Working Paper

Estimating and testing preferences for consumption, work hours and savings using the PSID, the profit function and the true dynamic budget constraint

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ABSTRACT

Five waves of the Panel Study of Income Dynamics (PSID), 1985-1989 including both wealth supplements, are used to construct an intertemporal budget constraint for selected single headed households. A new functional form of the dual consumer profit function rationalizing consumption, labor supply and savings is specified, estimated and used to test commonly maintained separability hypotheses. Both consumption-labor and time separability are rejected. Cross-price Frisch elasticities are found not to equal zero and this in turns affects all estimates of consumption, labor supply and saving elasticities.

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INTRODUCTION

In the field of commodity demand estimation, sophisticated representations of preferences are routinely employed. With the advent of duality and flexible functional forms, parameter rich models with sufficient flexibility to uniquely identify the full set of share, income and price cross-price elasticities are used. What is minimally required in a model of n-goods are (n+4)(n-1)/2 parameters to independently identify the (n-1) shares, (n-1) income elasticities and n(n-1)/2 price cross-price elasticities. More recently in this field, Banks, Blundell and Lewbel (1997) and Ryan and Wales (1999) have derived and estimated rank-3 demand systems with a further (n-1) parameters to capture a quadratic element in expenditure. As a broad characterization of this field, it has been found that more restrictive models of demand should be rejected in favor of greater generality.

In contrast, economists who model intertemporal choice commonly use rather crude functional forms. While data limitations, the necessity of maintaining regularity conditions, the stochastic environment or some other feature of the exercise have confined modelers to forms such as Cobb-Douglas, Constant Elasticities of Substitution and Linear Expenditure Systems, it can still be argued that these are early generations of models used by demand modelers who have by in large rejected such forms as being overly restrictive.

The purpose of this paper is to show the feasibility of modeling intertemporal choice in a quite general, utility consistent manner- much along the lines of demand modelers- and then test the necessity of doing so. While studies focusing jointly on consumption and labor supply quickly run into problems from the lack of data on the evolution of consumption, what appears to be overlooked has been the availability of panel data on labor *and* wealth. We derive consumption expenditure quite differently deriving this from the dynamic budget constraint. We calculate consumption expenditure by adding beginning wealth plus earned and unearned income and subtracting end wealth with adjustments for rates of return on household wealth portfolios.[1] Taking intertemporal maximization as a three good problem- how much to spend, how much to earn and how much to save- our specification is quite general and treats each of these choice variables in a symmetric and mutually consistent manner.

Our approach draws on what are known as Frisch demand systems or lamda (λ) constant estimation (MaCurdy, 1983 and Altonji, 1986) which can be derived from a consumer profit function (Browning, Deaton and Irish, 1985). Unlike the existing literature which has been confined to simple specifications to either difference away or treat as a fixed effect the unobserved price of utility, $\mu \equiv 1/\lambda$, we invert the budget constraint to determine this unobservable. This fresh approach both required and enabled us to apply a new functional form that has necessary and appealing properties. Our functional form is globally regular, flexible, rank 3 and provides an explicit expression of the price of utility.

We apply the profit function to test two commonly maintained, but restrictive, hypotheses:- consumption-labor additivity and time separability. Avoiding the separability inflexibility (Blackorby, Primont and Russel, 1977) that hampers analyses

[1] Asset, portfolio, savings and wealth (non-human) are used interchangeably.

based on flexible indirect utility or expenditure functions, the profit function can test these hypotheses in a straight forward manner and amount to whether various Frisch cross-price terms enter into the system of structural equations explaining consumption, labor and wealth. Although separability tests seems not to have been done in either the dynamic or consumer context, Barnett and Hahm (1994) and the many papers cited by them show its application in static producer contexts. Both restrictions are easily rejected in favor of the most general case. This implies that Frisch cross-prices enter into the each structural equation independent of its impact on λ: wages and interest rates enter into the consumption equation; prices and interest rates enter into the labor equation; and prices and wages enter into the savings equation.

We next compare the elasticities of the most general model to those derived by the more restrictive models to evaluate the impact of these maintained assumptions. We find, for example, that the Frisch elasticity of consumption with respect to interest rates to be substantial in a general setting whereas this is constrained to zero under time separability. This effect reverses the conclusion one would draw from a time separable model which finds that the effect of higher short term interest rates significantly reduce consumption to one where it increases consumption for wealthy households and reduces consumption for poorer households. Another finding is that interest rates have a significant impact on labor supply which is constrained to zero in restrictive models. Because homogeneity of the profit function implies certain adding up properties in the matrix of price cross-price elasticities, removing constraints on certain cross-price elasticities changes the entire set of price cross-price Frisch and Marshallian elasticities.

Our procedure is also able to discern how changes in wages and interest rates changes μ depending on whether this is evaluated in cross section or in time series. We argue that when the response of μ to wage increases is evaluated in cross section, this is best thought of as the effect of a transitory one year increase in wages. On the other hand, when this is evaluated in time series, this can best be though of as a change to the evolutionary path of wages which has some persistence. By comparing the two, we find that the wage effect in time series is 7 times stronger than it is for a transitory wage increase suggesting if wages revert to a mean geometrically, it does so at a little over 15% annually. Labor supply switches from inelastic but positively sloped in response to short term wage increases to one that is backward bending for long term wage increases due to the wealth effect from wage persistence.

The following section presents some theoretical discussion of the model and is followed by a section describing the data used for the analysis. This is followed by a discussion of results and a conclusion that emphasizes the many possible extensions to this basic framework that we hope economist find fruitful.

ECONOMIC MODEL

In this section, we present the intertemporal decision as a one year problem where households evaluate prices, wages and interest rates together with an initial level of wealth to plan their consumption, labor and savings choices. The emphasis here will be how a household's contemporaneous interest return can be seen as a price which enters into their *present* savings decision. The homogeneity of the profit function where interest rate enters as an argument is also addressed. As will be seen in the next section, the data available does not include information on beginning-of-year and end-of-year wealth but rather wealth at the beginning and end of a five-year span. The issue of matching this model with 5 year's of income data is addressed in the data section but it suffices here to state that this is considered as a sequence of 5 one-year optimizations with household re-optimizing with the realization of new wage and interest information each year.

Consider a common specification of the intertemporal budget constraint: $W^n_t + y_t + w_t h_t - p_t c_t - (1+i_t)^{-1} W^n_{t+1} = 0$ where W^n is the nominal value of wealth, y is tax adjusted unearned income, w is after-tax nominal wage, h is hours of work, p is consumer prices, c is consumption, i is nominal interest and subscripts denotes time. We have mixed real and nominal variables in this specification. Define the real wealth variable $W_t = W^n_t/p_t$. The budget constraint can now be written terms of real choice variables: $p_t W_t + y_t + w_t h_t - p_t c_t - r_t W_{t+1} = 0$ where $r_t = p_{t+1}/(1+i_t)$ is the present nominal price of future real consumption and wealth. One can now divide the LHS by p_t to obtain the budget constraint exclusively in real prices and quantities. The constraint, written in this form, alerts us to the fact that the three real choice variables; c_t, h_t and W_{t+1} will be determined by real income y_t /p_t, real wages w_t /p_t, real interest r_t/p_t and predetermined real assets W_t. Clearly then, p_t, w_t and r_t are the correct prices corresponding to consumption, labor supply and future real wealth.[2]

[2] While r_t is a nominal variable as described above, we shall refer to it as the interest rate because dividing it by prices gives real interest rates.

Consider the recursive value function

$$
\begin{aligned}
&V(W_t, p_t, w_t, r_t : c_{t-1}, h_{t-1}) = \\
&\max_{c_t, h_t, W_{t+1}} \left[U(c_t, c_{t-1}, h_t, h_{t-1}) + E_t \delta V(W_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t)\right] \\
&\text{s.t.} \quad p_t W_t + y_t + w_t h_t - p_t c_t - r_t W_{t+1} = 0
\end{aligned}
$$

where V is the value function, U is the utility function, E is the expectations operator with respect to future dated information and δ is the time discount factor. The appearance of past consumption in the utility function allows for durability in the case of $\partial^2 U / \partial c_t \partial c_{t-1} < 0$ and for habits in the case of $\partial^2 U / \partial c_t \partial c_{t-1} > 0$. Symmetric time dependence is allowed for in work hours. In the case of durability or habit formation, the utility function is not time separable and present consumption and work will affect future utility.

In the most restrictive case we consider where utility, $U = U^c(c_t) + U^h(h_t)$, is additive, the first order conditions to the value function are $U_c^c(c_t) = \lambda_t p_t$ and $U_h^h(h_t) = -\lambda_t w_t$ where subscripts to functions denote differentiation. For a monotonic utility function, the consumption and labor supply function are determined by inverting the first order conditions. This gives rise to the consumption function, $c_t^* = f^A(\lambda_t p_t)$, and the labor supply function, $h_t^* = g^A(\lambda_t w_t)$, where the asterisk denotes model predicted quantities. Note that wages do not enter into the consumption equation nor do prices enter into the labor supply equation. Relaxing additivity, the time separable utility function $U = U^{TS}(c_t, h_t)$ has first order conditions $U_c^{TS}(c_t, h_t) = \lambda_t p_t$ and

$U_h^{TS}(c_t, h_t) = -\lambda_t w_t$. With concavity of the utility function, inversion leads to the consumption function $c_t^* = f^{TS}(\lambda_t p_t, \lambda_t w_t)$ and the labor supply function $h_t^* = g^{TS}(\lambda_t p_t, \lambda_t w_t)$. Wages now appear in the consumption function and prices appear in the labor supply function.

For notational purposes, let U(t) denote period t utility $U(c_t, c_{t-1}, h_t, h_{t-1})$ and V(t) denote the corresponding period t value function $V(W_t, p_t, w_t, r_t : c_{t-1}, h_{t-1})$. In the time separable case with or without consumption labor additivity, the inversion of the first order condition, $E_t \delta V(t+1)_{Wt+1} = \lambda_t r_t$, gives rise to the wealth or savings function, $W_{t+1}^* = h(\lambda_t r_t)$.[3] In the most general case where time separability is relaxed, the appearance of prior consumption and labor supply, c_t and h_t, in the value function V(t+1) implies that, on inversion, the wealth function $W_{t+1}^* = h^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t,)$ contains time t prices and wages. Prices and wages now enter into the savings function. Additionally, intertemporally optimizing households will ensure the equality of discounted expected future marginal utility with present marginal utility, $\delta E_t \lambda_{t+1} = (1+i_t)^{-1} \lambda_t$. This intertemporal optimization can be re-expressed, taking the view that contemporaneous real interest is part of the information set of the household, as $\delta E_t U_{Ct+1}(t+1) = \lambda_t r_t$. The appearance of c_t in this expression for future marginal utility together with c_t in period t optimization shows that interest rates belong in our consumption equation. A similar argument applies to our labor equation. The most general case will have interest rates enter into the consumption and labor supply equation independently of λ. Thus our most general case will have the system $c_t^* = f^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t)$, $h_t^* = g^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t)$ and $W_{t+1}^* = h^G(\lambda_t p_t, \lambda_t w_t, \lambda_t r_t,)$. Our objective then is to develop a utility consistent system of equations for consumption, labor and savings and examine the significance of these cross-price terms.

The profit function associated with the above value function is

$$\pi(p_t, w_t, r_t, \mu_t) = \max_{c_t, h_t, W_{t+1}} \left[\mu_t U(c_t, c_{t-1}, h_t, h_{t-1}) + \mu_t E_t \delta V(W_{t+1}, p_{t+1}, w_{t+1}, r_{t+1} : c_t, h_t) + w_t h_t - p_t c_t - r_t W_{t+1}\right]$$

where $\mu_t = 1/\lambda_t$ is the inverse of time-t lagrangian multiplier of the constraint or the price of utility. Recognizing data limitations that lie ahead, prior consumption and work and future prices are subsumed in the profit function in accordance with our approach of determining current choice variables solely from current exogenous variables.[4] Properties of the consumer profit function are homogeneity and convexity, necessary conditions given maximization. Conveniently, the consumption, labor supply and savings functions are obtained simply by taking the derivative of the profit function with respect to own price, a result from the envelope theorem. Thus for example, estimated consumption, c* = $-\pi_p$ where the subscript denotes partial differentiation and estimated consumption expenditures pc*= $-p\pi_p$.[5]

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[3] We have ignored the mechanism whereby w_t and r_t conditions expectations of future w_{t+j} and r_{t+j}, j>0.
[4] Time subscripts are henceforth dropped unless needed for exposition.
[5] Note that consumption and assets are given by the negative of the partial derivative while labor hours are given by the positive.

One of the challenges of taking a new approach to the data is often developing a parametric specification to fully rationalize the data. In addition to the homogeneity and convexity of the profit function in the observable price variables, consideration needs to be given for the unobserved μ. We develop a model that is not only globally convex including μ, it lends itself to an explicit expression of it on inversion of the dynamic budget constraint. As far as we are aware, this is a new functional form. It is parameterized by $\pi(p, w, r, \mu : z) = A(P, \mu, \alpha)\mu + B(P, \beta : z) + C(P, \gamma)/\mu$

where P = (p_1, p_2, p_3)' = (p, w, r)' is a vector of prices, z=(age, age^2, age^3, number of dependents, sex of household head)' is a vector of demographic characteristics and α, β and γ are vectors of parameters to be estimated. The sub-functions $A(P, \mu, \alpha)$, $B(P, \beta : z)$ and $C(P, \gamma)$ are given by

$$A(P, \mu, \alpha) = \sum_{i=1}^{3} \alpha_{ii} \ln(\mu / p_i) + \sum_{i=1}^{3} \sum_{j>i} \alpha_{ij} \ln(\mu / (p_i + \alpha_{ijj} p_j))$$

$$B(P, \beta : z) = \sum_{i=1}^{2} d_i [\beta_{i10} p_1 + \sum_{j=2}^{3} p_j (\beta_{ij0} + \beta_{ij1} age + \beta_{ij2} age^2 + \beta_{ij3} age^3 + \beta_{ij4} dependents)]$$

where d_i is the indicator for the sex of the household head, i=1 indicating male and i=2 indicating female[6], and

$$C(P, \gamma) = [(\gamma_{11} p + \gamma_{12} w + \gamma_{13} r)^2 + (\gamma_{22} w + \gamma_{23} r)^2 + (\gamma_{33} r)^2]/2$$

It can be readily verified the profit function is homogeneous and that $\alpha_{ij} > 0$ and $\alpha_{ijj} > 0$ is sufficient for global convexity.

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[6] Additional terms with β parameters of the form $\beta_{ij} p_i^{0.5} p_j^{0.5}$ with convexity restriction $\beta_{ij} \leq 0$ were also

To illustrate some of the properties of this model, consider now the structural equation for saving of a male head of household. Our savings equation is found by differentiating the profit function with respect to interest to give

$$-W^{*} = \partial\pi / \partial r = A_r(P,\mu,\alpha)\mu + B_r(P,\beta : z) + C_r(P,\gamma)/\mu$$

with differentiated sub-functions

$$A_r(P,\mu,\alpha) = -\left(\frac{\alpha_{13}\alpha_{133}}{p+\alpha_{133}r} + \frac{\alpha_{23}\alpha_{233}}{w+\alpha_{233}r} + \frac{\alpha_{33}}{r}\right)$$

$$B_r(P,\beta : z) = \beta_{130} + \beta_{131}\text{age} + \beta_{132}\text{age}^2 + \beta_{133}\text{age}^3 + \beta_{134}\text{dependents}$$

$$C_r(P,\gamma) = \left(\left(\gamma_{11}p + \gamma_{12}w + \gamma_{13}r\right)\gamma_{13} + \left(\gamma_{22}w + \gamma_{23}r\right)\gamma_{23} + \gamma_{33}^2 r\right)$$

A total of 16 parameters determine the wealth equation of male headed households. By differentiating the profit function with respect to wages and interest rates, it is easily verified that 15 parameters determine the labor supply equation and 9 parameters determine the consumption equation.

In estimation, the set of β parameters act as intercept terms identifying the central location of the distribution of consumption, labor or wealth while the derivatives of the sub-functions A and C conditions on μ. Since one expects demographic and life cycle factors to have an important influence on preferences, a total of 22 β_{ijk} parameters are added to distinguish household heads by gender, age and number of dependents in the system of structural equations and their significance evaluated empirically.[7]

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tried in the regression. The constraint was binding in every regression performed.

[7] Age and dependents parameters were not added to the consumption equation. The reason for this is discussed in the next section.

It is important to note here that the structural equations are not linear in μ.[8] For large values of μ, the relevant derivative of the sub-function A give the proportion by which demand (or labor supply) changes with μ. Large values of μ of course imply small values of $\lambda = 1/\mu$ so the affect on the structural equation through the derivative of the sub-function C is minimal. Alternatively, small values of μ imply that the impact of the derivative of sub-function A on the structural equations will be small. Small values of μ of course imply large values of $\lambda = 1/\mu$ so the affect of the derivative of the sub-function C is large. In this way, the sub-function A can be thought of as apportioning increases in consumption, labor and wealth from levels determined by the sub-function B arising from increases in μ according to “rich” household preferences. Similarly, the sub-function C can be thought of as abating consumption, labor and wealth from levels determined by the sub-function B arising from increases in λ according to “poor” household preferences.

Each structural equation conditions multiplicatively on μ and $1/\mu$. Together with the derivative of the sub-function B which gives the third function, this Frisch system might be called rank 3 (Lewbel, 1991) drawing obvious analogies with the indirect utility functions of Banks, Blundell and Lewbel (1997) and Ryan and Wales (1999) where μ

[8] Indeed, how the structural equations are non-linear in μ is an important attribute of this model and solved a particular problem encountered with what might be called rank 2 models. Consider the case where C(P: γ)=0 and the structural equations are linear in μ. We found that the β parameters again estimated the location of the data but solved values of μ took positive and negative values as the regression pivots around the central location of the distribution. Negative values for the price of utility not only violate

replaces income. Additionally as μ and $1/\mu$ enters the structural equations, this can be considered as a first order Laurent approximation. As demonstrated theoretically by Barnett (1983), the Laurent series approximation has superior fit compared to the corresponding Taylor series approximation of the same order.

Our profit function has considerable generality which we highlight be drawing analogies to flexible functional forms. The unrestricted off-diagonal parameters γ_{12}, γ_{13} and γ_{23} identify cross-price responses. The unrestricted diagonal γ_{22} and γ_{33} parameters identify how the structural equations change with respect to changes in $1/\mu$ making these analogous to income responses.[9] The unrestricted β parameters identify levels with sufficient parameters for flexibility and additionally capture suspected demographic influences. The diagonal parameters α_{22} and α_{33} are analogous to the third rank in rank 3 systems[10] while the off-diagonal parameters α_{12}, α_{13} and α_{23} identify how the structural equations change with respect to cross-price terms for high levels of μ for additional generality.

The economic restriction of consumption-labor separability and time separability is now easily presented as simple parametric restrictions on our structural equations which we describe now in order of increasing generality. The first of these we call the basic regression, our most parsimonious case. This regression sets all cross terms $\alpha_{ij} = 0$ and $\gamma_{ij} = 0$ for $i \neq j$ and is implied by consumption-leisure additivity and time

regularity, it doesn't make economic sense. Negative values were found in approximately half the cross section irrespective of whether the sub-functions C or A were identically zero.

[9] We do not estimate γ_{11} as will be discussed later.

separability. Additionally, all β parameters except 6 β_{ij0}, i=1, 2 and j=1, 2, 3, are set to zero removing the impact of age and number of dependents from the structural equations.

The first generalization allows demographic variation to impact the levels c, h, and W. One expects labor supply and wealth demand to follow lifecycle patterns and this is accomplished by estimating an additional 16 β parameters associated with age,[11] age squared and age cubed and the number of dependent. We call this case the demographic regression.

The second generalization allows for consumption-leisure non-additivity. This is accomplished by allowing parameters α_{12} and γ_{12} to take on non-zero values. This case we call the time-separable regression. The third generalization allows for intertemporal non-separability. This is accomplished by allowing the remaining parameters α_{13}, α_{23}, γ_{13} and γ_{23} to take non-zero values. We call this the general regression.

We now discuss the determination of unobserved μ which follows Cooper, McLaren and Wong (2001) but uses the dynamic rather than static constraint. If beginning and ending wealth, exogenous income, wages and interest rates are known, the unobserved μ is then implicitly defined by the budget constraint: $pW + y + p\pi_p + w\pi_w + r\pi_r = 0$. However, for the specific functional form we use, a convenient explicit expression is available. For notational convenience, let $A^* = \sum_{i=1}^{3} p_i A_{pi}$,

[10] We do not estimate α_{11} for the same reason as the preceding footnote.

[11] The age variable we use is reported age of head minus 45 years in order to center the regression around prime aged heads.

$B^* = pW + y + \sum_{i=1}^{3} p_i B_{pi}$ and $C^* = \sum_{i=1}^{3} p_i C_{pi}$.[12] Then, the formula for the roots of the quadratic equation gives

$$\mu = \frac{-B^* - \sqrt{B^{*2} - 4A^*C^*}}{2A^*} \text{ and}$$

$$\lambda = 1/\mu = \frac{-B^* + \sqrt{B^{*2} - 4A^*C^*}}{2C^*}$$

where the positive root is used. Quite serendipitously, restrictions sufficient for convexity, $\alpha_{ij} \geq 0, \alpha_{ijj} \geq 0$, lead to A*<0 and C*>0 which is also sufficient for a globally positive discriminant and a real root. These expressions for μ and λ, which are now solely in terms of observable variables, are substituted into the structural equations.

Unfortunately, beginning and end wealth is not available for just one year as has been supposed here- but fortunately, this is not a major complication. We turn next to a description of the data and how this was made operational.

DATA

The data for this study comes from the Panel Study of Income Dynamics (PSID) a continuing study started in 1968 with approximately 4,800 households. Since that time, the panel has grown as numbers of new household formed have exceeded those that have attrited (Hill, 1992). Five waves of the family files from surveys fielded from 1985 to 1989 were used which each contained detailed income and work data for the previous

[12] The latter two have simple forms, B*=pW+y+B and C*=2C from homogeneity of degree 1 and 2

year. Additionally, these files include two wealth supplements that asked about wealth in the beginning and end of this span. By the time of interview, many households have had long standing participation with this survey.

To build a balanced panel across 5 years, single headed households who remained heads from 1984 to 1989 were selected. This choice does not allow for changes in what the PSID calls major adults but allows other changes to the households such as the birth or adoption of children or children leaving home to establish one of their own.

Next, a measure of real return on wealth for each year is calculated. We treat wealth as a single amorphous good (as we do for consumption and labor supply) and add all disparate returns to wealth into a single total. We derive real returns by adding all recorded sources of income from wealth: net of tax income received from financial assets such as rent, dividends and interests; and the asset portion of income from unincorporated businesses, farming, market gardening and roomers and boarders as determined by the PSID. To this, one fifth of the 5-year capital gains as determined by the PSID and an imputed rental services on owner occupied housing was added. This total return on assets was divided by an interpolated level of wealth based on net wealth in 1984 and in 1989. We use the variable net wealth as defined by the PSID which includes the main home, other real estate, farms or businesses, stocks, cash accounts and other items, but exclude the value of motor vehicles.

The income return on wealth, i_t, is given by

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respectively.

$$i_t = \frac{(1-mt_t)\text{total asset income}_t + 0.05\text{house value}_t + \text{capital gain}/5}{(W_t + W_{t+1})/2}$$

with $W_t = W_{1984} + (t\text{-}1984)(W_{1989}\text{-}W_{1984})/5$ and mt_t equal to time-t marginal federal income tax rate. In the money demand literature, for example Barnett, Fisher and Serletis (1992), expressions are sought for the return on different kinds of money assets and a household's return is based on their mix of these assets. Here, we use directly the assets or income and capital gain on assets to derive an individual specific return on their assets.

All households with starting or ending wealth of less that $500 were deleted. Some of these cases showed implausibly large values for i_t which is understandable given the denominator is small. Additionally, all households with i_t greater than 0.4 or less than -0.2 in any of the five years were also deleted.

Next, a value for 5 year consumption is calculated for each household. This is calculated as a residual from the budget constraint. The budget constraint for a one year period is $p_tW_t + y + w_th_t - p_tc_t - r_tW_{t+1} = 0$. By recursive substitution, the 5 year budget constraint is $p_{1984}W_{1984} + \sum_{t=1984}^{1988} rr_t(y_t + w_th_t - p_tc_t) - rr_{1988}r_{1988}W_{1989} = 0$ where $rr_{1984}=1$ and $rr_j = \prod_{t=1985}^{j} r_t / p_{t+1}$, j=1985,…1988. While it is not possible to calculate consumption in each period, we can calculate 5 year discounted consumption by $\sum_{t=1984}^{1988} rr_t p_t c_t = p_{1984}W_{1984} + \sum_{t=1984}^{1988} rr_t(y_t + w_th_t) - rr_{1988}r_{1988}W_{1989}$ where all terms on the LHS are observable.

Using this measure of consumption, several cases of negative consumption expenditures were observed. For these cases, final assets were too high given beginning assets and recorded incomes. These cases were deleted as were cases which had calculated 5 year consumption expenditures less than $5,000.

The term y_t which measures time t exogenous income was calculated as income from all public and private transfers plus inheritances plus federal marginal tax rates times pretax labor earnings less federal income taxes for year t. Since we wish to capture household decisions at the margins, it is appropriate to use $(1-mt_t)$pretax $wage_t$ as the real benefit of working the marginal hour in year t. However, since we have a progressive income tax system, marginal tax rates times labor earnings overstates the amount of tax paid on labor earnings. As the objective of the above equation is to calculate consumption, we add back marginal tax times earning and subtract federal income taxes since this information is available.

After removing observations as described, 525 observations were left. Summary statistics for both male and female headed households are recorded in the table 1.

Table 1. Summary statistics of selected sample in 1988 by gender.

	Male headed households (n=92)				Female headed households (n=433)			
Variable	mean	std. dev.	minimum	maximum	mean	std. dev.	minimum	maximum
Age of Head	56.5	17.6	27	90	63.1	16.2	25	97
# of dependents	1.3	0.75	1	6	1.3	0.81	0	8
Consumption ($)	80,178	45,055	10,802	209,461	61,130	39,823	6,397	270,458
Wealth ($)	73,950	79,257	1,500	439,000	69,587	91,392	1,200	825,000
Interest return (i)	0.071	0.086	-0.184	0.335	0.076	0.093	-0.199	0.338
Exogenous income ($y)	1,998	8,643	-21,918	34,320	5,172	15,116	-10,399	255,953
Head work hours *	1,907	586	120	2,880	1,666	611	10	2,975
Labor Earnings ($) *	16,144	13,243	441	66,861	10,243	8,120	65	42,947

* summary statistics reported only for the 61 males and 203 females working in 1989.

Our approach give rise to 7 structural equations that can be matched with the data: one for end wealth, one for 5 year consumption and 5 for labor supply in each of the years 1984 to1988. We multiply the wealth equation by r_{1988} and each period t labor supply equation by w_t, a practice common in the demand analysis field that allows for adding up of commodity expenditures in the budget constraint. For our purpose, it also has the effect of removing households with non-working heads from the regression.[13] We use 6 independent estimating equations;[14]

$w_t h_t = w_t \pi_w(t) + \varepsilon_t$ for t=1984,…1988, and

$r_{1989} W_{1989} = -r_{1989} \pi_r(1988) + \varepsilon_{1989}$

appending ε_t as period t error. We assume that $\varepsilon = (\varepsilon_{1984}, \varepsilon_{1985}, \varepsilon_{1986}, \varepsilon_{1987}, \varepsilon_{1988}, \varepsilon_{1989})'$ is multivariate normal and estimate parameters using the full information maximum likelihood procedure implemented in TSP version 4.5.

As the 6 estimating equations stand, they contain 5 unobserved arguments, μ_t while the single 5 year budget constraint allows us to solve only one additional variable. Our strategy is to use the budget constraint to determine an individual specific μ and use 5 additional equations which relate μ_t to μ. We offer three models that corresponds to three hypotheses about the evolution of μ_t. The first of these we call the fixed effect model and this corresponds to $\mu_t = \mu$. The second case we offer we call the perfect foresight model and is defined by $\mu_{1986} = \mu$ and $\mu_{t+1} = \delta p_t / r_t \mu_t$ which corresponds to a perfect foresight Euler equation with time discount parameter, δ, an additional parameter requiring estimation. The third case we offer we call the stochastic model where μ_t is

[13] Our purpose is not to explain the dichotomous decision to work or not for which a reservation wage needs to be constructed.

perturbed around μ by realizations of real time t variables. The stochastic model specifies $\mu_t = \exp(\theta_w (w_t / p_t - \overline{w/p}) + \theta_r (r_t / p_t - \overline{r/p}) + \theta_a (W_t - \overline{W}))\mu$ where θ_w, θ_r and θ_a are 3 additional parameters requiring estimation and $\overline{X/Y} = \sum_{t=1984}^{1988} \frac{X_t / Y_t}{5}$ is the individual average of real wage, interest or wealth over the 5 year span.

RESULTS

This section reports on some of the challenges of estimation, how these were overcome and results obtained. As is common in non-linear estimation, functions of parameters were often easier to identify that the parameters individually. We found that the remaining α_{ij} and γ_{ij} parameters were far easier to identify once α_{11} and γ_{11} were specified as constants. This is understandable if one considers the structural equations. Consider the (negative) wealth expenditure equation which is[15]

$$-rW^* = r\pi_r = rA_r(P)\mu + rB_r(P;z) + rC_r(P)/\mu$$

and the explicit function for μ,

$$\mu = (-B^* - \sqrt{B^{*2} - 4A^*C^*})/2A^* .$$

It can be seen on substitution that the wealth equation changes with the term $-B^* - \sqrt{B^{*2} - 4A^*C^*}$ in the proportion $rA_r/2A^*$ which is simply a ratio of parameters α. The same is also true of the γ parameters where the wealth equation changes with the

[14] A consumption expenditure estimating equation could be added to the list but the error from this equation is not independent of the other 6 errors as the sum of all 7 errors is identically zero.

term $-B^*+\sqrt{B^{*2}-4A^*C^*}$ in the proportion $rC_r/2C^*$. Except for the appearance of the α and γ parameters in the discriminant, the structural equations would be homogeneous of degree zero in these parameters. As it is not possible to simultaneously add both α_{11} and γ_{11} as parameters to be estimated we arbitrarily set α_{11}=10,000, an innocuous specification because multiplying the α_{ij} parameters and dividing the γ_{ij} parameters by any positive constant leaves the structural equations unchanged.

With the parameter α_{11} set to a constant, attempts were made to estimate the other parameters however this was often not successful. In some of the most restricted regressions, the parameter γ_{11} could be estimated however for some of the richer specifications, it was found that this parameter, together with the other γ_{ij} parameters in proportion, would uniformly converge to zero. This behavior increased the number of iterations and squeeze steps required before the program was terminated either successfully by meeting specified parameter convergence criteria or unsuccessfully by exceeded specified iteration or squeeze step limits within the TSP procedure.

Consider now the functional forms of μ and λ as the parameters γ_{ij} converge to zero. In this case, μ converges to min[B*,0]/A* and λ converges to max[B*,0]/C* as the discriminant converges to B^{*2}. This creates linear segments to the structural equations in μ and λ. Because it is arguably more appealing that the structural equations should be "smooth" in μ and λ and, in particular, because of the difficulties of obtaining

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[15] We use the one-year setting for purposes of exposition.

successful convergence as the γ_{ij}'s approached zero, γ_{11}=11.5575 was set. This value of γ_{11} maximized the likelihood of one of our regressions, the basic regression of the stochastic model we describe below, and was subsequently used in all of the regressions.

Other specifications that aided estimation were holding parameters γ_{11}, γ_{22} and γ_{33} to be non-negative. It can be seen from the structural equations that these parameters are squared. Consequently, it is innocuous for the fit of the regression to use the positive values of these diagonal γ_{ii} parameters or truncate it at zero if the likelihood function is decreasing in this parameter.[16] Without this specification, the parameter γ_{ii} often seemed to oscillate explosively around zero for cases where likelihood was maximized at γ_{ii}=0. This truncation was also applied to the α_{ij} parameters to ensure non-negativity. Despite these aids to estimation, we found that in no case were we able to estimate parameters α_{ijj} so these were arbitrarily left at one. Thus, in the results we report below, some of the parameters are unrestricted while others are bound by our convexity restrictions. Furthermore, the set of parameters that were binding differed depended on the regression performed. Nonetheless, the parameter estimates we report here are convergent and robust in the sense of being independent of the many starting values we tried. We describe these regressions next.

For each of the three models: fixed effect, perfect foresight, and stochastic models, we performed four regressions: the basic, demographic, time-separable and

[16] This was specified in TSP as $\gamma_{ii} = (\varphi_{ii} > 0)\varphi_{ii}$ where φ_{ii} was the parameter actually submitted into the regression. Within the parenthesis is a logical function that takes the value 1 if true and 0 if false.

general regressions. The log likelihoods of these 12 regressions and the number of free and constrained parameters are summarized by the 4x3 cells of table 2.

Table 2. Log likelihood fit of various regressions and models

	Fixed Effect Model	Perfect Foresight Model	Stochastic Model
Basic Regression	-29,982.8 9(10)	-29,947.9 10(11)	-29,735.0 12(13)
Demographic Regression	-29,550.6 24(26)	-29,543.8 26(27)	-29,370.5 29(29)
Time Separable Regression	-29,539.2 25(28)	-29,533.5 26(29)	-29,364.6 30(31)
General Regression	-29,533.0 26(32)	-29,528.4 26(33)	-29,306.3 30(35)

In each of the 12 cells of table 2, the top row gives the log likelihood of the corresponding model and regression. The second row of each cell gives two numbers: the number of free parameters estimated *outside* the parenthesis and the total number of parameters estimated *within* the parenthesis. The difference between the numbers inside and outside the parenthesis is the number of α_{ij} and γ_{ij} parameters constrained at zero due to convexity restrictions. The perfect foresight model involves one additional parameter over the fixed effect model, the time discount parameter δ. The stochastic model involves three additional parameters over the fixed effects model, the parameters θ_a, θ_r, and θ_w. While the perfect foresight model is not nested in the fixed effect model, the likelihood result suggests that it is a somewhat better fit than the fixed effects model. The estimate for the time discount factor parameter ranged from 0.61 to 0.91 in the four regressions with a value of 0.84 for the general regression.

The fixed effect model is nested in the stochastic model with θ_a, θ_r and θ_w all set to zero. It is clear on the basis of the likelihood ratio test that the hypothesis that these parameters are equal to zero is easily rejected in every regression. For this reason, and because the stochastic model allows for the determination of long-term effects we discuss later, we focus remaining discussion on the demographic, time separable and general regression of the stochastic model.

Table 3 reports parameter estimates for the demographic, time separable and general regressions for the stochastic model. To provide a graphical representation of life cycle effects, we use our model to estimate labor supply and wealth demand in figures 1.a and 1.b for an individual with no dependents, with $p = 1$, $w = 4.1$, $r = 0.93$ and $A = pW + y = 100{,}000$. The wage and interest rate here are at the 1988 sample average and prices normalized at 1 on 1988 CPI. This shows the labor supply of male headed households peak at age 37 while female have a less pronounced and earlier peak at age 32. There seems to be hump to wealth demands peaking at around 80 years of age.

Table 3. Parameter estimates of the stochastic model across 3 regressions

	Demographic		Time Separable		General	
Parameter	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
α_{12}			8485.75	2652.06	0*	
α_{13}					0*	
α_{22}	1808.59	354.07	0*		0*	
α_{23}					0*	
α_{33}	424833	84282.4	1860870	487094	4769540	2046540
β_{110}	-24084.9	1041.87	-27743.1	1185.82	-43730.8	3045.66
β_{210}	-25884.9	1024.03	-31614.1	1046.61	-57066.6	4440.23
β_{120}	1863.22	30.5587	1826.14	32.4791	1370.48	68.647
β_{121}	-33.4221	1.90699	-31.1563	1.99033	-38.0877	1.91713
β_{122}	-1.38281	0.126621	-1.40077	0.136657	-1.89486	0.140168
β_{123}	0.018931	5.26E-03	0.017638	5.33E-03	0.034528	5.27E-03
β_{124}	-9.92525	13.1198	-7.36611	12.4001	11.1614	11.8681
β_{220}	1432.25	29.4612	1338.11	32.3958	564.093	102.739
β_{221}	-19.7095	1.99692	-18.2984	1.95828	-17.6474	1.94663
β_{222}	-0.95516	0.122262	-0.882254	0.119571	-0.611815	0.123175
β_{223}	0.015583	5.40E-03	0.011555	5.16E-03	2.21E-03	5.27E-03
β_{224}	36.3409	15.1188	37.1858	15.2968	35.5548	15.477
β_{130}	-15198	14131.8	9963.97	18307.7	-88484.1	45188.2
β_{131}	-4144.91	692.667	-3856.06	886.885	-4706.07	840.721
β_{132}	-122.32	49.0145	-176.61	57.4116	-374.934	54.9323
β_{133}	3.55159	1.40508	4.48747	1.70999	9.11641	1.51552
β_{134}	11257.9	18231.8	11491.2	18663.1	26399	14860
β_{230}	-56860.1	5882.58	-59013.2	9017.75	-316884	52255.3
β_{231}	-2234.96	418.422	-2318.37	470.033	-3292.86	626.821
β_{232}	-114.794	29.1574	-108.269	31.8879	-88.184	41.0438
β_{233}	2.77098	0.65582	2.65586	0.703245	2.67919	0.860868
β_{234}	-6251.24	4268.23	-5094.28	4863.94	-2020.92	6479.76
γ_{12}			0.184139	0.037056	-0.125694	0.053804
γ_{13}					34.4974	11.8873
γ_{22}	0.905278	0.040897	0.607157	0.053627	0.396221	0.08324
γ_{23}					51.5958	4.40143
γ_{33}	26.6911	1.36798	24.3494	1.26526	0*	
θ_w	0.242007	0.010481	0.186174	0.01041	0.057391	5.89E-03
θ_r	0.991863	0.354554	0.981783	0.282012	0.874317	0.119846
θ_a	1.04E-05	1.83E-06	6.28E-06	1.37E-06	4.65E-06	5.90E-07

0* indicates a binding convexity restriction on the parameter.

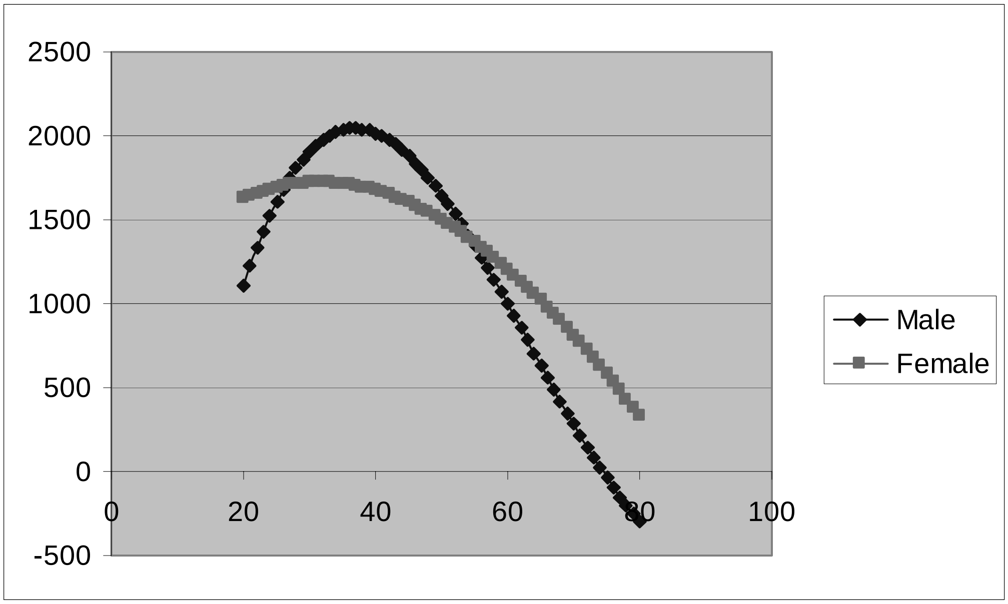

Figure 1.a. Male and female labor supply
(age of head and hours per annum on horizontal and vertical axis respectively)

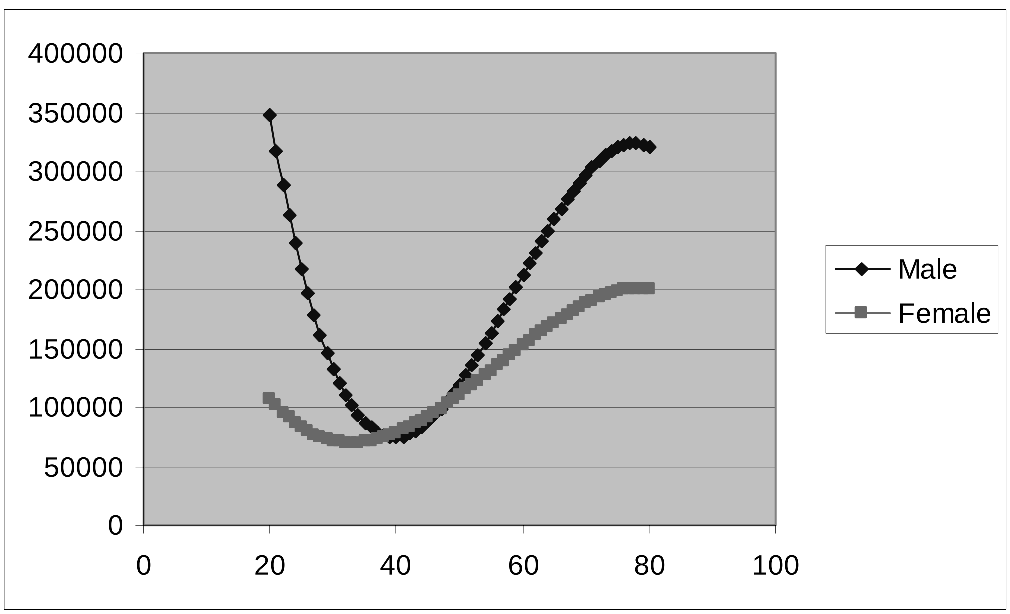

Figure 1.b. Male and female wealth demand
(age of head and nominal wealth on horizontal and vertical axis respectively)

Table 4 gives quantitative indicators of fit for the system of 6 equations for the demographic, time separable and general regressions for the stochastic model. The R-square statistic are for the labor earnings equation $w_t h_t = w_t \pi_w(t) + \varepsilon_t$ equation for t = 1984 to 1988 and wealth equation $r_{1988} W_{1988} = r_{1988} \pi_r(1988) + \varepsilon_{1988}$. Looking across the regressions, relaxing consumption-labor additivity improves the fit of the labor supply equations although the fit of the wealth equation falls. Despite this, it can be seen from the likelihoods and from table 3 that the two additional parameters, α_{12} and γ_{12}, are significant. By relaxing time separability, we have a comparatively larger increase in the likelihood which is mainly attributable to the improved fit of the wealth equation.

Table 4. Statistics of fit for labor earnings and wealth equations for stochastic model across demographic, time separable and general regressions.

Statistic of Fit	Demographic	Time Separable	General
R^2: labor 1984	0.880	0.883	0.883
R^2: labor 1985	0.891	0.893	0.891
R^2: labor 1986	0.918	0.919	0.916
R^2: labor 1987	0.923	0.927	0.928
R^2: labor 1988	0.931	0.933	0.932
R^2: wealth 1989	0.801	0.785	0.827
Log. Likelihood	-29,370.5	-29,364.6	-29,306.3

We show the qualitative fit of our consumption, labor and wealth equations for the general regression in figures 2.a, 2.b and 2.c respectively. In these figures, we plot actual values on predicted values. If our model predicted perfectly, all points would lie on a 45 degree line. We see here that the model fits fairly well without obvious systematic

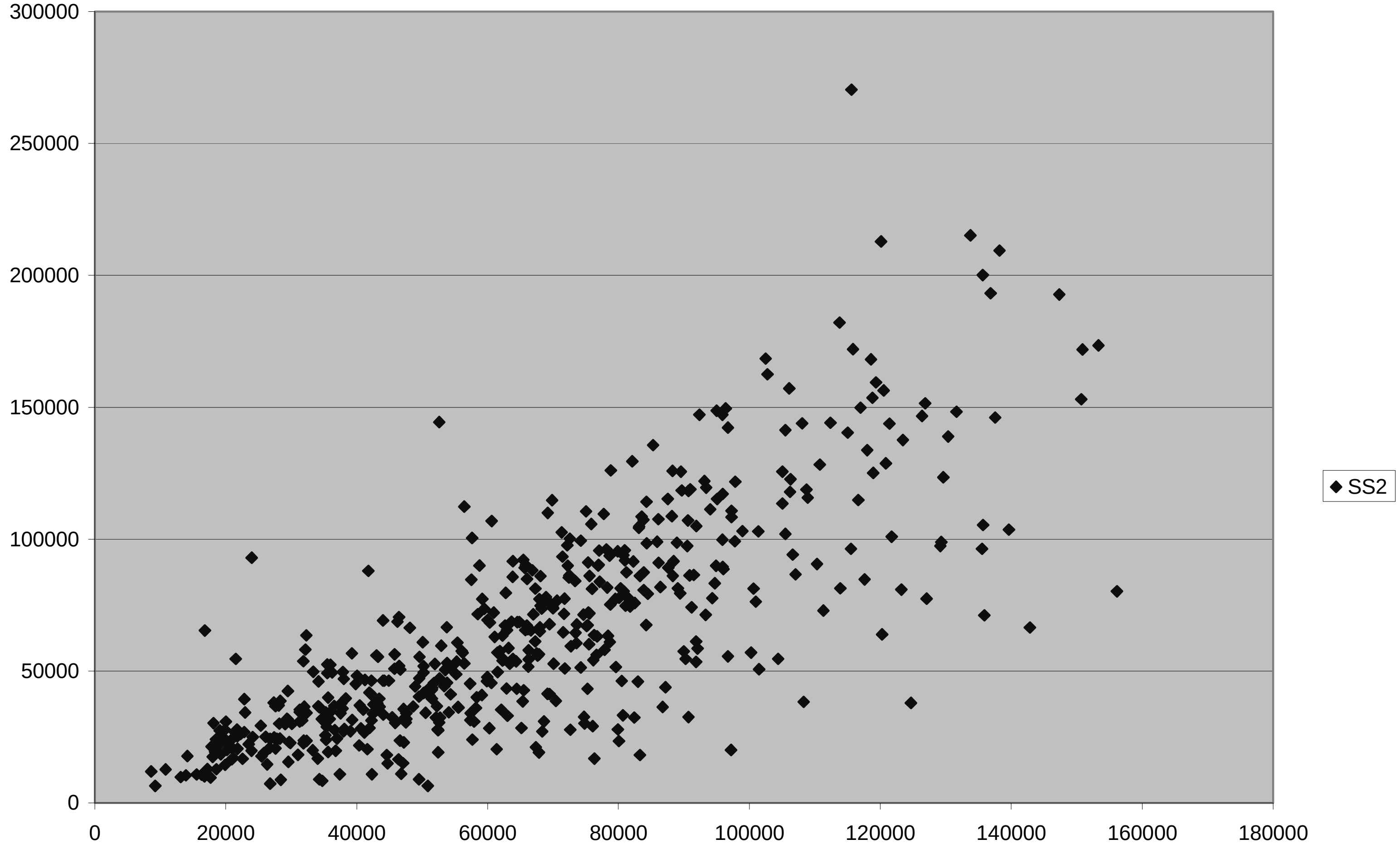

Figure 2.a. Actual 5 year consumption expenditure on predicted consumption in dollars

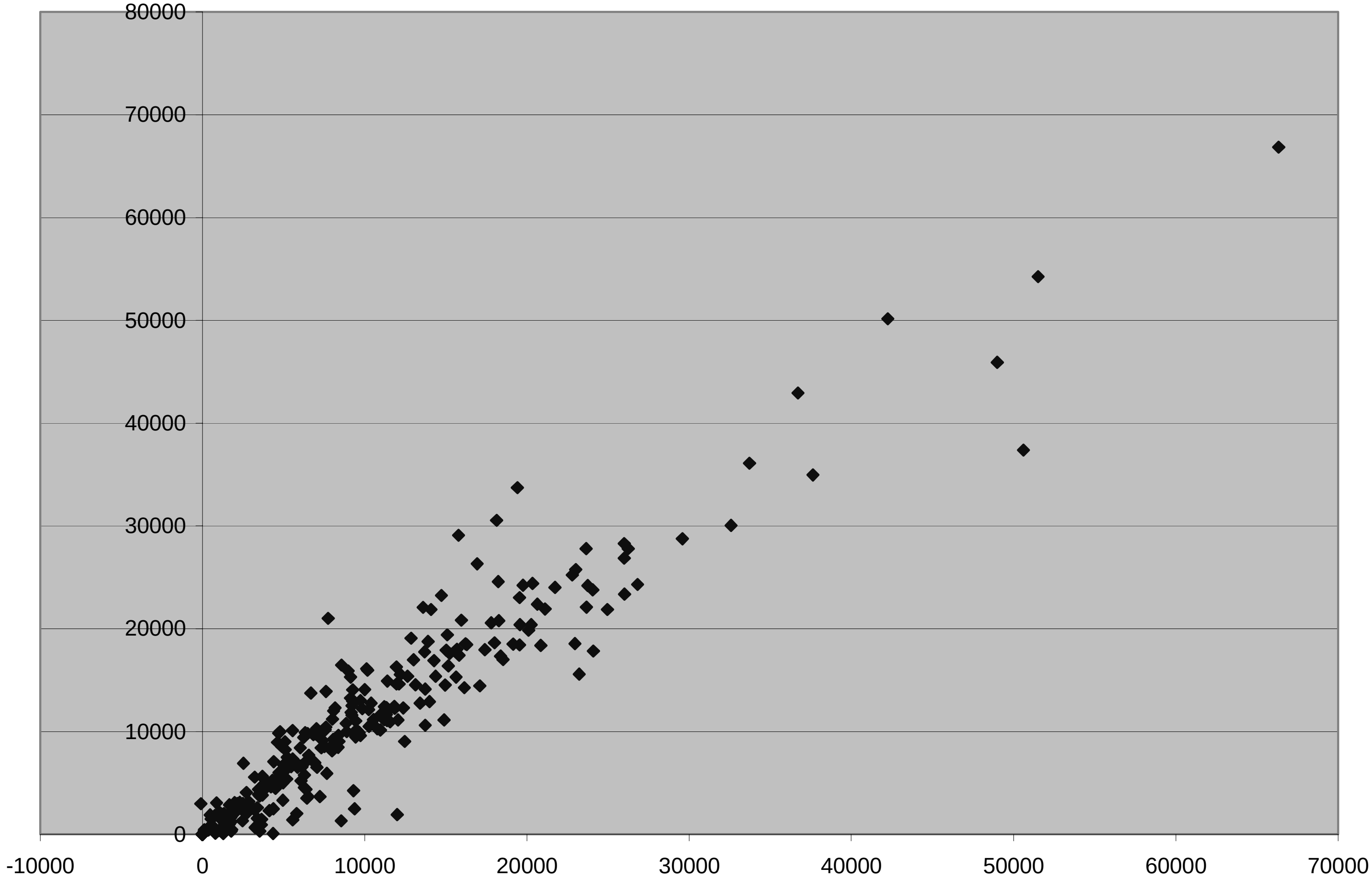

Figure 2.b. Actual labor earnings on predicted earnings in dollars

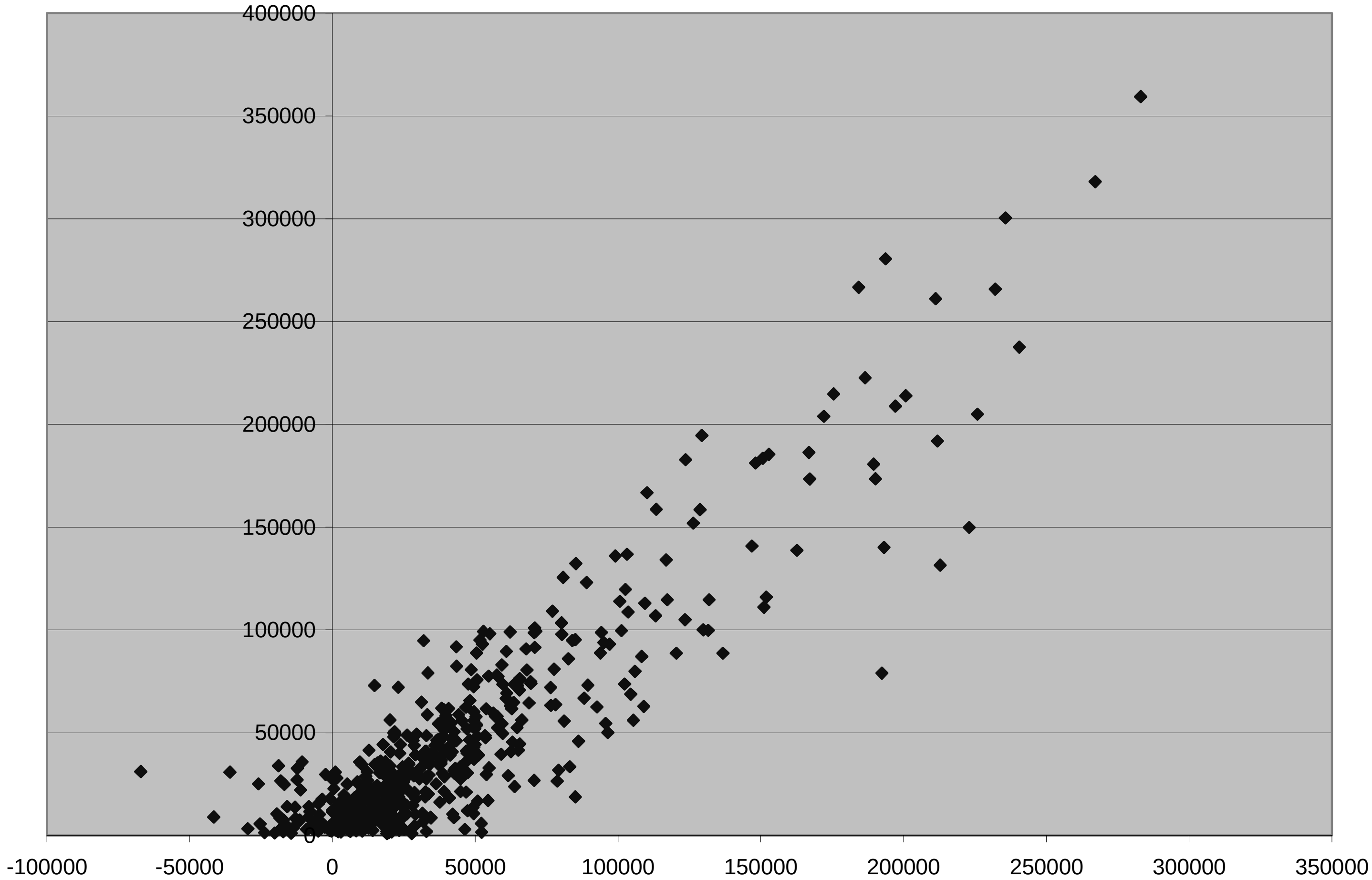

Figure 2.c. Actual wealth on pridicted wealth in dollars.

errors. There appears to be some heteroskedasticity for our consumption equation although this doesn't appear as strong in our labor earnings or wealth equation.

Our final analysis of this model looks at the impact of restrictions that are commonly employed in the literature and their impact on various estimated elasticities. For the structural equation using say wealth as the example we have $-W^* = \pi_r(p, w, r, \mu)$ for which we define Frisch elasticity as the derivative of the log of wealth with respect to the log of any of the arguments. We define short term Marshallian elasticities by substituting the expression for μ into the structural equations. Again using wealth as an example, with structural equation $-W^* = \pi_r(p, w, r, \mu(p, w, r, A))$, we have Marshallian elasticity of wealth with respect to p_i, $\varepsilon^M_{Wp_i} = \varepsilon^F_{Wp_i} + \varepsilon^F_{W\mu}\varepsilon^{\mu}_{\mu p_i}$ where ε_{ij} denotes the elasticity of i={c,h,W} with respect to j={p,w,r} and superscript M and F denote Marshallian and Frisch elasticities respectively and superscript μ denotes the elasticity of μ with respect to any of its arguments.

We report Frisch and Marshallian elasticities for rich which we define as those with beginning wealth and exogenous income, A = 100,000 and poor households which we define as those with A = 50,000. Other prices, held at sample means, are p = 1, w = 4.1 and r = 0.93 and we assume the individual is 45 years of age with no dependents. Table 5 reports predicted consumption, labor and wealth of our rich and poor householders which should be read in conjunction with table 1 which reports mean values of the sample for interpretation.[17]

Table 5. Predicted consumption, labor and wealth for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
Assets (1989 dollars)	88,629.3	840.335	87,817.1	929.616	87,620.7	1,025.03
Consumption (1989 dollars)	24,989.1	781.177	25,736.1	866.169	26,215.5	936.661
Annual Work Hours	1,808.38	25.4853	1,806.35	25.1778	1,878.74	29.0262
	Poor Household					
Assets (1989 dollars)	37,472.7	970.608	36,661.7	1,011.2	37,907.4	1,114.86
Consumption (1989 dollars)	22,861.7	880.241	23,659.1	917.786	22,839.2	1,005.65
Annual Work Hours	1,880.8	28.2984	1,891.35	30.066	1,973.91	31.0949

Table 5 can be evaluated in many respects. In comparing between rich and poor households we see that the main impact of greater wealth is the perpetuation of high wealth holdings. Rich household consume more than poor households although the increment is proportionately less than it is for wealth. Note also that rich work less than poor by about 80-100 hours annually.

In table 6 and 7, we compare the Frisch and Marshallian elasticities respectively for rich and poor households. We see across the regressions of table 6 the effect of excluding certain cross-prices in the estimation of elasticities and how, in the general model, the full set of elasticities can be estimated. Because of the interconnection

[17] The consumption figure in table 1 is actual 5 year consumption whereas the consumption figure here is predicted one year consumption. Otherwise, actual figures of table 1 and the predicted figures of table 5 are comparable.

Table 6. Frisch elasticities for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^F_{Cp}	-0.099	0.010	-0.118	0.016	-0.193	0.055
ε^F_{Cw}			-0.009	0.002	0.008	0.005
ε^F_{Cr}					-0.505	0.102
$\varepsilon^F_{C\mu}$	0.099	0.010	0.127	0.018	0.689	0.125
ε^F_{hp}			0.032	0.008	-0.028	0.015
ε^F_{hw}	0.052	0.006	0.057	0.008	0.013	0.003
ε^F_{hr}					0.285	0.038
$\varepsilon^F_{h\mu}$	-0.052	0.006	-0.088	0.014	-0.271	0.039
ε^F_{Ap}					-0.162	0.031
ε^F_{Aw}					-0.027	0.004
ε^F_{Ar}	-0.917	0.145	-1.346	0.198	-3.099	0.493
$\varepsilon^F_{A\mu}$	0.917	0.145	1.346	0.198	3.289	0.520
	Poor Household					
ε^F_{Cp}	-0.114	0.022	-0.191	0.039	-0.258	0.077
ε^F_{Cw}			-0.013	0.004	0.011	0.006
ε^F_{Cr}					-0.688	0.128
$\varepsilon^F_{C\mu}$	0.114	0.022	0.204	0.042	0.935	0.159
ε^F_{hp}			0.041	0.011	-0.031	0.018
ε^F_{hw}	0.042	0.005	0.050	0.006	0.015	0.004
ε^F_{hr}					0.322	0.037
$\varepsilon^F_{h\mu}$	-0.042	0.005	-0.091	0.014	-0.306	0.037
ε^F_{Ap}					-0.446	0.074
ε^F_{Aw}					-0.074	0.008
ε^F_{Ar}	-1.102	0.221	-2.214	0.409	-7.198	1.052
$\varepsilon^F_{A\mu}$	1.102	0.221	2.214	0.409	7.717	1.116

Table 7. Marshallian elasticities for rich and poor households.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^{M}_{Cp}	-0.128	0.010	-0.143	0.019	-0.212	0.052
ε^{M}_{Cw}	0.010	0.001	0.000	0.002	0.032	0.006
ε^{M}_{Cr}	-0.009	0.016	0.032	0.015	-0.059	0.042
ε^{M}_{CA}	0.127	0.016	0.111	0.021	0.239	0.025
ε^{M}_{hp}	0.015	0.002	0.049	0.009	-0.020	0.015
ε^{M}_{hw}	0.047	0.006	0.051	0.008	0.004	0.003
ε^{M}_{hr}	0.005	0.008	-0.022	0.012	0.110	0.019
ε^{M}_{hA}	-0.067	0.007	-0.078	0.010	-0.094	0.008
ε^{M}_{Ap}	-0.263	0.012	-0.266	0.014	-0.255	0.023
ε^{M}_{Aw}	0.091	0.002	0.095	0.002	0.085	0.002
ε^{M}_{Ar}	-0.997	0.006	-1.012	0.006	-0.971	0.013
ε^{M}_{AA}	1.169	0.012	1.182	0.014	1.141	0.016
	Poor Household					
ε^{M}_{Cp}	-0.170	0.037	-0.238	0.047	-0.263	0.071
ε^{M}_{Cw}	0.022	0.008	0.008	0.005	0.044	0.009
ε^{M}_{Cr}	0.010	0.018	0.104	0.023	0.060	0.058
ε^{M}_{CA}	0.138	0.045	0.126	0.035	0.158	0.017
ε^{M}_{hp}	0.021	0.004	0.062	0.012	-0.030	0.018
ε^{M}_{hw}	0.034	0.003	0.040	0.006	0.004	0.004
ε^{M}_{hr}	-0.004	0.007	-0.046	0.012	0.077	0.018
ε^{M}_{hA}	-0.051	0.012	-0.056	0.009	-0.052	0.004
ε^{M}_{Ap}	-0.540	0.052	-0.515	0.059	-0.484	0.066
ε^{M}_{Aw}	0.214	0.008	0.231	0.008	0.202	0.009
ε^{M}_{Ar}	-1.007	0.014	-1.083	0.019	-1.021	0.035
ε^{M}_{AA}	1.333	0.056	1.366	0.053	1.304	0.043

between Frisch and Marshallian elasticities given above, greater generality affect all estimates.

We draw attention to a few elasticities to illustrate the model and the effect of the restrictions. Consider the Frisch elasticity of consumption with respect to interest rate. Note that because $r_t = p_{t+1}/(1+i_t)$, a *one percentage point increase* in i_t will result in an approximate one percent *decrease* in r_t. With time separability, interest rates do not exert an independent effect conditional on μ as seen the Frisch elasticities of table 6. When this channel is allowed for in our general regression, we see a statistically significant and considerable effect which becomes stronger for poorer households. Going now to the same cells of table 7 which report Marshallian elasticities, we see with the time separable regression that a one percentage point increase in interest rates lead to approximately a 0.032% and 0.104% decrease in consumption for rich and poor households respectively. On the basis of t-scores, both these effects are significant. On the other hand, when this effect is estimated with the general regression, one finds that a one percentage point increase in interest rates has the effect of *increasing* rich household consumption by 0.059% but *decreasing* poor household consumption by 0.060%. The conclusions one draws about the impact of interest rates on consumption are reversed by our better fitting general regression. Other patterns that are noteworthy are that the positively sloped labor supply curve becomes appreciably more inelastic when estimated with the general regression and that the effect of increased wealth on labor supply differentially affects rich and poor households.

Our general model has one other feature which is noteworthy: the ability to distinguish between household variation in μ arising from cross sectional variation and within household across time variation in μ_t arising from innovations in wages, interest rates or wealth. From above, μ depends on prices, wages, interest rates, exogenous income and starting wealth and its *variation* among households is a function of cross-sectional household variation of real wages, interest rates, exogenous income and starting wealth. Consequently, an increase of the wage in say 1985 is combined with wages in other years, interest rates, exogenous income and so forth before it results in an increase in μ.

However, we can also compute a corresponding elasticity of μ_t with respect to real wages, real interest rates or real wealth as we have for μ. Given the expression $\mu_t = \exp(\theta_w (w_t / p_t - \overline{w/p}) + \theta_r (r_t / p_t - \overline{r/p}) + \theta_a (W_t - \overline{W}))\mu$, the elasticity of μ is $\varepsilon^{\mu}_{\mu i} = i\theta_i$ where index i= w, r and a and here denotes real wage, interest rates and wealth respectively. Table 8, divided into three panels, reports the elasticity of μ with respect to real wages, real interest rates and real wealth for rich and poor in the first and second panel and reports the similarly constructed within household across time elasticity of μ_t in the third panel.

Table 8. Cross sectional and time series μ elasticities.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
$\varepsilon^{\mu}_{\mu w}$	0.099	0.015	0.071	0.010	0.034	0.004
$\varepsilon^{\mu}_{\mu r}$	-0.087	0.165	0.248	0.106	0.647	0.046
$\varepsilon^{\mu}_{\mu A}$	1.275	0.197	0.878	0.126	0.347	0.055
	Poor Household					
$\varepsilon^{\mu}_{\mu w}$	0.194	0.034	0.104	0.017	0.036	0.004
$\varepsilon^{\mu}_{\mu r}$	0.086	0.171	0.511	0.085	0.800	0.020
$\varepsilon^{\mu}_{\mu A}$	1.209	0.199	0.617	0.100	0.169	0.024
	Within Elasticity					
$\varepsilon^{\mu}_{\mu w}$	0.992	0.043	0.763	0.043	0.235	0.024
$\varepsilon^{\mu}_{\mu r}$	0.922	0.330	0.913	0.262	0.813	0.111
$\varepsilon^{\mu}_{\mu A}$	1.043	0.183	0.628	0.137	0.465	0.059

The estimate of the elasticity of μ and μ_t with respect to increases in assets is comparable whether this is evaluated in cross section or in time series although this is derived from very different methodologies. The cross sectional variation of wealth for the sample gives us our estimate of this elasticity which is predominantly determined up by our β parameters. On the other hand, changes in the level of individual wealth over a five year span give us our estimate of time series elasticity which is solely a function of the parameter θ_a. Additionally, our procedure of interpolating wealth between the beginning and end of the 5 year span means that the change in wealth isn't truly a surprise. The consistency between these two estimates suggests this elasticity is estimated with some accuracy.

The elasticity of μ and μ_t with respect to wages however differs with the impact of wages in time series being almost 7 times stronger than it is in cross section. We account for this by the different construction of these elasticity estimates. The elasticity of μ with respect to wages combines many other variables into a single scalar measure. Although higher wages are likely to persist, no account of this is taken with our cross sectional measure of elasticity.

On the other hand, this restriction is not imposed in our time series estimate of elasticity. One can imagine, for example, an individual with low wages for the first 4 years of our sample with an unexpected increase in wages in the final year of our sample. This individual may expect this new wage to continue and adjust consumption, work hours and savings target by a larger amount than a similarly situated individual whose wages will fall to the lower level in subsequent years.

If such an interpretation is correct, we can now compute long term Marshallian elasticities. Given the relationship between Frisch and Marshallian elasticities, long term Marshallian elasticities defined by substituting the time series elasticity term $\varepsilon^{\mu}_{\mu Pi}$ reported in the third panel of table 8 into the expression $\varepsilon^{M}_{Wp_i} = \varepsilon^{F}_{Wp_i} + \varepsilon^{F}_{W\mu}\varepsilon^{\mu}_{\mu p_i}$ effectively capitalizing the impact of new and persistent prices into μ. Such defined long term Marshallian elasticities are reported in table 9.

Table 9. Long term Marshallian Elasticities.

	Demographic		Time Separable		General	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
	Rich Household					
ε^{M}_{Cw}	0.099	0.012	0.088	0.010	0.170	0.021
ε^{M}_{Cr}	0.092	0.035	0.116	0.037	0.056	0.094
ε^{M}_{CA}	0.104	0.022	0.080	0.018	0.320	0.043
ε^{M}_{hw}	0.000	0.002	-0.011	0.004	-0.050	0.005
ε^{M}_{hr}	-0.048	0.017	-0.081	0.025	0.065	0.033
ε^{M}_{hA}	-0.055	0.011	-0.056	0.013	-0.126	0.013
ε^{M}_{Aw}	0.910	0.150	1.028	0.135	0.747	0.082
ε^{M}_{Ar}	-0.071	0.302	-0.117	0.355	-0.425	0.392
ε^{M}_{AA}	0.956	0.213	0.846	0.185	1.528	0.137
	Poor Household					
ε^{M}_{Cw}	0.114	0.021	0.142	0.027	0.231	0.029
ε^{M}_{Cr}	0.106	0.041	0.186	0.067	0.072	0.129
ε^{M}_{CA}	0.060	0.015	0.064	0.018	0.217	0.031
ε^{M}_{hw}	0.000	0.002	-0.019	0.006	-0.057	0.005
ε^{M}_{hr}	-0.039	0.014	-0.083	0.026	0.073	0.037
ε^{M}_{hA}	-0.022	0.004	-0.028	0.006	-0.071	0.007
ε^{M}_{Aw}	1.094	0.220	1.690	0.273	1.742	0.167
ε^{M}_{Ar}	-0.086	0.363	-0.192	0.585	-0.923	0.912
ε^{M}_{AA}	0.575	0.141	0.696	0.159	1.793	0.148

The most notable feature of these estimates when compared to those of table 7 giving short term Marshallian elasticities is the effect of wages. For example, the short term response of a rich household to a one percentage increase in wages on consumption, labor supply and wealth is 0.032%, 0.004% and 0.085% respectively but when this evaluated over the long term, the corresponding change is 0.170%, -0.050% and 0.747%. The labor supply curve to long term wages is now backward bending. Looking across regressions, the slope of the more restrictive functional forms understate the extent of the backward bend in labor.

CONCLUSION

Our intertemporal model treats intertemporal choice as a three good problem with choice variables consumption, labor supply and savings subject to a dynamic budget constraint. The functional form used is globally regular and rank 3 and can be seen to fit the cross-sectional data well. With this model, we tested two commonly maintained hypotheses and decisively rejected both consumption-labor additivity and time separability. This cast doubt on contemporary elasticity estimates made using the more restrictive forms. In particular, the common assertion that consumption falls when interest rates rises holds only for households with low wealth. Also, the common assertion that labor supply is positively sloped was found only to apply to transitory wage increases. Should the wage increase be permanent or at least persistent, the wealth effect of higher wages causes labor supply to contract. These results have important policy implications, in the exercise of monetary wage policies for example.

However, more than the particulars of our findings, what we hope we offer to the profession is a fruitful approach that opens new areas in economics. To this end, we offer what we consider as worthwhile extensions starting with technical refinements and then moving towards using the information gained from this approach in other fields.

On a technical level, we estimated preferences for single headed households using what amounts to a single cross-section. Although the data was a panel, we do not have true within and between errors analogous to variance component models. To do this, a third wealth supplement is required. This would then allow analysis of household behavior across time and in cross-section. This was not done in this paper because, at the time of writing, final release PSID data for the next wealth supplement in 1994 was unavailable. The use of early release data would have allowed for analysis to include wealth from the 1994, 1999 and 2001 waves of the PSID however we relied on numerous constructed variables unavailable in the early release files.

Another important development would be extending this to couples. The number of married and joint households is 3 times bigger that single headed households which allows for a better analysis simply from having more data alone. The household intertemporal problem can be conceived of as a four good problem: how much to consume, how much to save, and the labor supply of the head and the spouse as separate choices. It would be interesting to allow for the important interactions between head and spouse and possibly dependents.

Another extension is to model the demand for particular classes of assets. We aggregated all forms of wealth and derived a composite return of this wealth. However, given that that the PSID has the individual return of many assets, it would seem possible to treat these as distinct goods. This would then allow the analysis of the substitutability or complementarity of the different assets. Other extensions and refinement would be to further generalize the functional form and to improve the efficiency of the estimation procedure which had been in this case a substantial challenge.

It would seem that once a good understanding of this model is developed and becomes widely know, this model, with the inherent marginal of utility of income assessable, is apt to make contributions in the fields of social welfare, static and dynamic general equilibrium, tax policy analyses among other. We offer this in the hope economist find this fertile ground.

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