The technological theory of production and a method of decomposition of the rate of GDP in terms of labour and capital services

Vladimir N. Pokrovski[*]
Moscow University of Economics and Statistics

Abstract

It is assumed that performance of production system can be described with the three variables: amount of production equipment – capital stock K and 'consumption' of labour L and capital services S. It is shown that the production function can be specified as the known Cobb-Douglas production function, in which capital services S stands instead of capital stock K, while the state of the production system itself is specified by the technological index α. Capital stock plays the role of the means through which the labour resource is substituted by capital services. A method for estimating of capital services and the technological index due to known time series of the output Y, capital stock K and labour L is developed which allows one to separate contributions from production factors and structural change. Empirical evidence for the US economy is used to estimate the validity of the proposed theory.

JEL - classification: C00, E10, O47.

Key words: Capital productivity; Economic Growth; Investment; Labour productivity; Production function; Solow residual; Technology

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[*]Visiting Research Fellow at University of Malta, Msida, MSD 06 MALTA. Address for correspondence: IDA House, Triq il-Ward, Madliena, NXR 09 MALTA. Tel/Fax: +356 21377446; e-mail address: vpok@waldonet.net.mt

1 Introduction

The conventional neo-classical approach to the problem of production distinguishes between influence of production factors and something which is connected with change (the structural or technological change) of the production system itself. The neo-classical solution of the problem, classically exposed in a famous work by Cobb and Douglas (1928), is to consider output Y to be determined by production equipment measured by its value K (capital stock) and work of labourers L measured, for example, in working hours per year. The dependence is conventionally approximated by function

$$Y = Y_0 \frac{L}{L_0} \left(\frac{L_0}{L} \frac{K}{K_0} \right)^{\alpha}, \tag{1}$$

where the index α is a characteristic of production system and has meaning of the share of expenses needed for utilisation of capital stock as a production factor in total expenses for production factors. The conventional decomposition of the growth rate

$$\frac{1}{Y}\frac{dY}{dt} = (1-\alpha)\frac{1}{L}\frac{dL}{dt} + \alpha \frac{1}{K}\frac{dK}{dt} + q \tag{2}$$

defines the total factor productivity or the Solow residual q, which includes both changes in capital and labour services and changes (technological and structural) of production system itself. Equations (1) and (2) are used as a base for interpretation of empirical facts, while the actual decomposition depends on empirical values of index α which are estimated as 0.3 - 0.4 for the US economy. However, it is known that

$$Y \sim K \tag{3}$$

so that index α in equations (1) and (2) must have value $\alpha \approx 1$. Consideration of this contradiction lead to introduction concepts of capital services and labour services (Solow, 1957). These quantities are connected with capital stock and use of labour, but different from them. Some scholars (Jorgenson and Griliches, 1967; Jorgenson and Stiroh, 2000) found that a concept of 'capital and labour services' could be very useful to explain the observed growth of productivity. Capital service S does not exist without capital stock K, but is different from it, so that, generalising the neo-classical approach, one can include capital service S in the list of production factors equally with production factors of conventional neo-classical economics – capital stock K and labour L. It is convenient to consider production of value Y to depend on the three production factors

$$Y = Y(K, L, S) \tag{4}$$

The proposed paper introduces capital services as an independent production factor into the theory of production of value. The theory begins with a description of a production system which may be viewed as a collection of equipment (measured by its value K), getting its ability to act from labour (L) and capital services (S) inputs. It will be shown that production function (4) can be specified in the form of two alternative lines

$$Y = \begin{cases} \xi K \\ Y_0 \dfrac{L}{L_0} \left(\dfrac{L_0}{L} \dfrac{S}{S_0} \right)^{\alpha} \end{cases} \tag{5}$$

The complementary descriptions of production of value can be traced back. The first line in formula (5) reminds us about Harod-Domar approach, while the function in the second line coincides with the Cobb-Douglas production function, in which productive energy S stands in the place of capital stock K. The productivity of capital stock ξ in equation (5) is an internal characteristic of the production system itself, which is connected with index α, so that one has the only characteristic of the production system itself.

Assuming that time series of the quantities of Y, K and L are known, a method for estimating of capital services and the index α is developed, so that the influence of the production factors and the structural change on the growth rate of output can be separated. In Section 2, the paper discusses the main principles of the theory, which were also discussed in a previous publication of the author (Pokrovski, 2003). In Section 3, the ability of the theory to describe a real situation is illustrated for the US economy.

2 Derivation of Basic Equations

Technology and Dynamics of Production Factors

From a material point of view the process of production is a process of transformation of raw materials into finished and semi-finished items, semi-finished items into other semi-finished and finished items and so on, until the finished items, which can be used by man, are made. A method of producing is technology that determines, first, what one needs to produce, that is the material side of the process of production. Different appliances are invented to perform transformations. This is a material realisation of technology, i.e. production equipment. The total collection of it is estimated by its value K that obeys to the balance relation

$$\frac{dK}{dt} = I - \mu K \tag{6}$$

where production investment I is a part of the output accumulated in a material form of production equipment.. The second term on the right side of Eq. (6) describes the decrease of capital due to the removal from service with the depreciation coefficient μ. The amount and the distribution of production capital with time are thus comprehensively determined by the history of investments. The major part of production capital comes from recent investments.

The applied technology determines that one needs a certain amount of efforts of human being L and a certain amount of capital services S to produce something, while human efforts can be substituted by capital services. The expansion of production, characterised by changes of the accumulated value, requires additional labour and capital service, so that dynamics of the production factors can be derived as the balance equations

$$\frac{dL}{dt} = \lambda I - \mu L = \left(\bar{\lambda}\frac{I}{K} - \mu\right) L, \qquad \frac{dS}{dt} = \varepsilon I - \mu S = \left(\bar{\varepsilon}\frac{I}{K} - \mu\right) S. \tag{7}$$

The first terms in the right side of these relations describe the increase in the quantities caused by gross investments I; the second terms reflect the decrease in the corresponding quantities due to the removal of a part of the production equipment from service. Coefficients λ and ε determine the required amount of labour and capital services per unit of investment and are characteristics of introduced technology: therefore they can be denominated as technological coefficients: the labour requirement (λ) and capital services requirement (ε). It is convenient to use the dimensionless technological variables

$$\bar{\lambda}(t) = \frac{K}{L}\,\lambda, \qquad \bar{\varepsilon}(t) = \frac{K}{S}\,\varepsilon.$$

If these quantities are less than unity, it means that labour-saving and capital-service-saving technologies are being introduced at the time. Due to Eqs. (6) and (7) the technological variables can be written as

$$\bar{\lambda} = \frac{\nu + \mu}{\delta + \mu}, \qquad \bar{\varepsilon} = \frac{\eta + \mu}{\delta + \mu}. \tag{8}$$

where notations for the growth rates of production factors are introduced

$$\delta = \frac{1}{K}\frac{dK}{dt}, \quad \nu = \frac{1}{L}\frac{dL}{dt}, \quad \eta = \frac{1}{S}\frac{dS}{dt} \tag{9}$$

The depreciation coefficient μ can be excluded from Eqs. (8). So, one can obtain the relation between different real rates of growth

$$\delta = \nu + \alpha\,(\eta - \nu), \quad \alpha = \frac{1 - \bar{\lambda}}{\bar{\varepsilon} - \bar{\lambda}}. \tag{10}$$

The quantity α – the technological index appears to be a very important characteristic of the production system. Equation (10) connects four quantities: δ, ν, η and α, while two of them (δ and ν) are considered known. So, Eq. (10) appears to be relation between the unknown growth rate of capital service η and the technological index α.

Let us note that the simple relation (10) was obtained at simplifying assumption that deterioration of labour and capital services coincides with deterioration of capital stock, that is the coefficients of depreciation in Eqs (7) are equal to the one in Eq. (6). It is not difficult to obtain a more general relation.

Production of Value

To determine production of value, we have to reduce function (4) to the form which is consistent with the above technological description of the production process. First, as there is a relation (10) among the growth rates of the production factors, the variables K, L and S appear to be interdependent: only two of the arguments of function (4) are independent. Then, the technological description assumes that one ought to consider capital services and labour inputs as substitutes to each other, and amount of production equipment, universally measured by its value K, has to be considered as a complement to L and S. All this urges us to write production function (4) in a form of the two alternative functions

$$Y = \begin{cases} Y(K) \\ Y(L,S) \end{cases}, \qquad dY - \Delta dt = \begin{cases} \xi(K)\,dK \\ \beta(L,S)\,dL + \gamma(L,S)\,dS \end{cases} \tag{11}$$

where Δdt is a part of an increment of production of value that is connected with change of characteristics of the production system (the structural change). When the structural change is zero, the marginal productivities ξ, β and γ correspond to the value produced by the addition of a unit of capital or by the addition of a unit of labour input at constant capital services or by the addition of a unit of capital services at constant labour input, respectively. In line with the existing practice these quantities can be labelled as marginal productivities of the corresponding production factors. We have to consider that all marginal productivities are positive. One uses production factors to create useful commodities and an addition of any production factor must increase in production of things – this is known as the productivity principle.

One can use equations for the production factors (6) and (7) to rewrite relations (11) for production of value in the form

$$\frac{dY}{dt} - \Delta = \begin{cases} \xi\,(I - \mu\,K) \\ (\beta\,\lambda + \gamma\,\varepsilon)\,I - \mu\,(\beta\,L + \gamma\,S) \end{cases} \tag{12}$$

The right-hand sides of these equations are equal to each other, so that one can write equations for the characteristic parameters of the system and discover relations among the marginal productivities

$$\beta = \xi\,\frac{\bar{\varepsilon} - 1}{\bar{\varepsilon} - \bar{\lambda}}\,\frac{K}{L}, \quad \gamma = \xi\,\frac{1 - \bar{\lambda}}{\bar{\varepsilon} - \bar{\lambda}}\,\frac{K}{S} \tag{13}$$

If the technological coefficients $\bar{\lambda}$ and $\bar{\varepsilon}$ take arbitrary values, Eqs. (13) permit that one of the marginal productivities, but not both, can be negative. One can see that, if relations

$$\bar{\lambda} < 1 < \bar{\varepsilon} \quad or \quad \bar{\lambda} > 1 > \bar{\varepsilon} \tag{14}$$

are valid, the marginal productivities are non-negative, so that these relations can be considered as an expression of the productivity principle.

The forms (11) can be considered as a generalisation and extension of the conventional neo-classical approach. The first line corresponds to so-called AK-theories, the second - to the Sollow's approach. Two roles of capital in the latter: a measure of an amount of capital equipment and a substitute for labour are separated and presented by two variables in our approach: capital stock K and capital services S, while the perfect substitution of labour and capital service does not lead to any discrepancies in contrast to assumption of substitution of capital stock and labour. Capital stock is considered the means of attracting labour and capital services to production, while human efforts and capital services are considered the true sources of value. Human work is replaced by work of different sophisticated appliances.

Now, one can write a simple approximation for the marginal productivities. So as the description ought to be valid for any initial point, one approximates production function, assuming also that production is homogeneous, as a power function

$$Y = Y_0 \frac{L}{L_0} \left(\frac{L_0}{L} \frac{S}{S_0} \right)^{\alpha}, \tag{15}$$

where α is a characteristic of the production system that, as shown below, coincides with the technological index introduced in the previous Subsection (see Eq. 10). Indeed, the above relations provide the following expressions for marginal productivities and the contribution from structural change

$$\beta = Y_0 \frac{1-\alpha}{L_0} \left(\frac{L_0}{L} \frac{S}{S_0} \right)^{\alpha}, \quad \gamma = Y_0 \frac{\alpha}{S_0} \left(\frac{L_0}{L} \frac{S}{S_0} \right)^{\alpha-1}, \quad \varDelta = Y \ln \left(\frac{L_0}{L} \frac{S}{S_0} \right) \frac{d\alpha}{dt} \tag{16}$$

where L_0 and S_0 are the values of labour and capital services in the base year. Having compared expressions (13) and (15) for marginal productivities, one obtains

$$\xi = Y_0 \frac{L}{L_0 K} \left(\frac{L_0}{L} \frac{S}{S_0} \right)^{\alpha}, \qquad \alpha = \frac{1-\bar{\lambda}}{\bar{\varepsilon}-\bar{\lambda}} \tag{17}$$

Thus, the index α in Eq. (15) is, indeed, the same quantity as introduced by Eq. (10). The productivity principle restricts values of the technological index, $0 < \alpha < 1$. Besides, all available information about the technological performance could be introduced at estimating of this quantity. Moreover, a condition regarding the optimal use of production factors enables us to establish a relation between the parameter α on one hand and the shared costs of production factors on the other. This provides the different means of estimating of the technological index.

Decomposition of the Growth Rate of Output

The previous results allow us to write the expression for production of value as

$$Y = \begin{cases} \xi K \\ Y_0 \dfrac{L}{L_0} \left(\dfrac{L_0}{L} \dfrac{S}{S_0} \right)^{\alpha} \end{cases} \tag{18}$$

so that the growth rate of output in terms of the present theory can be written as two alternative expressions

$$\frac{1}{Y} \frac{dY}{dt} = \begin{cases} \dfrac{1}{K} \dfrac{dK}{dt} + \dfrac{1}{\xi} \dfrac{d\xi}{dt}, \\ (1-\alpha) \dfrac{1}{L} \dfrac{dL}{dt} + \alpha \dfrac{1}{S} \dfrac{dS}{dt} + \ln \left(\dfrac{L_0}{L} \dfrac{S}{S_0} \right) \dfrac{d\alpha}{dt} \end{cases} \tag{19}$$

The first terms present contribution to growth due to growth of production factors: capital, labour and capital services, the last ones – due to the change of the production system itself; the changes of quantities ξ and α are connected with each other due to the formula, which follows the above relations,

$$\frac{1}{\xi} \frac{d\xi}{dt} = \ln \left(\frac{L_0}{L} \frac{S}{S_0} \right) \frac{d\alpha}{dt} \tag{20}$$

The growth rate of capital marginal productivity is determined by technological and structural changes and cannot be reduced to any function of production factors.

One can use relations (7) and (10) to rewrite expression for the growth rate of output in the form

$$\frac{1}{Y}\frac{dY}{dt} = \frac{\nu + (1-\bar{\lambda})\mu}{\bar{\lambda}} + \frac{1}{\xi}\frac{d\xi}{dt} \tag{21}$$

One can see that the growth rate of output is determined by four quantities:

- Productivity of capital stock ξ – this is a fundamental quantity which can be calculated at more detailed approaches. If a multi sector approach (input-output model) is applied, this quantity is connected with the fundamental technological matrixes. Technological and structural changes are being introduced via this quantity.
- The dimensionless technological coefficient $\bar{\lambda}$. If the quantity $\bar{\lambda} < 1$, the consumption (for unit of capital stock) of labour decreases and consumption of capital services (whatever it is) increases. The situation is opposite, if the quantity $\bar{\lambda} > 1$.
- The coefficient of depreciation μ. This quantity does not affect the rate of output growth, if $\bar{\lambda} = 1$.
- The rate of growth of labour ν. The rate of output growth coincides with this quantity, if $\bar{\lambda} = 1$.

Equation (19) can be compared with the conventional decomposition (2) of the growth rate. One can see that the total factor productivity or the Solow residual can be defined as

$$q = \alpha\left(\frac{1}{S}\frac{dS}{dt} - \frac{1}{K}\frac{dK}{dt}\right) + \ln\left(\frac{L_0}{L}\frac{S}{S_0}\right)\frac{d\alpha}{dt} \tag{22}$$

or, by using equation (10), as

$$q = (1-\alpha)\left(\frac{1}{K}\frac{dK}{dt} - \frac{1}{L}\frac{dL}{dt}\right) + \ln\left(\frac{L_0}{L}\frac{S}{S_0}\right)\frac{d\alpha}{dt} \tag{23}$$

The first terms of the last two equations are connected with difference between the growth rates of capital services and capital stock. The second terms present the technological and structural change. Decomposition of the Solow residual will be illustrated for the US economy in the next Section.

3 Application to the US economy

To illustrate the applicability of the theory to describe a real situation, we refer to the latest available time series for the US economy. Values of gross national product Y from year 1959 and capital K from year 1925 are available on a website of the US Bureau of Economic Analysis (http://www.bea.doc.gov). Capital K is understood, in terms of the US Bureau of Economic Analysis, as a sum of private fixed assets, government fixed assets and consumer durable goods. The time series for labour L for the latest decades (from year 1948) are found on a website of the US Bureau of Labour Statistics (http://www.stats.bls.gov). The series of relative quantities compiled by Scott (1989) are used to restore absolute values of quantities for earlier years. We use the series (the numbers can be found in the recent article (Pokrovski, 2003)) for illustration of methods of estimating of capital services and decomposition the growth rate of output.

The Technological Index and Personal Consumption

The technological index α appears to be a very important characteristic of the production system. The value of the technological index can be estimated at regarding optimal use of the production factors. One can assume that production factors L and S are chosen in such amounts, so as they must be the most effective in production, that is, values of production factors maximise production function (15) at given total expenses for production factors

$$cL + pS = V,$$

where c and p are cost of 'consumption' of the production factors, and V is a part of gross output which goes for maintenance of production factors.

One can follow a conventional method of finding a conditional extremum to determine the extremum point and to find that the technological index α can be expressed through prices and amounts of production factors

$$\alpha = \frac{pS}{cL + pS}. \tag{24}$$

So, the technological index α represents the share of expenses needed for utilisation of capital services as a production factor in total expenses for production factors. If production factors are chosen as optimal, then

$$0 < \alpha < 1$$

which coincides with conditions of positivity of marginal productivities (see formulae (14) in the previous Section).

Expression (24) allows one to estimate the technological index α due to estimates of cost of consumption of production factors. It is clear that amount of value needed

to support capital services S during a year is equal to μK, so that the cost of consumption of capital services is

$$p = \frac{\mu K}{S} \tag{25}$$

The current consumption $C = cL$ is value of the minimum amount of products, which are needed for the humans to subsist. Perhaps, the proper quantity to

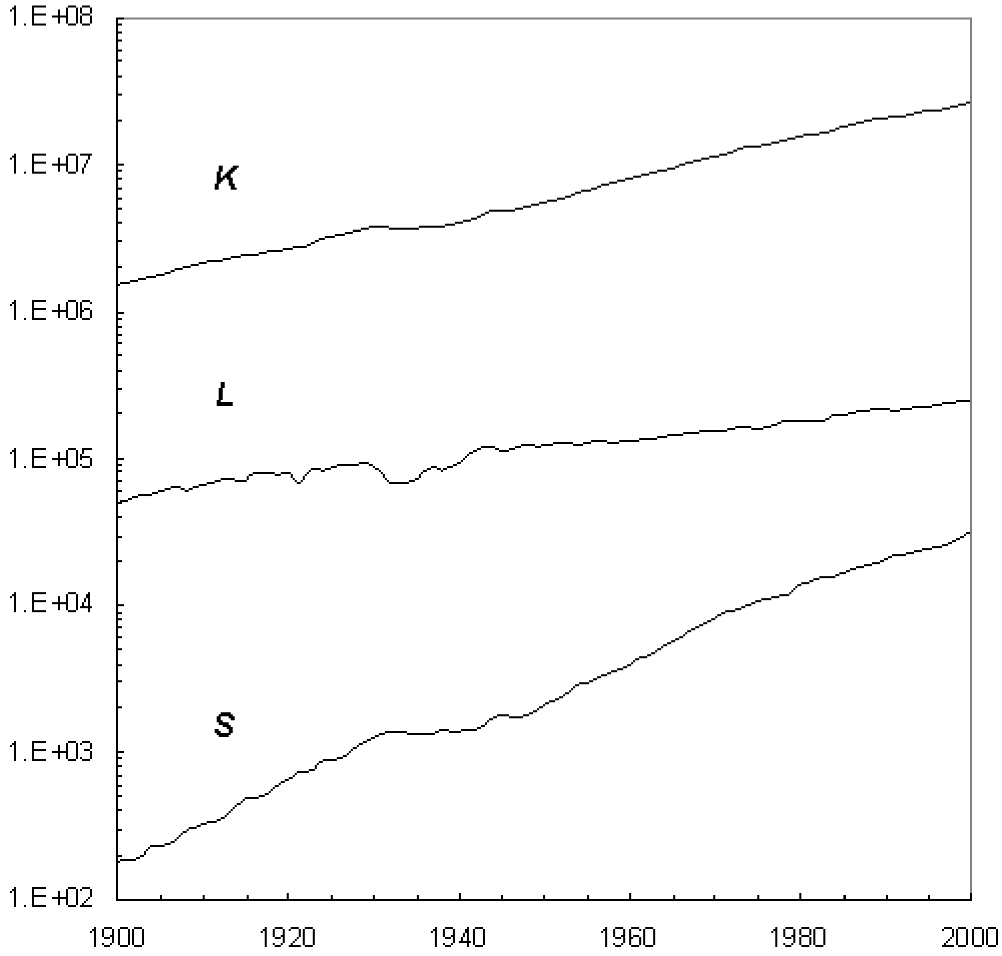

Figure 1. Production Factors in the US Economy

Capital stock (amount of basic production equipment) K in million 1996 dollars; consumption of labour L in million hours of labour per year and capital services S in arbitrary units.

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characterise the necessary consumption is the poverty threshold used in the US statistics. The estimates of these quantities for a person in different family situations since year 1959 can be found on the US Bureau of Census website (http://www.census.gov). One can consider the poverty threshold per a single person to be a realistic estimate of the current consumption. For year 1996, for example, this quantity is estimated as 7995 dollars per a person per year. This quantity ought to be multiplied by the number of population to get an estimate of the consumption in year 1996 as $C = 2,120,506$ million of the dollars compared with the expenses for maintenance of consumption of capital services $pS = \mu K = 1,378,880$ million of dollars (1996). So, for the latest decade of the twentieth century, one can get $\alpha \approx 0.4$.

Production Factors and the Technological Index

Empirical values of capital K and labour L are known and depicted on the plot in Fig. 1 with solid lines. The third production factor – capital services S – has to be

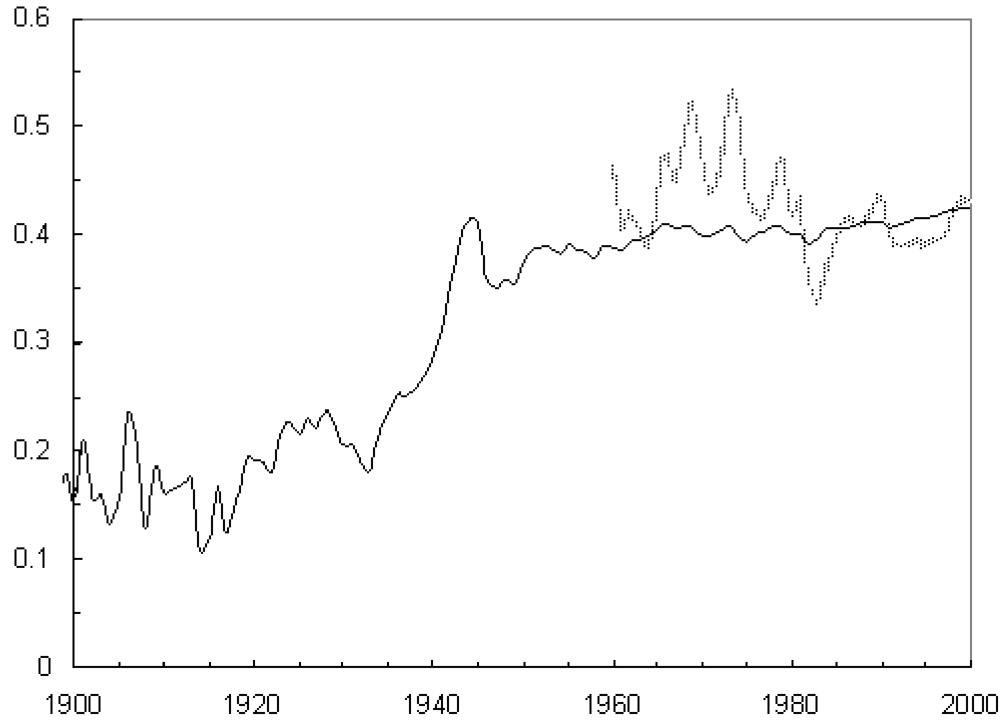

Figure 2. Technological Index

Solid line represents values of α calculated according to Eqs. (26) - (28). The dotted line depicts α estimated due to values of the poverty threshold.

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calculated as follows. The rate of growth of capital services, due to Eq. (10), is calculated through the growth rates of capital stocks and labour as

$$\eta = \frac{\delta - (1-\alpha)\nu}{\alpha}, \quad 0 < \alpha < 1 \tag{26}$$

and the time dependence of capital services can be restored by solving the equation

$$\frac{dS}{dt} = \eta(\alpha) S \tag{27}$$

The value of the technological index α can be calculated, due to equation (15) as

$$\alpha = \frac{\ln\left(\frac{Y}{Y_0}\frac{L_0}{L}\right)}{\ln\left(\frac{L_0}{L}\frac{S}{S_0}\right)} \tag{28}$$

However, amount of capital services S itself depends on the value of the technological index α. Equations (26) - (28) allow one to develop methods of estimating the technological index at given time series of Y, K and L. The results of calculation are depicted on the plot of Fig. 2 in line with the values of α calculated due to available data of the US Bureau of Census for the poverty threshold taken as personal consumption. Note that the choice of initial value of the technological index allows

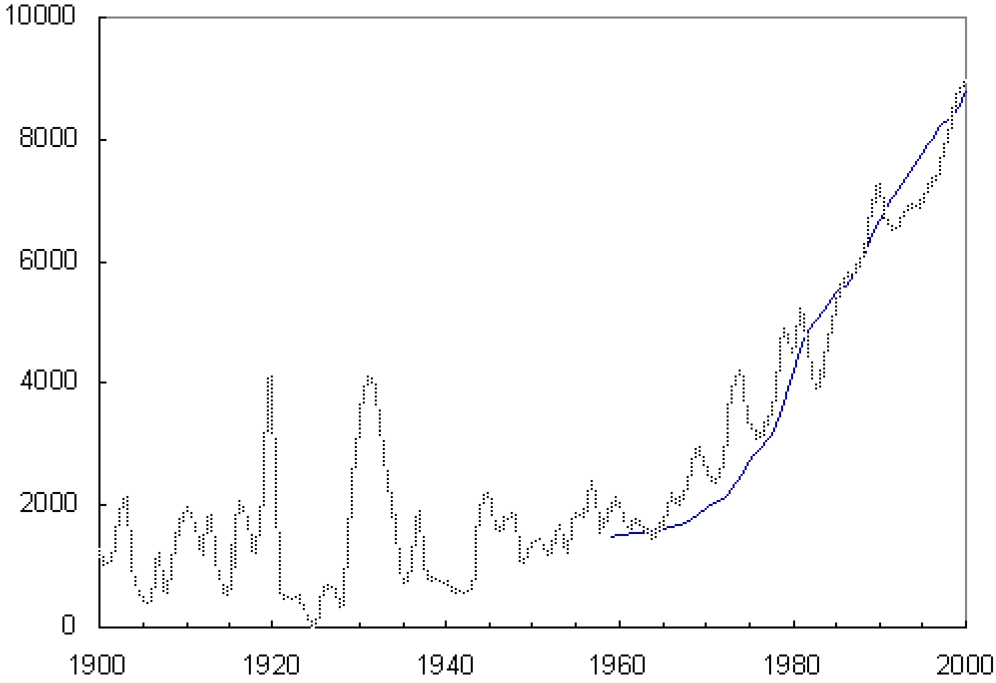

Figure 3. Personal Consumption in the US

Solid line presents values of poverty threshold according to US Bureau of Census. The dotted line presents personal consumption calculated according to estimated values of the technological index.

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us to move the whole curve of α up and down, so that it is important to have, at least, one point where absolute value of α is known which, according to the previous estimate, is taken as $\alpha \approx 0.4$ in year 1996. On the other hand, one can use the estimated from Eqs (26) - (28) values of the technological index to calculate the personal consumption. The results for the US in the twentieth century are shown in Fig. 3.

The values of the technological index allow us to calculate the growth rate of capital services η and to restore the time dependence of capital services. The results for S in arbitrary units are shown in Fig. 1 by solid curve in line with the other production factors for the US economy. One can see that the capital services is growing on average faster than the capital stock in years 1900 - 2000; however, there are some years of recession.

One can see that the value of the technological index for years 1950 – 2000 can be considered approximately to be constant $\alpha \approx 0.4$, and the time dependencies of the production factors for these years can be approximated by the exponential functions

$$K = K_0 e^{\delta t}, \qquad L = L_0 e^{\nu t}, \qquad S = S_0 e^{\eta t}. \tag{29}$$

where $\delta = 0.0316, \quad \nu = 0.0146, \quad \eta = 0.0588.$

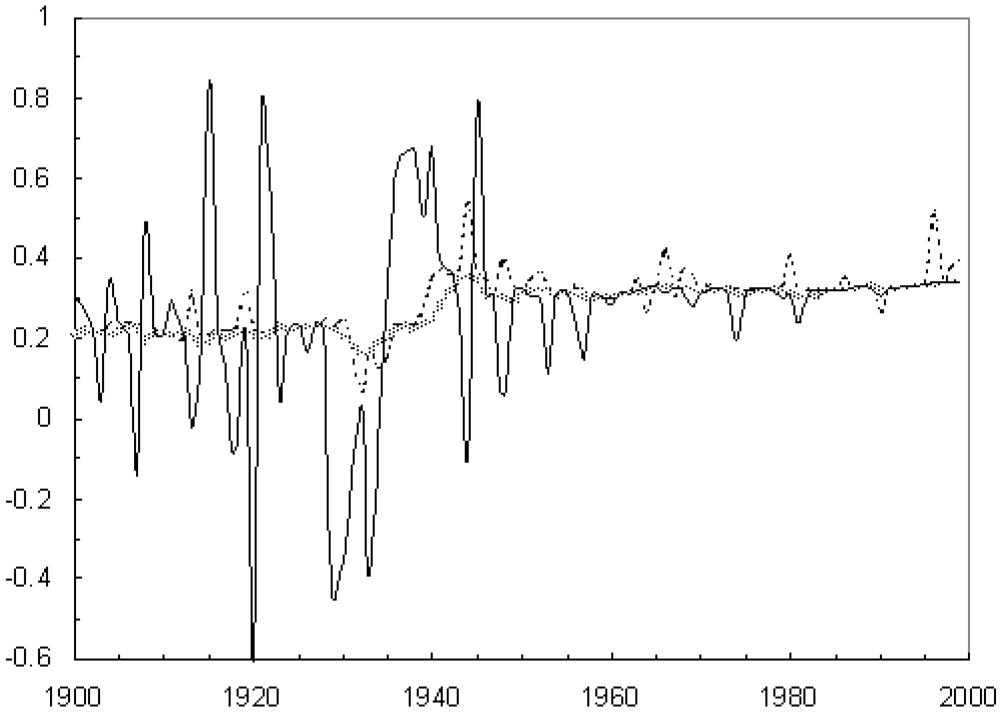

Figure 4. Productivity of Capital Stock in the US Economy

The solid line represents direct estimates of ξ from the empirical data and the equation $dY - \Delta dt = \xi\, dK$. The dashed line shows the marginal productivity calculated according to Eq. (31), while β and γ are estimated directly due to the empirical data and the equation $dY - \Delta dt = \beta\, dL + \gamma\, dS$. The dotted line represents the ratio Y/K.

Marginal Productivities

Production function (15), that is the Cobb - Douglas production function

$$Y = Y_0 \frac{L}{L_0} \left(\frac{L_0}{L} \frac{S}{S_0} \right)^{\alpha}, \tag{30}$$

in which capital services S stand in the place of capital stock K, allows one to calculate time dependence of output. At given time dependence of labour services and at calculated values of the technological index and capital services, time dependence of output identically coincides with empirical one.

Differential formulae (11) allow us to estimate the marginal productivities ξ, β and γ due to empirical data. These quantities are connected with each other by expressions (13) which are followed by one more simple relation for the marginal productivities

$$\xi = \beta \frac{L}{K} + \gamma \frac{S}{K} \tag{31}$$

One has a unique opportunity to confirm the consistency of the proposed description by testing relation (31). The comparison of results of calculation of the left and

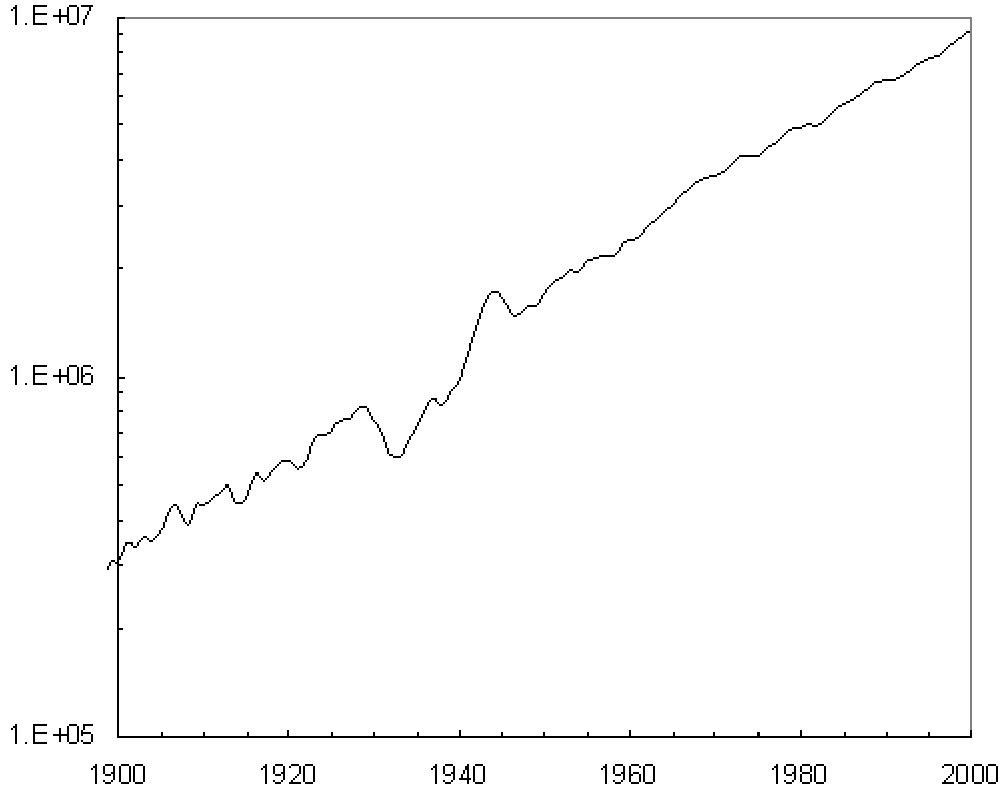

Figure 5. Production of value in the US economy

The coinciding empirical and calculated values of GDP in million of 1996 dollars.

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right sides of relation (31) for the US economy are depicted in Fig. 4. For the reliable years 1950 − 2000, the average value of the capital-stock marginal productivity is $(0.309 \pm 0.035)\ year^{-1}$, whereas average value of the right hand side of Eq. (31) is $(0.320 \pm 0.041)\ year^{-1}$. The values of the marginal productivity practically coincides with the averaged bulk productivity Y/K, which is $(0.318 \pm 0.010)\ year^{-1}$; this is an evidence that the capital marginal productivity does not depend on argument K. Thus, indeed, the marginal productivity of capital stock can be considered as the 'sum' of the marginal productivities of labour and capital services and no other factors are needed to include into the production function. Although one needs production equipment (capital stock) to attract extra amount of capital services to substitute labour, labour services are replaced only by capital services, not by capital stock.

Decomposition of the Growth Rate of Production of Value

Empirical values of GDP for the US economy for years 1890 - 1999 are depicted on the plot in Fig. 5 with solid line. At given time dependence of labour services L and at calculated values of the technological index α and capital services S, time dependence of output identically coincides with empirical one.

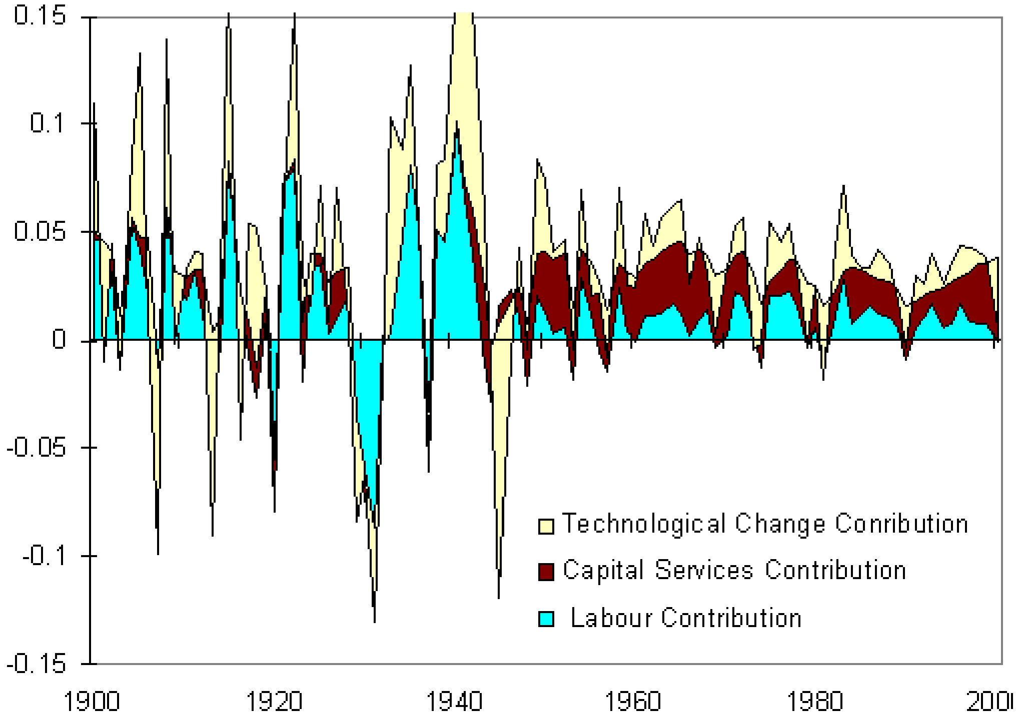

Figure 6. Decomposition of the Growth Rate of Output

There is some interest to write approximate relations for calm period of years 1950 - 2000 to describe the 'stylised' facts of economic growth, that is, exponential growth of output and production factors. On the base of relation (18), taking Eqs. (29) into account, the output can be written in the following form

$$Y = Y_0 e^{[\nu + \alpha(\eta - \nu)]t} = Y_0 e^{\delta\, t} \tag{32}$$

where, for this span of time, empirical value of α can be taken to be 0.4. The growth rate of output is equal to the growth rate of capital stock and is connected with the growth rates of labour and capital services. The empirical averaged growth rate of output 0.0329 is approximately equal to the growth rate of capital $\delta = 0.0314$. One can directly estimate contributions of labour and capital services in the growth of output. The contributions to the growth of output are $(1-\alpha)\nu \approx 0.0088$ from the labour growth and $\alpha\eta \approx 0.0235$ from the capital services growth on average. Though capital stock is the means of attracting the production factors to production, increase in consumption of the production factors is connected with an increase in capital stock. One can separate the growth rate of capital stock δ in the growth rate of capital services η to get the breakdown of the growth rate of output in conventional terms: the contribution from the labour growth $(1-\alpha)\nu \approx 0.0088$, the contribution from the capital growth $\alpha\delta \approx 0.0126$, and the contribution from the total factor productivity $\alpha(\eta - \delta) \approx 0.0109$.

The decomposition of the growth rate of output is shown in Fig 6 according to equation (19). The decomposition of the Solow residual is shown in Fig 7.

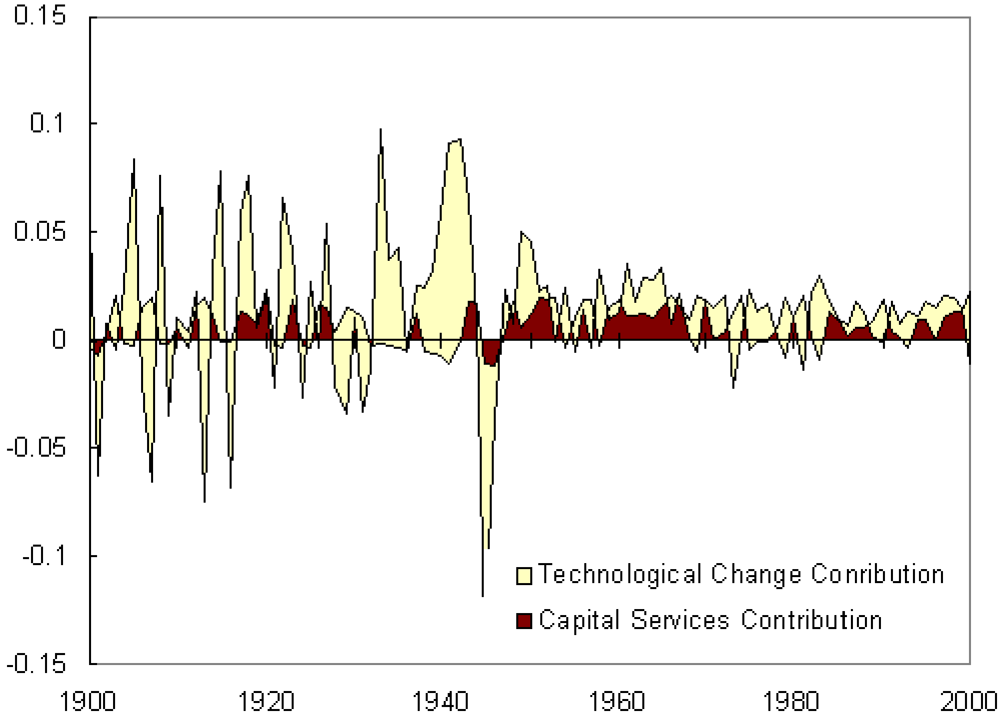

Figure 7. Decomposition of the Solow Residual

4 Conclusion.

The distinguishing between capital stock and capital services allows one to get the proper schematisation of production process and to reach a consistent formalisation of well-known neo-classical concepts and ideas. So, the three production factors: capital stock K, labour L and capital services S are used to explain the process of production of value. The production system itself is characterized by the index α which is a share of expenses needed for utilisation of capital services in total expenses for production factors. It was shown that this quantity is also a combination of the technological characteristics λ and ε, so that the index α is connected with properties of the introduced production equipment. The growth rate of output is determined by the growth rate of labour consumption and changes of technology of the production system. No arbitrary parameters are included in the theory, so that the decomposition of the growth rate of output does not depend on the arbitrariness in estimation of the index α. Comparison with empirical data for the US economy allows us to conclude that the described three-factor model can be acceptable for macroeconomic (phenomenological) description of production process.

We believe that the new formulation of the theory has some advantages, because it discloses the mechanism of growth by referring to the mechanism of the utilisation of labour and capital services and allows us, if the words of Solow (1994) can be used, "*to model the endogenous component of technological progress as an integral part of the theory of economic growth*".

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