

"Political Competition in Government Formation: the Effect of Simultaneous Policy Bidding on the Political Outcome"

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February 1, 2004

Abstract

We present an alternative model of government formation in which two parties simultaneously and independently announce their policy proposals through a take-it-or-leave-it offer, to a third party - the formateur -, which picks the one that maximizes its own utility. As a consequence, the chosen policy proposal is implemented by a government coalition composed of the formateur and the party associated with the selected policy proposal. The model purportedly captures the political competition arising among the parties other than the formateur for the partnership in the governing coalition. The political equilibria resulting from the model confirm that the intensification of political competition among the parties, implied by the present framework, is beneficial for the formateur.

1 Introduction

Proportional representation systems raise a theoretical question: How do parties with different interests and electorates form coalition governments in post-election periods? The literature on government formation points out

two classes of issues to be determined through this formation process. The first one concerns the distribution of governmental benefits among governing parties. The second one consists in the identification of the policies implemented in the aftermath of government formation. A variety of papers, following the path traced by Baron and Ferejohn (1988), limited their analysis to the allocation of benefits, while others, in the spirit of Baron's (1991) and Laver's and Shepsle's works (1990, 1996), devoted exclusive attention to the policies stemming from the formation process. Finally, a number of studies, among which Austen-Smith and Banks' (1988) and Morelli's (1999), addressed both issues. All the aforementioned authors regard government formation as the result of bargaining mechanisms involving the parties, and show that political outcomes largely result from political institutions, which affect the set of both admissible and optimal actions available to the parties, and ultimately represent the driving force of the political outcome.

The most significant institutional feature within the largely adopted bargaining framework turns out to be the order in which parties are called upon to take their decision, or, in game theory words, the sequence of moves. Innovating with respect to the previous literature, Morelli (1999) endogenizes the sequence of moves by introducing a mechanism¹ that triggers the selection of the party whose preferences are closer to the median voter's as the first mover. Notwithstanding this relevant improvement, Morelli's findings are not entirely satisfactory, and appear to rely too heavily on the *ad hoc* mechanism constructed to allow the first move to the party closest to the median voter; indeed, the equilibrium policy are those proposed by the first mover. Furthermore, as far as we know, all the studies delving into both the distribution of private benefits and the policy outcomes seem to display an overwhelming prevalence of the benefits *versus* the policies in the outcome determination.

In our view, the relative importance assigned by the parties to the policy outcomes offset that attributed to the benefit distribution *per se*. The latter are hence neglected in the following analysis, *a fortiori* since a large fraction of them is tied to the policy outcome, and hence does not require separate consideration. More generally, it is appropriate to assess that the present work does not account for the private benefits *per se*, while it captures those

¹The mechanism is justified by the following story. The Head of the State is entitled to select the first mover, and his preferences are analogous to the median voter's. As a consequence, the closest party to the median voter is selected as the first mover.

embodied in the policy dimensions. The core of this paper lies in the exploration of the consequences of an enhanced competition between parties. In our opinion, the sequential bargaining setup employed in the mainstream literature on government formation excessively constrains the parties and restrains their potential for competition. The present work purportedly expands the room for such competition by framing a mechanism, in which all the political groups except for one simultaneously make a take-it-or-leave-it offer, and one of the parties exogenously chosen as the formateur optimally selects his coalition partners. Simultaneity represents the key novelty that widens the scope for inter-parties competition.

Finally, the model provides for the possibility that the relative ranking, in terms of importance, of the different policy issues are not homogenous across parties. This assumption implicitly incorporates the private benefits allocation into the model. For instance, it may be plausible that environmental policies are deemed the most relevant by the green party. However, it may be as much true that one of the party overcares about a norm to incentivate automobile purchases simply because the major automobile companies are among its financial supporters.

The paper proceeds as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 5 concludes.

2 Model

Consider the process of constitution of a government among three parties, denoted $i = \{j, k, f\}$, within an assembly in which none of them is majoritarian. Suppose the government has to agree on n policy issues, denoted as $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$, to be implemented. Define a government protocol as an n -tuple $\mathbf{t}^* = (t^{1*}, \dots, t^{n*})$ of equilibrium policies, one for each policy issue. The preferences of the parties are Euclidean, and they are defined over the vector of policy dimensions $\boldsymbol{\pi}$. Each party has an ideal policy over each policy dimension, and the ideal policy vector for party i is denoted by $\mathbf{x}_i = (x_i^1, \dots, x_i^n)$. Each party ranks differently the various policy issues. Define $\boldsymbol{\nu}_i = (\nu_i^1, \dots, \nu_i^n)$ as the vector of relative weights attributed by party i to each of the n policy dimensions. Assume $\sum_{\pi=1}^n \nu_i^\pi = 1, \forall i$; since weights represent a relative measure of the policy ordering for each party, such normalization is legitimate. Write $\mathbf{x}_M = (x_M^1, \dots, x_M^n)$ the vector of the median

ideal policies, also said the multidimensional median; along each dimension π , $x_M^\pi \in \{x_f^\pi, x_j^\pi, x_k^\pi\}$. In this perfect information framework, both the matrix

of ideal policies, $\mathbf{X}_{3xn} = \begin{matrix} x_f^1 & \dots & x_f^n \\ x_j^1 & \dots & x_j^n \\ x_k^1 & \dots & x_k^n \end{matrix}$, and of weights, $\mathbf{V}_{3xn} = \begin{matrix} \nu_f^1 & \dots & \nu_f^n \\ \nu_j^1 & \dots & \nu_j^n \\ \nu_k^1 & \dots & \nu_k^n \end{matrix}$,

are common knowledge. Because of the previously illustrated motives, parties do not bargain over private benefits *per se*. We assume the following utility specification for each party:

$$U(\mathbf{x}_i, \boldsymbol{\nu}_i, \mathbf{t}^*) = - \sum_{\pi=1}^n \nu_i^\pi |t^{*\pi} - x_i^\pi|, \quad \forall i \in \{f, j, k\} \quad (1)$$

The bargaining proceeds as follows. In the first stage, a party is selected as the formateur. Without loss of generality, say f is picked in this capacity. In the second stage, the two remaining parties, j and k , simultaneously and independently submit a vector of policies each, $\mathbf{t}_j = (t_j^1, \dots, t_j^n)$ and $\mathbf{t}_k = (t_k^1, \dots, t_k^n)$. \mathbf{t}_j (\mathbf{t}_k) should be interpreted as the vector of policies that the government will implement should j (k) be selected as the coalition partner by the formateur. In the final stage, the formateur chooses either j or k (and the associated \mathbf{t}_j or \mathbf{t}_k) to maximize its own utility, and the government is constituted.

Intuitively, despite the implemented proposal is actually proposed by a different party than the formateur, still the competition between the two parties, j and k , to be part of the government, should intuitively drive the result closer to the formateur's ideal policy.

3 Equilibrium characterization

3.1 Single policy dimension

At first, we consider a simple case of competition arising on a single policy dimension. The first result establishes the obvious irrelevance of weights under these circumstances.

Lemma 1 *The weights \mathbf{V} are irrelevant when the government formation process contemplates a bargaining over a single policy issue.*

Proof. Having established, in the previous section, the legitimacy of the normalization $\sum_{\pi=1}^n \nu_i^\pi = 1, \forall i$, the result follows by noticing that in this case $\pi = \{1\}$, that implies that the only possibility is $\nu_f^1 = \nu_j^1 = \nu_k^1 = 1$. ■

The next discussion is devoted to the characterization of the equilibria. There are two possible cases: $x_M = x_f$, or $x_M \neq x_f$. We start by examining the former, in which the median ideal policy is the formateur's.

Proposition 2 *In a political game characterized by three parties, a single policy dimension, and $x_M = x_f$, the ideal policy of the formateur is implemented.*

Proof. We start by showing the non optimality of all the non-equilibrium outcomes. Without loss of generality, assume that the ideal policies are such that: $x_k \leq x_f \leq x_j$. Suppose a policy different than the median one is implemented. There are two possibilities:

- 1) $t^* > x_f$. In this case, k prefers to propose $x_f < t_k < t^*$, thus being selected by the formateur and implementing a policy closer to x_k ;
- 2) $t^* < x_f$. Now, j prefers to propose $t^* < t_j < x_f$, thus being selected by the formateur, and implementing a policy closer to x_j .

Now, we prove the optimality of the equilibrium strategy. For $t^* = x_f$, we need either that $t_k = x_f$, or that $t_j = x_f$, or both. Say $t^* = t_k = x_f$. If j deviates, whatever the original t_j was, either it is not chosen, or it can be chosen if it proposes $t_j = x_f$. However, by this proposal, it gets exactly the same utility as under t^* , so the deviation is not profitable. If $t^* = t_j = x_f$, the choice of the coalition partner is irrelevant for both the formateur, and the other two parties, neither of which, by the previous argument, has a profitable deviation. ■

The result shows that competition between parties j and k allows the formateur to get x_f , its ideal policy, as the unique equilibrium outcome as long as x_f is median.

We now pass to the second case, and examine the situation in which the formateur's ideal policy is not the median one.

Proposition 3 *A political game characterized by three parties, a single policy dimension, and $x_m \neq x_f$, has multiple Nash equilibria outcomes, represented by the set of policies lying the interval between the formateur's ideal policy and the median party's ideal policy. Formally, $x_M = x_k(x_j) \Leftrightarrow t^* \in$*

$[x_M, x_f]$. However, for any equilibrium such that $t^{**} \in (x_M, x_f]$, there exists a single strategy implementing (t_j, t_k) it, given by $t_j = t_k = t^{**}$; on the other hand, there exist multiple strategies implementing the equilibrium $t^{***} = x_M$, given by the set $t_j = t^{***}, t_k \leq t^{***}$.

Proof. Suppose without loss of generality that $x_k \leq x_j \leq x_f$. The proposition assesses that there are multiple equilibria, and all equilibria belong to the set $[x_j, x_f]$. We now rule out all other equilibria, specifically all the policy belonging to the interval $[x_k, x_j)$. Assume that $t^* \in [x_k, x_j)$. Then, j finds it profitable to propose t_j such that $t^* < t_j < x_j$. He will thus be selected by the formateur and will improve upon his initial utility. All the remaining policies outside the Pareto set cannot clearly be equilibrium policies. Now, we prove that all $t^{**} \in (x_j, x_f]$ are equilibrium outcomes resulting from a strategy (t_j, t_k) prescribing $t^{**} = t_j = t_k \in (x_j, x_f]$. Then, if j (or k) deviates by proposing t_j (or t_k) $< t^{**}$, then its proposal will be rejected by the formateur, and thus it is indifferent between the two situations. If on the other hand j (or k) deviate by proposing t_j (or t_k) $> t^{**}$, its offer will be accepted, but it will yield a lower utility than the utility achieved under t^* . Finally, we show that $t^{***} = x_j$ is an equilibrium outcome resulting from a larger set of strategies $(t_j, t_k) : t_j = t^{***}, t_k \leq t^{***}$. Such a strategy set implements x_j . Hence, j would not have a profitable deviation, while k 's only relevant deviation - which would entail a different policy outcome - happens if $t_k > x_j$. In this case, t_k would be implemented, causing k 's utility to dwindle. The deviation is therefore not profitable. ■

In conclusion, if the formateur's ideal policy is not the median one, the competition between parties is not sufficient to drive the policy to the first best for the formateur. Furthermore, if we are willing to share the idea that, in the instance of multiple equilibria, an equilibrium is relative more likely to arise if the set of strategies that implements it is larger, it is possible to claim that the implementation of x_M is relatively more likely than the implementation of any other equilibrium outcome. A stronger result holds under the assumption that neither of the parties' proposals is outside the Pareto set.

Proposition 4 *Under the assumption the strategy set for j and k is limited to the Pareto set, a political game characterized by three parties, a single policy dimension, and $x_m \neq x_f$, yields a unique Nash equilibria outcome not*

involving the play of weakly dominated strategy, represented by the median party's ideal policy. Formally, $(t_j, t_k) \in (a, b)$, $a = \min \{x_i, x_k, x_f\}$, $b = \max \{x_i, x_k, x_f\}$, $x_M = x_k(x_j) \Rightarrow t^{TH} = x_M$.

Proof. We stick to the notation used in the previous proof, and prove that all the equilibria implementing a policy different than the median ideal policy involve the play of weakly dominated strategies even when the admissible strategy set is limited to the Pareto set. Suppose without loss of generality that $x_k \leq x_j \leq x_f$. Given any $t^{**} \in (x_j, x_f]$, the only strategy (t_j, t_k) implementing t^{**} is the one prescribing $t^{**} = t_j = t_k \in (x_j, x_f]$, as previously illustrated. However, playing t_k is weakly dominated by playing $t'_k < t_k$ for k . There are three possibilities to consider:

1) $|x_f - t_k| < |x_f - t_j| < |x_f - t'_k|$. In this case, when k proposes t_k , it is chosen by the formateur, while when it proposes t'_k , the formateur prefers j . However, $t_j \succ_k t_k$, thus $t'_k \succ_k t_k$.

2) $|x_f - t_j| < |x_f - t_k| < |x_f - t'_k|$. In this case, k is not chosen by the formateur in either case; hence, $t'_k \sim_k t_k$.

3) $|x_f - t_k| < |x_f - t'_k| < |x_f - t_j|$. In this case, the formateur chooses k under both proposals. However, t'_k yields k a higher utility; therefore, $t'_k \succ_k t_k$.

In conclusion, $\forall t_j \in (a, b)$, $a = \min \{x_i, x_k, x_f\}$, $b = \max \{x_i, x_k, x_f\}$, $t'_k \succeq_k t_k$. Therefore, t_k is weakly dominated. On the other hand, the implementation of $t^{TH} = x_M$ does not involve the play of weakly dominated strategy by any of the two players. ■

3.2 Double policy dimension

We assume now that there are two policy issues, denoted by π_1 and π_2 , on which the two parties j and k compete to be selected as government's partner by the formateur. By now, we assume that all parties have the same valuation for each policy dimension, say $v_i^\pi = \frac{1}{2}$, $i = \{j, k, f\}$, $\pi = \{1, 2\}$. In one dimension, we found that when the formateur's ideal policy is median, the unique Nash equilibrium implements the formateur's ideal policy; otherwise, when the formateur's ideal policy is not median, there exist multiple Nash equilibria spanning the whole set of policies located between the median ideal policy and the formateur's ideal policy. As intuitively plausible, it turns out that the same logic applies in two dimensions. For the policy dimension(s) in which the median ideal policy is the formateur's, the unique Nash equilibrium

prescribes the implementation of the formateur's policy in that dimension(s); on the other hand, for the policy dimension(s) for which the median ideal policy is not the formateur's, there exist multiple Nash equilibria for that dimension(s), spanning the set of policies located between the median ideal policy and the formateur's. In conclusion, we have the following general result:

Proposition 5 *In a political game characterized by three parties, two policy dimensions and equal weights across policies and across parties, the set of Nash equilibrium outcomes is the following: for the policies for which the formateur has the median ideal policy, the unique equilibrium outcome is the ideal formateur's policy, while for the policies for which the formateur has not the median ideal policy, there are multiple equilibrium outcomes, belonging to the set between the formateur's ideal policy and the median ideal policy. Formally, denote $\pi' \in \boldsymbol{\pi}$ each policy dimension with the property that $x_M^{\pi'} = x_f^{\pi'}$, and $\pi'' \in \boldsymbol{\pi}$ each policy dimension with the property that $x_M^{\pi''} \neq x_f^{\pi''}$. Then, for any π' we have an unique equilibrium policy $t^{*\pi'} = x_M^{\pi'} = x_f^{\pi'}$; on the other hand, for any π'' , there exist multiple equilibrium policies $t^{*\pi''} \in [x_f^{\pi''}, x_k^{\pi''}(x_j^{\pi''])]$.*

Proof. The proof proceeds by a number of steps.

Step 1: Preliminarily, we point out that the utility of each party is the negative of the distance between the implemented point, and the party's ideal point in the two dimension space. The distance is measured as the sum of the distances between the two policies on each of the two dimensions. Formally, given $a = (a^1, a^2)$, and $b = (b^1, b^2)$, where the superscripts still indicate the dimension, $d(a, b) = |a^1 - b^1| + |a^2 - b^2|$. For example, given $a = (3, 6)$ and $b = (11, 7)$, we have that $d(a, b) = |3 - 11| + |6 - 7| = 8 + 1 = 9$. This definition of distance stems from the utility specification, which is the negative of the sum of the distances between the ideal policy and the implemented one in each dimension. The following proof relies on the separability on the utility on each dimension, and on the fact that no interaction exists between the two policies. In other words, a two-dimensional policy is a Nash equilibrium if and only if both of the policies are equilibrium policies along their dimension.

Step 2: We now provide a characterization of the equilibrium points. The equilibrium vectors t^* are those in which it is not possible to increase at the same time the utility of the formateur and the utility of either j or k or both.

To prove that, note that if this was not true, it would be possible for j or k to propose \widehat{t}_j (\widehat{t}_k) that increases both j (k)'s utility and the formateur's. This implies that \widehat{t}_j (\widehat{t}_k) would be chosen by the formateur, and \widehat{t}_j (\widehat{t}_k) $\succ_{j(k)} t^*$. Therefore, t^* cannot be an equilibrium. Given the measurement of distance, and as a consequence of utility, in any Nash equilibrium of the game there must not exist a possibility to move along either policy dimension in a direction that makes both the formateur and at least one of the two parties closer to the ideal point.

Step 3: If the formateur's ideal policy is median in both dimensions, then its ideal median policy is implemented in equilibrium. Formally, $x_M^\pi = x_f^\pi, \boldsymbol{\pi} = \{\pi_1, \pi_2\} \Rightarrow t^{*\pi} = x_M^\pi = x_f^\pi, \boldsymbol{\pi} = \{\pi_1, \pi_2\}$, or, equivalently, $\pi_1, \pi_2 = \pi' \Rightarrow t^{*\pi'} = x_M^{\pi'} = x_f^{\pi'}$. To see it, consider any other \widehat{t} such that $\widehat{t}^{\pi'} \neq x_M^{\pi'} \neq x_f^{\pi'}$, π' being either π_1 or π_2 . Without loss of generality, consider $\widehat{t}^{\pi'} \neq x_M^{\pi'} \neq x_f^{\pi'}$, $\pi' = \pi_1$, while $\widehat{t}^{\pi'} = x_M^{\pi'} = x_f^{\pi'}$, $\pi' = \pi_2$. For $\pi' = \pi_1$, we have that either \widehat{t}^{π_1} is on the right of the formateur's ideal point, or it is on its left. Since we know that $x_M^{\pi_1} = x_f^{\pi_1}$, it has to be that $x_M^{\pi_1}$ is located between $x_j^{\pi_1}$ and $x_k^{\pi_1}$. Without loss of generality, suppose $x_j^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1}$. We now have four possible locations of \widehat{t}^{π_1} :

1) $\widehat{t}^{\pi_1} < x_j^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1}$. In that case, both j and k have an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_j^{\pi_1} > \widehat{\widehat{t}}^{\pi_1} > \widehat{t}^{\pi_1}$ for π_1 , and $\widehat{\widehat{t}}^{\pi_2} = \widehat{t}^{\pi_2}$ for π_2 . The vector $\widehat{\widehat{t}}$ will be selected by the formateur, and the overall policy will yield a higher utility

2) $x_j^{\pi_1} < \widehat{t}^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1}$. In this case, k has an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_f^{\pi_1} > \widehat{\widehat{t}}^{\pi_1} > \widehat{t}^{\pi_1}$ for π_1 , and $\widehat{\widehat{t}}^{\pi_2} = \widehat{t}^{\pi_2}$ for π_2 . The vector $\widehat{\widehat{t}}$ will be selected by the formateur, making the deviation profitable.

3) $x_j^{\pi_1} < x_f^{\pi_1} < \widehat{t}^{\pi_1} < x_k^{\pi_1}$. In this case, j has an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_f^{\pi_1} < \widehat{\widehat{t}}^{\pi_1} < \widehat{t}^{\pi_1}$ for π_1 , and $\widehat{\widehat{t}}^{\pi_2} = \widehat{t}^{\pi_2}$ for π_2 . The vector $\widehat{\widehat{t}}$ will be selected by the formateur.

4) $x_j^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1} < \widehat{t}^{\pi_1}$. In this case, both j and k have an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_k^{\pi_1} < \widehat{\widehat{t}}^{\pi_1} < \widehat{t}^{\pi_1}$ for $\pi = 1$, and $\widehat{\widehat{t}}^{\pi_2} = \widehat{t}^{\pi_2}$ for $\pi = 2$. The vector $\widehat{\widehat{t}}$ will be selected by the formateur.

Consider instead the following situation: $x_j^{\pi'} < t^{*\pi'} = x_f^{\pi'} < x_k^{\pi'}, \pi_1, \pi_2 = \pi'$. Obviously, since the vector t^* is a satiation point for the formateur, no offer can make it better off, therefore t^* must be an equilibrium.

Step 4: We can generalize the previous result to show that any policy dimension in which the median ideal policy is the formateur's, the implemented policy along that dimension is indeed the formateur's. Formally, $\pi' \Rightarrow t^{*\pi'} = x_M^{\pi'} = x_f^{\pi'}$. To see this, consider the case in which $x_M^\pi = x_f^\pi$ along $\pi_1, x_M^\pi \neq x_f^\pi$ along π_2 . Equivalently, $\pi_1 = \pi', \pi_2 = \pi''$. Suppose $\widehat{t}^{\pi_1} \neq x_M^{\pi_1} \neq x_f^{\pi_1}$, and repeat all the five cases presented in step 3, to show that there exists a profitable deviation along π_1 , no matter what happens along π_2 . This in turn implies that $\widehat{t}^{\pi_1} \neq x_M^{\pi_1} \neq x_f^{\pi_1}$, cannot be the equilibrium policy along $\pi_1 = \pi'$. The generalized result is therefore that:

$$t^{*\pi'} = x_f^{\pi'} \quad (2)$$

Step 5: Now, we show that the dimensions along which the formateur has not the median ideal policy feature multiple equilibrium outcomes, belonging to the set stretching between the formateur's ideal policy and the median ideal policy. Formally, $\pi'' \Rightarrow t^{*\pi''} \in [x_f^{\pi''}, x_k^{\pi''}(x_j^{\pi''})]$. Without loss of generality, assume that along the dimension π_1 , the median ideal policy ordering is such that $x_j^{\pi_1} < x_k^{\pi_1} < x_f^{\pi_1}$, so that $\pi_1 = \pi''$. To see that no other policy may be an equilibrium one, consider an alternative point \widehat{t}^{π_1} along π_1 not belonging to the equilibrium set $[x_k^{\pi_1}, x_f^{\pi_1}]$. \widehat{t}^{π_1} may have the three following locations:

1) $\widehat{t}^{\pi_1} < x_j^{\pi_1} < x_k^{\pi_1} < x_f^{\pi_1}$. In this case, both j and k have an incentive to offer \widehat{t}^{π_1} such that $\widehat{t}^{\pi_1} < \widehat{\widehat{t}}^{\pi_1} < x_j^{\pi_1}$. This increases the utility of the formateur, meanwhile increasing theirs.

2) $x_j^{\pi_1} < \widehat{t}^{\pi_1} < x_k^{\pi_1} < x_f^{\pi_1}$. In this case, k has an incentive to offer $\widehat{\widehat{t}}^{\pi_1}$ such that $\widehat{t}^{\pi_1} < \widehat{\widehat{t}}^{\pi_1} < x_k^{\pi_1}$. This increases the utility of the formateur, meanwhile increasing k 's utility.

3) $x_j^{\pi_1} < x_k^{\pi_1} < x_f^{\pi_1} < \widehat{t}^{\pi_1}$. In this case, both j and k have an incentive to offer $\widehat{\widehat{t}}^{\pi_1}$ such that $\widehat{t}^{\pi_1} > \widehat{\widehat{t}}^{\pi_1} > x_f^{\pi_1}$. This increases the utility of the formateur, meanwhile increasing theirs.

Now, we show that all $t^{*\pi_1}$ such that $x_j^{\pi_1} < x_k^{\pi_1} < t^{*\pi_1} < x_f^{\pi_1}$ are indeed Nash equilibria, resulting from a strategy prescribing to both parties to play $t^{*\pi_1}$. Say that k wants to deviate from $t^{*\pi_1}$. If it deviates to $\widehat{t}^{\pi_1} < t^{*\pi_1}$, the deviation is irrelevant, since the formateur will select j , and the outcome will still be $t^{*\pi_1}$. If k deviates to $\widehat{t}^{\pi_1} > t^{*\pi_1}$, it will be chosen by the formateur, but \widehat{t}^{π_1} will yield k a lower utility than $t^{*\pi_1}$, thus the deviation is not going

to be profitable. The same exact argument can be applied for the possible deviations of j . Since the assumption on the location of ideal policies along π_1 is without loss of generality as long as the median ideal policy is not the formateur's, the result can be generalized to any π'' . We thus have that

$$t^{*\pi''} \in \left\langle \begin{array}{l} [\max(x_k^{\pi''}, x_j^{\pi''}), x_f^{\pi''}] \Leftrightarrow x_f^{\pi''} = \max(x_k^{\pi''}, x_j^{\pi''}, x_f^{\pi''}) \\ [x_f^{\pi''}, \min(x_k^{\pi''}, x_j^{\pi''})] \Leftrightarrow x_f^{\pi''} = \min(x_k^{\pi''}, x_j^{\pi''}, x_f^{\pi''}) \end{array} \right\rangle \quad (3)$$

are Nash equilibria for any π'' , while all other points $\widehat{t^{*\pi''}}$ outside this region are not. On the contrary,

As usual, it is worth remarking that the policy dimensions are equilibrium-independent, in the sense that the equilibrium outcomes in one dimension do not affect the equilibrium outcomes along the other. In other words, what counts to determine the equilibrium outcomes on one dimension is only the location of preferences on that dimension. ■

As a consequence of the equilibrium characterization in two dimensions, we have some interesting implications on the relation between equilibrium outcomes and Pareto sets.

Corollary 6 *All equilibrium vectors t^* are included in the Pareto set generated by the three parties.*

Proof. Say they were not in the Pareto set. Then there would exist a vector t' that Pareto dominates t^* . The fact that it is preferred by the two parties implies that the parties will propose it, the fact that it is preferred by the formateur implies that the formateur will accept it. Therefore, there would exist a profitable deviation from t^* . But then t^* cannot be an equilibrium. Thus, a Nash equilibrium has to be in the Pareto set. ■

At this point, we have all the ingredients to generalize the results for a wider set of possible weights. The following proposition shows that the results obtained for two dimensions are robust to any distribution of weights across the parties and the policies that assigns a strictly positive weight to every policy of every party.

Corollary 7 *As long as all the weights assigned by all the parties are strictly positive, for any arbitrary set of weights, the equilibrium outcome is a (not necessarily proper) subset of the equilibria under the case of homogenous*

weights across policies and across parties . Formally, denoting the equilibrium with arbitrary weights t^+ , we have that as long as $v_i^\pi > 0, i = \{i, k, f\}, \boldsymbol{\pi} = \{\pi_1, \pi_2\}$:

$$\begin{aligned} t^{+\pi''} &\subseteq t^{*\pi''} \in \left\langle \begin{aligned} &[\max(x_k^{\pi''}, x_j^{\pi''}), x_f^{\pi''}] \Leftrightarrow x_f^{\pi''} = \max(x_k^{\pi''}, x_j^{\pi''}, x_f^{\pi''}) \\ &[x_f^{\pi''}, \min(x_k^{\pi''}, x_j^{\pi''})] \Leftrightarrow x_f^{\pi''} = \min(x_k^{\pi''}, x_j^{\pi''}, x_f^{\pi''}) \end{aligned} \right\rangle \\ t^{+\pi'} &\subseteq t^{*\pi'} = x_f^{\pi'} \end{aligned}$$

Proof. Consider a new distribution of weights along a dimension π' , in which the median ideal ideal policy is the formateur's, such that $v_i^{\pi'} > 0, \forall i$. Then, no matter how small the valuation for the policy π' is for the three parties, if the parties have a profitable deviation, they will still want to deviate. Specifically, we now show that all points $\widehat{t}^{\pi'} \neq x_f^{\pi'}$ that differ from the formateur's ideal point are indeed not equilibria. Without loss of generality, consider $\widehat{t}^{\pi'} \neq x_M^{\pi'} \neq x_f^{\pi'}, \pi' = \pi_1$. For $\pi' = \pi_1$, we have that either \widehat{t}^{π_1} is on the right of the formateur's ideal point, or it is on its left. Since we know that $x_M^{\pi_1} = x_f^{\pi_1}$, it has to be that $x_M^{\pi_1}$ is located between $x_j^{\pi_1}$ and $x_k^{\pi_1}$. Without loss of generality, suppose $x_j^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1}$. We now have four possible locations of \widehat{t}^{π_1} :

1) $\widehat{t}^{\pi_1} < x_j^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1}$. In that case, both j and k have an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_j^{\pi_1} > \widehat{\widehat{t}}^{\pi_1} > \widehat{t}^{\pi_1}$ for $\pi = \pi_1$, and the formateur will select the party associated with $\widehat{\widehat{t}}^{\pi_1}$, thus achieving a Pareto improvement for the three parties. Indeed, for whatever positive value of v_k, v_j , and for $i = \{j, k, f\}$, $U_i(\widehat{\widehat{t}}^{\pi_1}) = -v_i^1 \left| \widehat{\widehat{t}}^{\pi_1} - x_i \right| > U_i(\widehat{t}^{\pi_1}) = -v_i^1 \left| \widehat{t}^{\pi_1} - x_i \right|$. Intuitively, even if the policy is valued very little, it is still profitable for each party to deviate to a policy closer to its ideal point.

2) $x_j^{\pi_1} < \widehat{t}^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1}$. In this case, k has an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_f^{\pi_1} > \widehat{\widehat{t}}^{\pi_1} > \widehat{t}^{\pi_1}$ for $\pi = \pi_1$. By the same token, the vector $\widehat{\widehat{t}}$ will be selected by the formateur, making the deviation profitable.

3) $x_j^{\pi_1} < x_f^{\pi_1} < \widehat{t}^{\pi_1} < x_k^{\pi_1}$. In this case, j has an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_f^{\pi_1} < \widehat{\widehat{t}}^{\pi_1} < \widehat{t}^{\pi_1}$ for $\pi = \pi_1$. Again, the vector $\widehat{\widehat{t}}$ will be selected by the formateur.

4) $x_j^{\pi_1} < x_f^{\pi_1} < x_k^{\pi_1} < \widehat{t}^{\pi_1}$. In this case, both j and k have an incentive to propose $\widehat{\widehat{t}}^{\pi_1}$ such that $x_k^{\pi_1} < \widehat{\widehat{t}}^{\pi_1} < \widehat{t}^{\pi_1}$ for $\pi = \pi_1$. The vector $\widehat{\widehat{t}}$ will be selected by the formateur.

The same argument as before can be applied to show that for $\nu_i^{\pi''} > 0, i = \{j, k, f\}$, $t^{+\pi''} \subseteq t^{*\pi''} \left\langle \begin{array}{l} [\max(x_k^{\pi''}, x_j^{\pi''}), x_f^{\pi''}] \Leftrightarrow x_f^{\pi''} = \max(x_k^{\pi''}, x_j^{\pi''}, x_f^{\pi''}) \\ [x_f^{\pi''}, \min(x_k^{\pi''}, x_j^{\pi''})] \Leftrightarrow x_f^{\pi''} = \min(x_k^{\pi''}, x_j^{\pi''}, x_f^{\pi''}) \end{array} \right\rangle$, remembering that no matter how little the valuation of that policy is, the parties still prefer to deviate to a policy closer to its ideal points. The proof is not repeated here because of its perfect analogy with the proof of proposition 4. ■

3.3 Multiple policy dimensions

In spite of being referred to a bidimensional policy space, the previous analysis has shown that with the present specification of the utility function, the equilibrium outcomes on a policy dimension are independent of the equilibrium outcomes on all other policy dimensions, except for the interaction between them represented by the weights v_i^{π} . However, we have shown that as long as $v_i^{\pi} > 0, \forall i, \forall \pi$, the results obtained for constant weights across policies and across parties hold unchanged. We are now ready to present a general result, valid on multiple dimensions under the condition that all parties attribute positive weights to all policies.

Proposition 8 *In a political game characterized by three parties, n policy dimensions and equal weights for all policies and all parties, the set of Nash equilibrium outcomes is the following: for the policies for which the formateur has the median ideal policy, the unique equilibrium outcome is the ideal formateur's policy, while for the policies for which the formateur has not the median ideal policy, there are multiple equilibrium outcomes, belonging to the set between the formateur's ideal policy and the median ideal policy. Formally, denote $\pi' \subset \pi$ the subset of policy dimension with the property that $x_M^{\pi'} = x_f^{\pi'}$, and $\pi'' \subset \pi$ the subset of policy dimensions with the property that $x_M^{\pi''} \neq x_f^{\pi''}$. Then, for any π' we have an unique equilibrium policy $t^{*\pi'} = x_M^{\pi'} = x_f^{\pi'}$; on the other hand, for any π'' , there exist multiple equilibrium policies $t^{*\pi''} \in [x_f^{\pi''}, x_k^{\pi''}(x_j^{\pi''})]$.*

Proof. As the weights are homogenous across policies and across parties, the only way for a party to achieve a profitable deviation consists in the implementation of a policy closer to its ideal point. Therefore, the analysis performed for the bidimensional case may be readily extended, and the results follow. ■

In analogy with the bidimensional case, for all the dimensions for which the median ideal policy is not the formateur's, the only equilibrium outcome achievable without the play of weakly dominated strategies is the one prescribing the implementation of the median ideal policy.

Furthermore, a second analogy with the previously examined situations is worth being pointed out. Allowing for arbitrary weights in the multidimensional case weakly shrinks the set of Nash equilibria.

3.4 Case of null valuations

We now briefly extend our analysis to capture the possibility of null valuations for some of the parties. We do it for the single dimensional case, knowing that the results can be easily extended to the bidimensional and multidimensional case. We find that the presence of null valuations affects the equilibrium outcomes in a significant way.

Proposition 9 *For any policy $\pi''' \subset \pi'$ for which the formateur is median and either j or k have a null valuation, there are multiple equilibrium outcomes, included in the set between the formateur's ideal policy and the positive valuation party's ideal policy. Formally,*

$$\begin{aligned} \forall \pi''' & : [(x_k^{\pi'''} < x_f^{\pi'''} < x_j^{\pi'''}) \cup (x_k^{\pi'''} < x_f^{\pi'''} < x_j^{\pi'''})] \cap \\ & \quad [(\nu_j = 0, \nu_k > 0) \cup (\nu_j > 0, \nu_k = 0)] \cap [(\nu_f > 0)], \\ t^{*\pi'''} & \in [x_f^{\pi'''}, x_{i:\nu_i>0}^{\pi'''}], i \in \{x_j, x_k\}. \end{aligned}$$

Proof. Say k has a null valuation, and $x_j < x_f$. Then all the policies outside the interval $[x_f, x_j]$ are not equilibria, as for each of them j can profitably deviate, and propose a policy within the interval $[x_f, x_j]$. On the other hand, a strategy set (t_j, t_k) prescribing $t_j = t_k \in [x_f, x_j]$ is indeed an equilibrium, since k would have no interest in deviating, because of his null valuation, whereas j would want to deviate to $t'_j < t_j$, but not to $t''_j > t_j$. However, if he deviates to t'_j , $t_k \succ_f t'_j$; hence, the formateur chooses t_k ; since $t_k \sim_j t_j$, the deviation to t'_j is not profitable. The result is thus proved. ■

4 Conclusion

The paper develops a stylized model aimed at capturing the political competition involving the parties willing to participate in a government. While

a more complex model may describe more accurately the sequence of actions that parties can take, our view is that the main effect of a more articulated string of possible actions lies in the enhancement of political competition between parties; such competition can be approximated by a framework of simultaneous and independent take-it-or-leave offers whose details have been previously discussed.

The political equilibria implied by the model show, not surprisingly, that the simultaneous and independent take-it-or-leave offers benefit the formateur. The force of competition between parties is strong enough to unambiguously foster the prevalence of the formateur's ideal policy for all the dimensions along which the latter is located in-between the other two parties' ideal policies; if, on the other hand, the formateur's ideal policy is not median, the model predicts multiple equilibria. Competition *per se* does not insure in this case the unambiguous prevalence of the formateur's ideal policy on the dimensions along which the median ideal policy is not median.

The competition is driven not only by differences in ideal policies, but also by heterogeneity in weights. When weights differ, the set of equilibrium policies, for the dimension along which the median ideal policy is not the formateur's, shrinks toward the formateur's ideal point. Basically, the weights play the role of an additional source of competition.

The paper neglects the distribution of benefits; in spite of the previously discussed political reason for their exclusion, a possible extension of this model could consist in the check of the robustness of the findings to the introduction of private benefits into the game. Moreover, general results applicable to an arbitrary number of parties would be desirable. Finally, a more general specification of the utility function may serve as a useful test for the robustness of the outcomes.

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