Notes on an Environment with Debt-Settlement Intermediation

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1 Introduction

These notes concern a model of an economy where the following four things happen in an equilibrium.

- Fiat money is useful as a medium of exchange. Consequently it has value.
- Some trades are also financed by the issuance of private IOUs, and money must be used to pay these off. The use of money for settling debt is conceptually distinct from its direct use as a medium of exchange. In this equilibrium, one can identify separate transactions where the two types of use occur.
- Besides there being transactions in which money is exchanged for a good, there are also transactions in which money is exchanged for an IOU that has not yet matured. That is, the IOUs are a form of *circulating* debt—a privately issued analogue of government bonds.
- Relative to maintaining a fixed supply of fiat currency, efficiency can be increased by having a "monetary authority" that engages in some pattern of transactions that causes the quantity of fiat money to fluctuate.

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The present model owes a great deal to recent work of Scott Freeman (1995). The equilibrium that Freeman studies has all four of the attributes that I have just listed. The present model is simpler than Freeman's, making some aspects of the definition of equilibrium more straightforward. It is easier here to characterize the feasible transactions in the economy, and also to see clearly that the equilibrium concept for the economy involves parametric price-taking on the part of all agents rather than involving any strategic interaction among them.

Freeman concentrates on transactions that might best be regarded as representing discount-window policy, but it is clear that there are others (such as open-market operations—which I will outline in this note—and provision of float in the checking system) that would have identical implications in the context of this model.

The aim is to study how allocations that are Pareto-efficient in the set of all allocations satisfying materials-balance constraints can be supported as equilibria of trading institutions by the use of money and credit, and specifically by the nontrivial involvement of the monetary authority. This question can be divided into two parts, by considering the market system as a kind of communication network in which both goods and messages are passed among traders. If traders meet one another only in a specified sequence of combinations, then a paucity of trading opportunities can impose a binding logistical constraint on moving goods from the traders who are endowed with them to the traders who can best use them. So the first question is, for which networks can this logistical problem of distributing goods efficiently be solved? If a network does solve this problem then the second question is whether rules for message-passing in the network can be devised so that agents have incentive to transfer goods among themselves in the efficient pattern rather than in some other, inefficient one. Fiat money and IOUs are examples of communication in this sense. To give a dollar to a merchant is to announce to him, "I have provided a dollar's worth of goods or services to someone already, which has given me a right to take a a dollar's worth of them from someone else, and now I am exercising that right."

2 The population

At each date $0, 1, 2, \ldots$, a set $A_t = C_t \cup D_t$ of agents is born. C_t and D_t each have N agents. Each of these agents lives for two periods (dates t and $t+1$). Furthermore there is a set C^* of agents, the "initial old," who live only at date 0. Define $C = C_0 \cup C_1 \cup \ldots$ and $D = D_0 \cup D_1 \cup \ldots$

Each agent in A_t is endowed with one unit of a perishable good at date t, and with nothing at date $t+1$. Agents in C and D are endowed different goods.

Each agent in C^* is endowed with one unit of fiat money but with no consumption good.

Let $x_{c\tau}$ (resp. $x_{d\tau}$ be an agent's consumption of the endowment good of agents in C (resp. D) at date τ.

An agent must consume a nonnegative quantity of each good at each date. Let the utility

function of an agent be

$$W(x_{ct}, x_{dt}, x_{c(t+1)}, x_{d(t+1)}) = \begin{cases} u^c(x_{ct}) + v^c(x_{d(t+1)}) & \text{if the agent is in } C; \\ u^d(x_{ct}) + v^d(x_{dt}) & \text{if the agent is in } D; \\ v^*(x_{d0}) & \text{if the agent is in } C^*. \end{cases} \quad (1)$$

Assume that all of the functions on the right side are strictly increasing and strictly concave, and that no agent can consume his endowment in an efficient and individually rational allocation.[1]

Note that agents in D wish to trade with members of their own age cohort in C, while agents in C wish to trade with members of the next age cohort in D. Thus, as in the standard overlapping-generations model of money (as well as most other models in which fiat money is endogenously valued in equilibrium), there can be no mutually advantageous trades unless fiat money has value.

3 A variant of the standard OG structure: valued fiat money

Suppose that, at each date t, all of the traders currently alive are able to trade among themselves in the following pattern. First, the agents in C_{t-1} (or C^* if $t = 0$) trade with those in D_t. Subsequently the agents in D_t trade with those in C_t.

There is a trading pattern for goods that can achieve efficiency in this market structure. Young D agents give some of their endowment to old C agents, and subsequently they receive some of the endowment of the young C agents. If the entire money stock is passed in the opposite direction to goods at each stage, then the old C agents will be the money holders at the beginning of each period. If prices are set appropriately, markets clear and all agents have incentive to make the efficient trades.

Because the C agents closely resemble the agents in the standard OG model, and the D agents want only to trade their endowment good for a contemporaneous good, it is not surprising that the efficient equilibrium here bears very close resemblance to the efficient OG equilibrium. In particular, money has value but there is no credit and there is no role for a monetary authority.

4 Reversing the order of transactions within a period: IOUs

Now consider the opposite order of transactions. That is, suppose that first the agents in D_t trade with those in C_t and subsequently the agents in C_{t-1} (or C^* if $t = 0$) trade with those in D_t.

For fiat money to be passed from the old C agents to the young ones, it would have to pass through the hands of the young D agents. But since those agents do not meet the old C agents until it is too late to deal with the young ones, that cannot happen. The solution is for the D agents to use IOUs to finance their consumption of goods purchased from young C agents, to give some

[1] This last condition will be satisfied if all of the functions on the right side satisfy the Inada condition that the limit of the derivative as the argument tends to 0 from the right is infinite.

of their endowments to old C agents in return for their fiat money, and to carry the money into the next period and then use it to repay the holders (who will still be alive since they are young when the IOUs are issued).

The efficient equilibrium in this transactions structure involves use of both valued fiat money and IOUs, but the IOUs still do not circulate and there is no role for a monetary authority.

5 Separation within a cohort: circulating IOUs

In the preceding market structures, the young D agents have to trade separately with the young and old C agents. Now, in order to provide a reason for IOUs to circulate, consider a structure in which not all agents of the same cohort can communicate directly with one another in the second period of their lives. Specifically, let $C'_t \subseteq C_t$, $D'_t \subseteq D_t$, and consider the following sequence of trading-opportunity stages within each period $t > 0$. (Only the first and last stages occur for $t = 0$.)

1. All agents in A_t trade with one another.
2. All agents in C_{t-1} enter the market. Agents in D'_{t-1} also enter the market, and have the opportunity to pay the IOUs to their creditors.
3. Agents in C_{t-1} can trade IOUs for money with one another.
4. Agents in C'_{t-1} leave the market. Agents in D_{t-1}/D'_{t-1} enter and have the opportunity to pay their IOUs to anyone in C_{t-1}/C'_{t-1} who is holding them.
5. Agents in C_{t-1} trade with agents in D_t.

When he is young, an agent's incentive to trade with another member of his cohort is evidently affected by what he knows or believes about both his own subgroup and his trading partner's subgroup in the market structure when they are old. These considerations are made as simple as possible by assuming that no information about these matters is available until the second period of agents' lives.

Another question that comes to the fore now concerns the structure of IOU issuance. Is trade bilateral, so that each young D agent issues one IOU to a single young C agent, or does each young D agent make small purchases from many young C agents, so that each C agent holds a diversified portfolio of small-denomination IOUs afterwards? Risk-diversification considerations would seemingly lead the C agents to prefer the latter arrangement, if it is feasible. Thus, if the former, bilateral arrangement is what one intends to have emerge as an equilibrium trading pattern, there must be some constraint on (or cost of) IOU issuance to induce it. In that case, the terms of trade would be negotiated by bargaining within each two-member trading coalition, rather than taken by agents as parametrically determined by an economy-wide price (to which

Freeman refers). Thus the diversified, non-strategic trading arrangement, which seems to be the one that Freeman has in mind, will be adopted here.

Now consider what sort of trading pattern will emerge in the transaction structure posited here. Let there be Γ traders in C'_{t-1} and Δ traders in D'_{t-1}. Suppose that, in Stage 1 at date $t-1$, each agent in C_{t-1} has acquired IOUs for f units of fiat money to be delivered at date t. Note that market clearing in that stage implies that each agent in D_{t-1} owes f units of fiat money at date t. By diversification, in Stage 2 at date t, each agent in C_{t-1} receives a total of $\Delta f/N$ units of fiat money from the traders in D'_{t-1}, and is still owed $(N-\Delta)f/N$ from the remaining traders in D_{t-1}. Traders in C'_{t-1} will not be able to collect their payments from those debtors in Stage 4, though, so in Stage 3 they will sell the IOUs still in their possession to other creditors who will participate in Stage 4.

Agents in C'_{t-1} regard their IOUs as worthless except in trade at Stage 3, so they will trade away their full inventories at any positive price. The IOUs are not subject to any payment risk in this model, and creditors do not need to use fiat money until Stage 5, so creditors who will be market participants at stage 4 regard an IOU and the amount of fiat money specified for its repayment as being equally valuable. so they will be willing to pay up to the face value of an IOU to obtain it. At the beginning of Stage 3, the total face value of the IOUs in the possession of agents in C'_{t-1} is $\Gamma(N-\Delta)f$. The total amount of fiat money in the possession of agents in C_{t-1}/C'_{t-1} is $(N-\Gamma)\Delta f$. Thus the competitive price at Stage 3 of an IOU with face value 1 is

$$\min\left[1, \frac{(N-\Gamma)\Delta f}{\Gamma(N-\Delta)f}\right] = \min\left[1, \frac{(N-\Gamma)\Delta}{\Gamma(N-\Delta)}\right]. \tag{2}$$

If $\Gamma > \Delta$, then this minimum is strictly less than 1. and in this case the random timing of trading opportunities induces consumption risk. Specifically, an agent in C_{t-1} will be advantaged at date t if he turns out not to be in C'_{t-1}, because he will be able to purchase IOUs below par that will be fully repaid before he needs to spend money, but he will be disadvantaged if he does turn out to be in C'_{t-1}, because he will have to sell his IOUs below par before settlement. "Open-market operations" at Stage 3–4, involving the monetary authority purchasing securities at par from agents in C'_{t-1} and then re-absorbing this money when the agents in D_{t-1}/D'_{t-1} settle their debts, can eliminate this risk and thus can increase welfare. Equivalently in this environment, the monetary authority could issue new money at Stage 3 to agents in C_{t-1}/C'_{t-1} in return for IOUs that would be settled at Stage 4. This is the analogue of discount-window policy that Freeman considers.

Reference

Freeman, Scott, "Currency Shortages and the Discount Window," University of Texas working paper, 1995.