INFORMATION REVELATION, POST-BID MARKET INTERACTION AND AUCTION CHOICE

By Hongjun Zhong*

Nuffield College, OXFORD

Abstract

We study the influence of product market competition forms on the first-price sealed auction and the English ascending auction with independent cost types. Bidders, valuing the license basing on the information released in the first stage license bidding game and the possible game they will play in the product market, care about not only whether they can win and thus how much to bid, but also the information released in the auction when they win. As in the English ascending auction, all bidders are able to constantly adjust their belief about their potential rival's cost distribution, and the higher the bid goes, the lower the potential rival's cost, the lower the expected gain from winning a license, thus bidders will keep downgrade the value of license and bid more conservatively and the government will generate lower expected revenue from the English auction than in first-price sealed auction. In particular, if the government uses the English ascending auction while the Bertrand price-cutting game being played in the product market, then all bidders except the two lowest cost type bidder will quit the bidding game sequentially and the expected revenue will be close to zero. Furthermore, as the Bertrand competition is more intensive than the Cournot competition, the government's expected revenue is lower when the product market game played as a Bertrand game.

Keywords: license auction, Information, oligopoly, revenue comparison.

JEL Classification: D44

---

* This paper is a revised version of the second chapter of my MPhil thesis. I benefited greatly from discussions with Ian Jewitt. Paul Klemperer and Erik Eyster provided invaluable comments.
Corresponding address: Nuffield College, New Road, Oxford, OX1 1NF, United Kingdom. Tel: (+44)7786-437615. Email: hong.zhong@nuf.ox.ac.uk

1. Introduction

The past few years have seen the big roles played by economic analyses in many cases. FCC spectrum auction and the recent European 3G mobile license auction raised people's great interest in auction theory and their application to the practical auction design issues. Many works have been done to study and evaluate the auction scheme proposed by theorists. Recently, auction theorists' interests have shifted to multi-object endogenous value model, largely due to the 3G license auction issue. In the classical auction model, it is usually assumed that all bidders' value are exogenously given, which implies all bidders know their valuations of objects and they cannot adjust their valuations of objects and that the auction is a one-shot game. There is no strategic behavior that can affect their valuation of objects, neither pre-auction nor post-auction. However, this is clearly not the case in license auction and procurement contracting.

In license auctions, designers have to consider not only how to sell these licenses, but also what kind of impact the selling scheme has on the valuation of the licenses to potential buyers. In particular, a license auction is essentially a dynamic game. On the one hand, when attending the bidding game, all potential buyers have to consider how much to pay, given their belief of likely final product market competition if they win. On the other hand, they have to take account of the fact that how much they bid in the license auction will inevitably reveal information about their types (cost or capital ability) and how this information will change competitors' belief about other bidders' types, thus change the game they are going to play. A rational player in this game must be able to look forward and backward consistently.

This kind two-way interaction is a common phenomenon in many situations. We can easily observe these phenomena in many industries such as the transportation industry, the telecommunication industry and the banking industry. In the transportation industry, government or its agency (industry regulator) usually issues more than one license to enhance the competition. For example, there are currently two companies providing coaches services between Oxford and London. Obviously, if the government issues more than two licenses, then the licenses' valuations will be much lower and make potential bidders bid more conservatively in license bidding game. Same things happen to the telecommunication and the water industry. How to take this factor into account and design a good selling scheme, while a lot of other things to be considered, is vital to the success of public policy.

Myerson's path-breaking work (Myerson (1981)) contains the seeds of endogenous determined value problem, yet few noticed this point. Probably shadowed by Myerson's optimal design result which has great generality and the methodology merits, the *quality uncertainty* issue raised in this paper has unfortunately failed to attract much attention. Milgrom and Weber (1982) studied an affiliated value model in which the winning bidder's payoff may depend upon his personal preferences and the preferences of his opponents. The basic result of this model is that an English auction generates higher expected revenue than does a second-price auction, which in turn dominates the Dutch auction and first-price sealed bid auction. The intuition is straightforward. In the English auction with affiliation value, higher opponents' bid means higher value of the object, which makes all bidders update upward their valuation of the object. This process is essentially a process of information release which reduces winners' curse phenomenon thus makes the English auction superior to other auction formats.

The affiliation assumption plays a key role in deriving this revenue ranking. If one bidder's good news is bad news for his opponents, then this conclusion will hold no longer. This is exactly the case we will consider in the next chapter. The point is, if higher bids submitted by opponents are signals bad news to all other bidders, then they will adjust downward their valuation of the objects and thus bid more conservatively. As the English auction enables all bidders to update their belief, while first-price sealed auction cannot, thus first-price sealed auction will generate higher expected revenue for sellers.

Jehiel and Moldovanu (2000) studied an auction whose outcome affects the future interaction among bidders. The point in their work is that in the case of externality, bidders' bidding behaviors will be adjusted accordingly, depending on whether the externality is positive or negative. They build up a single-object auction model in which allocating the object to a buyer will induce externality to other potential buyers. As this kind externality can be either positive or negative, it is unclear which auction format will be better. The point of value in this paper is the impact of externality on bidders' strategy behavior.

However, they ignored the fact that in many cases, the bids at the auction can serve as signals that will influence beliefs in the post-bid interaction, which will be studied here in the next chapter. Another question in this paper is that they consider only the single-object auction rather than the multiple-object auction, which is more important to understand license auction and more related to externality.

Other studies on externalities are Krishna (1993; Jehiel and Moldovanu (1996); Jehiel, Moldovanu and Stacchetti (1996). The theme of these studies is all bidders' *willingness* to pay for the objects depend on how they anticipate the allocation of other objects and who will win the objects. The term endogenous valuations come from the fact that bidders' valuations are determined endogenously from the post-bid market interaction, which depends on the post-bid market and information structure, thus post-bid behavior and performance. We will discuss these problems in the following section.

Another problem related to multi-object auction is the post-bid market structure. While the studies of industrial organization (IO) and auction theory have made great advances in the past thirty years due to the development of game theory, the marriage of IO and auction theory is still in its early stage[1,2].

In the monopoly case, this problem is relatively simple. As issuing a monopolist license is essentially equivalent to auction a single object valued by different bidders with signals drawn independently from assumed identical assumption, most of results for IPV model and Common value model can be carried over here directly. In particular, the efficient allocation requirement is easily satisfied. However, for the revenue ranking, as the value of a monopoly license essentially depends on the market demand and the winner's operating cost, so the value of monopoly license contains both private and common value factors, we would not be surprise if we find out that an ascending English auction generates higher revenue than the first-price sealed bid auction does Milgrom and Weber (1982).

In the multi-license case, things become much more complicated. This point can be seen clearly from the European 3G license auction. Most of countries issued more than one license. At the same time, there exist incumbent operatorsin all these countries. Another point worthy mentioned here is that entering into this market usually requires huge sunk cost investment, which restricts the government or regulators' ability to enhance

[1] However, this does not mean that economists failed to see this point and its importance. In an influential paper, Demsetz (1968) discusses nicely the possibility of using auction to substitute for regulating utilities that are seen as natural monopoly industry then. The insight in this paper is that, even the utilities industry has monopoly characteristics; the government still does not need to regulate this kind of monopoly industry. Instead, government can introduce competition into this monopoly industry by holding a monopoly license auction.

[2] Recently held license auctions in Europe proved the importance of IO issues in designing auction schemes. Klemperer (2002), Binmore and Klemperer (2002) gave detailed expositions of how IO issues affect the auction design.

competition by issuing as many license as possible[3]. This point is explained and examined in Jehiel and Moldovanu (2002). They find that the bidding among incumbents displays 'war of attrition' features'.[4] If only one license is licensed, then war of attrition may lead to the entry. Contrary to common sense, this features will be alleviated if several licenses are auctioned, resulting in less entry. In particular, if the number of license is equal to the number of incumbent, then each of them can purchase one license and thus block new entry completely. Related evidence can be found in Klemperer (2002).

A corollary deduced from the result above is that auctioning the maximum possible number of licenses need not induce a higher degree of competitiveness. Restricting supply is a way to combat tacit collusion and to induce more entry. Another possibility is introducing supply uncertainty about the number of licenses to encourage entry. The Germany 3G license auction scheme can be regarded as a variant of this possibility. However, many economists criticized this scheme as it provided many chances for bidders to collude. (see Jehiel and Moldovanu (2001); Klemperer (2002))

Their analysis also point out that the induced entry and revenue obtained in various auction formats crucially depends on the relation between the number of incumbents and the number of auctioned licenses. Furthermore, the competitiveness and revenue may be either positively or negatively correlated, depending upon the setting we are considering.

All these point to the necessity of further study of auction design and bidding behavior. In particular, the interdependence of valuations and the post-bid market competition deserve our careful consideration. In the next chapter, we will study the possible affect of the post-bid market interaction on the bidders' bidding behavior and thus the auction choice.

---

[3] Duplication of sunk cost in the same industry will inevitably give rise to welfare loss, which should be traded off with the welfare improvement induced by introducing competition.
[4] For the war of attrition, please see Fudenberg (1991), page 216-219; Bishop (1978); and Bulow (1999).

This paper focuses on the interplay between license auction and market competition from an information perspective. As license auctions essentially evolve multi-stage game among all bidders, from incumbents to potential entrants and license winners. Not only have the bidders (firms) to decide how much to bid in the license auction game, but also what kind of game they are going to play in the final product market (if they win). When they decide how much to bid for the license, they should take the following factors into account:

1.Different auction forms adopted in the first stage game will reveal different information to license winners (and losers, but they are not relevant any more in the product market), thus affect their expected profit in the second stage product market competition.

2. Different games played in the second stage product market competition will result in different expected profit, thus affect their valuations of license and their bidding behaviors in the first stage license auction game.

3. As bidders' valuations of licenses are based mainly on their market share size and efficiency factors (costs of providing service in the final market),their biddings are strategic substitute, as higher biddings offered by rivals means their servicing cost is low thus will lower bidders' valuations of license. So bidding process is in fact an information revelation process.

Combined consideration of the factors above implies government may prefer one auction form to another one, depending on the game played in the product market.

The key point in this story is that, if players are strategic and fully rational, then when they bid for the licenses, they should take into account not only how much to pay, but also how much they can get if they win the licenses. Only by taking both effects into account then can we tell a good story about license auctions.

This paper is organized as follows. Section one is an introduction, I briefly explain the characteristics of license auction and some facts which have been widely observed in the recently held European 3G license auction. We set up a simple model in the section two. Section three is the analysis of the model. As the post-bid competition may be of many kind forms, we have to distinguish them and see if this difference will influence our result or not. In particular, we discuss two representative cases respectively. We first discuss the case when post-bid game is played as a Cournot quantity game, and then we will discuss the case when post-bid game is played as a Bertrand price competition. We find out that if the post-bid competition is the Bertrand game, then a first-price sealed bid auction generate higher expected revenue for the government than does an English ascending auction. In the Bertrand price competition, we also get a rather extraordinary result, which is similar to the toehold effect (Bulow, Huang et al. (1999)). However, if the post-bid competition is the Cournot game, the revenue comparison is not clear, though we have proved that the symmetric solution is unique.

Section four focuses on the efficiency problem. As entering utilities industry usually requires huge sunk-cost investment and the cost structure of utilities industry is usually characterized by the high fixed cost and low marginal cost, we will study their impacts on the license auction and the action the seller may take when he cares for the efficiency. The last section concludes.

2. The Model

Consider an industry in which only two firms can survive due to market size or technology constraints. The government decides to issue entry permit by holding a license auction[5]. This is essentially a two-stage game. In the first-stage game, the government, who cares about revenue, chooses either an ascending English auction or a first-price sealed bid auction to sell 2 licenses[6]. All potential buyers, labeled $i \in N = \{1, \cdots, n\}$, decide how much to bid basing on their expected profit, which depends on their costs, here their types, and the game they are going to play in the product market, if they win the license; and 2 license-winners enter into the final market. In the second stage, 2 license-winners compete with each other by playing the Cournot quantity or the Bertrand price competition. Bidders' costs are drawn independently from the identical, continuous uniform distribution, i.e. $c_i \in [0,1]$ for any bidder $i \in N$.,

To simplify the analysis, we make the following assumptions.

<u>Assumption 1</u>: Government (the seller) is risk-neutral and aims to raise as much revenue as possible from selling licenses.

Remark: Theoretically, the government should be benevolent and cares the weighted sum of consumer surplus and firms' (here the bidders) surplus. But in the real world, this consideration may not work very well, as government usually hire its agent to sell the license and agents' reward will be based on their performance which is related to the auction result. Thus, even the government is benevolent; the agent may care only for the revenue.

<u>Assumption 2</u>: Each bidder has a unit demand. And all licenses are homogenous. The services they may provide in the final market are also homogenous.

---

[5] Of course, the government can allocate license by beauty contest. However, the France experience shows that it is not a good way to either allocate license or raise money. French telecommunication regulator, after the British and Germany license auction, decided to set a price for each of four licenses rather than to hold an auction. It turned out only two firms paid the price and got license, two other license remained unsold.

[6] Whether two or more licenses being sold is not important here. The important thing here is that there exists competition between winners, i.e., winners have to play another game after winning the license.

Remark: This is plausible due to the following two reasons. Firstly, to enhance competition, the government may wish to restrict each bidder to hold only one license at most. Secondly, due to capacity constraint or other reasons, any potential services provider is unable to operate more than one license.

Some scholars may question the homogeneity assumption. But this is true for the case of selling transportation license, for example, from Oxford to London. Even in the 3G license auction, we also have evidence that similar slots are very similar and thus generate very close revenue. So this assumption is reasonable and we should feel comfortable with it.

Assumption 3: All N potential buyers' costs are subject to constant return to scale. Buyer i has marginal cost (also average unit cost under constant return to scales assumption) c_i, which here is buyer i's type, is drawn from the independent and identical uniform distribution ranging from 0 to 1, i.e., $c_i \in [0,1]$ for bidder $i \in N$. c_i is buyer i's private knowledge, but the distribution of it is common knowledge.

Remark: This assumption is quite standard, yet not without its problem. The cost symmetry assumption will be appropriate if the government is going to issue licenses to a new market. In many utilities industry, there exists usually some incumbent, thus the symmetry assumption of cost distribution across all bidders may not be very appropriate.

Assumption 4: The market demand is linear. Specifically, it can be written as $p = k - Q$. Furthermore, market demand is common knowledge. k, a parameter characterizing market size, is sufficient large such that all problem we considered later have interior solutions. There is no uncertainty about demand.

<u>Remark</u>: This assumption is mainly for tractability and simplicity. The specific form of demand curve comes from the fact that there is a one-one mapping between any two linear demand functions. For any linear demand function, we can always transform it to the specific form we assumed.

<u>Assumption 5</u>: All bidders are risk-neutral. They care only for the expected profit they can make if they win a license. Buyer i's valuation of license is his conditional expected profit given he enters the market, i.e., $v_i \equiv E_{c_j}\pi_i\left(c_i \mid I\right) = \max_{s_i} E_{c_j}\pi_i\left(c_i, s_i \mid I\right)$, where I is the information revealed at the first stage license auction, s_i is buyer i's strategy, which can be either quantity or price. All bidders know what kind of game they will play if they win a license.

<u>Remark</u>: This is also a quite standard assumption. The assumption of exogenously given form of the post-bid game is fairly standard in the IO literature. However, whether the Cournot game or the Bertrand will be played in the post-bid competition may be endogenously determined rather than exogenously given.

<u>Assumption 6</u>: There is no entry cost or reserve price.

<u>Remark</u>: No entry cost is only for convenience and simplicity here, yet not without reason. Basically, entry cost plays a role to block the entry of those high cost bidders. We can always normalize the highest cost type of entrant to be the highest cost type in our setting, so the main result will not be changed due to this simplification. The no-reserve-price assumption is based on its equivalence to entry cost and on the fact that in the practical auction design, it is very difficult to set an appropriate reserve price. This can be seen from the European 3G license auction.

Now we turn to the analysis of the model. We will find out bidders' bidding strategies in different licenses auction scheme and different post-bid product market competition.

3. The Analysis

As the existence of competition in the product market and different game played in the product market will end up with different expected profit thus affect bidders' valuation of license and their bid in the first stage license auction game, it is very likely that this impact will change the revenue equivalence result. As this is a dynamic game, we analyze first license-winning bidders' competition in the product market, and then we will consider their bidding strategies in the first stage bidding game and the governments' optimal selling scheme. We will consider the Cournot game and the Bertrand game separately.

3.1 Cournot Quantity Competition in The Product Market

In the Cournot game, two licenses winners will set their quantity simultaneously, holding each other's quantity constant. The optimal quantity depends on their cost type and the information released in the bidding game. Different first stage bid game will produce different information to the winners, so affect their quantity decision. We consider the case of an English auction in the bid game first.

A. ENGLISH AUCTION

Here, we consider the Japanese style ascending auction. At the right beginning, the price of license is set to zero and all bidders raise their hand. Then price goes up step by step, bidders quit when the price reaches their expected valuation of the license. The auction will end when there are only two bidders remained.

Let two winners be bidder i and j, and their cost be c_i and c_j respectively. Let the last quit bid be $\bar{b} = \bar{b}(c)$, suppose at the moment that it has inverse function, and denote

$\overline{c} = b^{-1}(\overline{b}(c))$. Both bidders know that, each other's cost is drawn from the conditional identical independent (*i.i.d*) uniform distribution from 0 to $\overline{c}$. Let $F\left(c_j \middle| c_j \le \overline{c}\right)$ ($F\left(c_i \middle| c_i \le \overline{c}\right)$) denote bidder i (j)'s belief about j's (i's) cost. Then bidders i and j's profit are respectively,

$$p_i(\overline{c}, c_i) \equiv \max_{q_i} \int_0^{\overline{c}} \left(k - q_i - q_j(c_j) - c_i\right) q_i dF\left(c_j \middle| c_j \le \overline{c}\right),$$

and

$$p_j(\overline{c}, c_j) \equiv \max_{q_j} \int_0^{\overline{c}} \left(k - q_j - q_i(c_i) - c_j\right) q_j dF\left(c_i \middle| c_j \le \overline{c}\right). \quad (1)$$

First of all, we establish the following lemma.

LEMMA 1: In the product market, the expected profit is a non-increasing function of license winner's cost and non-decreasing function of the final quit bid in the first stage license auction. Mathematically, we have:

(1.a) $\dfrac{\partial p_i(\overline{c}, c_i)}{\partial c_i} \le 0$; (1.b) $\dfrac{\partial p_i(\overline{c}, c_i)}{\partial \overline{c}} \ge 0$; and (1.c) $\dfrac{\partial p_i(\overline{c}, c_i)}{\partial k} \ge 0$

PROOF: From (1), we know that

$$p_i(\overline{c}, c_i) \equiv \max_{q_i} \int_0^{\overline{c}} \left(k - q_i - q_j(c_j) - c_i\right) q_i dF\left(c_j \middle| c_j \le \overline{c}\right).$$

The envelope theorem implies that,

$$\frac{\partial p_i(\overline{c}, c_i)}{\partial c_i} = -\int_0^{\overline{c}} q_i(c_i) dF\left(c_j \middle| c_j < \overline{c}\right) = -q_i(c_i) \le 0, \quad (2)$$

here $q_i(c_i) \in \underset{q_i}{\operatorname{argmax}}\, p_i(\overline{c}, c_i, q_i)$. So we proved part (1.a).

Now we prove part (1.b). To see this, noticing that

$$p_i(\overline{c}, c_i) \equiv \max_{q_i} \int_0^{\overline{c}} \left(k - q_i - q_j(c_j) - c_i\right) q_i dF\left(c_j \middle| c_j \le \overline{c}\right)$$

$$= \max_{q_i} \left(k - q_i - c_i\right) q_i - q_i \int_0^{\overline{c}} q_j\left(c_j\right) dF\left(c_j \middle| c_j \le \overline{c}\right), \tag{3}$$

differentiating it with respect to q_i and let it equal to zero, as sufficient large k guarantees the existence of interior solution, we ha ve

$$2q_i = k - c_i - \int_0^{\overline{c}} q_j\left(c_j\right) dF\left(c_j \middle| c_j \le \overline{c}\right).$$

By same token, we have

$$2q_j = k - c_j - \int_0^{\overline{c}} q_i\left(c_i\right) dF\left(c_i \middle| c_i \le \overline{c}\right).$$

Naturally, we look for symmetric reaction function. Furthermore, the quadratic from of profit function implies that the reaction function must be linear. To find out the reaction function, we can use the undetermined coefficient method. Assume that $q_i = a + bc_i$ and $q_j = a + bc_j$, substituting them back to the first order conditions, we have $a = \frac{k}{3} + \frac{1}{12}\overline{c}$ and $b = -\frac{1}{2}$. So the winner i's optimal quantity and expected profit are respectively

$$q_i = \frac{k}{3} + \frac{1}{12}\overline{c} - \frac{1}{2}c_i, \tag{4}$$

and $$p_i\left(\overline{c}, c_i\right) = \left(\frac{k}{3} + \frac{1}{12}\overline{c} - \frac{1}{2}c_i\right)^2. \tag{5}$$

Similarly for bidder j.

Now it is self-evident that $\frac{\partial p_i\left(\overline{c}, c_i\right)}{\partial \overline{c}} \ge 0$ and $\frac{\partial p_i\left(\overline{c}, c_i\right)}{\partial k} \ge 0$, hence the proof of part (1.b) and (1.c). Noticing that $\overline{b} = \overline{b}\left(\overline{c}\right) = \overline{p}_i\left(\overline{c}, \overline{c}\right) = \left(\frac{k}{3} - \frac{5}{12}\overline{c}\right)^2$, it is indeed a monotonic function of $\overline{c}$.

What implied by lemma 1 is straightforward. The intuition behind it is that, the more efficient the bidders are, the more valuable the licenses to them; the higher the quit bid, the more efficient the other winning rivals are, so the less valuable the license to the bidder.

In the Japanese auction, when the bid goes up, the corresponding $\bar{c}$ is getting lower and lower. For the last quit bidder, the on-going bid must be equal to his expected profit if he wins. The last bid is $\bar{b} \equiv \bar{b}(\bar{c})$. The last quit bidder's expected profit is $p_i(\bar{c}, c_i) = \left(\frac{k}{3} + \frac{1}{12}\bar{c} - \frac{1}{2}c_i\right)^2$. As he is the last bidder, his cost must be equal to $\bar{c}$. Hence his expected profit must be $p_i(\bar{c}, \bar{c}) = \left(\frac{k}{3} + \frac{1}{12}\bar{c} - \frac{1}{2}\bar{c}\right)^2 = \left(\frac{k}{3} - \frac{5}{12}\bar{c}\right)^2$. As the last bid must be equal to the last quit bidder's expected, we must have

$$\bar{b} = \bar{b}(\bar{c}) = p_i(\bar{c}, c_i = \bar{c}) = \left(\frac{k}{3} - \frac{5}{12}\bar{c}\right)^2, \tag{6}$$

which is obviously a non-increasing function of cost.

As the government is going to sell two licenses, so the last quit bid will be the expected profit of the third lowest cost bidder if he wins. As the third lowest cost has density function $f(x) = \frac{n!}{2!(n-3)!}x^2(1-x)^{n-3}$, the governments' expected revenue from each license will be

$$ER^E = \int_0^1 \left(\frac{k}{3} - \frac{5}{12}x\right)^2 \frac{n!}{2!(n-3)!} x^2 (1-x)^{n-3} dx$$

$$= \frac{k^2}{9} - \frac{5k}{6(n+1)} + \frac{25}{12} \cdot \frac{1}{(n+1)(n+2)}. \tag{7}$$

We summarize the above analysis as the following proposition.

<u>**PROPOSITION 2**</u>: If the licenses seller chooses an ascending English auction, and the license winners will play the Cournot quantity game in the product market, then

(2.a) A typical bidder with cost type c will bid up to $b(c)=\left(\frac{k}{3}-\frac{5}{12}c\right)^2$;

(2.b) The seller's expected revenue from each of license will be $ER^E=\frac{k^2}{9}-\frac{5k}{6(n+1)}+\frac{25}{12}\cdot\frac{1}{(n+1)(n+2)}$.

In the English auction, when the bidding price increases, there are two factors affect those still staying bidders' bidding strategy. On the one hand, the increase of bid means less net surplus for him if he wins, we called this effect direct effect, which is common to all kind auction. On the other hand, the increase of bidding is in fact a signal to all still remaining bidders, which implies rivals will be more competitive, and the value of license will be even smaller, so all bidders have to take account of this effect, which we called indirect effect.

REMARK: In the perfect information Cournot game, if there are two firms compete in the product market, then it is easy to find out that bidder i's optimal quantity reaction and profit function is $q_i(c_i,c_j)=\left(\frac{k-2c_i+c_j}{3}\right)$ and $p_i(c_i,c_j)=\left(\frac{k-2c_i+c_j}{3}\right)^2$ respectively. So the proposition 2.2 simply says in the ascending English license auction followed by the Cournot competition in the product market, firm i will bid up to the profit level as if his competitor has cost equal to $\frac{3}{4}c_i$[7], rather than c_i. This is because an ascending English provides bidders opportunities to update their information; the increase of bids implies competitors will be more strong even if them win, so all bidders will on average regard competitors be more competent.

[7] This is from the equation $\left(\frac{k}{3}-\frac{5}{12}c_i\right)^2=\left(\frac{k-2c_i+c_j}{3}\right)^2$, so we get $c_j=\frac{3}{4}c_i$.

B. FIRST-PRICE SEALED BID AUCTION

In the first price sealed auction, every bidder submits his bid to the seller and the seller chooses 2 bidders who have offered the two highest prices as the licenses winners, each of them pay what they bid. In this case, the seller can choose to announce or not, after he received all bid, the submitted bids. This decision on whether or not to reveal information about bid may affect bidders' valuation thus their bidding decision. To simplify the analysis, we assume here the seller hide the information about the bid and inform bidders only whether he win or lose. In this case, for a typical bidder with cost c_i, if he wins a license, then his updated belief about another winner's cost is a conditional Bayesian distribution, which can be shown to be

$$\Pr\left(c_j \le c \middle| c_i win\right) = \begin{cases} \dfrac{(n-1)c}{(1-c_i)+(n-1)c_i}, if: c \le c_i \\ \dfrac{(n-1)c_i}{(1-c_i)+(n-1)c_i} + \dfrac{1-c_i}{(1-c_i)+(n-1)c_i}\left[1-\left(\dfrac{1-c}{1-c_i}\right)^{n-1}\right], if: c_i < c \le 1 \end{cases} \quad (8)$$

This distribution comes from the following facts. The probability of c_i is one of two lowest cost amongst n bidders is $(1-c_i)^{n-1} + (n-1)\, c_i (1-c_i)^{n-2}$, the first part of which is the probability of c_i being the lowest cost type, and the second part is the probability of c_i being the second lowest type. So bidder i 's belief about the other winner's cost being lower than his is

$$\frac{(n-1)\, c_i (1-c_i)^{n-2}}{(1-c_i)^{n-1} + (n-1)\, c_i (1-c_i)^{n-2}}.$$

Given this belief, the other winner j's cost is uniformly distributed on $[0, c_i]$. Thus we get the conditional distribution of the other winner's cost, given bidder i with cost c_i is one of the winners,

$$\frac{(n-1)\, c_i\, (1-c_i)^{n-2}}{(1-c_i)^{n-1}+(n-1)\, c_i\, (1-c_i)^{n-2}} \cdot \frac{c}{c_i}.$$

Simplifying it, we get,

$$\frac{(n-1)c}{(1-c_i)+(n-1)c_i}.$$

On the other hand, winner j's cost may be higher than winner i's cost, that is, $c_i < c_j \le 1$, this happens with probability $\frac{(1-c_i)^{n-1}}{(1-c_i)^{n-1}+(n-1)\, c_i\, (1-c_i)^{n-2}}$. Besides, in this case, the conditional probability of $c_i < c_j \le c$ is

$$\Pr\left(c_j \le c \middle| c_i = c_{(1)}\right) = 1-\Pr\left(c_j > c \middle| c_i = c_{(1)}\right) = 1-\left(\frac{1-c}{1-c_i}\right)^{n-1}.$$

Here $\Pr(\cdot|\cdot)$ represents conditional probability, $c_{(1)}$ represents the lowest cost type of all bidders. So for any $0 \le c \le 1$, the conditional probability of $c_j \le c$, given bidder i with cost c_i is one of winners, is

$$\Pr\left(c_j \le c \middle| c_i win\right) =$$

$$\begin{cases} \dfrac{(n-1)c}{(1-c_i)+(n-1)c_i}, if : c \le c_i \\ \dfrac{(n-1)c_i}{(1-c_i)+(n-1)c_i} + \dfrac{1-c_i}{(1-c_i)+(n-1)c_i}\left[1-\left(\dfrac{1-c}{1-c_i}\right)^{n-1}\right], if : c_i < c \le 1 \end{cases}$$

Hence the conditional distribution (8).

For the winner with cost c_i, his expected profit from the product market given he wins a license is,

$$p\left(c_i\right)=E_{c_j}p_i\left(c_j,c_i\right)=\max_{q_i}\int_0^1\left(k-q_i-q_j\left(c_j\right)-c_i\right)q_i d\Pr\left(c_j<c\left|c_i win\right.\right). \qquad (9)$$

Differentiating it with respect to q_i, and equalising it to 0, we have

$$k-2q_i-c_i-\int_0^1 q_j\left(c_j\right)dF\left(c_j\left|c_i win\right.\right)=0. \qquad (10)$$

Similarly, for the winner with cost c_j, we have the first order condition

$$k-2q_j-c_j-\int_0^1 q_i\left(c_i\right)dF\left(c_i\left|c_j win\right.\right)=0. \qquad (11)$$

From (10) and (11), we know there is a symmetric solution. So we can find out the solution of the following integral equation

$$k-2q\left(x\right)-x-\int_0^1 q\left(t\right)dF\left(t\left|x\in\left\{type-of-winner\right\}\right.\right)=0,$$

which can be rewritten as

$$q\left(x\right)=\frac{1}{2}\left(k-x\right)-\frac{1}{2}\int_0^1 q\left(t\right)dF\left(t\left|x\in\left\{type-of-winner\right\}\right.\right),\forall x\in\left[0,1\right]. \qquad (12)$$

Furthermore, we can also prove it has unique solution.

To see this, we firstly give out the following lemma.

LEMMA 3: Given an integral equation

$$y\left(x\right)=w\left(x\right)+l\int_a^b m\left(x,t\right)y\left(t\right)dt, \qquad (13)$$

where $w\left(x\right)$ is a known continuous function defined on $\left[a,b\right]$, $m\left(x,t\right)$ is a known continuous function defined on $\left[a,b\right]^2$, then when $\left|l\right|$ is sufficiently small (l is a constant here), $y\left(x\right)$ has a unique continuous solution on $\left[a,b\right]$. [8]

PROOF: See Wang, Zhou, Zhu and Wang (1978).

[8] See Wang (1987), page 79.

Let $w(x)=\frac{1}{2}(k-x)$, $l=\frac{1}{2}$, $[a,b]=[0,1], y(x)=q(x)$, $m(x,t)dt=dF$, then from lemma 3, we know that equation (12) has a unique solution. Besides, differentiating (12), we find that $q'(x)<0$

Now we have the following proposition.

PROPOSITION 4: (4.a) There is a unique symmetric solution in the first-price sealed bid auction followed by the Cournot quantity competition. (4.b) The bidders' optimal production level is a decreasing function of their cost type; the expected profit is also a decreasing function of cost type, i.e., $\frac{\partial p_i(c_i)}{\partial c_i}<0$.

(4.b) implies that in a first-price sealed bid auction followed by the Cournot quantity competition, the licenses will go to the most efficient bidders. This is a very nice property, which guarantees the allocation efficienc y.

To find out bidders' explicit bidding function and compare seller's expected revenue under two different auction formats, we must solve integral equation (12), then find out expected profit as a function of cost. However, to give an analytical solution of integral equation (12) will be very cumbersome. Lemma 3 implicitly gives us a way to find the approximate solution of the integral equation (12). After we solve out $q(x)$ from (12), we can solve out bidders' bidding strategies in the first-price sealed bid auction and then compare the expected revenue with the expected revenue under the ascending English auction. Since this involves only calculation, there is no much point to pursuit this further in detail.

3.2. Bertrand Price Competition in The Product Market

C. ENGLISH AUCTION

Follow the same method used in the Cournot game case above, let $\bar{c}$ denote the last quit bidder's cost type, then the two license winners' cost must be drawn from the uniform distribution from 0 to $\bar{c}$. Let two license winners be bidder i and j with cost c_i and c_j. In the product market with Bertrand price competition, bidder i will win the price cutting competition if and only if his cost is lower than bidder j's cost, which happens with probability $\Pr\left(c_i < c_j \middle| i, jwin\right) = \frac{\bar{c} - c_i}{\bar{c}}$. Furthermore, given bidder i wins, his profit will be

$$\int_{c_i}^{\bar{c}} (k-c)(c-c_i)\frac{1}{\bar{c}-c_i}dc. \qquad (14)$$

This is because, in the Bertrand game, the price-cutting competition will push the market price down to level of the bidder j's cost. The market demand then will be $q = k - c_j$. Also, bidder j's cost follows uniform distribution from c_i to $\bar{c}$ with density $\frac{1}{\bar{c}-c_i}$. Thus bidder i's expected profit from the Bertrand price competition is

$$p_i(\bar{c}, c_i) = \frac{\bar{c}-c_i}{\bar{c}} \cdot \int_{c_i}^{\bar{c}} (k-c)(c-c_i)\frac{1}{\bar{c}-c_i}dc$$

$$= \frac{1}{6\bar{c}}\left[3k - 2\bar{c} - c_i\right]\left(\bar{c}-c_i\right)^2. \qquad (15)$$

As we assumed, it turned out that $\frac{\partial}{\partial c_i} p_i(\bar{c}, c_i) < 0$ holds.

At the beginning of the English auction, $\bar{c}_0 = 1$, so for a typical bidder with cost c_i, his valuation of license is $p_i(1, c_i) = = \frac{1}{6}\left[3k - 2 - c_i\right]\left(1 - c_i\right)^2$. Now suppose that the bid goes up a little bit from 0 to e, what will happen?

First of all, all bidders with valuation less than e will quite immediately, as they can not make positive profit even if they win a license, so there is no point to stay on. Put it another way, all bidders with cost type satisfying $\frac{1}{6}[3k-2-c](1-c)^2 \le e$ will quit. Taking Taylor expansion of $\frac{1}{6}[3k-2-c](1-c)^2$ around $c=1$, we get

$$p(1,c) \approx \frac{1}{2}(k-1)(1-c)^2 \le e .$$

As $c \le 1$, we can rewrite the equation above as

$$c \ge 1-\sqrt{\frac{2e}{k-1}} \equiv \bar{c}_1. \tag{16}$$

Since all bidders with cost greater than $\bar{c}_1$ will quit immediately, all remaining bidders will adjust their belief about all other remaining bidders' cost distribution accordingly. What will happen now? Now all remaining bidders know that all other remaining bidders' cost are drawn from the uniform distribution ranging from 0 to $\bar{c}_1$, for a typical remaining bidder with cost c_i, his expected valuation of license is

$$p_i(\bar{c}_1, c_i) = = \frac{1}{6\bar{c}_1}[3k-2\bar{c}_1-c_i](\bar{c}_1-c_i)^2 . \tag{17}$$

Now any bidders with cost satisfying $\frac{1}{6\bar{c}_1}[3k-2\bar{c}_1-c_i](\bar{c}_1-c_i)^2 \le e$ will quit the competition, as they can not make any money even if they win a license. Taking Taylor expansion around $\bar{c}_1$, we have

$$p(\bar{c}_1, c) \approx \frac{1}{2}\frac{k-\bar{c}_1}{\bar{c}_1}(c-\bar{c}_1)^2 \le e . \tag{18}$$

Again, as now $c \le \bar{c}_1$, we can rewrite (18) as

$$c \ge \bar{c}_1 - \sqrt{\frac{2\bar{c}_1 e}{k-\bar{c}_1}} \equiv \bar{c}_2 . \tag{19}$$

(19) simply says any bidder with cost greater than $\overline{c}_2$ will quit the auction following the first quit flow of bidders with cost greater than $\overline{c}_1$. All remaining bidders after the second quit flow will adjust their belief about all other remaining bidders cost distribution again. Their new belief is that all remaining bidders must have cost type drawn from $\left[0,\overline{c}_2\right]$.

Repeating the above procedure, we get a series of $\overline{c}_1 > \overline{c}_2 > \cdots > \overline{c}_m > \cdots \geq 0$, until there are only two remaining bidders, the bidding game will end, and the government will raise revenue of e. All bidders except the two lowest cost type bidders will quit the bidding game sequentially. So we get a dramatic result, which we summarize as the following proposition.

PROPOSITION 5: In an ascending English auction with license winners play the Bertrand price competition in the product market, all bidders except the two lowest cost type bidders will quit sequentially after the bid goes up a little bit above 0; license will be allocated to the most efficient bidders, but the government will raise almost no money from selling licenses.

This result is extraordinary in terms of the revenue raised in this scheme. Similar results can be found in Klemperer (1998), where he find that in the 'Almost-common value' environment, bidders with a tiny advantage will bid much more aggressively than tiny disadvantage bidders. Here tiny disadvantage bidders quit very quickly after a small increase of bid. The underlying rationale behind these two phenomenon are same, which is basically that, as all bidders are fully rational, they will keep introspecting and correctly adjust their belief and make right bidding decision.

D. FIRST-PRICE SEALED BID AUCTION

In a first price sealed bid auction, if bidder i is told he wins a license after submitting his bid, then his updated belief about another winner's cost is same as in (8), that is

$$F\left(c_j \le c \middle| c_i win\right) = \begin{cases} \dfrac{(n-1)c}{(1-c_i)+(n-1)c_i}, if: c \le c_i \\ \dfrac{(n-1)c_i}{(1-c_i)+(n-1)c_i} + \dfrac{1-c_i}{(1-c_i)+(n-1)c_i}\left[1-\left(\dfrac{1-c}{1-c_i}\right)^{n-1}\right], if: c_i < c \le 1 \end{cases}$$

Bidder i's expected profit is

$$p_i(c_i) = P(c_i \le c_j) \cdot \int_{c_i}^{1} (k-c)(c-c_i)\, d\left(\frac{-(1-c_i)}{(1-c_i)+(n-1)c_i}\left(\frac{1-c}{1-c_i}\right)^{n-1}\right)$$

$$= -\left[\frac{1-c_i}{(1-c_i)+(n-1)c_i}\right]^2 \cdot \int_{c_i}^{1} (k-c)(c-c_i)\, d\left(\left(\frac{1-c}{1-c_i}\right)^{n-1}\right)$$

$$= \frac{(1-c_i)^3}{\left[(1-c_i)+(n-1)c_i\right]^2} \cdot \frac{(n+1)k-2-(n-1)c_i}{(n+1)(n+2)} \equiv v^B{}_i(c_i), \qquad (20)$$

which is a decreasing function of c_i, as the denominator increases with c_i and the numerator decrease with c_i, this property guarantees the monotonicity of $v^B{}_i(c_i)$.

Being one of winner implies that the cost is one of two lowest cost among all bidders. Let $G_i(c_i)$ denote the winning probability of bidder with cost type c_i, then we know $G_i(c_i) = \left[(1-c_i)^{n-1} + nc_i(1-c_i)^{n-2}\right]$. Bidder i's surplus when reporting c is

$$S^B{}_i(c_i, c) = \left[v_i^B(c_i) - b_i^B(c)\right] G_i(c). \qquad (21)$$

The envelope theorem implies,

$$\frac{\partial S_i}{\partial c_i}\Big|_{c=c_i} = v_i'(c_i) G_i(c_i). \qquad (22)$$

Integrating, we have

$$S_i^B(c_i, c_i) = S_i^B(1,1) - \int_{c_i}^{1} \left(\frac{\partial}{\partial x} v^B{}_i(x) \right) \left[(1-x)^{n-1} + nx(1-x)^{n-2} \right] dx, \qquad (23)$$

So the bidding function will be,

$$b^B{}_i(c_i) = v^B{}_i(c_i) - \frac{S^B{}_i(c_i, c_i)}{G_i(c_i)}. \qquad (24)$$

For the government, the expected revenue from each license will be,

$$ER_B^F = \int_0^1 v^B(x) \frac{n!}{2!(n-3)!} x^2 (1-x)^{n-3} dx, \qquad (25)$$

where $v^B(x)$ is given by (20).

Though this equation is very complicated, but we need not to know the exact result to compare it with the expected revenue from an ascending English auction, which generates revenue of zero. As the expected revenue from a first-price sealed bid auction in this case is strictly greater than zero, we thus have the following proposition.

PROPOSITION 6: If license-winners are going to play the Bertrand price-cutting competition in the product market, then in the first stage license bidding game, the government should choose a first-price sealed bid auction rather than an ascending English auction. In particular, if the government adopt an ascending English auction, then the expected revenue raised from license is close to zero.

The similar analysis can be found in Klemperer (1998; Bulow and Klemperer (1999). The main idea in their work is that tiny asymmetry across bidders may give rise to dramatic different results. In Bulow and Klemperer (1999), they studied a model of the generalized war of attrition, the main result obtained there is the so-called 'instant sorting' proposition which states that all players will quit at time zero instantly except one more bidder than the number of prizes. Klemperer (1998) shows that , in an ascending English auction, a bidder with slight advantage will find it optimal, by retrospective, to overbid his rivals who is in a slightly disadvantage position, while rival will never gain by bid more than his value. On the other hand, first-price sealed bid auction will not provide any bidders such chance to update their information. Thus the result will be very close to the case where there is no such asymmetry. The conclusion is that a first-price sealed bid auction is better than an ascending English auction for sellers.

While our results are similar to these findings, the rationales behind them are different. Our findings depend on the post-bid competition. The zero revenue result only occurs in the case of an ascending English auction followed by the Bertrand price competition between license-winners. The furious competition in the post-bid game makes all bidders predict correctly that the expected profit will be low, and more importantly, the ascending English auction enable them to update their beliefs and thus all but two lowest cost bidders will quit the bid sequentially.

In the analysis above, we assume constant return to scale. Under this assumption, we can find out that allocation efficiency is easily satisfied, as the licenses always go to the bidders with lowest costs. However, in the utilities industry, the cost structure has often been characterized by increasing return to scale. This fact has two effects. On the one hand, it may change their post-bid strategy given they enter into the licensed market. On the other hand, it may also influence their possibility of winning license and entering into the licensed market. This is the topic of the next section.

4. Cost Structure and Efficiency

A characteristic of public utility industries is that they usually require high-level sunk investment and display increasing return to scale. How it affect the allocation efficiency? We examine this question now.

From the analysis in the section 2.3, we find that the bidding function is decreasing with bidders' cost type in either ascending English auction or first-price sealed bid auction. This finding ensures allocation efficiency of license auction, as bidder with the lowest cost will submit the highest bid in the auction. So efficiency is not a problem in the case where there is no sunk cost investment.

However, if entering into the market requires huge sunk investment, things will be different. Let us check what will happen in this case.

To see this, let F_i be bidder i's fixed sunk cost. As this is sunk cost, it will not change the post-bid competition and hence not the bidders' expected profit (gross of sunk cost) from entering into the market, and thus the expected valuation of license. Accordingly, it will not change all bidders' bidding strategy. However, ex ante, a typical bidder with cost c_i will make expected profit from winning a license[9]

$$v_i(c_i, F_i) = p_i(c_i) - F_i. \tag{26}$$

If any license-winners will iccur same sunk cost, i.e., $F_i = F$, then

$$v_i(c_i, F_i) = p_i(c_i) - F.$$

As in this case, the bidder with lowest cost has the highest valuation, $v_i(c_i, F_i)$, so the license will go to those bidder who have lowest costs. Allocation efficiency is guaranteed again in this case. If the sunk cost is heterogeneous, then the story will be different. If the sunk cost F_i is non-decreasing with marginal cost, i.e., $\frac{\partial F_i}{\partial c_i} \geq 0$, then $v_i(c_i, F_i)$ will be a monotonically decreasing function of cost c_i, and again, allocation efficiency is not a problem.

[9] As our study shows that first-price sealed bid dominates ascending English auction, we assume the seller will use first-price sealed bid auction to sell the license. Also, we assume the post-bid competition will be the Cournot competition for simplicity. Same conclusion can be easily found for the Bertrand case.

However, if the sunk cost F_i is decreasing with marginal cost, which is more likely the case, then $v_i(c_i, F_i)$ will not be a monotonic function of marginal cost anymore. In this case, we have to consider allocation problem. And it may happen that the lowest marginal cost bidder can not win a license due to its relatively higher fixed cost. We summarize the analysis above as the following proposition.

PROPOSITION 7: Sunk cost may or may not affect the allocation efficiency of license auction, depending on how sunk cost changes with marginal cost:

(7.a) If the sunk costs are symmetric or non-decreasing with marginal cost, then license will still go to the bidder with lowest marginal cost. Allocation efficiency is guaranteed in this case.

(7.b) If the sunk costs are asymmetric and they are decreasing with marginal cost, then license may or may not go to the bidder with lowest marginal cost.[10] In this case, government may need to take some action to intervene with it to guarantee allocation efficiency.

This proposition raises the interesting question of how to sell the licenses and whether the government should 'level the playing field'. If the government cares only the revenue, then the case (7.b) is just a variation of irregular cases considered in MyersonMyerson (1981) and we can use the optimal auction method developed there. If we are interested in the social welfare, then the best policy is to assist 'entry' or to subsidize all bidders' sunk cost investment. In practice, the incumbent in utilities industry may have advantage of established network and thus need to invest much less in sunk cost investment if they win licenses. On the other hand, the new entrant may have to spend huge money to set up their new network. In this case, even if the potential entrant has operation (marginal) cost advantage; he may not be able to win a license due to the sunk cost investment expenditure. This point can explain why government should assist entry in certain circumstances.

[10] This point is actually very close to Che's study of multidimensional auction design problem. For details, see Che (1993).

5. Conclusion

We have studied how the auction format affects the potential license holders' belief through the bidding process and how this information changes their evaluation of license and thus their bidding strategy. We discuss separately different auction format followed by different post-bid competition.

In the case where licenses winners are going to play the Cournot game in the post-bid market, I have showed that there is a unique symmetric solution in the post-bid market. Furthermore, the quantity and the expected profit are both decreasing function of cost type, which guarantees the allocation efficiency. However, I failed to give an explicit solution thus failed to compare the expected revenue for the seller under an ascending English auction and a first-price sealed bid auction. This problem is mainly due to the complexity of the integral equation (12).

In the case where licenses winners are going to play the Bertrand game, things are more interesting. The basic finding here is that a first-price sealed bid auction generate higher expected revenue than an ascending English auction. The reason is simple, and also very similar to Milgrom's argument (Milgrom and Weber (1982)). An ascending English auction enables all bidders to update their belief and adjust their evaluation of license continuously. The only difference from Milgrom's argument is that, in his model, one's good news is also good news for his rival, so higher bid of others makes all other bidder to bid higher. In the story told in this chapter, one's good news is bad news for his rival, so higher bid of others makes all other bidder to bid lower.

In the first-price sealed bid auction, all bidders are not able to update their belief and evaluation of license. So on average, from the perspective of government who want to raise more money, a first-price sealed bid auction is better than an ascending English auction.

Besides, I find also an extraordinary result which is very interesting and similar to some other findings. In the Bertrand competition case, if an ascending English auction is adopted, then the evaluation adjust process will be such dramatic that all except two lowest cost bidders will quit the auction sequentially. The result is that in this case, government will get nothing from selling licenses. Interestingly, though no revenue is raised, allocation efficiency is still guaranteed.

BIBLIOGRAPHY

Armstrong, M. (1996). "Multiproduct Nonlinear Pricing." *Econometrica* 64: 51-76.

Armstrong, M. (1998). "Optimal Multi-unit Auctions." *Review of Economics Studies*.

Binmore, K. and P. D. Klemperer (2002). "The Biggest Auction Ever: the Sale of the British 3G Telecom Licenses." *Economic Journal*.

Bulow, J. I., M. Huang and P. D. Klemperer (1999). "Toehold and Takeovers." *Journal of Political Economy* 107: 427-454.

Bulow, J. I. and P. D. Klemperer (1999). "The Generalized Wars of Attrition." *American Economic Review* 89: 175-189.

Bulow, J. I. and J. Roberts (1989). "The Simple Economics of Optimal Auctions." *Journal of Political Economy* 97: 1060-1090.

Che, Y.-K. and I. L. Gale (1998). "Standard Auctions with Financially Constrained Bidders." *Review of Economic Studies* 65: 1-21.

Demsetz, H. (1968). "Why Regulate Utilities?" *Journal of Law and Economics* 11: 55-65.

Dewatripont, M. (1988). "Commitment Through Renegotiation-Proof Contracts with Third Parties." *Review of Economic Studies* 55(3): 377-389.

Harris, M. and A. Raviv (1981). "A Theory of Monopoly Pricing Schemes with Demand Uncertainty." *American Economic Review* 71(3): 347-365.

Harsanyi, J. C. (1967). "Games with Incomplete Information Played by Bayesian Players: Part 1." *Management Science* 14(3): 159-82.

Hendricks, K. and R. H. Porter (1989). "Collusion in Auctions." *Annales D'conomie et de Statistique* 15: 217-230.

Jehiel, P. and B. Moldovanu (1996). "Strategic Nonparticipation." *Rand Journal of Economics* 27: 84-98.

Jehiel, P. and B. Moldovanu (2000). "Auctions with Downstream Interaction among Buyers." *Rand Journal of Economics* 31(4): 768-791.

Jehiel, P. and B. MoldovanuJehiel, P. and B. Moldovanu (2001). The European UMTS/IMT-2000 License Auctions.

Jehiel, P. and B. Moldovanu (2002). "License Auctions and Market Structure." *Econometrica*.

Jehiel, P., B. Moldovanu and E. Stacchetti (1996). "How (Not) to Sell Nuclear Weapons." *American Economic Review* 86: 814-29.

Klemperer, P. D. (1998). "Auctions with Almost Common Values." *European Economic Review* 42: 757-69.

Klemperer, P. D. (2002). "What Really Matters in Auction Design." *Journal of Economic Perspective* Forthcoming.

Krishna, K. (1993). "Auctions with Endogenous Valuations: The Persistence of Monopoly Revisited." *American Economic Review* 83: 147-60.

Li, H. and J. G. RileyLi, H. and J. G. Riley (1999). Auction Choice. Department of Economics, Hongkong University of Science and Technology.

Maskin, E. S. and J. G. Riley (1984). "Optimal Auctions with Risk Averse Buyers." *Econometrica* 52: 1473-1518.

Maskin, E. S. and J. G. Riley (1999). "Optimal Multi-Unit Auctions." *Review of Economic Studies*.

Maskin, E. S. and J. G. Riley (2000). "Asymmetric Auctions." *Review of Economic Studies* 67: 413-438.

Milgrom, P. R. (2000). "Putting Auction Theory to Work: The Simultaneous Ascending Auction." *Journal of Political Economy* 108(2): 245-272.

Milgrom, P. R. and R. J. Weber (1982). "A Theory of Auctions and Competitive Bidding." *Econometrica* 50: 1089-1122.

Mirrlees, J. A. (1971). "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies* 38: 175-208.

Myerson, R. B. (1981). "Optimal Auction Design." *Mathematics of Operations Research* 6: 58-73.

Riley, J. G. and W. F. Samuelson (1981). "Optimal Auctions." *American Economic Review* 71: 381-92.

Robinson, M. S. (1985). "Collusion and the Choice of Auction." *Rand Journal of Economics* 16(1): 141-145.

Tan, G. (1992). "Entry and R&D in Procurement Contracting." *Journal of Economic Theory* 58: 41-60.

Vickrey, W. (1961). "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance* 16: 8-37.

Wang, G., Z. Zhou, s. Zhu and S. Wang (1978). *Ordinarly Differential Equation*. Beijing, Advanced Education Press.

Weber, R. J. (2000). Multiple-object Auctions. In *The Economic Theory of Auctions*. P. D. Klemperer. Cheltenham, Edward Elgar. 2: 241-266.

Wilson, R. (1977). "A Bidding Model of Perfect Competition." *Review of Economic Studies* 44: 511-18.