

Joseph Schumpeter and Modern Nonlinear Dynamics

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Abstract

This paper explores the conceptual links between Joseph Schumpeter's theory of instability under capitalism and both theoretical and empirical research that has been done over the past fifteen years in nonlinear dynamics. Recent work related to chaos and bifurcation theory is shown to be consistent with Schumpeter's view that instability is an inherent feature of capitalism, and that there is a positive, though difficult, role for stabilization policy as a result. The strong claim that modern research has proven Schumpeter correct is not made, but rather that existing recent research is not inconsistent with his views.

Introduction

Decades ago, in his monumental analyses of business cycles and the capitalist order, Schumpeter asserted that capitalism is an inherently unstable system (Schumpeter, 1939, 1942). He believed that the sources of economic fluctuations are technological innovations, which are the results of profit-maximizing agents taking advantage of opportunities for improvement of either products or the processes used for making

products. Technological innovation, in this view, is a defining feature of capitalism, an inevitable result of entrepreneurs attempting to improve their firms' outcomes. Because, under capitalism, innovations are continually being devised and implemented, and because the acceptance and spread of innovations are taken to be the immediate causes of business cycles, for Schumpeter instability is an unavoidable and essential aspect of capitalism.

More recently, research in nonlinear dynamics has provided theoretical evidence that instability can be derived from even very simple economic models for certain sets of initial parameter values. Empirical work involving the estimation of continuous time macroeconomic models using data from a variety of countries has found that their parameters are indeed from within the set of parameters that induce unstable solution paths for those models. These findings lend credence to Schumpeter's contention that business cycles are endogenous phenomena rather than the result of external shocks.

The purpose of this paper is to present a brief overview of the theoretical and empirical results that support the presence of macroeconomic instability due to parameter values being within unstable subsets of the possible parameter space for a given system. These results are then related to Schumpeter's view that business cycles are caused by the process of technological innovation, which he took to be not an external force but the driving mechanism of the capitalist order itself. The paper will also compare the role of stabilization policy as seen by Schumpeter and more modern researchers. The intent is not to present the results as evidence of Schumpeter's view, but merely as consistent with it, suggesting that his view may not be dismissed out of hand.

Schumpeter's Perspective

Although a full exploration of Schumpeter's view is not necessary for the purpose of this paper, a brief summary is in order here. Classical economic analyses developed before Schumpeter emphasized the stability of the capitalist system, focussing on the forces that compel prices and quantities to their equilibrium levels. Schumpeter was quite clear in his view that instability is inherent to capitalism and that the source of that instability, technological innovation, resides within the system rather than outside of it. In searching for ways to increase profits by producing goods at lower costs or producing new or improved goods, entrepreneurs employ ideas, materials, or processes that have not before been used in production. Successful innovations allow the entrepreneurs to realize positive profits by producing rival goods at lower costs than do firms using their industries' standard practices. The lure of profits inspires others to follow the trail thus blazed, copying the changes previously made or seizing new opportunities that the original innovations make possible. Each such innovation or cluster of innovations may have spillover effects, both positive and negative, that benefit some firms while creating impediments to others. A particular innovation, such as a cost-saving improvement in the production of a single consumer good, may have effects isolated to a single industry, or it could have far-reaching, economy-wide impacts, as the development and expansion of the railroad in the U.S. did. Firms (or entire industries) that are not able to adapt to the new methods and circumstances fall behind and disappear, while those able to successfully adopt the innovations survive in a new economy in which the innovations have been assimilated into new standard practices.

In Schumpeter's analysis, business cycles come about as a direct result of this process of innovations being implemented and then copied, relative prices shifting and eliminating profit opportunities, firms unable to adjust to the new conditions dying off, and the survivors participating in what is essentially a new economy characterized by improved products or production techniques. All of this happens as a result of motives and behaviors found within the capitalist system itself, without regard to any external forces or events. In *Capitalism, Socialism, and Democracy*, Schumpeter (1942) summarized this view as follows:

Capitalism, then, is by nature a form or method of economic change and not only never is but never can be stationary.... The fundamental impulse that sets and keeps the capitalist regime in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates.... The opening up of new markets, foreign or domestic, and the organizational development from the craft shop and factory to such concerns as U.S. Steel illustrate the same process of industrial mutation – if I may use that biological term – that incessantly revolutionizes the economic structure *from within*, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism.¹

Creative destruction is in this view what accounts for capitalism's success in terms of rising productivities and standards of living. Without the instability caused by innovation, economies would remain stagnant with no development or improvement.

In *Business Cycles*, Schumpeter (1939) explicitly described his ideas of innovation in terms of production functions. An innovation represents a new way of combining factors of production, or in his own words, "the setting up of a new production function."² Innovations are then taken to be shifts from one production function to a higher one, or more precisely a change in the parameters of a firm's production function,

¹ Schumpeter (1942), pp. 82-83.

allowing the firm to produce more for any given set of inputs. Schumpeter also stated the same concept using cost functions, writing that “[w]henver at any time a given quantity of output costs less to produce than the same or a smaller quantity did cost or would have cost before, we may be sure, if prices of factors have not fallen, that there has been innovation somewhere.”³ Again, the crucial point of his argument was that these shifts in cost and production functions do not come about as a result of exogenous shocks, but rather through a process of innovation that is contained and motivated within the system itself. The interpretation of innovations as parameter shifts will be a useful one when Schumpeter’s perspective is related to more recent theoretical and empirical research below.

Viewing instability as a permanent feature of capitalism, Schumpeter did see a role for stabilization policy to temper the potentially convulsive effects of business cycles. While he did not advocate attempts to make firms or industries that were unable to adapt permanently viable, he did recognize a benefit to policy interventions designed to moderate dramatic changes and allow the inevitable evolution of an economy to run its course at a more gradual pace. Schumpeter did not believe in government intervention as a means of completely controlling business cycles, acknowledging the extreme difficulty of even measuring in a timely way all of the relevant variables, let alone arriving at the correct diagnosis for every problem. He did support using the economic tools of government as known in his day to address what were perceived as acute crises.⁴ As will be seen below, the recognition of a positive role for stabilization policy in the face of

² Schumpeter (1939), p. 87.

³ Schumpeter (1939), pp. 88-89.

instability puts Schumpeter well in line with modern theorists examining instability through nonlinear dynamics.

Theoretical Evidence

Schumpeter's disagreement with the prevailing opinion of his day that economies are basically stable was expressed largely in descriptive terms, supported by the relatively limited analytical tools available to him. Today, developments in modern nonlinear dynamics have opened a new arena for the ongoing debate over whether economies are stable or unstable. This round of the debate is carried on in terms of bifurcation theory, chaos, and nonlinear dynamics of lower orders. The lines of argument being used now of course are quite distinct from those used by Schumpeter, but the evidence recently produced in support of instability is not inconsistent with the basic thrust of Schumpeter's view.

An early landmark work in this area was that of Grandmont (1985). Grandmont used a classical, Cobb-Douglas model of one firm and one consumer, and assumed a stationary environment. Most economists assumed that without exogenous shocks to disrupt its natural working, such a model would always yield a stable system. Grandmont proved that in even such a straightforward model, certain parameter settings would lead to chaotic solution paths that change direction so fast as to appear stochastic.

More explicitly, Grandmont showed that his parameter sets exhibited bifurcation, meaning that as the initial parameter settings were shifted from one subset, or bifurcation

⁴ For Schumpeter's discussions of the limits of government intervention and regulation, see Schumpeter (1942), pp. 61 and 91. For an example of his proposing specific policy interventions in response to a perceived economic crisis, see Schumpeter (1951), pp. 236-247.

area, to the next, there was a period doubling in the solution path. As the initial parameter settings continued to be changed, passing from one subset to the next, the periodicity of the solution path continued to increase until he arrived at settings that produced a solution path that changed direction exponentially fast, or, in another word, chaos.

Grandmont's paper helped ignite the current resurgence in the controversy over instability by showing that even with all of the assumptions of classical economists intact, there is no guarantee of stability. His analysis went on to imply that the subset of parameters that would lead to stable outcomes is not especially large, and more disturbingly that the subset leading to chaotic solutions is a sizable one. Nothing in Grandmont's results implies that parameters will be such that a given economy will exhibit stability.

Some have argued that economies simply ought to be stable, that there are probably benefits to stability even if we do not quite know what they are, and that there exists an as yet unknown mechanism in the economy that keeps the parameters within the parameter subset that produces stable solutions. For example, agents and institutions may have somehow arranged for parameters to fall in the stable region, perhaps through a process of natural selection. There does not, however, seem to be a compelling reason to believe this to be the case. Schumpeter's view provides a plausible rationale for believing just the opposite, that there is a strong, indeed critical, benefit gained from instability. Instability is a crucial aspect of the process of technological innovation, which is what allows economies to evolve in the sense of improving economic outcomes by introducing advancements, weeding out inefficiencies, and redirecting resources to

those firms capable of adapting to changing circumstances. An unstable economy that never reaches equilibrium is from this perspective preferred to a stable one because strict stability prevents progress. As Schumpeter wrote, “A system – any system, economic or other – that at *every* given point of time fully utilizes its possibilities to the best advantage may yet in the long run be inferior to a system that does so at *no* given point of time, because the latter’s failure to do so may be a condition for the level or speed of long-run performance.”⁵

To further relate Schumpeter’s view to Grandmont, it is useful to note that taste and technological parameters are among those described in the parameter space examined by Grandmont. When Schumpeter referred to a shift from one production function to another in response to innovation, we may therefore interpret it as a shift from one point within the parameter space to another. Given Schumpeter’s position that capitalism is always inherently unstable, we must also interpret such a shift as being from one point in the unstable region of the parameter space to another unstable point. Shifting within the stable region would be inconsistent with the presence of the kind of business cycles hypothesized by Schumpeter. Without any instability, shifts in parameters would only involve moves from one stable point to another. This would cause changes in relative prices, relative quantities, and allocations of goods, but would not generate cycles of the sort that would be consistent with Schumpeter’s views on welfare-improving policy and on innovation.

Grandmont proved that both periodic and chaotic self-sustaining fluctuations can be derived from a well-formulated competitive economic model, but other researchers,

⁵ Schumpeter (1942), p. 83.

notably Woodford (1989), pointed out that Grandmont required fairly extreme parameter specifications to produce such instability. Woodford employed a model that was largely competitive but lacked complete financial markets, specifically a loan market capable of linking present and future consumption goods for certain agents. He thus incorporated rigidities where Grandmont had a completely classical model. Using his model, Woodford found that solution paths exhibiting chaos could be derived under far less extreme parameter settings than were necessary under Grandmont's analysis. He concluded that when taken in conjunction with rigidities or market imperfections, parameter settings that could more feasibly be observed in the real world than Grandmont's can still lead to instability. Stated another way, under such circumstances parameter settings do not need to be "too unstable" in order to produce unstable outcomes.

Although a formal model will not be attempted here, it is possible that in a similar fashion, plausible parameter settings combined with innovation might also lead to unstable solution paths. The argument would run along the lines of noting that innovation, like rigidity, tends to prevent outcomes from reaching a stable competitive market equilibrium. Just as Woodford found that parameter settings that do not lie too deeply within the unstable region of the parameter set can, when combined with rigidities, lead to significant fluctuations, it could be that the same is true when such parameters are combined with innovation. That would be consistent with Schumpeter's view that innovation is the endogenous source of the business cycles that are actually observed.

The possibility that parameters do not need to be very extreme in order to support significant instability fits well with both a feature of bifurcation theory mentioned above and Schumpeter's theory of business cycles. It was noted that as initial parameter settings are moved from one subset in the unstable region to the next, the number of cycle frequencies superimposed on each other increases until all possible frequencies are present at once. If it is true that in the presence of innovation, parameters do not need to be deep within the unstable region to derive instability, then the number of superimposed cycles need not be very large. In his examination of American, English, and German data spanning the nineteenth century, Schumpeter arrived at a three-cycle scheme to best describe what he found, suggesting that the relevant parameters for those economies were not too far removed from the stable region of the parameter set.⁶

Grandmont and Woodford both saw a role for stabilization policy. Woodford, in particular, found that the fluctuations derived from having parameters from the chaotic subset as well as small market imperfections produced large Pareto welfare losses, justifying government intervention. Grandmont also supported the use of stabilization policy, although he noted that if the government is incorrect in its analysis of a particular situation, its intervention could cause even greater losses than those due to the instability. These judgments are thoroughly consistent with Schumpeter's as described above, both in terms of the possibility of beneficial stabilization policy and the difficulties in actually implementing such policy.

Theoretical work in nonlinear dynamics and its implications for stabilization policy thus pose no direct contradiction to arguments Schumpeter put forth nearly fifty

⁶ It should be stressed, as Schumpeter (1939, pp. 169-170) himself did, that his selection of a three-cycle

years prior. It should be stated again that this is not to suggest that the more recent theoretic work provides evidence that verifies his arguments. The point being made here is that advances being made in theory today in no way rule out the broader framework laid out many years ago, and can in fact be seen to be quite compatible with it.

Empirical Evidence

After recognizing that Schumpeter's view of instability is consistent with modern nonlinear dynamic theory when parameters are within the subset that produces unstable solution paths, a natural next issue is to explore whether parameters of real world economies actually do fall within such regions. Several studies have examined precisely that question and have found significant evidence of parameter values that would lead to instability.

Böhm and Kaas (2000) examined a neoclassical one-sector growth model with differential saving rates between shareholders and workers. They calculated bifurcation diagrams to determine which subsets of the parameter space would produce cyclical and chaotic dynamic behavior. Their results showed that given sufficient variation in income distribution and shareholders saving more than workers, plausible parameter settings cause their model to exhibit endogenous unstable steady states and fluctuations. This result is precisely what would be expected if Schumpeter's views were correct.

An often cited model is that of Bergstrom, Nowman, and Wymer (1992), who used a continuous time second order differential equation macroeconomic model of the

scheme was not based on theory, but rather on a practical desire to balance the additional explanatory power of more cycles against the resulting increasing complexity.

United Kingdom to generate point estimates for the parameters of that economy. Their model is described by the following fourteen equations:

$$D^2 \log C = \gamma_1(\lambda_1 + \lambda_2 - D \log C) + \gamma_2 \log \left[\frac{\beta_1 e^{-\{\beta_2(r - D \log p) + \beta_3 D \log p\}} (Q + P)}{T_1 C} \right] \quad (1)$$

$$D^2 \log L = \gamma_3(\lambda_2 - D \log L) + \gamma_4 \log \left[\frac{\beta_4 e^{-\lambda_1 t} \{Q^{-\beta_6} - \beta_5 K^{-\beta_6}\}^{-1/\beta_6}}{L} \right] \quad (2)$$

$$D^2 \log K = \gamma_5(\lambda_1 + \lambda_2 - D \log K) + \gamma_6 \log \left[\frac{\beta_5 (Q/K)^{1+\beta_6}}{r - \beta_7 D \log p + \beta_8} \right] \quad (3)$$

$$D^2 \log Q = \gamma_7(\lambda_1 + \lambda_2 - D \log Q) + \gamma_8 \log \left[\frac{\{1 - \beta_9 (qp/p_i)^{\beta_{10}}\} (C + G_c + DK + E_n + E_o)}{Q} \right] \quad (4)$$

$$D^2 \log p = \gamma_9(D \log(w/p) - \lambda_1) + \gamma_{10} \log \left[\frac{\beta_{11} \beta_4 T_2 w e^{-\lambda_1 t} \{1 - \beta_5 (Q/K)^{\beta_6}\}^{-(1+\beta_6)/\beta_6}}{p} \right] \quad (5)$$

$$D^2 \log w = \gamma_{11}(\lambda_1 - D \log(w/p)) + \gamma_{12} D \log(p_i/qp) + \gamma_{13} \log \left[\frac{\beta_4 e^{-\lambda_1 t} \{Q^{-\beta_6} \beta_5 K^{-\beta_6}\}^{-1/\beta_6}}{\beta_{12} e^{\lambda_2 t}} \right] \quad (6)$$

$$D^2 r = -\gamma_{14} D r + \gamma_{15} \left[\beta_{13} + r_f - \beta_{14} D \log q + \beta_{15} \frac{p(Q+P)}{M} - r \right] \quad (7)$$

$$D^2 \log I = \gamma_{16}(\lambda_1 + \lambda_2 - D \log(p_i I/qp)) + \gamma_{17} \log \left[\frac{\beta_9 (qp/p_i)^{\beta_{10}} (C + G_c + DK + E_n + E_o)}{(p_i/qp)I} \right] \quad (8)$$

$$D^2 \log E_n = \gamma_{18}(\lambda_1 + \lambda_2 - D \log E_n) + \gamma_{19} \log \left[\frac{\beta_{16} Y_f^{\beta_{17}} (p_f/qp)^{\beta_{18}}}{E_n} \right] \quad (9)$$

$$D^2 F = -\gamma_{20} D F + \gamma_{21} [\beta_{19} (Q + P) - F] \quad (10)$$

$$D^2 P = -\gamma_{22} DP + \gamma_{23} \{ [\beta_{20} + \beta_{21}(r_f - D \log p_f)] K_a - P \} \quad (11)$$

$$D^2 K_a = -\gamma_{24} DK_a + \gamma_{25} \{ [\beta_{22} + \beta_{23}(r_f - r) - \beta_{24} D \log q - \beta_{25} d_x] (Q + P) - K_a \} \quad (12)$$

$$D^2 \log M = \gamma_{26} (\lambda_3 - D \log M) + \gamma_{27} \log \left[\frac{\beta_{26} e^{\lambda_3 t}}{M} \right] + \gamma_{28} D \log \left[\frac{E_n + E_o + P - F}{(p_i / qp) I} \right] + \gamma_{29} \log \left[\frac{E_n + E_o + P - F - DK_a}{(p_i / qp) I} \right] \quad (13)$$

$$D^2 \log q = \gamma_{30} D \log(p_f / qp) + \gamma_{31} \log \left[\frac{\beta_{27} p_f}{qp} \right] + \gamma_{32} D \log \left[\frac{E_n + E_o + P - F}{(p_i / qp) I} \right] + \gamma_{33} \log \left[\frac{E_n + E_o + P - F - DK_a}{(p_i / qp) I} \right] \quad (14)$$

where t is time, D is the derivative operator, $Dx = dx/dt$, $D^2x = d^2x/dt^2$, and $C, E_n, F, I, K, K_a, L, M, P, Q, q, r, w$ are endogenous variables whose definitions are listed below.

C	real private consumption
E_n	real non-oil exports
F	real current transfers abroad
I	volume of imports
K	amount of fixed capital
K_a	cumulative net real investment abroad (excluding changes in official reserve)
L	employment
M	money supply
P	real profits, interest and dividends from abroad
p	price level
Q	real net output
q	exchange rate (price of sterling in foreign currency)
r	interest rate
w	wage rate

The variables $d_x, E_o, G_c, p_f, p_i, r_f, T_1, T_2, Y_f$ are exogenous variables with the following definitions:

$d_x =$	dummy variable for exchange controls ($d_x = 1$ for 1974-79, $d_x = 0$ for 1980 onwards)
$E_o =$	real oil exports
$G_c =$	real government consumption

- p_f = price level in leading foreign industrial countries
- p_i = price of imports (in foreign currency)
- r_f = foreign interest rate
- T_1 = total taxation policy variable defined by Bergstrom et al. (1992, p. 317)
- T_2 = indirect taxation policy variable defined by Bergstrom et al. (1992, p. 317)
- Y_f = real income of leading foreign industrial countries.

Bergstrom et al. (1992) estimated the structural parameters $\beta_i, i = 1, 2, \dots, 27, \gamma_j, j = 1, 2, \dots, 33,$ and $\lambda_k, k = 1, 2, 3,$ using quarterly data from the U.K. from 1974 to 1984. They found that the parameters lie within the unstable region of the parameter space, but not very far from the stable region. Although the full covariance matrix was not provided with the point estimates, confidence intervals around the individual estimates could be calculated from their standard errors. The point estimates themselves were not in the stable region of the parameter space, but their confidence intervals in many cases did extend into that region. Similar studies performed by Gandolfo, Padoan, Arcangelils, and Wymer (1996) and Donaghy (1993) yielded comparable results for continuous time differential equation models of the Italian and American economies, respectively. Just as in the U.K. model, the estimated parameters of the other economies were found to be in the unstable region of their parameter sets, but not deep inside those regions.⁷

These results connect well with the relationship between Schumpeter's view of instability and theoretical work in nonlinear dynamics described above. The estimated parameters of models depicting three developed economies all appear to be within the unstable region of their respective parameter sets, as is required for instability to be found inherently within the system as described by Schumpeter. Further, the proximity of those

⁷ It must be noted that there is a theoretical problem with using historical time series data to determine the location of parameters which under Schumpeter's view of instability are not constant. Still, for the limited purposes of this paper, the parameter point estimates can be considered in a crude way to indicate the average position of those parameters over time. The important point is simply that the empirical tests lend support to the contention that the parameters lie within the unstable region.

estimates to the stable regions of those economies is consistent with the point made above regarding the number of cycle frequencies described by Schumpeter. The fact that he chose a three-cycle scheme to characterize his data would suggest that for each economy, few period doublings of the solution path from a stable path would be present.

The formulation of effective stabilization policies is a very complex issue in the context of nonlinear dynamic models, but connections can still be made between the implications of modern research and Schumpeter's position on such policies. The following discussion of one example of such research will make these connections apparent.

Barnett and He (1999) examined the problems of describing bifurcation boundaries numerically and determining how boundaries shift in response to stabilization policies. Much of their analysis focussed on the complicated geometry of bifurcation boundaries, particularly Hopf bifurcations, the type of bifurcation boundary that is thought to be most relevant to economics. Hopf bifurcations occur in systems with a dimension of at least two, at points at which the system has a non-hyperbolic equilibrium with a pair of purely imaginary eigenvalues, but without zero eigenvalues (Guckenheimer and Holmes, 1983).⁸ A standard form of such systems is

$$Dx = -y + x(\theta - (x^2 + y^2)),$$

$$Dy = x + y(\theta - (x^2 + y^2)).$$

Hopf bifurcations are of special interest in economics because of the ways in which systems behave when their parameters are on or in the vicinity of this type of boundary. In these circumstances, systems behave with cyclical behavior of just the sort

that is seen in actual economies and in Schumpeter's views on instability. Barnett and He's (1999) findings of Hopf bifurcations within the U.K. model are supportive of Schumpeter's view.

Using the model from Bergstrom et al. given above, Barnett and He explored the ability of policy control rules to move bifurcation boundaries so as to include given parameter point estimates within the stable region of the parameter set. Specifically, they examined a policy control rule put forth by Bergstrom, Nowman, and Wandasiewicz (1994) and found it unlikely to stabilize the model. That policy does produce shifts in bifurcation boundaries, but overall the feasible stable region is smaller under the control policy than it is without it, suggesting that the control policy is not likely to succeed.

Barnett and He (1999) then applied optimal control theory in order to select feedback rules for the Bergstrom et al. model and determine whether this approach could yield a more successful stabilization policy. They found that optimal control theory could successfully be used to stabilize the model, but the policy equation they derived was much too complicated to be of practical use in real world situations. Furthermore, it was heavily dependent on the specification of the model itself.

These results are supportive of Schumpeter's view concerning stabilization policy. As was described above, Schumpeter recognized a positive role for government intervention to temper the effects of instability, but also acknowledged that an economy is complex, and that determining appropriate policies for different perceived problems is an extremely difficult task. Optimal control theory was found by Barnett and He to be capable of deriving successful stabilization policy rules, but due to their intricacy those

⁸ Additional transversality conditions must also be satisfied for a Hopf bifurcation to exist. These

rules may be beyond the scope of action (or understanding) for authorities responsible for fiscal and monetary policy in a given economy. Just as in the case of the purely theoretical work in nonlinear dynamics, the evidence and implications from empirical research in the field lends credence to Schumpeter's view on both the inherent nature of instability under capitalism and the ability of government intervention to cope with that instability.

Conclusion

Over half a century ago, Schumpeter advanced his theory of business cycles, positing that instability, driven by technological innovation, is an essential fact of capitalism, in contrast to the more classical theories prevalent in his day. Modern nonlinear dynamics has produced theoretical as well as empirical evidence that has revitalized the debate over the stability of economic systems, and that supports the contention that today's economies do indeed exhibit instability endogenously. This modern evidence alone is not sufficient to verify Schumpeter's view – for instance, it says nothing about the underlying source of instability. What it does do is expand the body of knowledge that is consistent with that view. It remains possible that Schumpeter's view is incorrect, and it may also come to pass that future research will dispute the modern findings discussed here. However, in the light of this limited corroborative evidence, Schumpeter's views on dynamics continue to be relevant and ought not be rejected relative to the current availability of evidence.

conditions may be found in Glendinning (1994).

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