consumption (20 year planning window)

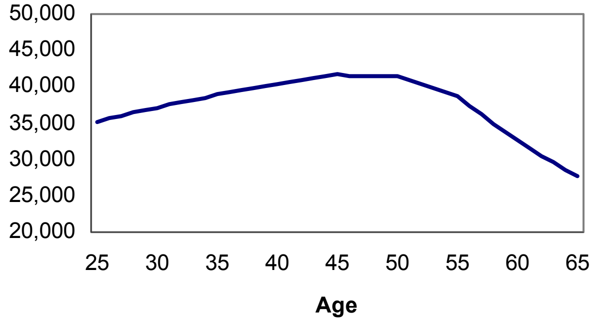

Figure 4. Aggregate work life consumption