History of consumer demand theory 1871-1971: A Neo-Kantian rational reconstruction

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Abstract

This paper examines the history of the neoclassical theory of consumer demand from 1871 to 1971 by bringing into play the knowledge theory of the Marburg School, a Neo-Kantian philosophical movement. The work aims to show the usefulness of a Marburg-inspired epistemology in rationalizing the development of consumer analysis and, more generally, to understand the principles that regulate the process of knowing in neoclassical economics.

Keywords

Consumer Theory, Demand Theory, Utility Theory, Neo-Kantianism, Marburg School

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The neoclassical theory of consumer demand emerged, developed and reached its current standard form over the course of a century, from the time it appeared in the works of C. Menger and W.S. Jevons in 1871 up to the 1971 volume *Preferences, Utility, and Demand* edited by J.S. Chipman and others, which fine-tuned it in many ways. Several scholars have already studied the phases of this century-long development of consumer demand theory[1]. My analysis of the history of consumption theory between 1871 and 1971 focuses on three central aspects: the acceptance of a general utility function in place of the original additive utility function, the superseding of the cardinal utility framework according to either a behaviorist approach or an ordinal utility approach, and the axiomatic foundation of the utilitarian-ordinalist analysis[2].

My historiographical reconstruction belongs to the genre of "rational reconstructions". Since this expression originally coined by Lakatos (1971) has been interpreted in different and often pejorative ways, a terminological qualification is in order. First, by rational reconstruction I mean a narrative which considers the development of economic science as fundamentally driven by theoretical questions rather than by personal, academic, social, political, or economic factors. Second, my historical research is characterized by an epistemological approach which aims to understand the cognitive principles that seem to guide the development of neoclassical consumption theory. In this respect, the story I propose is a rational reconstruction in the sense that it attempts to rationalize the development of consumer demand theory as regulated by such cognitive principles.

What are these principles? Like many other branches of theoretical economics, consumption analysis faces a trade-off between the "realism" of a theory and its "systematicity". I understand a theory's realism to be the extent to which its assumptions and empirical implications are consistent with commonsensical, statistical or experimental evidence. If any assumption or implication of the theory does not correspond to commonsense evidence, statistical market data or the experimental findings obtained in a controlled environment, the theory is not wholly "realistic" or, put another way, it is to some extent "unrealistic". In

particular, one theory is "more realistic" than another if, *ceteris paribus*, the former has at least an assumption or an implication that corresponds better to commonsensical, statistical or experimental evidence[3]. By systematicity of a theory I mean its power to derive the largest possible set of exact implications concerning a certain phenomenon from the smallest possible set of exact assumptions.

Now, my historical inquiry suggests that more realistic assumptions are introduced into consumption analysis only if they do not weaken the systematicity of the theory already achieved. If the inclusion of assumptions that correspond better to commonsense evidence, statistical data or experimental findings jeopardize the systematicity of the theory, they are simply set aside. This does not mean that the search for realism is irrelevant in neoclassical theorizing about consumption, but rather that the pursuit of systematicity is stronger than that of realism. One can say that neoclassical economics tries to maximize the realism of consumer theory under the constraint of preserving its systematic character.

This conclusion suggests a more general set of questions. The effort of capturing the available evidence without blurring the systematicity of the theory appears to be an epistemic principle not only of neoclassical consumption theory, but of neoclassical economics in general. In fact, this principle seems to correspond to the method of successive approximations which neoclassical economists – at least from Menger's defense of theoretical economics against the Historical School during the *Methodenstreit* – have repeatedly upheld in response to the criticisms of lack of realism raised by various heterodox schools. The epistemological question is therefore: Why is the systematic nature of the theory so important for neoclassical economics? Is the hierarchy between systematicity and realism (in the sense that when the commitment to realism conflicts with the commitment to systematicity, the latter prevails) an idiosyncrasy of neoclassical economics, or is it instead rooted in a broader feature of scientific knowledge? Is there any relation between the central role that systematicity had and still has in neoclassical theorizing, and the fact that the neoclassical theory displaced and superseded the other approaches, and became the prevailing one?

As I shall suggest, the available methodological theories of economics do not offer a satisfactory response to these questions. However, there is an epistemological perspective which is virtually unexplored in the literature on economic methodology and appears useful in this respect. The epistemological perspective at issue is that of the so-called Marburg School, a Neo-Kantian philosophical movement which attained a leading position in German philosophy during the late 19th and early 20th century, and whose main exponents were H. Cohen, P. Natorp and E. Cassirer[4]. My appraisal of the Marburg epistemology will stress the tenets of it that make the cognitive principles of neoclassical economics more comprehensible. Therefore, I choose to refer to a "Marburg-inspired" theory of knowledge when referring to the epistemological view I present and support in this paper.

The aim of this work is therefore threefold. First, I claim that a Marburg-inspired theory of knowledge is useful in understanding the historical development of neoclassical consumer theory. Second and more generally, I argue that the Marburg epistemological view allows us to grasp the principles that regulate the process of knowing in neoclassical economics and does so better than other epistemological theories. Third, I suggest that a Marburg theory of knowledge not only allows us to comprehend the actual cognitive logic of neoclassical economics but also gives it a philosophical justification.

This does not mean that the economists who played a part in developing consumer theory were somehow inspired by the Marburg epistemology: to my knowledge, none of them refer to it anywhere, nor do Cohen, Natorp, or Cassirer refer to economic science in their writings. Therefore, I am not claiming that the Marburg School's epistemology is the hidden "muse-philosophy" of neoclassical economists. Moreover, neoclassical economists have professed and still profess different methodological views. Despite these differences, consumer theory seems to have stable, specific principles that regulate its development. These regulative principles, and not the possible, hidden roots of the methodological views of neoclassical economists are the subject of the paper. The point I would like to make is that a Marburg-inspired theory of knowledge explains such principles well. Consequently,

the reconstruction of consumption theory I propose is not an investigation of hidden intellectual backgrounds nor a search for possible influences, but rather a case for the general validity of the Marburg theory of knowledge.

Even if the rediscovery of the Marburg epistemology was originally stimulated by abductive reasoning ("Which theory of knowledge can rationalize the way consumer demand theory developed?"), the presentation approach I adopt in this paper is more deductive. Therefore, in Section 1 I illustrate what I consider to be the main tenets of the Marburg theory of knowledge, and in the following sections I attempt to show how these tenets shed light on the historical development of neoclassical consumer theory. Accordingly, Section 2 is devoted to the acceptance of a general utility function in the theory; Section 3 examines how the cardinal framework was superseded, and Section 4 considers the axiomatic foundation of the ordinalist analysis. Finally, Section 5 briefly compares the Marburg perspective with some other trends in economic methodology and illustrates why the history of consumer theory can be best thought of in Neo-Kantian terms.

1. The Marburg epistemological view

Among the different trends included in Neo-Kantianism, the Marburg School is characterized by its logicist development of Kant's theory of knowledge. Its main epistemological tenets can be outlined as follows:

1) Things are never given in and of themselves, but are always mind-correlative. This means that things are always given within some kind of "sense" of them: perception, common sense, scientific knowledge, practical use, moral commitment, aesthetic experience, etc. Of all these kinds of experiencing things, however, scientific knowledge holds a position of prominence, in the sense that ultimately something is "real" exactly as science takes it to be.

2) Since things are given within the knowledge of them, the *a priori* principles of cognition determine the things as given. Kant distinguished between two types of *a priori* princi-

ples, the "constitutive" and the "regulative" ones. The constitutive principles determine the basic features and laws of a certain domain of phenomena, and are expressed by synthetic *a priori* judgments. In particular, Kant claimed that the fundamental statements of Euclidean geometry and Newtonian physics were synthetic *a priori* judgments expressing the *a priori* constitutive principles of natural phenomena. During the late 19th and early 20th century the success of non-Euclidean geometries and of A. Einstein's theory of relativity challenged Kant's claim and, more generally, undermined his concept of constitutive principles. As regards the regulative principles, they are epistemic criteria that guide scientific research and hence shape the development of science. Marburg Neo-Kantianism plays down the constitutive principles, focuses on the regulative principles of cognition, and sees in the actual history of science an a posteriori manifestation of them. Cohen, for example, affirms that "we *objectify* […] *reason in science*" (Cohen 1984 [1883]: 6, my translation).

3) What Cohen calls "the fact of science" therefore becomes the starting point for any epistemological investigation regarding the principles that regulate the process of scientific knowledge. With reference to economic science, this means that a Marburg-inspired epistemology aims to understand economics as it developed and as it is, without dispensing prescriptions on how economics should be done, without attempting to decide whether economics meets some alleged "right" methodological rule, and without dealing with demarcation issues ("Is economics *really* a science?") of a positivistic flavor. In this sense, the Marburg approach to science is quite in accord with what in contemporary epistemology is called the "naturalistic approach"[5].

4) What are then the principles that regulate the process of scientific knowledge? According to Marburg Neo-Kantianism, the scientific understanding basically endeavors to make experience intelligible by rationalizing the diverse features of a phenomenon as the outcome of as few explanatory factors as possible. Scientific thought reconstructs a phenomenon at the conceptual level by connecting its diverse elements in an ordered and exact network of entailment relations, so that what happens in any circumstance is wholly deter-

mined by the theory itself. In the terminology previously introduced, scientific thought primarily attempts to get a systematic theory of the phenomena under investigation.

5) This implies that the first goal of scientific understanding is not to attain a perfect copy of what is presented in the kinds of experiences (perception, common sense, etc.) that precede scientific knowledge. In its fundamental tendency to design a systematic unity of the given, theoretical thought can lose the characteristics of things as they are immediately offered in pre-scientific experiences. In particular, scientific concepts need not correspond to objects of perception or common sense, nor need they be abstractions maintaining the essential features of the "real world". Consistence with common sense is not the right criterion to evaluate scientific concepts since they are products of thought through which understanding reconstructs experience as a system of dependencies and so makes it intelligible (cf. *e.g.* Cassirer 1953 [1910]: 156 ff., 302 ff.).

6) It is worth noting that there is some affinity between the instrumentalist viewpoint, which in economics is usually associated with the positions of Milton Friedman and Fritz Machlup, and the Marburg perspective[6]. Both epistemological views acknowledge that, in the first instance, scientific theories do not aim to be exact copies of "reality". However, there is also an essential difference between the instrumentalist fictionalism and the Marburg cognitive fictionalism, since the former regards a theory as a useful tool for practical ends (typically for empirical prediction), whereas the latter regards a theory as primarily useful for cognitive ends, that is, in order to make experience intelligible. In particular, Friedman's position is most distant from the Marburg one since Friedman rather univocally stresses the idea that scientific theories are tools used to make predictions, and famously claims that "in general, the more significant the theory, the more unrealistic [its] assumption" (Friedman 1953: 14). This thesis regarding the unrealism of assumptions is alien to the Marburg view, which simply claims that assumptions can be unrealistic if they make it possible to obtain a systematic picture of the phenomenon at issue. Machlup is less distant from the Marburg view since he does not even mention the unrealism-of-assumptions the-

sis, and points out that theories serve not only for prediction but also "*as an instrument of explanation*" (Machlup 1955: 12).

7) It is also noteworthy that the Marburg epistemological view does not prescribe what the "right" theory for a particular field of phenomena should be. As a matter of fact, the intellectual pursuit of a systematic synthesis of a given domain is just a formal one, which produces scientific systems with different contents. Therefore, two divergent theories of a certain phenomenal domain can coexist if they both connect the diverse elements of the domain in an evenly ordered and exact system of entailment relations. Conversely, if the system of dependencies set forth by one theory is more complete than the one set forth by another, scientific understanding will endorse the former.

8) The idea that there can be a multiplicity of theories to give the manifold phenomena a synthetic unity links the Marburg epistemology to the conventionalism of H. Poincaré (1902) and especially of P. Duhem (1906). In fact, the Marburg view shares with Duhem's conventionalism the concept of scientific theories as systematic wholes, whose parts cannot be separately subject to empirical verification, and the subsequent idea that a theory can be evaluated simply with reference to another theory and not with reference to the observational data alone.

9) A last specification is necessary. In economic methodology the label "Neo-Kantian" is sometimes attributed to L. von Mises' approach to economic theory[7]. Mises (1949) claims to derive economic propositions from a basic statement regarding human action, namely that men consciously act in order to attain their goals. In particular, Mises claims that this statement about human action is a synthetic *a priori* judgment in a Kantian sense. Mises' doctrine bears little relationship to the Marburg epistemology. Basically, whereas the former is a theory of action and ultimately of economic phenomena, the latter is a theory of knowledge. Moreover, whereas Mises invokes the action as the *a priori* constitutive principle of the economic domain, the Marburg approach refers only to the regulative principles of scientific cognition and is quite suspicious about claims concerning constitutive princi-

ples. Indeed, such claims implicitly prescribe what the right theory for a certain domain should be, and often end up being discounted by subsequent scientific developments, as in the case of Kant's claims on Euclidean geometry and Newtonian physics.

The Marburg theory of knowledge just outlined offers a philosophical justification for what I claimed is the main factual rule characterizing the development of consumption theory, namely that more realistic assumptions are introduced into the theory only if they do not weaken the systematicity of the picture already achieved. According to the Marburg view, such a rule is indeed the outcome of the basic regulative principle of the scientific understanding, which is to make phenomena intelligible by connecting them in a systematic theory even at price of unrealism. I will now examine the history of consumption analysis, trying to show that the rule "more realism, but without loss of systematicity" actually regulates its development.

2. From the additive to the general utility function

2.1 The additively separable utility function

At the starting point of the present narrative are the theories of Menger (1871), Jevons (1871) and L. Walras (1874), since these authors lay down the basic marginalist framework on which neoclassical consumer demand theory was subsequently built[8]. According to Menger, Jevons and Walras, the economic value of a commodity depends on the evaluation that the subjects give to its marginal units. This evaluation is called "final degree of utility" by Jevons, "Grenznutzen" by the Austrian School, "rareté" by Walras and then simply "marginal utility". Even though Jevons acknowledges that utility is not measurable, he first writes total utility as a cardinal function $u(x)$ of the quantity consumed of the good, and marginal utility as the function obtained by differentiating total utility. In addition, Jevons assumes that marginal utility is a positive and decreasing function, and that total utility is an additively separable function. This last assumption means that, if more commodities are consumed, the total utility obtained from any one of them is independent of the quantities

consumed of all the others or, in other words, that the marginal utility of a commodity depends only on the quantity consumed of that commodity[9]. Based on these hypotheses about total utility (additively separable) and marginal utility (positive and decreasing), Jevons obtains his famous "equations of exchange" for the two commodities/two agents case. The equations state that, for the maximization of the traders' utility under the budget constraints and without trades at disequilibrium exchange ratios, for each agent the ratio between the marginal utilities of the commodities must be equal to the exchange ratio between the commodities (Jevons 1871: 95 ff.).

Based on the same assumptions as Jevons, in the *Élements* Walras systematizes the subjective value theory in a coherent and general price theory, and first fixes the exact relationship between the marginal utility of a good and its demand (Walras 1874: 77 ff.). In his *Mathematical Psychics*, F.Y. Edgeworth introduces into the theory a more general utility function, that is, a function that need not be an additively separable one. This "general" function would make it possible to capture the apparent phenomenon of the utility interdependence of goods (Edgeworth 1881: 20, 104). In order to analyze the exchange when utilities are interdependent, Edgeworth also invented the indifference lines.

2.2. Problems with the general utility function

However, introducing a general utility function represents a tricky theoretical problem. In fact, the assumption that the marginal utility of any commodity is positive, decreasing, and depends only on the quantity consumed of that commodity makes it possible to univocally determine the relationships between the demand for goods and their prices, as well as ensure that the second order conditions for the constrained maximization of utility are satisfied[10]. As regards the first issue, if the utilities of the goods are independent, the demand for a good definitely decreases (increases) as its price rises (falls) (*i.e.* $\partial x_i / \partial p_i < 0$). In other words, with additive utility the so-called "law of demand" holds, and demand curves are always downward sloping. With a general utility function this is no longer univocally true:

demand for a good might also increase as its price rises (this is what will be called the Giffen case)[11]. As regards the utility maximization, additive utility together with positive and decreasing marginal utility ensure that the second order conditions for the constrained maximization problem of the agent's utility are satisfied. Therefore, if all commodities are purchased, first order conditions are sufficient for a maximum. In geometrical terms, Jevons' assumptions imply that the agent's indifference curves are strictly convex, so that tangency conditions are sufficient to discover the optimal combination of commodities. With a general utility function, this is no longer true. Even with decreasing marginal utilities, plausible substitution relationships between commodities could cause indifference curves to be concave, so that a point that satisfies the tangency conditions can also be of minimum, not maximum, utility (in the literature of the period such points are called "unstable equilibria"). Moreover, if the indifference curves are convex in some tracts and concave in others, there can be not one, but a multiplicity of tangency points, some of maximum and others of minimum utility.

Edgeworth aims to introduce a general utility function, but without losing the convexity of indifference curves. Therefore, introducing a general utility function raises two problems: that of providing the analytical condition for convexity (*i.e.* the second order condition for the constrained maximization problem), and that of justifying why such condition should hold. Edgeworth (1881: 34 ff. and 108 ff.) provides the condition at issue for the two-commodities case, and claims that it is, by and large, satisfied, albeit with a deceptive argument[12]. As regards the sign of $\partial x_i / \partial p_i$ in a non-additive framework, Edgeworth does not investigate such an issue. In conclusion, Edgeworth draws attention to the unrealistic nature of the additive utility assumption (unrealistic in the sense that it is at odds with the commonsensical perception of the utility interdependence of commodities), but does not solve all the analytical problems raised by the adoption of general utility.

In this state of affairs, the introduction of the more realistic general utility function into the hypotheses of the theory would have caused the loss of the determinateness of the de-

pendencies between demand and prices obtained by Jevons and Walras. With a general utility function the demand curve can be downward or upward sloping (as well as horizontal) and indifference curves can be convex or concave. This larger set of possible implications appears desirable just to the extent to which it is possible to understand and exactly determine under which theoretical circumstances each case occurs, otherwise the theory loses its explanatory power and vacuously states that "anything can occur".

The determinants of $\partial x_i/\partial p_i$ and the analytical conditions for convex indifference curves in a non-additive framework will become clear with the works of V. Pareto, W.E. Johnson and E. Slutsky at the beginning of the 20th century. Until that moment, introducing general utility just means introducing unpredictability and indetermination into the picture. According to the Marburg epistemological theory, economics is expected to hold to the systematic theory based on the unrealistic additive utility assumption until the difficulties raised by the introduction of general utility are resolved. And this is what happens.

2.3 Marshall's demand theory

Marshall basically rejects Edgeworth's suggestion of a general utility function and never utilizes indifference lines which are the typical analytical tools that make it possible to take into account the interdependence of commodity utilities. In all the editions of the *Principles* (1890-1920), Marshall's consumption theory relies on the additive utility assumption in a crucial, though subtle, way. Without this assumption, "the one universal rule to which the demand curve conforms [*i.e.*] that it is *inclined negatively* throughout the whole of its length" (Marshall 1961: 99, note) breaks down. However, this means that Marshall's partial equilibrium analysis, based on the intersection of a downward sloping demand curve and of an upward sloping supply curve, falters. As regards the admission of the interdependence of utilities, Marshall's attitude is therefore rather inconsistent. Although in the first edition of the *Principles* (1890), he explicitly deals with rival commodities, in the second edition (1891) he tries to defend the additive utility assumption against Edgeworth (Marshall 1961:

845). Subsequently, in the third edition (1895), he openly admits that "we cannot say that the total utility of [two commodities which contribute to the same purpose] is equal to the sum of the total utilities of each separately" and afterwards introduces the Giffen goods case. This can be explained only if the additive utility assumption is removed, but fortunately such a case is "rare" (Marshall 1961: 131 f.). These are all surface adjustments which do not modify the analytical groundwork of Marshall's demand theory, which continues to be implicitly based on an additive utility function until the final 1920 edition of the *Principles*[13].

Apart from the question of additive utility, Marshall is a noteworthy figure in our narrative because he introduces a way of dealing with consumption theory, different from that of Jevons, Walras and Edgeworth. First, Marshall considers purchasing by an isolated agent rather than the exchange among many agents. Second, consumer prices are fixed by the market and are not the result of the exchange process. Third, the household does not have an endowment of commodities to trade but a monetary endowment with which to purchase commodities. In this way, Marshall limits consumption analysis to the current boundaries and separates exchange theory from consumption theory.

2.4 The situation in the 1890s

In the 1890s, in addition to the unrealistic nature of the additive utility assumption, another serious limitation of the classical utilitarian theory is highlighted: the absence of a measure of utility. It is not at all clear what the values that the function $u(x)$ associates to the commodity quantities x mean, how these values could be measured, or, least of all, what the unit of measure of utility is. In his *Mathematical Investigation,* I. Fisher tries to find an empirical measure of utility based on the economic choices of agents. Nevertheless, Fisher's attempt also collides with utility non-additivity: he himself recognizes that his method of measurement works only if commodity utilities are independent (Fisher 1926 [1892]: 64-7). G. Cassel also attempts to overcome the problems related to utility by adopting an alterna-

tive demand theory independent from the utility notion, but obtains no satisfactory results (Cassel 1899: 413 ff.).

Therefore, at the end of the 19th century, the unrealistic character of the classical – additive, cardinal – utility is quite clear. Classical utility is unrealistic in at least three ways: *i*) as an assumption in itself, since the utilities of commodities appear to be interdependent; *ii*) for its empirical implications, which conflict with Giffen goods; *iii*) in the sense that it does not correspond to anything actually measurable. However, despite the recognized unrealistic character of classical utility, the neoclassical economists of the period hold to it. The subsequent editions of Walras' *Élements* (1889-1900) follow the first on this critical point. As already mentioned, not even Marshall changes the groundwork for his theory in the subsequent editions of the *Principles*. P.H. Wicksteed (1888), K. Wicksell (1893), E. Barone (1894) and, although in a non-mathematically explicit form, M. Pantaleoni (1889 and 1898), F.F. von Wieser (1889), and J.B. Clark (1899) maintain the cardinal additive utility assumption[14].

2.5 The method of successive approximations

The Marburg explanation for this state of affairs is that the neoclassical economists of the end of the 19th century hold themselves to the unrealistic classical utility since it allows them to construct a systematic theory of consumer demand. In this case, the commitment to realism conflicts with the commitment to systematicity, and the latter prevails. This does not mean that grasping the observed phenomena ceases to be the task of these economists. The point is that this understanding must be a theoretical one, that is, the phenomena at issue (*e.g.* those related to the individual demand of commodities) have to be understood in a system of exact entailment relations. If, however, capturing empirical evidence requires the introduction of assumptions that preclude precise theoretical implications and open the door to an "anything goes" theory, then they are temporarily put aside.

The method of successive approximations basically consists in this effort of "approximating" the available evidence as much as possible by means of successive theories based on progressively more realistic assumptions which must not, however, blur the theoretical system. Neoclassical economists have repeatedly upheld such a method in response to the criticisms of the lack of realism and empirical relevance often addressed to them by the German Historical School, the American Institutionalists and other heterodox schools. The Marburg theory of knowledge legitimizes the successive approximation method by tracing it back to the regulative principles of scientific understanding, and, accordingly, offers a vindication of neoclassical economists' methodological common sense.

2.6 Pareto's step forward

In his first cardinalist phase, Pareto builds up a general utility analysis more comprehensive than those of his predecessors, without, however, using his own results. In fact, in the five-part article, "Considerazioni" he recognizes the soundness of Edgeworth's generalization, states the exchange analysis in general terms, but then develops it within the additive framework (Pareto 1892-93, parts I-III). He subsequently resumes the general utility analysis and first provides the exact expression of $\partial x_i / \partial p_i$ in the general utility case and for any number of goods, but then continues by using the additive utility which he declares "approximately true" (Pareto 1892-93, part V: 306-7)[15]. Similarly, in the *Cours*, Pareto develops the entire analysis with additive utility functions and discusses the general case only in a footnote (Pareto 1896-97: 332-4). After 1900 Pareto changes his approach to the topic. In the *Manual of Political Economy* (Italian edition 1906; French edition 1909) he recognizes that the additive assumption is an approximation made "in order to simplify the problems" but affirms that "it is time now to take a step forward and also consider the case in which the ophelimity of a good depends on the consumption of all other goods" (cf. Pareto 1906: 241, and 1909: 253)[16].

Accordingly, in the Appendix to the *Manual*, Pareto provides a sufficient condition for the convexity of the indifference curves in the general two-commodity case. Besides, he discusses what sort of relationships between the commodities support the condition at issue, and points out that it is surely fulfilled when the utilities of the commodities are independent (cf. Pareto 1906: 504 ff., and 1909: 572 ff., 651 ff). He also provides a different, sufficient convexity condition in the case with any number of goods (Pareto 1906: 550, and 1909: 577). Moreover, in the Appendix to the French edition of his work, Pareto reintroduces the general expression of $\partial x_i/\partial p_i$ that he already obtained in his 1892 "Considerazioni" (cf. Pareto 1909: 580-1)[17]. Although Pareto's conditions for convexity are too strong and the economic meaning of the Paretian formula for $\partial x_i/\partial p_i$ is not clear, these results offer the first satisfactory solution to the problems induced by a general utility function. The sufficient conditions for convexity identify a set of theoretical circumstances which ensure that a commodity bundle actually is of maximum utility, and the analytical expression of $\partial x_i/\partial p_i$ makes it possible to state when the law of demand holds.

The impact of Pareto's non-additive treatment is quite apparent in the literature of the period. At the very beginning of the 20th century, the contributions in demand theory were still developed within the additive framework, as the works of A. Aupetit (1901), C. Colson (1901), U. Ricci (1904), P. Boninsegni (1904), A.W. Flux (1904) and H. Cunynghame (1904) show. After the publication of Pareto's *Manual* (particularly the French edition), general utility became the starting point of the analysis. In the 1910s the main contributions that built on consumption theory in a general utility framework were those of Johnson (1913) and, above all, Slutsky (1915).

2.7 The completion of general utility analysis: Johnson and Slutsky

Johnson analyzes the effects on demand of a variation in individual income as well as in commodity prices in the two-commodity case[18]. At a geometrical level, he constructs the income-consumption curve as well as the price-consumption one. At a mathematical level, he

provides an analytical treatment of "the case in which an increased price leads to an *increase* of the amount of the commodity bought (*i.e.* [the] Giffen's paradox)" (Johnson 1913: 484) and demonstrates that the Giffen goods are a subset of the inferior goods.

The definitive systematization of consumer demand theory in a general utility framework is accomplished by Slutsky. In 1915, Slutsky publishes his fundamental article in the *Giornale degli Economisti*, the Italian journal where Pareto had published most of his contributions[19]. However, as is well known, Slutsky's paper will remain basically ignored until the 1930s (see below). In his article, Slutsky provides the following results for any number of goods and for a general utility function (cf. Slutsky 1915: 4-19):

1) The second order, necessary and sufficient conditions for the constrained maximization of utility.

2) The exact mathematical expressions of $\partial x_i/\partial p_j$, $\partial x_i/\partial p_i$ and $\partial x_i/\partial I$.

3) By introducing the notion of "compensated variation of price", Slutsky (1915: 14) decomposes the effect of a price change on the demand for goods into the part due to the substitution effect $\hat{\partial} x_i/\hat{\partial} p_i$ and into the one due to the income effect, thereby obtaining what will be called "The Slutsky Equation"[20].

4) If the second order condition is satisfied, the substitution effect is always non-positive. That means that for the compensated demand, the "law of demand" holds.

5) The compensated variations of the demand for good *i* when the price of commodity *j* varies, that is, the cross-price substitution effect $\hat{\partial} x_i/\hat{\partial} p_j$, is symmetric. If we call "substitution matrix" the matrix of the terms $\hat{\partial} x_i/\hat{\partial} p_j$, this means that the substitution matrix of the demand function is symmetric. Moreover, if the second order condition holds, the substitution matrix is negative semi-definite[21].

6) Although the substitution effect is always non-positive, the income effect can be either negative or positive and the Giffen case is due to a positive income effect which is greater in absolute value than the substitution effect.

The works of Pareto, Johnson and, after its re-discovery, of Slutsky show that the introduction of general utility does not imply introducing unpredictability into the theoretical picture of consumer demand. It becomes clear that not only with additive utility but also with general utility it is possible to specify in all theoretical circumstances the exact conditions for the maximization of consumer utility, as well as to determine the precise interrelations between demand, prices, and income. Indeed, in the 1910s general utility became the standard whereas additive utility became a special case, as the works of Wicksteed (1910), G. Borgatta (1911-12a and 1911-12b), A. Osorio (1913) and W. Zawadzki (1914) show.

2.8 The acceptance of general utility and the Marburg epistemological view

The story of the acceptance in consumer demand theory of a general utility function corroborates the Marburg-inspired claim that the rule "more realism, but without loss of systematicity" actually regulates the development of consumption analysis. Let us sum up this story. In the 1870s Jevons and Walras elaborated a systematic exchange and demand theory that crucially depended on the additive utility assumption. In 1881 Edgeworth pointed out the unrealistic character of this assumption, but did not elaborate an evenly systematic theory based on the more realistic assumption of general utility. The problems related to the introduction of general utility were resolved about 30 years later by Pareto, Johnson, and Slutsky. In the meanwhile, neoclassical economists generally recognized that the additive utility assumption was unrealistic, but held to it in all their works (cf. the works listed at the end of Sections 2.4 and 2.6). Only when Pareto, Johnson, and Slutsky showed how to construct a systematic theory of consumer demand in a general utility framework did general utility supersede additive utility as the standard assumption in consumption analysis.

3. Superseding the cardinal utility framework

3.1 Pareto the behaviorist, Pareto the utilitarian

Pareto is not usually remembered for having introduced a general utility function in demand theory, but for his landmark attempt to restate consumer equilibrium analysis without reference to cardinally measurable utility, whose empirical meaning appeared highly problematic at the turn of the 20th century. Such an attempt – the so called "Paretian revolution" – is carried out by Pareto after 1900 along two different lines of attack. According to the behaviorist approach, outlined in the "Sunto" (Pareto 1900), every hint of utility has to be set aside, and demand theory should be based only on the observable consumer choice behaviour. According to the ordinalist approach, more evident in the *Manual*, economic theory can still be based on utility, which is nonetheless regarded as a ranking index with just an ordinal meaning. From a mathematical viewpoint, this is tantamount to saying that a general utility function is unique only up to a strictly increasing transformation of it.

Pareto's post-1900 work is as rich as it is ambiguous, and actually includes both the behaviorist and the ordinalist approach. However, as observed by many authors from the 1930s on, Pareto's superseding of cardinal utility appears problematic on both fronts: his behaviorist demand theory actually implies many implicit references to utility, and such references are not only to ordinal but also to cardinal utility[22]. For example, utilitarian elements are evident in the way Pareto-as-behaviorist justifies convexity and the negative slope of indifference curves. He also refers to decreasing marginal utility, even if the sign of the second-order derivative is not invariant with respect to a monotone transformation of the utility function[23]. Similarly, the notion of complementary goods in terms of second-order cross-partial derivatives that Pareto shares with R. Auspitz and R. Lieben (1889) as well as with Edgeworth (1897), is cardinal[24]. As regards additivity, Pareto (1906: 501) points out that this is in fact a cardinal property of utility functions: a generic monotone transformation can convert an additive utility function into a non-additive utility function, and only linear transformations preserve additive separability. Still, on some occasions Pareto keeps assigning a

special status to additive utility functions[25]. Lastly, Pareto does not examine the question of whether the second order conditions for the utility maximization and the formula for $\partial x_i / \partial p_i$ he has derived in a cardinal framework are invariant to a monotone transformation of the utility function[26].

The contributions of Allen, Hicks and Samuelson in the 1930s all aim to carry out the Paretian revolution in demand theory by solving the problems the Italian economist left open. In their attempts to supersede cardinal utility, such economists follow one of the two approaches originally delineated by Pareto: Hicks and Allen in their joint paper as well as Samuelson adopt the behaviorist approach, whereas Hicks in *Value and Capital* adopts the utilitarian-ordinalist one. Before discussing the celebrated developments of the 1930s, I will briefly survey the works that appeared in the 1910s and 1920s, with special attention to their cardinal or ordinal character.

3.2 Ordinal insights, cardinal treatments

Returning to Johnson's and Slutsky's papers, although they contain important ordinal insights, many cardinal concepts remain in them. In his paper, Johnson points out the non-cardinal nature of the indifference map and observes that consumption theory does not need to know the marginal utility of a commodity but only the ratio of one marginal utility to another. However, he also talks about "*equal* additional increments of net utility" stemming from the continually increasing acquisition of a good (Johnson 1913: 485), and attributes meaning to the distance between indifference curves. Moreover, he makes assumptions regarding the sign and the magnitude of the second-order and cross-partial derivatives of the utility function without seeming to realize that these properties are not invariant to an increasing transformation of the utility function (cf. Johnson 1913: 490 ff.).

As for Slutsky, he takes Pareto's *Manual* as the starting point of his analysis and endorses the Paretian definition of utility as a purely ordinal index. Slutsky is interested in the empirical implications of the utility analysis, and in particular he seeks to determine the

sign of $\partial^2 u/\partial^2 x_i$ and of $\partial^2 u(x)/\partial x_i\,\partial x_j$ starting from the empirically observable values of $\partial x_i/\partial p_j$, $\hat{\partial} x_i/\hat{\partial} p_i$ and $\partial x_i/\partial I$. However, he comes to the conclusion that "it is impossible to deduce from the facts of conduct the character (that is, the sign) of the second derivatives of utility" (Slutsky 1915: 25), and recognizes that the traditional definition of complementarity, *i.e.* the Auspitz–Lieben–Edgeworth–Pareto definition, cannot be justified within a purely ordinal framework. He then suggests that the subjects may be able to tell whether two goods are complementary by means of "internal evidence" (Slutsky 1915: 25). Slutsky briefly discusses the empirical implications of this hypothesis which, however, is cardinal in nature. Furthermore, despite the fact Slutsky carries out all his analysis in terms of the second-order and cross-partial derivatives of the utility function, he does not attempt to verify whether the results obtained are invariant to an increasing transformation of the utility function.

In 1924, A.L. Bowley publishes his *Mathematical Groundwork of Economics* in which he attempts "to reduce to a uniform notation, and to present as a properly related whole" the main parts of the mathematical utility theories elaborated "by Cournot, Jevons, Pareto, Edgeworth, Marshall, Pigou, and Johnson" (Bowley 1924: v)[27]. Bowley's work presents one of the most comprehensive mathematical treatments of consumer demand theory of the 1920s. It also contains some ordinal insights, which however are not fully worked out. In fact, on the one hand, he explicitly points out that propositions depending on the sign or magnitude of the second-order derivative of the utility function embody the assumption that utility is cardinally measurable (Bowley 1924: 2). Accordingly, among the results obtained, he clearly indicates those which rely on assumptions about the utility second-order derivatives, namely those regarding the sign of $\partial x_i/\partial p_i$ (cf. Bowley 1924: 13, 15, 55-6). On the other hand, he maintains the cardinal notions of complementarity and substitutability in terms of the sign of $\partial^2 u(x)/\partial x_i\,\partial x_j$ and does not examine what happens in an ordinal framework to the formula $\partial x_i/\partial p_i$.

Other major contributions of the period 1920-33 either continue consumer demand analysis in an explicitly cardinal framework or contain some ordinal insights, which are not suitably exploited. The main examples are L. Amoroso (1921 and 1928), P. Boninsegni (1925), M. Fanno (1925-26), R. Frisch (1926), A. de Pietri-Tonelli (1927), F. Divisia (1928), G.C. Evans (1930), O. Weinberger (1930), A. Bordin (1932), V. Dominedò (1933) and H. Schultz (1933).

3.3 Hicks, Allen and the marginal rate of substitution

In 1934, Hicks and Allen publish a famous paper which aims to work out a theory of consumption choices along a Paretian line but "free of the inconsistencies detected in Pareto" (Hicks–Allen 1934: 55). Even though widespread interpretation puts the Hicks–Allen paper at the origin of utilitarian ordinalism, it belongs to the behaviorist camp: in fact, they start by admitting the cardinal immeasurability of utility and note that "if total utility is not quantitatively definable, neither is marginal utility" (Hicks–Allen 1934: 55). Hicks and Allen do not replace total and marginal utility with ordinal utility, but with the marginal rate of substitution (MRS). The MRS, defined as the amount of good *j* which substitutes a marginal unit of good *i* for the individual, is a quantitatively definable entity which can be empirically observed, and whose notion is independent from the notion of utility. Starting from the MRS, Hicks and Allen: 1) determine the relationships between the demand for goods, their price, and the consumer income in elasticity terms; 2) decompose the effect of a price change on demand, into income and substitution effect; 3) provide a new, non-cardinal definition of complementary and competitive goods which is equivalent to the current standard definition in terms of the cross effect of a compensated price change.

A problem with the Hicks-Allen construction is that all their results rely on two crucial assumptions: first, that the MRS is negative (which means decreasing indifference curves) and second, that the MRS is decreasing (which means convex indifference curves). As in the case of Pareto's behavioral approach, the tricky issue is that of justifying such assump-

tions without any reference to utility or psychological concepts. As to the first item, Hicks and Allen postulate the MRS negativity without any particular explanation. This statement can seem natural only on the basis of an unspoken endorsement of the hedonistic non-satiation principle. With reference to the decreasing MRS assumption, Hicks and Allen only assert that it is not falsified by experience, without putting forward any positive justification for it. Therefore, as Samuelson (1938: 61-2) will promptly underline, the convexity hypothesis once more seems ultimately to rely on psychological-utilitarian considerations.

In the end, this third attempt (after those of Cassel and Pareto) to set aside every reference to utility does not succeed. The Hicks-Allen restatement of consumer theory in terms of the MRS fails to become the standard neoclassical one for two reasons. From a methodological standpoint, the superseding of the utility notion appears debatable since the crucial assumptions of the MRS rely on postulations that are utilitarian in nature. In spite of this, the Hicks-Allen paper presents the most advanced results on the relationships between price, income and demand from a theoretical viewpoint. However, with the rediscovery of Slutsky's article and with the publication of *Value and Capital* by Hicks in 1939, it becomes clear that the results obtained on the basis of the MRS can be obtained through ordinal utilities in a much simpler way. Therefore, there are no methodological or theoretical reasons to adopt the Hicks-Allen approach, and it is indeed thereafter abandoned.

3.4 Hicks the orderer

At the beginning of the 1930s, Slutsky's fundamental article is rediscovered: Dominedò (1933) in Italy, and later Schultz (1935) and Allen (1936) call attention to it[28]. In particular, Allen shows that Slutsky's results are independent from cardinal measurability assumptions on utility and can be obtained entirely within an ordinalist framework (Allen 1936: 191-2). This opens the way to the ordinalist restatement of Slutsky's findings and to the formulation of the standard ordinalist neoclassical consumer theory.

Although for a while Allen (1938: 344 ff.) continues to maintain consumer choice analysis in terms of the MRS as a primary concept, Hicks in 1937 has already set forth consumer theory in terms of ordinal utility (Hicks 1937) and fine-tunes his formulation in *Value and Capital* (1939). In this work, the MRS ceases to be the basic element of the model and is regarded as the opposite of the ratio between the partial derivatives of the ordinal utility function. In any case, the required features for the MRS are the same as in the paper written together with Allen: the MRS has to be negative and decreasing. Negativity is still upheld on the basis of the principle of nonsatiation which makes marginal utilities positive. The case for the decreasing MRS is more complex than in the article written with Allen but remains basically the same: the assumption of a decreasing MRS (*i.e.* of indifference curve convexity) is required for the determinateness of the theory and does not seem to be disproven by experience (Hicks 1939: 13 ff.). In the end, Hicks re-presents Slutsky's results in a systematic and mathematically clear way, and demonstrates, more thoroughly than Allen and Schultz, how these findings are invariant to strictly increasing transformations of a general utility function. After *Value and Capital*, the Slutsky-Hicks ordinal theory becomes the standard one in consumer demand analysis.

3.5 The introduction of ordinal utility and the Marburg epistemological view

The story of the introduction into consumer demand theory of ordinal utility also seems to be well rationalized by the Marburg epistemology. Let us recapitulate this story. From the 1890s on, the cardinal measurability of utility which underpins the theories of Jevons, Walras, Edgeworth and Marshall began to appear as an unrealistic assumption to neoclassical economists. Cardinal utility looked unrealistic since it did not seem to correspond to anything empirically observable. Attempts to find a cardinal measure of utility (Fisher) or to develop an alternative demand theory independent from the utility notion (Cassel) failed. In this state of affairs, removing the assumption of cardinal measurability of utility meant undermining the existing theoretical construction without having a practicable alternative. Ac-

cording to the Marburg view, neoclassical economists were then expected to stick temporarily to cardinal utility, and this is what in fact happened until the publication of Pareto's *Manual* (cf. the works listed at the end of Sections 2.4 and 2.6).

In the *Manual*, Pareto elaborated a quite systematic analysis of consumer equilibrium in an ordinal framework. However, he did not solve all the problems raised by the superseding of cardinal utility. In this situation, still characterized by a certain theoretical ambiguity, the main books and papers published after the *Manual* (cf. the listings at the end of Sections 2.7 and 3.2) either simply held to the cardinal framework, or contained some ordinal insights which nonetheless were not worked out. After 1934 the rediscovery of Slutsky's paper and the contributions of Allen and Hicks showed that the introduction of ordinal utility did not imply introducing unpredictability into the theoretical picture of consumer demand. In fact, not only with cardinal utility but also with ordinal utility was it possible to determine exactly the conditions for the utility maximization, the interrelations between demand, prices, and income, as well as to set univocally the substitution and complementary relations between goods. Hence, when it turned out that the cardinal measurability assumption could be set aside without any loss of systematicity, the ordinal utility framework became the standard in consumer demand analysis.

3.6 The Chicago discussion of the Slutsky-Hicks demand theory

The establishment of the Slutsky-Hicks theory as the new orthodox one is not hindered by the fact that it is difficult to apply empirically, as some scholars at the University of Chicago point out. Schultz (1935, 1938) tries to test the empirical implications of Slutsky's analysis by comparing them with the data from the statistical demand curves of some commodities. Although Schultz's findings do not refute Slutsky's results, they certainly do not corroborate them[29]. Other Chicago scholars, such as G.J. Stigler (1939), W.A. Wallis and M. Friedman (1942), and F. Knight (1944), subsequently point out the difficulty of reconciling the demand curves of the ordinal Slutsky-Hicks theory with the statistical demand

curves. In particular, Wallis and Friedman question the possibility of empirically deriving the indifference function, either through laboratory experiments, like that performed by L.L. Thurstone (1931), or through a statistical estimation[30]. This makes the Slutsky-Hicks framework useless for empirical evaluations and for predicting the effect of changes in economic conditions on the consumption of various goods (Wallis–Friedman 1942: 177 ff.).

However, once again these criticisms of lack of realism and empirical relevance have no effect. As a matter of fact, neoclassical consumer theory does not take them into account at all, and instead develops in two different directions, with divergent aims but a similar approach. On the one hand, Samuelson strives to obtain analytical results comparable to those of Slutsky-Hicks, but on the basis of a set of postulates different from the utilitarian ones. On the other hand, first H. Wold and then G. Debreu, maintain the reference to utility but seek to rigorously ground the ordinal utility function and its features on an axiomatic theory of consumer preference.

3.7 Samuelson and the behaviorist-ordinalist equivalence

In an often quoted passage from his 1938 note on consumer theory, Samuelson (1938: 71) says that he seeks to "develop the theory of consumer behaviour freed from any vestigial traces of the utility concept". This does not mean that Samuelson wants to preclude the introduction of the utility notion or to contradict the results obtained by using related constructs (as shown by his work of the late 1930s on welfare economics with A. Bergson), but merely that he thinks that consumer behavior analysis "can be carried on more directly […] from a different set of postulates" (Samuelson 1938: 62)[31]. In fact, Samuelson (1938, 1947, 1953) proves that almost all the restrictions on the demand functions that derive from the constrained maximization of an ordinal utility function can also be obtained starting from a simple coherence assumption on consumer behavior. This assumption is what will be later termed the Weak Axiom of Revealed Preference.

The only restriction on demand functions that derives from the utility maximization hypothesis but not from the Weak Axiom is the symmetry of the cross-price substitution effects, *i.e.* the symmetry of the substitution matrix. This property of the substitution matrix is important since it is necessary (together with the matrix's negative semi-definiteness) for the existence of a utility function whose maximization generates the consumption choices expressed by a given demand function. The problem of defining under which conditions such a generating utility function exists, is the so-called "integrability problem", first studied by G.B. Antonelli (1886) and conclusively solved in some of the works collected in Chipman et al. (1971)[32].

For our purposes, it suffices to say that in 1950 H.S. Houthakker introduces the Strong Axiom of Revealed Preference (an iteration of the Weak Axiom, which rules out the possibility of cyclical choices) and shows that it is sufficient for the integrability of demand functions. On the basis of Houthakker's contribution, Samuelson (1950) proves that the symmetry and negative semi-definiteness of Slutsky's substitution matrix are not only necessary but also sufficient conditions for the integrability of demand functions (Samuelson 1950: 376-85). Samuelson's demonstration will be perfected by L. Hurwicz and H. Uzawa (1971)[33].

In conclusion, it turns out that the Slutsky-Hicks framework and the Samuelson-Houthakker one lead to identical restrictions on demand functions. This fulfills Samuelson's program of carrying out consumption theory from a non-utilitarian set of postulates, and at the same time makes the opposition between the ordinalist and the behaviorist approach vanish. After 1950 further results showing the substantial equivalence between the ordinalist and the behaviorist approaches are obtained by K. Arrow (1959), H. Uzawa (1960), M.K. Richter (1971) and others. As a matter of fact, since the 1950s the two approaches have coexisted in consumer demand analysis. This state of affairs is well in accord with the Marburg claim that two different theories can be considered both valid if they con-

nect the diverse elements of a given domain in an evenly exact system of entailment relations (see above Section 1, points 7 and 8).

4. Axiomatization and formalism

4.1 The axiomatic foundation of Slutsky-Hicks' theory

From the 1940s on, utilitarian consumer theory strives to find a sounder and more rigorous basis for its fundamental analytical tools, preferences and ordinal utility functions. In this direction, the pioneering attempt was made by Frisch (1926), but it is Wold who, in 1943-44, gives the first axiomatic treatment of the utilitarian theory of demand. In particular, Wold states axiomatically the general properties of the preference/indifference relations that allow us to represent "every indifference map […] as the level map of a continuous, non decreasing function" (Wold 1943-44, II: 223). Furthermore, Wold proposes a method to construct such a function which is ordinal in nature. Unfortunately, Wold's analysis is not completely correct.

The axiomatic treatment of preferences spreads into neoclassical economics a very few years later through *Theory of Games and Economic Behavior*, the book written by J. von Neumann and O. Morgenstern (1944). Their axiomatic handling of preferences is adopted by J. Marschak (1950), employed by Houthakker (1950), and perfected by Arrow (1951). Preference is formally conceived as a binary relation between two generic alternatives, whose basic properties are "completeness" and "transitivity" (cf. Arrow 1951: 11 ff.). In 1954, the so-called "paradox of lexicographic preferences" introduced by N. Georgescu-Roegen (1954) makes it apparent that it is not always possible to define a real valued order-preserving function on a set of alternatives ordered by the preferences of some agent. In the same year, Debreu specifies assumptions under which the representation of a preference ordering, by means of a real-valued continuous function, is possible and demonstrates the consequential representation theorem (Debreu 1954). In *Theory of Value* Debreu carries out the axiomatic analysis and determines the assumptions on both the consumption set and the

preference relation which are sufficient to derive a continuous quasi-concave utility function representing the preferences (Debreu 1959: 52 ff.)[34].

Yet, a continuous quasi-concave utility function is exactly the ordinal utility function that the Slutsky-Hicks consumer theory needs, since such a function ensures that the second order conditions for the constrained maximization of utility are satisfied, *i.e.* that the indifference curves are convex. In this way, it is possible to recover all the results already obtained by ordinal utility theory, but now starting from a basis fully specified in axiomatic terms. Debreu therefore provides the Slutsky-Hicks theory with a sound axiomatic foundation and brings neoclassical consumer theory to its current standard form.

4.2 Formalism as neoclassical methodological awareness

The axiomatic or formalist method stresses the logical rigor of the theory. In an oft-quoted passage of *Theory of Value*, Debreu peremptorily states that: "Allegiance to rigor dictates the axiomatic form of the analysis" (Debreu 1959: *x*). In another text, Debreu observes that the aim of the axiomatization of economic theory is the "full specification of the assumptions under which any one of its conclusions is asserted", and that "the complete specification of assumptions, the exact statement of conclusions, and the rigor of the deductions of an axiomatized study provide a secure foundation on which the construction of economic theory can proceed" (Debreu 1983 [1977]: 5-6).

From the formalist standpoint, the assumptions of the theory do not aim primarily to grasp the features of the "real world" or the "actual human beings", but have basically a logical-systematic function: they are axioms that ensure the determinateness and the connectedness of the theoretical construction. The interpretation and justification of an axiom by referring to commonsensical, statistical or experimental evidence (*e.g.* the psychological interpretation of the convexity of the indifference curves as a consequence of the human preference for variety) is of secondary import. If an interpretation makes some axiom appear more realistic, or provides a handy intuition to it, the interpretation is welcome. On the

contrary, if an axiom appears unacceptably unrealistic, the theorist's task is to either remove it ("to relax the assumptions") without letting the edifice of the theory collapse, or to replace the entire theoretical construction with another one which ought to be comparably systematic. Notably, within the axiomatic approach even previously abandoned assumptions can be recovered and analyzed as special cases valid under specific conditions. This is in fact what happens to the assumptions of additive separability and cardinal measurability of utility[35].

Now, the methodological tenets of the formalist approach are clearly in accord with those of the Marburg view, and the formalist motto "allegiance to rigor" may be easily translated in a Neo-Kantian "allegiance to systematicity". As I have tried to demonstrate, however, the priority assigned to the pursuit of systematicity over the pursuit of realism is not a formalist novelty. On the contrary, this is the same methodological principle that has tacitly guided the development of neoclassical consumer theory from its very beginnings. Sometimes it is instead claimed that the formalist approach would have changed the aims and epistemic rules of economic science: after World War II economic theory would have become more and more directed at achieving internal consistency to the detriment of factual relevance; the empirical element would have vanished in economic model-building, and a permissive attitude to unrealistic assumptions would have pervaded economic theorizing[36]. In our terms, the realism of the theory would have been sacrificed to its systematicity. I disagree with this claim and argue that, at least for consumption theory, no substantial change in the epistemic principles of neoclassical analysis took place after World War II: the sacrifice of realism on behalf of systematicity has been the rule from the very beginnings of consumption theory. What happened with the formalist approach is not that the epistemic principles of neoclassical analysis have changed, but that neoclassical analysis has become aware of those which already were its internal epistemic laws, and has followed them more consistently.

5. Marburg and the other trends in economic methodology

In this last section I briefly illustrate why some other epistemological theories rationalize the history of consumption analysis less persuasively than the Marburg one. A more general and thorough confrontation between the Marburg epistemology and the other trends in the philosophy of economics will be made in a companion paper.

As regards the methodologies that apply to economics logical empiricism or Popperian falsificationism, they are largely unable to rationalize a history in which the accepted theories have little empirical relevance and are regularly in contrast with commonsense evidence, observational data or experimental tests. As a consequence, the fundamental attitude of such methodologies towards economics is that of complaint and normative advice, which however do not appear useful in understanding economic science as it developed over time.

Lakatos's methodology of scientific research programs has surely more affinities with the Marburg view. In particular, Lakatos also takes carefully into account the actual history of the sciences and maintains that an epistemological theory that fails to rationalize this history "must be rejected" (Lakatos 1971: 124). However, his notion of progressive series of theories does not square with what has been regarded as progress in the history of consumer theory, namely the steps from additive to general utility, then from cardinal to ordinal utility, and finally the axiomatic foundation of ordinal utility. According to Lakatos, a theory is progressive with respect to another if the former "has some excess empirical content over its predecessor, that is, if it predicts some novel, hitherto unexpected fact" (Lakatos 1970: 33). Yet, the theory of consumption based on general utility predicts no novel fact with respect to the theory grounded on additive utility. On the contrary, the general utility apparatus accounts for some already known facts (interdependence of commodity utilities, Giffen goods, inferior goods) which could not be explained within the additive framework. Similarly, the theory based on ordinal utility has no excess empirical content, but just fits better with the perceived non-measurability of utility. Therefore, the development of consumer theory seems better rationalized as an endeavor to make intelligible the already known phe-

nomena related to consumer demand by connecting them in a systematic framework, rather than as an attempt to predict some novel, hitherto unexpected fact[37].

A similar point holds for the instrumentalist viewpoint. My historical reconstruction suggests that the theories successively accepted as standard in the history of consumption analysis provide few empirical predictions and are quite useless for practical ends. Therefore, it is hard to think of the development of consumption theory as driven by the attempt to make reliable predictions of consumer demand.

Finally, with respect to the methodological trends that attempt to explain the historical developments of economic science on the basis of extra-scientific factors – as in the sociology or scientific knowledge, the discourse analysis or the economics of scientific knowledge[38] – I simply contend to have shown that an internal narrative is still a sufficiently powerful resource to account for the history and the logic of economics without resorting to extrinsic explanations.

Notes

[1] See among others Stigler 1950, Houthakker 1961, Chipman 1971, Hurwicz 1971, Ekelund–Furubotn–Gramm 1972, Samuelson 1974, Chipman 1976, Chipman 1982, Mirowski 1989, Blaug 1992, Ellingsten 1994, Montesano 1996, Hands–Mirowski 1998, Hurwicz 1998, Mirowski–Hands 1998, Mornati 1999, Weber 1999a and 1999b, Mongin 2000, Weber 2001, Mirowski 2002, Chipman–Lenfant 2002, Giocoli 2003, Montesano 2004.

[2] Throughout the paper, "utilitarianism" is meant to be related to individual choice theory, not to the social choice and ethical doctrine of J. Bentham, J.S. Mill or H. Sidwick.

[3] U. Mäki (1989) points out that the term "realism" has two different meanings: it designates an ontological doctrine about the external world (as opposed, *e.g.*, to idealism), or it is used as an attribute of a scientific theory (as in the phrase "the realism of neoclassical economics"). Throughout the paper, I employ the term in the latter meaning and always refer to the "realism of a theory." Mäki suggests reserving the term "realism" for ontological doctrines, and introduces the expression "realisticness" as an attribute of theories. However, since the expression "realisticness" has not gained general acceptance, I prefer to maintain the more familiar term "realism" to indicate a property of scientific theories.

[4] For a general introduction to the Neo-Kantian movement, see Ferrari 1997, Ollig 1998, and Michael Friedman 2004. Relevant to the questions discussed in this paper are also Michael Friedman 1999, 2000, 2001 and 2005.

[5] On the naturalistic approach with particular reference to economics, see Hands 2001: 128 ff.

[6] The main areas of research of F. Machlup (1902-83) were international monetary economics and industrial organization, but he also contributed to the methodology of economics. In particular, Machlup (1946) took part in a famous debate on the theory of the firm and defended the neoclassical profit-maximization assumption against the criticisms of lack of realism raised by R.A. Lester (1946). Later on, Machlup (1964) critically discussed P. Samuelson's methodological stance against unrealistic assumptions in economics (cf. Samuelson 1963). Machlup showed that Samuelson as a methodologist was contradicting Samuelson as a theorist, and argued that Samuelson's best theoretical work was in fact based on unrealistic assumptions.

[7] See *e.g.* Parsons 1990, Barrotta 1996, Parsons 1997 and Hands 2001: 41 ff.

[8] Shortly before 1871, A. Cournot (1838) developed an articulated theory of demand that, however, was not grounded in the utility concept. J. Dupuit (1844) analyzed total and marginal utility, but identified the de-

mand curve with the marginal utility curve. H.H. Gossen (1854) first observed that, for utility maximization, the marginal utility obtained by spending money upon different commodities must be the same, but his awkward book was rediscovered only at the end of the 1870s. On the history of demand theory before 1871, see Stigler 1950 and Ekelund–Furubotn–Gramm 1972.

[9] In mathematical terms, Jevons postulates that $u(x_1, x_2, ..., x_n) = u_1(x_1) + u_2(x_2) + ... + u_n(x_n)$, where x_i is the quantity consumed of commodity *i*, and $u_i(x_i)$ is the total utility function relative to commodity *i*, with $i = 1,2,...,n$. The marginal utility of commodity *i* is given by $\partial u/\partial x_i$, which is assumed to be positive ($\partial u/\partial x_i > 0$) and decreasing ($\partial^2 u/\partial^2 x_i < 0$) for each *i*. Additive separability implies that, for $i \neq j$, $\partial^2 u/\partial x_i \, \partial x_j = 0$.

[10] Additive separability of total utility and decreasing marginal utility are not invariant to a strictly increasing transformation of the utility function, that is, they both are cardinal and not ordinal properties of the utility function. However, the problems related to the ordinal measurability of utility are extraneous to this early phase of utility theory and will become relevant only later. See below.

[11] If the utilities of the goods are independent (and marginal utilities are positive and decreasing), it can also be shown that demand for a good definitely increases (decreases) as consumer income *I* rises (falls). In mathematical terms $\partial x_i/\partial I > 0$, which means that all goods are normal. With a general utility function, even this statement is no longer true. However, this problem has not been very relevant in the development of demand theory.

[12] Edgeworth's misleading argument is pointed out by Dominedò 1937: 227, 286-8.

[13] Regarding the methodological reason why Marshall rejected Edgeworth's suggestion of a general utility function, see Dardi 1991: 96-101.

[14] Regarding the use of the additive utility assumption among leading neoclassical economist at the end of the 19th century, see also Stigler 1950: 326-7.

[15] On the relationship between Pareto's expression of $\partial x_i/\partial p_i$ and those later obtained by Slutsky and Hicks–Allen, see Schultz 1935, Chipman 1976, Dooley 1983 and Weber 1999b.

[16] As is well known, "ophelimity" is the Paretian term for utility.

[17] The analytical expression of $\partial x_i/\partial I$ is not present in Pareto's works. In fact, in accordance with Walras' approach, in Pareto's general theory of exchange, income is not regarded as a distinct exogenous variable, but is endogenously given as the value of the agent's endowment at market prices. More on this in Weber 1999b.

[18] Johnson does not cite Pareto. On the relationship between Johnson and Pareto see Moscati 2005.

[19] Slutsky refers admiringly to Pareto but does not cite Johnson.

[20] Slutsky's compensated variation of price is a change in consumer income accompanying a change in price which makes the consumer's initial consumption bundle just affordable at new prices.

[21] Actually, Slutsky does not demonstrate this result, which will be first proved by Samuelson 1938: 69. On the importance of the negative semi-definiteness of the substitution matrix see below, section 3.7.

[22] For the 1930s, see Allen 1934, Hicks–Allen 1934, Lange 1934, Georgescu-Roegen 1936, Samuelson 1938 and Hicks 1939: 18-9.

[23] Let us consider $f(u(x_1, x_2, ..., x_n))$ with $f' > 0$ and denote $u_i = \partial u / \partial x_i$ and $u_{ij} = \partial^2 u / \partial x_i \partial x_j$. Even if $u_{ii} < 0$, the second-order derivative $\partial^2 f(u(x)) / \partial^2 x_i = f' u_{ii} + f'' u_i^2$ could be positive if $f'' u_i^2$ is large "enough".

[24] According to the Auspitz–Lieben–Edgeworth–Pareto notion, two goods *i* and *j* are complementary if $u_{ij} > 0$, and substitute if $u_{ij} < 0$. However the sign of $\partial^2 f(u(x)) / \partial x_i \partial x_j = f' u_{ij} + f'' u_i u_j$ can be different from that of u_{ij}.

[25] On Pareto's position concerning additive separability and cardinal measurability of utility see Chipman 1971 and 1976.

[26] On the ambiguities contained in Pareto's work and the possible explications for them see Chipman 1976, Gross–Tarascio 1998, Ranchetti 1998, Bruni-Guala 2001, Weber 2001, Bruni 2002.

[27] Bowley is evidently unaware of Slutsky's paper.

[28] For a detailed analysis of the rediscovery of Slutsky's article see Chipman–Lenfant 2002.

[29] More on the problems related to Schultz's findings in Hands–Mirowski 1998. Hands and Mirowski propose a story of the American demand theory centered on Schultz's statistical findings and on H. Hotelling's demand model (Hotelling 1932, 1935). Although Hands–Mirowski's reconstruction is replete with interesting, new information, in my opinion it is untenable, as Hurwicz 1998 convincingly shows.

[30] On Thurstone's and other experiments in consumer theory between 1930 and 1970, see Moscati 2004.

[31] P. Mongin (2000: 1135-9) persuasively demonstrates that, already in 1938 and not later, Samuelson was not interested in eliminating utility from microeconomic theory.

[32] A reconstruction of the history of the integrability problem can be found in Hurwicz 1971. See also Chipman 1971 and 1982.

[33] To complete the picture of the relationships between the axioms of revealed preference and the properties of the substitution matrix, it must be added that R. Kihlstrom, A. Mas-Colell, and H. Sonnenshein (1976) show that the negative semi-definiteness of the substitution matrix is equivalent to a weakened version of the Weak Axiom, which they term Weak Weak Axiom. L. Hurwicz and M.K. Richter (1979) prove that the symmetry of Slutsky's matrix is equivalent to a weakened version of the Strong Axiom, originally stated by J. Ville (1946).

[34] Quasi-concave is a function $u(x)$ for which $u(x_i) \geq \min\{u(x_j), u(x_z)\}$ for each $x_i = \lambda x_j + (1-\lambda)x_z$, with $\lambda \in [0,1]$. Quasi-concavity is invariant to a monotonic transformation of $u(x)$.

[35] For the axiomatic treatment of additive utility see among others Debreu 1960 and Luce–Tukey 1964. For the axiomatic treatment of cardinal utility, see among others Suppes–Winet 1955 and Chipman 1960. On these topics see Chipman 1971 and Ellingsten 1994.

[36] See *e.g.* Ward 1972, Blaug 1999, Hutchison 2000.

[37] More on the difficulties of Lakatos' approach in rationalizing the history of economics in Hands 1985.

[38] For a detailed and supportive assessment of these trends see Hands 2001.

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