Avoiding the Pitfalls:

Can Regime-Switching Tests Detect Bubbles?

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Abstract:

Work on testing for bubbles has caused much debate, much of which has focused on methodology. Monte Carlo simulations reported in Evans (1991) showed that standard tests for unit roots and cointegration frequently reject the presence of bubbles even when such bubbles are present by construction. Evans referred to this problem as the pitfall of testing for bubbles.

Since Evans' note, new tests for rational speculative bubbles that rely on regime switching have been proposed. van Norden and Schaller (1993) and van Norden (1996) use a switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental price. Hall and Sola (1993) and Funke, Hall and Sola (1994) test whether asset prices seem to switch between explosive growth and stationary behaviour.

Our paper does Monte Carlo experiments using Evan's data generating process to gauge the performance of these two kinds of regime-switching tests. These rely heavily on new fast robust programs developed at the Bank of Canada for the estimation of switching regression models, which make Monte Carlo studies of such estimators practical. We find that for some (but not all) parameter values, regime-switching tests have significant amount of power to detect periodically collapsing bubbles. We also compare and contrast the performance of the two different regime-switching tests.

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1.0 Introduction

Work on testing for bubbles has caused much debate, much of which has focused on methodology. Early work used variance-bound tests until various econometric problems with this approach were noted. (See LeRoy 1989.) The misspecification test suggested by West (1987) has fallen out of favour since misspecified fundamentals should cause it to detect bubbles and there is little agreement on how to specify the fundamentals. (For example, see Flood and Hodrick (1990).) Diba and Grossman (1988) and Hamilton and Whiteman (1985) recommend the use of tests for stationarity and for cointegration to test for the absence of rational speculative bubbles. However, Monte Carlo simulations reported in Evans (1991) showed that standard tests for unit roots and cointegration frequently reject the presence of bubbles even when such bubbles are present by construction.[1] Evans referred to this problem as the "pitfall" of testing for bubbles.

Since Evans' note, new tests for rational speculative bubbles that rely on regime switching have been proposed. van Norden and Schaller (1993a,b) and van Norden (1996) use a switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental price. Hall and Sola (1993) and Funke, Hall and Sola (1994) test whether asset prices seem to switch between explosive growth and stationary behaviour.

A potential flaw of this new approach is that the regime-switching estimators may not be well behaved. There are two plausible grounds for concern.[2]

- The presence of rational speculative bubbles implies that the data are nonstationarity, but the properties of regime-switching estimators in such a case are unknown. Since this nonstationarity exists only under the alternative hypothesis of bubbles, this raises the question of whether the regime-switching tests have the power to detect bubbles when they exist. This is similar to the pitfall that Evans (1991) found with the unit-root and cointegration tests.[3]
- Little is known about the finite-sample properties of regime-switching estimators. In particular, little has been done to determine whether the use of tests whose distribution is known only asymptotically leads to reliable inference. It is conceivable that asymptotically-correct tests could experience size distortion in small samples, which would tend to produce evidence of speculative bubbles even when none are present.

1. Charemza and Deadman (1995) show that this problem extends to a broader range of processes than those considered by Evans (1991).

2. The problems of directly testing for the presence of regime-switching have recently become better understood. However, both of the above approaches circumvent these complications by testing a regime-switching alternative against a regime-switching null.

3. Both problems could lead us to conclude that bubbles are absent when they are in fact present. The difference is that with the cointegration and unit-root tests, this is caused by size-distortion while with the regime-switching tests this is caused by a lack of power. This difference arises because the two kinds of tests reverse the null and alternative hypotheses.

Our paper is a first step in addressing these questions. We examine the power properties of regime-switching bubble tests via Monte Carlo experiments using Evan's data generating process. Our work relies on new, fast, robust programs developed at the Bank of Canada for the estimation of switching regression models, which make Monte Carlo studies of such estimators practical.[4] We find that for some (but not all) parameter values, regime-switching tests have significant amount of power to detect periodically collapsing bubbles. We are also the first to compare and contrast the performance of the two different kinds of regime-switching tests.

In the next section, we explain the relationship between speculative bubbles and regime switching, and then review the tests proposed by Hall and Sola, and by van Norden. Section 3.0 explains the design of our Monte Carlo experiments, and their results are discussed in Section 4.0. Section 5.0 concludes and gives several suggestions for further research.

2.0 Tests for Rational Speculative Bubbles

2.1 Bubbles and Regime Switching

Consider a simple asset-pricing model, which only requires that

$$p_t = f(X_t) + a \cdot E_t(p_{t+1}) \tag{EQ 1}$$

where p_t is the logarithm of the asset price, E_t is the operator for expectations conditional on information at time t, $0 < a < 1$, and X_t is a vector of other variables. Solving the equation forward gives the general result

$$p_t = \left(\sum_{j=0}^{T} a^j \cdot E_t(f(X_{t+j})) \right) + a^{T+1} \cdot E_t(p_{T+1}) . \tag{EQ 2}$$

One solution to equation (EQ 1), which we will denote p_t^*, occurs when

$$\lim_{T \to \infty} a^{T+1} \cdot E_t(p_{T+1}) = 0 , \tag{EQ 3}$$

so

$$p_t^* = \sum_{j=0}^{\infty} a^j \cdot E_t(f(X_{t+j})) . \tag{EQ 4}$$

4. See van Norden and Vigfusson (1996).

We refer to (EQ 4) as the fundamental solution, since it determines the asset price solely as a function of the current and expected behaviour of other macroeconomic variables.

However, equation (EQ 4) is not the only solution to (EQ 1). We define bubble solutions to be any other set of asset prices and expected asset prices that satisfy equation (EQ 1) but where $p_t \neq p_t^*$. We define the size of the bubble B_t as

$$B_t \equiv p_t - p_t^* . \qquad \text{(EQ 5)}$$

Note that since p_t^* satisfies equation (EQ 1), it follows from (EQ 1) and (EQ 5) that

$$B_t = a \cdot E_t(B_{t+1}) . \qquad \text{(EQ 6)}$$

Since $a < 1$, this means the bubble must be expected to grow over time.

A considerable literature exists on the conditions under which such bubbles are feasible rational expectations solutions. Important contributions to this debate include Obstfeld and Rogoff (1983),(1986), Diba and Grossman (1987), Tirole (1982), (1985), Weil (1990), Buiter and Pesenti (1990), Allen and Gorton (1991), and Gilles and LeRoy (1992). In single-representative-agent models, a truly rational agent cannot expect to sell an over-valued asset (one with a positive bubble) before the bubble bursts. Therefore, bubbles should exist in such models only if they can be expected to grow without limit. Some researchers, such as Froot and Obstfeld (1991), have therefore suggested interpreting empirical tests for bubbles as tests of whether agents are fully rational, or whether they exhibit some form of myopia when considering events that are either very far in the future, or occur with only very low probabilities. An alternative interpretation would be to consider evidence of bubbles as suggesting that non-representative-agent models (such as those of De Long et al. (1990), Allen and Gorton (1991) or Bulow and Klemperer (1991)) are required.

Nothing in the above model has any implications for regime switching. Some of the early literature on rational speculative bubbles even considered purely deterministic bubbles. Regime switching stems from the descriptions of asset market behaviour (for example, those surveyed in Kindleberger (1989)) to which the above model of bubbles is often applied. The first example of regime switching in the rational speculative bubble framework is Blanchard(1979), who proposes a bubble that moves randomly between two states, *C* and *S*. In state *C*, the bubble will collapse, so[5]

$$E_t(B_{t+1} | C) = 0 . \qquad \text{(EQ 7)}$$

5. The notation $E_t(X_j | C)$ (or $E_t(X_j | S)$) denotes the expectation of X_j conditioning on the fact that the state at *t* is *C* (or *S*) and on all other information available at time *t*.

State *S*, where the bubble survives and continues to grow, occurs with a fixed probability *q*. Since

$$E_t(B_{t+1}) = (1-q) \cdot E_t(B_{t+1}|C) + q \cdot E_t(B_{t+1}|S) \quad , \qquad \text{(EQ 8)}$$

it follows from (EQ 6) that

$$E_t(B_{t+1}|S) = \frac{B_t \cdot (1+r)}{q} \qquad \text{(EQ 9)}$$

This model was subsequently generalized by Evans (1991) and van Norden and Schaller (1993) to consider the case where both the size of collapses and their probability were functions of the size of the bubble.

The distinguishing feature of these regime switching models is that the behaviour of the asset price is now state dependent, and that the state itself is unobservable. However, these models may differ in the way the probability of observing a given regime varies over time. In the Blanchard (1979), this is simply a constant. In the van Norden bubble test, the probability of observing the collapsing regime is assumed to be an increasing function of the size of the bubble. In the Hall and Sola test, this probability is assumed to follow a first-order Markov process, where the probability of remaining in a given regime is constant.[6] To distinguish these two kinds of switching models, we will refer to the case where the probability of observing a given state is independent of past states as "simple switching." In the case of a two-state model, the simple switching model is simply the special case of the Markov-switching model where

$$Pr(S_t = 0|(S_{t-1} = 0)) = 1 - Pr(S_t = 1|(S_{t-1} = 1)) \qquad \text{(EQ 10)}$$

where $Pr(S_t = k|S_{t-1} = k)$ is the probability of remaining in state *k* given last period's state was *k*.

2.2 The Hall and Sola Test for bubbles

As mentioned earlier, Diba and Grossman (1988) suggested using tests for stationarity to rule out the existence of bubbles. This method could be useful in the case of a non-collapsing bubble but, as shown in Evans(1991), these tests tend to reject the presence of bubbles when regime-switching bubbles are present. Hall and Sola (1993) address this problem by extending the standard ADF test

6. As noted by Evans and Lewis (1995), a 2-state first-order Markov process is not compatible with (EQ 6). They reconcile this by modifying the usual 2-state Markov model to allow for jumps in asset prices when the regime changes.

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{k=1}^{n} \psi \Delta y_{t-k} + v_t$$

$$\text{where } v_t \sim N(0, \sigma) . \qquad \text{(EQ 11)}$$

to allow the parameters vary between two regimes, giving

$$\Delta y_t = \alpha_i + \beta_i y_{t-1} + \sum_{k=1}^{n} \psi_i \Delta y_{t-k} + v_{t,i}$$

$$\text{where } v_{t,i} \sim N(0, \sigma_i) . \qquad \text{(EQ 12)}$$

The slope coefficients β_S and β_C are the basis of bubble test. Evidence that one regime is non-stationary (i.e. $\beta_S > 0$) while the other is stationary (i.e. $\beta_C < 0$) is evidence of the presence of a bubble. However, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the *S* and *C* regimes. This means that one should find $\beta_S > 0, \beta_C < 0$ or $\beta_S < 0, \beta_C > 0$ unless some other means is used to identify the surviving and collapsing regimes.

Our application of the Hall and Sola test (below) will be conducted on artificial data for the bubble, while the original authors' tested asset prices (i.e. the bubble term plus the fundamental term.) Since both must satisfy the same dynamic relationships, this change should be innocuous. Funke, Hall, and Sola (1994) use the Markov-Switching ADF test to find evidence for bubbles in the Polish economy in the late 1980s and early 90s. Hall and Sola (1993) did a brief study of the test's properties. However, they only estimated a single realization of each of five different data generating processes, including Evans bubble process with $\pi = 0.75$. [7]

2.3 van Norden Bubble Test

van Norden (1993) and van Norden and Schaller (1993a,b) modify the Blanchard model to allow for the possibility that the bubble is expected to collapse only partially in state *C* by replacing (EQ 7) with

$$E_t(B_{t+1} | C) = u(B_t) \qquad \text{(EQ 13)}$$

7. Note that Hall and Sola(1993) arbitrarily multiply the bubble term by twenty in constructing their simulated asset prices. This seems to imply, unlike the case mentioned above, that their bubble will have a rate of return twenty times greater than that of the fundamental.

where *u(.)* is a continuous and everywhere differentiable function such that $u(0) = 0$ and $1 \geq u' \geq 0$ .Hence, the expected size of collapse will be a function of the relative size of the bubble, B_t, and that the bubble is not expected to grow (and may be expected to shrink) in state *C*. They also suggest that the probability of the bubble's continued growth falls as the bubble grows, so that[8]

$$q = q(B_t) \qquad \frac{d}{d|B_t|}q(B_t) < 0 \tag{EQ 14}$$

van Norden (1993) and van Norden and Schaller (1993b) show that a first-order Taylor-series approximation of this process gives the following 2-state switching-regression system[9]

$$\begin{aligned} E_t(\Delta B_{t+1}|S) &= \gamma_S + \psi_S B_t \\ E_t(\Delta B_{t+1}|C) &= \gamma_C + \psi_C B_t \\ Pr(\text{State}_{t+1} = S) &= \Phi(\lambda + \eta B_t) \\ Pr(\text{State}_{t+1} = C) &= 1 - Pr(\text{State}_{t+1} = S) \end{aligned} \tag{EQ 15}$$

where the model implies that $\psi_S > 0$, $\psi_C < 0$ and $\eta < 0$, and $\Phi(x)$ is the Gaussian cdf function.[10] Again, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the *S* and *C* regimes. In this case, this equivalence implies that

$$\text{llf}(\gamma_S, \psi_S, \gamma_C, \psi_C, \lambda, \eta, \sigma_S, \sigma_C)$$

$$= \text{llf}(\gamma_C, \psi_C, \gamma_S, \psi_S, -\lambda, -\eta, \sigma_C, \sigma_S) \tag{EQ 16}$$

where llf() is the log-likelihood function, so these alternative parameterizations cannot be distinguished without additional information. The van Norden bubble model implies that

8. Since we will only consider positive bubbles in this paper, the use of the absolute value in the derivative in (EQ 14) is not strictly necessary.

9. The original model has the exchange rate innovation R_{t+1} as being the dependent variable. It consists of innovations in fundamentals ε'_{t+1} and innovations in the bubble. Hence $R_{t+1} = \varepsilon'_{t+1} + B_{t+1} - E_t(B_{t+1})$. If we assume that in this model $\varepsilon'_{t+1} = 0$ and use (EQ 6) then $R_{t+1} = \Delta B_{t+1} - rB_t$. Since r is small the use of ΔB_t as the dependent variable is a good approximation of the earlier model.

10. This model differs trivially from that considered in van Norden (1993) and van Norden and Schaller (1993). The former assumed that $\Phi(x)$ was the logistic cdf rather than the gaussian. Both papers also used slightly different classifying equations (using either $|B_t|$ or B_t^2) to allow for the possibility of negative bubbles.

one should find either [$\psi_S > 0$, $\psi_C < 0$, $\eta < 0$], or [$\psi_S < 0$, $\psi_C > 0$, $\eta > 0$], unless some other means is used to identify the surviving and collapsing regimes.

In addition to testing the above restrictions implied by the bubble model, van Norden (1993) and van Norden and Schaller (1993a,b) test whether the bubble-motivated switching-regression model gives significantly more information about the behaviour of ΔB_{t+1}, than two simpler models[11]. Significant evidence of bubbles requires that the switching-regression model can reject these simpler models. One of these is the normal-mixture model

$$\begin{aligned} \Delta B_{t+1} &\sim N(\gamma_S, \sigma_S) \qquad \text{when State}_{t+1} = S \\ \Delta B_{t+1} &\sim N(\gamma_C, \sigma_C) \qquad \text{when State}_{t+1} = C \quad . \\ Pr(\text{State}_{t+1} &= S) = \Phi(\lambda) \end{aligned} \qquad \text{(EQ 17)}$$

which is simply the special case of (EQ 16) where $\psi_S = \psi_C = \eta = 0$. A rejection of this null hypothesis implies that there is a significant link between B_t and the behaviour of the mixing distributions, either because it captures shifts in their means, or in their mixing probabilities, or both.[12]

(EQ 16) also nests the linear regression model as the special case where $\psi_S = \psi_C, \gamma_S = \gamma_C$ and $\eta = 0$ giving[13]:

$$\begin{aligned} \Delta B_{t+1} &= \gamma + \psi B_t + e_{t+1} \\ e_{t+1} &\sim N(0, \sigma_S) \text{with prob} \Phi(\lambda_q) \quad . \\ e_{t+1} &\sim N(0, \sigma_C) \text{with prob} 1 - \Phi(\lambda_q) \end{aligned} \qquad \text{(EQ 18)}$$

Any rejection of this model can be interpreted as evidence of non-linear predictability in asset prices. Note that if the variances differ across the two regimes, all parameters will be identified under the null.

van Norden and Schaller (1993a) use this test framework to show evidence of bubbles in monthly returns from the Toronto Stock Exchange, van Norden (1996) looks for evidence

11. van Norden (1993) also considers a third model. Since it nests within the normal-mixture model, rejections of the normal-mixture model imply a rejection of the third model.

12. van Norden (1993) also notes the relationship of the time-varying transition probabilities to Markov-mixture models. Schaller and van Norden (1994) consider generalizations of (EQ 16) to allow for Markovian state-dependent transition probabilities.

13. We also examined a similar model where we drop the restriction that $\eta = 0$. We found that this model was on average somewhat more problematic to estimate and more likely to statistically reject than the two models considered above.

of bubbles in post-Bretton-Woods floating exchange rate data, and van Norden and Schaller (1993b) examine the behaviour of NYSE monthly stock returns from 1926-1989. The latter paper also presents extensive analysis on whether regime-switching in fundamentals can account for the evident regime-switching in stock returns.

3.0 Experimental Design

As we noted in the introduction, the purpose of this paper is to examine the behaviour of regime-switching bubble tests described in the preceding section. Specifically, we want to use Monte Carlo experiments to evaluate the power of the tests and to compare the two testing methodologies. This involves specifying a data generating process that creates bubbles, generating multiple time series from this process, estimating the regime-switching models and applying the tests described above. All of our estimation is done by maximum-likelihood methods using the programs documented in van Norden and Vigfusson (1996).[14]

We decided to use various parameterizations of the Evans (1991)'s bubble model as our data generating process. This choice has several attractive features. First, the problems of unit-root and cointegration-based tests on this data set are well-documented, so this facilitates a comparison of the regime-switching tests with earlier tests. Second, Charemza and Deadman (1995) study the performance of the earlier tests on other data-generating processes and reach conclusions broadly similar to those of Evans, which suggests that the Evans process might not produce atypical results. Third, as we explain below, the Evans model is not precisely nested within either the Hall and Sola or the van Norden bubble testing models. We think this introduces an interesting amount of misspecification into the experimental design and may give a better indication of how the tests are likely to perform when confronted with real data that may not nest perfectly within either model. We also felt that it offered a neutral "middle-ground" on which to compare the performance of the two tests.

As we noted in Section 2.0, the Evans model is a generalization of the Blanchard (1979) model where both the size of collapses and their probability are functions of the size of the bubble; it incorporates partial rather than total collapses and sets the probability of collapse equal to zero when $B_t \leq \alpha$.

Initially the bubble grows at an average rate 1+r, but the realized rate of growth differs from the expected value by serially uncorrelated mean zero errors. We will refer to this phase of steady expected growth as Regime G. Once the bubble's size reaches a threshold level of α, its behaviour changes. It continues to grow at an expected rate of 1+r but there

14. We made minor modifications to the code to improve its ability to find convergent solutions for hard-to-fit data sets. This mainly consisted of improving the error-trapping in the original programs, and when both gradient-based and EM-based maximization strategies seemed to be failing, using a few iterations of a simple simulated annealing procedure to get new starting values for maximum likelihood estimation.

is now a probability $1-\pi$ of collapse to a level δ. (Regime C) To compensate, if the bubble does not collapse (Regime E) it is expected to grow at a rate greater than $1+r$.

This model can be written as;

$$B_{t+1} = (1+r)\,B_t u_{t+1} \qquad \text{For } B_t \le \alpha$$
$$B_{t+1} = \left(\delta + \theta_{t+1}\pi^{-1}(1+r)\left(B_t - \delta(1+r)^{-1}\right)\right)u_{t+1} \qquad \text{For } B_t > \alpha \qquad \text{(EQ 19)}$$

where α and δ are positive parameters with $\delta < (1+r)\,\alpha$, u_t is an exogenous independently and identically distributed strictly positive random variable with $E_t u_{t+1} = 1$ and θ_t is an exogenous independently and identically distributed Bernoulli process which takes the value 1 with probability π and 0 with probability $1-\pi$. Evans' bubble satisfies (EQ 6).

There are two points to note about this model. First, since u_t is strictly positive, the bubble will never change sign and will never entirely vanish. Second, regime G is only distinguished from the mixture of the other two regimes by the distribution of innovations in the bubble. For a particular distribution of u_t, the innovations in the mixture of regimes C and E will simply appear to be more volatile than in G.

For estimation, we rewrite (EQ 19) in first differences as

G	$\Delta B_t = \{(1+r)\,u_t - 1\}\,B_{t-1}$	For $B_{t-1} \le \alpha$
E	$\Delta B_t = \{\frac{(1+r)}{\pi}u_t - 1\}\,B_{t-1} + \frac{(\pi-1)\,\delta u_t}{\pi}$	For $B_{t-1} > \alpha$ and $\theta_t = 1$ (EQ 20)
C	$\Delta B_t = \delta u_t - B_{t-1}$	For $B_{t-1} > \alpha$ and $\theta_t = 0$

For our Monte Carlo experiments, we generate 5000 draws of the above process, each with 100 observations. We use the same parameter values as Evans, setting $r = 0.05$, $\alpha = 1$, $\delta = 0.5$, $B_1 = \delta$, and $u_t = exp\left(y_t - \frac{\tau^2}{2}\right)$ where $y_t \sim IIN\left(0, \tau^2\right)$ and $\tau = 0.05$. We allow the probability of the bubble continuation, π, to vary over the same interval as Evans: [0.999,0.25].

To simplify estimation, all data series were standardized to have a mean of zero and a variance of one. (For graphing, they were also centered at (0,0).) The relationship between ΔB_t and B_t can be seen in Figure 1. At high levels of π the graph appears to be composed of two branches. The left branch corresponds to State C where the bubble collapses and the right with States G and E where the bubble continues to grow. As π decreases the

State G becomes more distinct from State E. State G can be identified as the large mass centered at 0 on the horizontal axis. It is most prominent when $\pi = 0.25$. The decrease in π also causes a change in the slopes of the two branches. This is because the growth rate in State E increases as π decreases. This increase in the growth rate results in the decline in the slope of the right branch.

FIGURE 1. Place About Here

The data were generated using the random number generators found in Gauss. As a check on the construction of these series, we examined whether we could reproduce Evan's results using the Bhargava (1986) N_1 and N_2 unit-root tests. For values of π less then 0.99, the stationarity tests failed to detect the bubble for a very large percentage of the draws; the same results as Evans.[15]

4.0 Monte Carlo Results

A standard problem in performing Monte Carlo or other simulation experiments with iterative estimators is that some fraction of the estimates will typically fail to converge. This in turn puts limits on the confidence we should attach to our experimental results. Fortunately, this was not a serious problem in practice. Table 1 shows that for the Hall and Sola test, we achieved convergence for 90% or more of the simulated data, regardless of which parameterization of the DGP we consider. For the van Norden test, we needed to estimate as many as 3 switching models on each data sample. Fortunately, convergence rates were generally higher than for the Hall and Sola model, as shown in Table 1. We also occasionally have the problem that the restricted models give values of the higher likelihood function than the unrestricted model (which may due to false convergence or the presence of multiple local maxima.) As shown in Table 2, this problem was also rare, except when $\pi = 0.5$. (We explore the case where $\pi = 0.5$ in greater detail, below.) In all subsequent tables, when report the fraction of cases in which bubbles were detected, this includes as non-detection the cases whether either some models failed to converge or where restricted models gave the highest values of the likelihood function.

4.1 Hall and Sola Test

We consider our results for the Hall and Sola test using two different levels of rigor. First, we examine whether the switching model gets the signs of the two autoregressive coefficients right. (Note that the two regimes were normalized by setting regime 1 to be the regime with the greater slope coefficient.) Next, we test whether these estimates are significantly different from zero. We also do a Wald test to see whether the two coefficients are jointly different from each other.

15. The greatest difference between a percentage that we report and Evans is less than five per cent.

TABLE 1. % of Draws that Failed to Converge: Hall and Sola (HS) van Norden (SW,NM,EC)

π	0.999	0.99	0.95	0.85	0.75	0.50	0.25
HS	NA[a]	7.600	10.78	4.800	3.400	3.460	10.42
SW	3.320	5.320	3.300	3.820	4.620	4.820	2.920
NM	0.9200	3.560	1.740	1.220	0.9000	0.5800	0.5800
EC	3.400	4.420	3.780	2.780	3.380	2.920	2.760
SW & NM	0.1400	0.6200	0.06000	0.1800	0.1200	0.08000	0.02000
SW & EC	0.5200	0.4600	0.3600	0.6400	0.5000	0.1800	0.2200
NM & EC	0.06000	0.2200	0.3200	0.1800	0.1800	0.08000	0.06000
HS $\cup$ SW	NA	11.98	13.78	8.320	7.880	8.160	13.06
HS & SW	NA	1.00	0.34	0.32	0.16	0.18	0.32

a. The Hall and Sola test has not been done for $\pi = 0.999$.

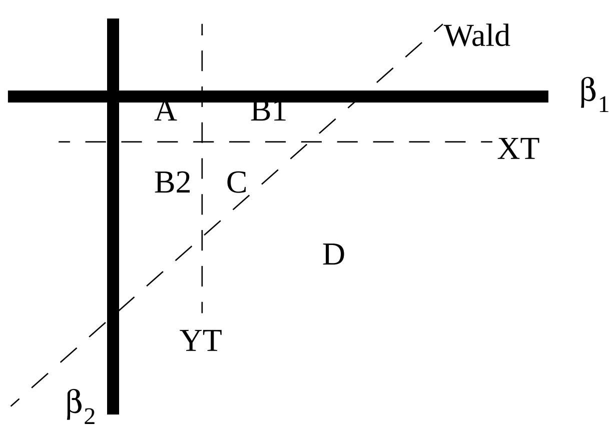
TABLE 2. % of Draws with Restricted LLF Greater than Unrestricted: van Norden

π	**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.5**	**0.25**
NM > SW[a]	1.122	0.4862	0.04138	0.2080	5.789	88.33	2.514
EC > SW	6.594	3.974	1.179	1.414	8.578	87.87	2.267

a. The NM > SW etc. are those draws who returned likelihood function values higher for the restricted model than the unrestricted case.

The relationships between these tests can be seen in Figure 2. The test of the coefficient signs is equivalent to the entire area either right of the Y-axis or below the X-axis. The t-stats restricts this area to right of the YT-line or below the XT-axis. Testing to see that they are both the right signs reduces the area to just the bottom quadrant below the X-axis and right of the Y-axis. Using the t-stats reduces the area to the area of C+D. Finally using the Wald Test eliminates area C leaving only D. Results for these tests are reported in Table 3.

FIGURE 2.



The individual tests (Table 3) show considerable power to detect bubbles. The autoregressive coefficients are significantly different 40-70% of the time, and the individual coefficients are significantly difference from zero and have the correct sign even more frequently (with one exception.) The test seems to have the greatest power when the probability of the bubble surviving is around 80%. At the highest probabilities, the estimated coefficient in the collapsing regime performs poorly, perhaps because so few collapses would be observed in a sample of 100 observations. At the lowest probabilities, the estimated coefficient in the surviving regime performs poorly (presumably for different reasons).

TABLE 3. Percentage of Draws Passing Single Test

	Regime 1 Explosive		Regime 2 Stable or Collapsing		Regimes not Equal
π	$\beta > 0$	t-stat > 2	$\beta < 0$	t-stat < –2	Wald Test
0.25	88.77	46.13	82.21	65.24	50.17
0.50	93.76	68.95	89.60	71.00	65.07
0.75	94.04	76.15	89.50	75.18	67.43
0.85	95.97	84.58	82.52	71.20	66.45
0.95	98.25	84.90	69.46	53.75	53.22
0.99	98.93	89.78	40.38	27.32	42.29

The joint tests (Table 4) results also show that the bubble test is most successful for midrange levels of π. If we simply require that the estimated autoregressive coefficients have the correct signs, then we find evidence of bubbles 40-80% of the time. Even the most stringent tests, which require that all the coefficients are both statistically significant and that they jointly not be equal, find significant evidence of bubbles as much as 43% of the time. However, it should be noted that this power again drops off considerably as π approaches 0 or 1

TABLE 4. Percentage of Draws Passing Multiple Tests

	Regime 1 Explosive and Regime 2 Stable or Collapsing		Regime 1 Explosive Regime 2 Stable or Collapsing and Regimes Not Equal	
π	$\beta > 0$ $\beta < 0$	t-stat > 2 t-stat < –2	$\beta > 0$ $\beta < 0$ Wald Test	t-stat > 2 t-stat < –2 Wald Test
0.25	70.98	34.78	35.72	17.26
0.50	83.36	57.97	54.63	37.79

TABLE 4. Percentage of Draws Passing Multiple Tests

	Regime 1 Explosive and Regime 2 Stable or Collapsing		Regime 1 Explosive Regime 2 Stable or Collapsing and Regimes Not Equal	
π	$\beta > 0$ $\beta < 0$	t-stat > 2 t-stat < –2	$\beta > 0$ $\beta < 0$ Wald Test	t-stat > 2 t-stat < –2 Wald Test
0.75	83.54	63.79	55.94	42.51
0.85	78.49	64.37	52.48	42.82
0.95	67.70	50.16	36.44	27.03
0.99	39.30	24.91	15.96	10.10

4.2 van Norden Bubble Test

Table 5 shows the results of the likelihood ratio tests for bubbles, which compare the fit of the regime-switching model (SW) (EQ 15) to that of the two simpler models NM (EQ 17) and EC (EQ 18). For both high and low values of π, both nulls are almost always rejected in favor of the switching model. (Remember that those draws where the restricted models had likelihood function values greater than the unrestricted models were included as non-rejections of the null! Including these draws did not have a great effect on the rejection rates except for $\pi = 0.5$.) However, the case where $\pi = 0.5$ stands out as an important exception. Here, there were very few rejections of the null, reflecting the very high (over 85%) frequency with which the restricted models gave higher values of the likelihood function than the unrestricted model.

TABLE 5. Adjusted LR Tests.% Rejections

Restriction	**NM**		**EC**	
π	**5%**	**1%**	**5%**	**1%**
.999	98.03	97.45	91.47	90.62
.99	99.24	98.90	94.97	93.81
.95	99.96	99.96	98.45	97.66
.85	99.73	99.73	98.02	97.44
.75	91.00	89.26	87.44	84.92
.5	11.46	11.40	12.01	11.96
.25	97.49	97.49	97.73	97.73

We next examined the parameters of our estimated model (Table 6.) To avoid the identification problem noted in (EQ 16), the parameters are normalized by setting regime 2to be the regime with the greater slope coefficient.The bubble model therefore implies that one should find [$\psi_1 < 0$, $\psi_2 > 0$, $\eta > 0$].

When $\pi = 0.999$ or $\pi = 0.99$ we may never observe a bubble collapse in our relatively small sample of 100 observations. This could cause the estimates to miss the behaviour of regime C and instead make a classification based upon the slight difference between Regimes E and G. This is consistant with the Monte Carlo results. Using the median of the distribution, we find $\psi_1 < 0$ and $\psi_2 > 0$ except for $\pi = \{0.999, 0.99\}$.

TABLE 6. Median Value of Parameters

π Parameter	**.999**	**.99**	**.95**	**.85**	**.75**	**.5**	**.25**
γ_1	-0.0023	0.126	0.552	0.468	-0.541	-0.338	1.02
ψ_1	0.363	0.221	-0.667	-0.941	-1.09	-0.937	-1.03
γ_2	0.0971	0.101	0.136	0.174	0.205	0.242	0.0897
ψ_2	0.749	0.628	0.221	0.339	0.470	0.854	0.0438
λ	0.211	-1.47	-2.69	-3.11	-3.23	-2.31	-185.
η	2.54	2.01	1.76	1.45	1.18	1.10	351.
σ_1	1.05	1.24	2.23	2.21	0.0987	0.0567	2.25
σ_2	0.154	0.0947	0.0643	0.0726	0.107	0.212	0.0914

As shown in Table 7, the t-statistics also provide evidence of bubbles. Independent of the level of π, regime two usually has significantly positive slope and intercept terms, while the slope term in the equation for the probability of being in regime one is usually (correctly) negative for all values of π. However the actual percentage varies greatly. The change in π greatly affects regime one's slope. At high levels of π the slope is often found to be significantly greater than zero. The lack of actual collapses at high levels of π may be responsible for this failure to detect a collapsing regime. As π decreases the slope is found to be significantly less than zero. This corresponds well with the presence of a bubble.

TABLE 7. Percentage of Draws with Parameters Significantly Different From Zero.[a]

π Parameter	**.999**	**.99**	**.95**	**.85**	**.75**	**.5**	**.25**
γ_1	26.17	16.19	7.98	23.1	53.9	81.58	10.28
ψ_1	55.54	47.50	16.9	51.2	84.20	96.00	95.98
γ_2	36.02	56.98	86.3	96.98	99.56	98.89	98.8
ψ_2	95.8	94.54	96.7	98.33	97.97	93.35	48.83
λ	16.58	44.1	76.9	90.52	94.3	52.35	8.02
η	47.7	60.4	83.6	91.88	90.7	29.24	6.76
σ_1	100	99.95	100	99.9	99.9	99.5	100
σ_2	99.8	99.97	100	100	100	99.9	100

a. Shaded are those that are significantly less than zero and Unshaded are significantly greater than zero. Being significantly different was to have a t-statistic greater than two or less than negative two. Both percentages were calculated for each parameter. The greater percentage was reported.

As mentioned earlier, the van Norden test for bubble consists of testing for three coefficients being the correct signs: $\psi_1 < 0$, $\psi_2 > 0$, and $\eta < 0$. When all three coefficients are the correct sign, the series is classified as having a bubble. Requiring that the coefficients only be the correct signs implies that the van Norden test classifies almost all the series as being bubbles for values of π less than 0.99 (Table 8). Requiring statistical significance causes the level of detection to drop. The series for which π equal to 0.25 sees the greatest decrease. This large drop is due to the transition equation's slope coefficient η being found statistically significant only six percent of the time (Table 7).

TABLE 8. Joint Bubble Tests $\psi_1 < 0$ $\psi_2 > 0$ and $\eta < 0$

π	**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
correct signs	8.294	39.51	82.45	95.59	98.87	89.30	94.93
t-test	0.7478	4.883	16.01	48.40	76.78	28.47	2.824

To better understand the behaviour of the test, we also examined the ex post probability that an observation was generated by regime 1 (i.e. the probability conditional on ΔB_{t+1}.)We would expect to find many periods classified as growing and few classified as collapsing. If we identify the collapsing regime with regime one then this is supported by Figure 3. The smaller the value on the horizontal axis, the smaller the number of draws that are classified as regime one. For π between 0.95 and 0.75 regime one is found to be somewhat infrequent, and at π equal to 0.50 there are a number of cases where regime one is very frequent and regime two is very infrequent. The majority of draws lie on the 45 degree line implying that all the periods are clearly distinguished between regimes one and two. Each period has either a very low or very high probability of being in regime one. The exception is when π equals 0.50. Some draws contain a large number of periods whose probability of being in regime one is neither very high, (greater then 0.9), nor very low, (less than 0.1). This may be due to the misspecification of not having a separate regime for State G, the growing state.

FIGURE 3. Place Around Here

4.3 Comparing van Norden and Hall and Sola.

Having already examined the two tests individual performance, we now use the two tests together. First, we examine how often the tests both agree that a bubble is present. Second, we examine how often one test confirms the presence of a bubble already detected by the other test.

In Table 9 using the least stringent conditions that the coefficients are of the correct sign, bubbles are found over 50 percent of the time for values of π equal to 0.85 and 0.75. Even the most stringent test that all the coefficients are statistically significant gives a positive find over one third of the time for the case where $\pi = 0.75$.

TABLE 9. Percentage of draws that agree with both Tests

Tests		π					
Hall and Sola	**van Norden**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
correct signs and Wald test	correct signs	14.02	33.14	50.74	55.38	49.04	34.18
	t-test	1.545	7.172	26.27	43.36	15.05	0.9432
correct t-tests and Wald test	correct signs	10.20	24.51	41.58	42.16	33.80	16.59
	t-test	1.318	5.661	22.01	33.56	10.34	0.5291

Table 10 gives the marginal benefit of running the second test: the percentage of bubbles found by test B given that test A has already found a bubble.

TABLE 10. Percentage of Times Test B agrees with a finding of a bubble by Test A[a]

A	B	0.99	0.95	0.85	0.75	0.50	0.25
Hall and Sola	van Norden	10.94	21.00	51.61	79.00	27.36	3.071
van Norden	Hall and Sola	30.85	36.08	45.49	43.80	36.15	17.97

a. Agrees and finding of a bubble implies that for the van Norden test all three slope coefficients are statistically significant and that for the Hall and Sola test that the t-tests and the Wald test are statistically significant.

For a positive finding by the van Norden test, the Hall and Sola test is more likely to disagree rather than agree for all values of π. The van Norden is more likely to agree rather than disagree for a finding of bubble by the Hall and Sola test only for the cases where π is equal to 0.85 or 0.75.

5.0 Conclusions:

6.0 Bibliography

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Avoiding the Pitfalls:

Can Regime-Switching Tests Detect Bubbles?

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Abstract:

Work on testing for bubbles has caused much debate, much of which has focused on methodology. Monte Carlo simulations reported in Evans (1991) showed that standard tests for unit roots and cointegration frequently reject the presence of bubbles even when such bubbles are present by construction. Evans referred to this problem as the pitfall of testing for bubbles.

Since Evans' note, new tests for rational speculative bubbles that rely on regime switching have been proposed. van Norden and Schaller (1993) and van Norden (1996) use a switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental price. Hall and Sola (1993) and Funke, Hall and Sola (1994) test whether asset prices seem to switch between explosive growth and stationary behaviour.

Our paper does Monte Carlo experiments using Evan's data generating process to gauge the performance of these two kinds of regime-switching tests. These rely heavily on new fast robust programs developed at the Bank of Canada for the estimation of switching regression models, which make Monte Carlo studies of such estimators practical. We find that for some (but not all) parameter values, regime-switching tests have significant amount of power to detect periodically collapsing bubbles. We also compare and contrast the performance of the two different regime-switching tests.

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1.0 Introduction

Work on testing for bubbles has caused much debate, much of which has focused on methodology. Early work used variance-bound tests until various econometric problems with this approach were noted. (See LeRoy 1989.) The misspecification test suggested by West (1987) has fallen out of favour since misspecified fundamentals should cause it to detect bubbles and there is little agreement on how to specify the fundamentals. (For example, see Flood and Hodrick (1990).) Diba and Grossman (1988) and Hamilton and Whiteman (1985) recommend the use of tests for stationarity and for cointegration to test for the absence of rational speculative bubbles. However, Monte Carlo simulations reported in Evans (1991) showed that standard tests for unit roots and cointegration frequently reject the presence of bubbles even when such bubbles are present by construction.[1] Evans referred to this problem as the "pitfall" of testing for bubbles.

Since Evans' note, new tests for rational speculative bubbles that rely on regime switching have been proposed. van Norden and Schaller (1993a,b) and van Norden (1996) use a switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental price. Hall and Sola (1993) and Funke, Hall and Sola (1994) test whether asset prices seem to switch between explosive growth and stationary behaviour.

A potential flaw of this new approach is that the regime-switching estimators may not be well behaved. There are two plausible grounds for concern.[2]

- The presence of rational speculative bubbles implies that the data are nonstationarity, but the properties of regime-switching estimators in such a case are unknown. Since this nonstationarity exists only under the alternative hypothesis of bubbles, this raises the question of whether the regime-switching tests have the power to detect bubbles when they exist. This is similar to the pitfall that Evans (1991) found with the unit-root and cointegration tests.[3]
- Little is known about the finite-sample properties of regime-switching estimators. In particular, little has been done to determine whether the use of tests whose distribution is known only asymptotically leads to reliable inference. It is conceivable that asymptotically-correct tests could experience size distortion in small samples, which would tend to produce evidence of speculative bubbles even when none are present.

1. Charemza and Deadman (1995) show that this problem extends to a broader range of processes than those considered by Evans (1991).

2. The problems of directly testing for the presence of regime-switching have recently become better understood. However, both of the above approaches circumvent these complications by testing a regime-switching alternative against a regime-switching null.

3. Both problems could lead us to conclude that bubbles are absent when they are in fact present. The difference is that with the cointegration and unit-root tests, this is caused by size-distortion while with the regime-switching tests this is caused by a lack of power. This difference arises because the two kinds of tests reverse the null and alternative hypotheses.

Our paper is a first step in addressing these questions. We examine the power properties of regime-switching bubble tests via Monte Carlo experiments using Evan's data generating process. Our work relies on new, fast, robust programs developed at the Bank of Canada for the estimation of switching regression models, which make Monte Carlo studies of such estimators practical.[4] We find that for some (but not all) parameter values, regime-switching tests have significant amount of power to detect periodically collapsing bubbles. We are also the first to compare and contrast the performance of the two different kinds of regime-switching tests.

In the next section, we explain the relationship between speculative bubbles and regime switching, and then review the tests proposed by Hall and Sola, and by van Norden. Section 3.0 explains the design of our Monte Carlo experiments, and their results are discussed in Section 4.0. Section 5.0 concludes and gives several suggestions for further research.

2.0 Tests for Rational Speculative Bubbles

2.1 Bubbles and Regime Switching

Consider a simple asset-pricing model, which only requires that

$$p_t = f(X_t) + a \cdot E_t(p_{t+1}) \qquad \text{(EQ 1)}$$

where p_t is the logarithm of the asset price, E_t is the operator for expectations conditional on information at time t, $0 < a < 1$, and X_t is a vector of other variables. Solving the equation forward gives the general result

$$p_t = \left(\sum_{j=0}^{T} a^j \cdot E_t(f(X_{t+j})) \right) + a^{T+1} \cdot E_t(p_{T+1}) . \qquad \text{(EQ 2)}$$

One solution to equation (EQ 1), which we will denote p_t^*, occurs when

$$\lim_{T \to \infty} a^{T+1} \cdot E_t(p_{T+1}) = 0 , \qquad \text{(EQ 3)}$$

so

$$p_t^* = \sum_{j=0}^{\infty} a^j \cdot E_t(f(X_{t+j})) . \qquad \text{(EQ 4)}$$

4. See van Norden and Vigfusson (1996).

We refer to (EQ 4) as the fundamental solution, since it determines the asset price solely as a function of the current and expected behaviour of other macroeconomic variables.

However, equation (EQ 4) is not the only solution to (EQ 1). We define bubble solutions to be any other set of asset prices and expected asset prices that satisfy equation (EQ 1) but where $p_t \neq p_t^*$. We define the size of the bubble B_t as

$$B_t \equiv p_t - p_t^* \quad . \qquad \text{(EQ 5)}$$

Note that since p_t^* satisfies equation (EQ 1), it follows from (EQ 1) and (EQ 5) that

$$B_t = a \cdot E_t(B_{t+1}) \quad . \qquad \text{(EQ 6)}$$

Since $a < 1$, this means the bubble must be expected to grow over time.

A considerable literature exists on the conditions under which such bubbles are feasible rational expectations solutions. Important contributions to this debate include Obstfeld and Rogoff (1983),(1986), Diba and Grossman (1987), Tirole (1982), (1985), Weil (1990), Buiter and Pesenti (1990), Allen and Gorton (1991), and Gilles and LeRoy (1992). In single-representative-agent models, a truly rational agent cannot expect to sell an over-valued asset (one with a positive bubble) before the bubble bursts. Therefore, bubbles should exist in such models only if they can be expected to grow without limit. Some researchers, such as Froot and Obstfeld (1991), have therefore suggested interpreting empirical tests for bubbles as tests of whether agents are fully rational, or whether they exhibit some form of myopia when considering events that are either very far in the future, or occur with only very low probabilities. An alternative interpretation would be to consider evidence of bubbles as suggesting that non-representative-agent models (such as those of De Long et al. (1990), Allen and Gorton (1991) or Bulow and Klemperer (1991)) are required.

Nothing in the above model has any implications for regime switching. Some of the early literature on rational speculative bubbles even considered purely deterministic bubbles. Regime switching stems from the descriptions of asset market behaviour (for example, those surveyed in Kindleberger (1989)) to which the above model of bubbles is often applied. The first example of regime switching in the rational speculative bubble framework is Blanchard(1979), who proposes a bubble that moves randomly between two states, *C* and *S*. In state *C*, the bubble will collapse, so[5]

$$E_t(B_{t+1} | C) = 0 \quad . \qquad \text{(EQ 7)}$$

5. The notation $E_t(X_j | C)$ (or $E_t(X_j | S)$) denotes the expectation of X_j conditioning on the fact that the state at *t* is *C* (or *S*) and on all other information available at time *t*.

State S, where the bubble survives and continues to grow, occurs with a fixed probability q. Since

$$E_t(B_{t+1}) = (1-q) \cdot E_t(B_{t+1}|C) + q \cdot E_t(B_{t+1}|S) \quad , \qquad \text{(EQ 8)}$$

it follows from (EQ 6) that

$$E_t(B_{t+1}|S) = \frac{B_t \cdot (1+r)}{q} \qquad \text{(EQ 9)}$$

This model was subsequently generalized by Evans (1991) and van Norden and Schaller (1993) to consider the case where both the size of collapses and their probability were functions of the size of the bubble.

The distinguishing feature of these regime switching models is that the behaviour of the asset price is now state dependent, and that the state itself is unobservable. However, these models may differ in the way the probability of observing a given regime varies over time. In the Blanchard (1979), this is simply a constant. In the van Norden bubble test, the probability of observing the collapsing regime is assumed to be an increasing function of the size of the bubble. In the Hall and Sola test, this probability is assumed to follow a first-order Markov process, where the probability of remaining in a given regime is constant.[6] To distinguish these two kinds of switching models, we will refer to the case where the probability of observing a given state is independent of past states as "simple switching." In the case of a two-state model, the simple switching model is simply the special case of the Markov-switching model where

$$Pr(S_t = 0|(S_{t-1} = 0)) = 1 - Pr(S_t = 1|(S_{t-1} = 1)) \qquad \text{(EQ 10)}$$

where $Pr(S_t = k|S_{t-1} = k)$ is the probability of remaining in state k given last period's state was k.

2.2 The Hall and Sola Test for bubbles

As mentioned earlier, Diba and Grossman (1988) suggested using tests for stationarity to rule out the existence of bubbles. This method could be useful in the case of a non-collapsing bubble but, as shown in Evans(1991), these tests tend to reject the presence of bubbles when regime-switching bubbles are present. Hall and Sola (1993) address this problem by extending the standard ADF test

6. As noted by Evans and Lewis (1995), a 2-state first-order Markov process is not compatible with (EQ 6). They reconcile this by modifying the usual 2-state Markov model to allow for jumps in asset prices when the regime changes.

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{k=1}^{n} \psi \Delta y_{t-k} + v_t$$

$$\text{where } v_t \sim N(0, \sigma) . \qquad \text{(EQ 11)}$$

to allow the parameters vary between two regimes, giving

$$\Delta y_t = \alpha_i + \beta_i y_{t-1} + \sum_{k=1}^{n} \psi_i \Delta y_{t-k} + v_{t,i}$$

$$\text{where } v_{t,i} \sim N(0, \sigma_i) . \qquad \text{(EQ 12)}$$

The slope coefficients β_S and β_C are the basis of bubble test. Evidence that one regime is non-stationary (i.e. $\beta_S > 0$) while the other is stationary (i.e. $\beta_C < 0$) is evidence of the presence of a bubble. However, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the *S* and *C* regimes. This means that one should find $\beta_S > 0, \beta_C < 0$ or $\beta_S < 0, \beta_C > 0$ unless some other means is used to identify the surviving and collapsing regimes.

Our application of the Hall and Sola test (below) will be conducted on artificial data for the bubble, while the original authors' tested asset prices (i.e. the bubble term plus the fundamental term.) Since both must satisfy the same dynamic relationships, this change should be innocuous. Funke, Hall, and Sola (1994) use the Markov-Switching ADF test to find evidence for bubbles in the Polish economy in the late 1980s and early 90s. Hall and Sola (1993) did a brief study of the test's properties. However, they only estimated a single realization of each of five different data generating processes, including Evans bubble process with $\pi = 0.75$. [7]

2.3 van Norden Bubble Test

van Norden (1993) and van Norden and Schaller (1993a,b) modify the Blanchard model to allow for the possibility that the bubble is expected to collapse only partially in state *C* by replacing (EQ 7) with

$$E_t(B_{t+1} | C) = u(B_t) \qquad \text{(EQ 13)}$$

7. Note that Hall and Sola(1993) arbitrarily multiply the bubble term by twenty in constructing their simulated asset prices. This seems to imply, unlike the case mentioned above, that their bubble will have a rate of return twenty times greater than that of the fundamental.

where $u(.)$ is a continuous and everywhere differentiable function such that $u(0) = 0$ and $1 \geq u' \geq 0$ .Hence, the expected size of collapse will be a function of the relative size of the bubble, B_t, and that the bubble is not expected to grow (and may be expected to shrink) in state C. They also suggest that the probability of the bubble's continued growth falls as the bubble grows, so that[8]

$$q = q(B_t) \qquad \frac{d}{d|B_t|}q(B_t) < 0 \tag{EQ 14}$$

van Norden (1993) and van Norden and Schaller (1993b) show that a first-order Taylor-series approximation of this process gives the following 2-state switching-regression system[9]

$$\begin{aligned} E_t(\Delta B_{t+1}|S) &= \gamma_S + \psi_S B_t \\ E_t(\Delta B_{t+1}|C) &= \gamma_C + \psi_C B_t \\ Pr(\text{State}_{t+1} = S) &= \Phi(\lambda + \eta B_t) \\ Pr(\text{State}_{t+1} = C) &= 1 - Pr(\text{State}_{t+1} = S) \end{aligned} \tag{EQ 15}$$

where the model implies that $\psi_S > 0$, $\psi_C < 0$ and $\eta < 0$, and $\Phi(x)$ is the Gaussian cdf function.[10] Again, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the S and C regimes. In this case, this equivalence implies that

$$\text{llf}(\gamma_S, \psi_S, \gamma_C, \psi_C, \lambda, \eta, \sigma_S, \sigma_C)$$

$$=\text{llf}(\gamma_C, \psi_C, \gamma_S, \psi_S, -\lambda, -\eta, \sigma_C, \sigma_S) \tag{EQ 16}$$

where llf() is the log-likelihood function, so these alternative parameterizations cannot be distinguished without additional information. The van Norden bubble model implies that

8. Since we will only consider positive bubbles in this paper, the use of the absolute value in the derivative in (EQ 14) is not strictly necessary.

9. The original model has the exchange rate innovation R_{t+1} as being the dependent variable. It consists of innovations in fundamentals ε'_{t+1} and innovations in the bubble. Hence $R_{t+1} = \varepsilon'_{t+1} + B_{t+1} - E_t(B_{t+1})$. If we assume that in this model $\varepsilon'_{t+1} = 0$ and use (EQ 6) then $R_{t+1} = \Delta B_{t+1} - rB_t$. Since r is small the use of ΔB_t as the dependent variable is a good approximation of the earlier model.

10. This model differs trivially from that considered in van Norden (1993) and van Norden and Schaller (1993). The former assumed that $\Phi(x)$ was the logistic cdf rather than the gaussian. Both papers also used slightly different classifying equations (using either $|B_t|$ or B_t^2) to allow for the possibility of negative bubbles.

one should find either [$\psi_S > 0$, $\psi_C < 0$, $\eta < 0$], or [$\psi_S < 0$, $\psi_C > 0$, $\eta > 0$], unless some other means is used to identify the surviving and collapsing regimes.

In addition to testing the above restrictions implied by the bubble model, van Norden (1993) and van Norden and Schaller (1993a,b) test whether the bubble-motivated switching-regression model gives significantly more information about the behaviour of ΔB_{t+1}, than two simpler models[11]. Significant evidence of bubbles requires that the switching-regression model can reject these simpler models. One of these is the normal-mixture model

$$\begin{gathered}\Delta B_{t+1} \sim N(\gamma_S, \sigma_S) \qquad \text{when State}_{t+1} = S \\ \Delta B_{t+1} \sim N(\gamma_C, \sigma_C) \qquad \text{when State}_{t+1} = C \\ Pr(\text{State}_{t+1} = S) = \Phi(\lambda)\end{gathered} \quad . \qquad \text{(EQ 17)}$$

which is simply the special case of (EQ 16) where $\psi_S = \psi_C = \eta = 0$. A rejection of this null hypothesis implies that there is a significant link between B_t and the behaviour of the mixing distributions, either because it captures shifts in their means, or in their mixing probabilities, or both.[12]

(EQ 16) also nests the linear regression model as the special case where $\psi_S = \psi_C, \gamma_S = \gamma_C$ and $\eta = 0$ giving[13]:

$$\begin{gathered}\Delta B_{t+1} = \gamma + \psi B_t + e_{t+1} \\ e_{t+1} \sim N(0, \sigma_S) \text{with prob} \Phi(\lambda_q) \\ e_{t+1} \sim N(0, \sigma_C) \text{with prob} 1 - \Phi(\lambda_q)\end{gathered} \quad . \qquad \text{(EQ 18)}$$

Any rejection of this model can be interpreted as evidence of non-linear predictability in asset prices. Note that if the variances differ across the two regimes, all parameters will be identified under the null.

van Norden and Schaller (1993a) use this test framework to show evidence of bubbles in monthly returns from the Toronto Stock Exchange, van Norden (1996) looks for evidence

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11. van Norden (1993) also considers a third model. Since it nests within the normal-mixture model, rejections of the normal-mixture model imply a rejection of the third model.

12. van Norden (1993) also notes the relationship of the time-varying transition probabilities to Markov-mixture models. Schaller and van Norden (1994) consider generalizations of (EQ 16) to allow for Markovian state-dependent transition probabilities.

13. We also examined a similar model where we drop the restriction that $\eta = 0$. We found that this model was on average somewhat more problematic to estimate and more likely to statistically reject than the two models considered above.

of bubbles in post-Bretton-Woods floating exchange rate data, and van Norden and Schaller (1993b) examine the behaviour of NYSE monthly stock returns from 1926-1989. The latter paper also presents extensive analysis on whether regime-switching in fundamentals can account for the evident regime-switching in stock returns.

3.0 Experimental Design

As we noted in the introduction, the purpose of this paper is to examine the behaviour of regime-switching bubble tests described in the preceding section. Specifically, we want to use Monte Carlo experiments to evaluate the power of the tests and to compare the two testing methodologies. This involves specifying a data generating process that creates bubbles, generating multiple time series from this process, estimating the regime-switching models and applying the tests described above. All of our estimation is done by maximum-likelihood methods using the programs documented in van Norden and Vigfusson (1996).[14]

We decided to use various parameterizations of the Evans (1991)'s bubble model as our data generating process. This choice has several attractive features. First, the problems of unit-root and cointegration-based tests on this data set are well-documented, so this facilitates a comparison of the regime-switching tests with earlier tests. Second, Charemza and Deadman (1995) study the performance of the earlier tests on other data-generating processes and reach conclusions broadly similar to those of Evans, which suggests that the Evans process might not produce atypical results. Third, as we explain below, the Evans model is not precisely nested within either the Hall and Sola or the van Norden bubble testing models. We think this introduces an interesting amount of misspecification into the experimental design and may give a better indication of how the tests are likely to perform when confronted with real data that may not nest perfectly within either model. We also felt that it offered a neutral "middle-ground" on which to compare the performance of the two tests.

As we noted in Section 2.0, the Evans model is a generalization of the Blanchard (1979) model where both the size of collapses and their probability are functions of the size of the bubble; it incorporates partial rather than total collapses and sets the probability of collapse equal to zero when $B_t \leq \alpha$.

Initially the bubble grows at an average rate 1+r, but the realized rate of growth differs from the expected value by serially uncorrelated mean zero errors. We will refer to this phase of steady expected growth as Regime G. Once the bubble's size reaches a threshold level of α, its behaviour changes. It continues to grow at an expected rate of 1+r but there

14. We made minor modifications to the code to improve its ability to find convergent solutions for hard-to-fit data sets. This mainly consisted of improving the error-trapping in the original programs, and when both gradient-based and EM-based maximization strategies seemed to be failing, using a few iterations of a simple simulated annealing procedure to get new starting values for maximum likelihood estimation.

is now a probability $1 - \pi$ of collapse to a level δ. (Regime C) To compensate, if the bubble does not collapse (Regime E) it is expected to grow at a rate greater than $1 + r$.

This model can be written as;

$$B_{t+1} = (1+r)B_t u_{t+1} \qquad \text{For } B_t \le \alpha$$
$$B_{t+1} = \left(\delta + \theta_{t+1}\pi^{-1}(1+r)\left(B_t - \delta(1+r)^{-1}\right)\right)u_{t+1} \qquad \text{For } B_t > \alpha \qquad \text{(EQ 19)}$$

where α and δ are positive parameters with $\delta < (1+r)\alpha$, u_t is an exogenous independently and identically distributed strictly positive random variable with $E_t u_{t+1} = 1$ and θ_t is an exogenous independently and identically distributed Bernoulli process which takes the value 1 with probability π and 0 with probability $1 - \pi$. Evans' bubble satisfies (EQ 6).

There are two points to note about this model. First, since u_t is strictly positive, the bubble will never change sign and will never entirely vanish. Second, regime G is only distinguished from the mixture of the other two regimes by the distribution of innovations in the bubble. For a particular distribution of u_t, the innovations in the mixture of regimes C and E will simply appear to be more volatile than in G.

For estimation, we rewrite (EQ 19) in first differences as

G $\quad \Delta B_t = \{(1+r)u_t - 1\}B_{t-1} \qquad \text{For } B_{t-1} \le \alpha$

E $\quad \Delta B_t = \{\frac{(1+r)}{\pi}u_t - 1\}B_{t-1} + \frac{(\pi - 1)\delta u_t}{\pi} \qquad \text{For } B_{t-1} > \alpha \text{ and } \theta_t = 1$ (EQ 20)

C $\quad \Delta B_t = \delta u_t - B_{t-1} \qquad \text{For } B_{t-1} > \alpha \text{ and } \theta_t = 0$

For our Monte Carlo experiments, we generate 5000 draws of the above process, each with 100 observations. We use the same parameter values as Evans, setting $r = 0.05$, $\alpha = 1$, $\delta = 0.5$, $B_1 = \delta$, and $u_t = exp\left(y_t - \frac{\tau^2}{2}\right)$ where $y_t \sim IIN\left(0, \tau^2\right)$ and $\tau = 0.05$. We allow the probability of the bubble continuation, π, to vary over the same interval as Evans: [0.999,0.25].

To simplify estimation, all data series were standardized to have a mean of zero and a variance of one. (For graphing, they were also centered at (0,0).) The relationship between ΔB_t and B_t can be seen in Figure 1. At high levels of π the graph appears to be composed of two branches. The left branch corresponds to State C where the bubble collapses and the right with States G and E where the bubble continues to grow. As π decreases the

State G becomes more distinct from State E. State G can be identified as the large mass centered at 0 on the horizontal axis. It is most prominent when $\pi = 0.25$. The decrease in π also causes a change in the slopes of the two branches. This is because the growth rate in State E increases as π decreases. This increase in the growth rate results in the decline in the slope of the right branch.

FIGURE 1. Place About Here

The data were generated using the random number generators found in Gauss. As a check on the construction of these series, we examined whether we could reproduce Evan's results using the Bhargava (1986) N_1 and N_2 unit-root tests. For values of π less then 0.99, the stationarity tests failed to detect the bubble for a very large percentage of the draws; the same results as Evans.[15]

4.0 Monte Carlo Results

A standard problem in performing Monte Carlo or other simulation experiments with iterative estimators is that some fraction of the estimates will typically fail to converge. This in turn puts limits on the confidence we should attach to our experimental results. Fortunately, this was not a serious problem in practice. Table 1 shows that for the Hall and Sola test, we achieved convergence for 90% or more of the simulated data, regardless of which parameterization of the DGP we consider. For the van Norden test, we needed to estimate as many as 3 switching models on each data sample. Fortunately, convergence rates were generally higher than for the Hall and Sola model, as shown in Table 1. We also occasionally have the problem that the restricted models give values of the higher likelihood function than the unrestricted model (which may due to false convergence or the presence of multiple local maxima.) As shown in Table 2, this problem was also rare, except when $\pi = 0.5$. (We explore the case where $\pi = 0.5$ in greater detail, below.) In all subsequent tables, when report the fraction of cases in which bubbles were detected, this includes as non-detection the cases whether either some models failed to converge or where restricted models gave the highest values of the likelihood function.

4.1 Hall and Sola Test

We consider our results for the Hall and Sola test using two different levels of rigor. First, we examine whether the switching model gets the signs of the two autoregressive coefficients right. (Note that the two regimes were normalized by setting regime 1 to be the regime with the greater slope coefficient.) Next, we test whether these estimates are significantly different from zero. We also do a Wald test to see whether the two coefficients are jointly different from each other.

15. The greatest difference between a percentage that we report and Evans is less than five per cent.

TABLE 1. % of Draws that Failed to Converge: Hall and Sola (HS) van Norden (SW,NM,EC)

π	0.999	0.99	0.95	0.85	0.75	0.50	0.25
HS	NA[a]	7.600	10.78	4.800	3.400	3.460	10.42
SW	3.320	5.320	3.300	3.820	4.620	4.820	2.920
NM	0.9200	3.560	1.740	1.220	0.9000	0.5800	0.5800
EC	3.400	4.420	3.780	2.780	3.380	2.920	2.760
SW & NM	0.1400	0.6200	0.06000	0.1800	0.1200	0.08000	0.02000
SW & EC	0.5200	0.4600	0.3600	0.6400	0.5000	0.1800	0.2200
NM & EC	0.06000	0.2200	0.3200	0.1800	0.1800	0.08000	0.06000
HS ∪ SW	NA	11.98	13.78	8.320	7.880	8.160	13.06
HS & SW	NA	1.00	0.34	0.32	0.16	0.18	0.32

a. The Hall and Sola test has not been done for $\pi = 0.999$.

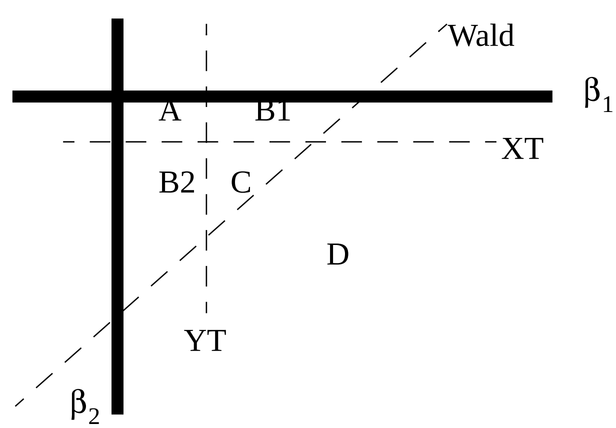
TABLE 2. % of Draws with Restricted LLF Greater than Unrestricted: van Norden

π	**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.5**	**0.25**
NM > SW[a]	1.122	0.4862	0.04138	0.2080	5.789	88.33	2.514
EC > SW	6.594	3.974	1.179	1.414	8.578	87.87	2.267

a. The NM > SW etc. are those draws who returned likelihood function values higher for the restricted model than the unrestricted case.

The relationships between these tests can be seen in Figure 2. The test of the coefficient signs is equivalent to the entire area either right of the Y-axis or below the X-axis. The t-stats restricts this area to right of the YT-line or below the XT-axis. Testing to see that they are both the right signs reduces the area to just the bottom quadrant below the X-axis and right of the Y-axis. Using the t-stats reduces the area to the area of C+D. Finally using the Wald Test eliminates area C leaving only D. Results for these tests are reported in Table 3.

FIGURE 2.



The individual tests (Table 3) show considerable power to detect bubbles. The autoregressive coefficients are significantly different 40-70% of the time, and the individual coefficients are significantly difference from zero and have the correct sign even more frequently (with one exception.) The test seems to have the greatest power when the probability of the bubble surviving is around 80%. At the highest probabilities, the estimated coefficient in the collapsing regime performs poorly, perhaps because so few collapses would be observed in a sample of 100 observations. At the lowest probabilities, the estimated coefficient in the surviving regime performs poorly (presumably for different reasons).

TABLE 3. Percentage of Draws Passing Single Test

	Regime 1 Explosive		Regime 2 Stable or Collapsing		Regimes not Equal
π	$\beta > 0$	t-stat > 2	$\beta < 0$	t-stat < –2	Wald Test
0.25	88.77	46.13	82.21	65.24	50.17
0.50	93.76	68.95	89.60	71.00	65.07
0.75	94.04	76.15	89.50	75.18	67.43
0.85	95.97	84.58	82.52	71.20	66.45
0.95	98.25	84.90	69.46	53.75	53.22
0.99	98.93	89.78	40.38	27.32	42.29

The joint tests (Table 4) results also show that the bubble test is most successful for midrange levels of π. If we simply require that the estimated autoregressive coefficients have the correct signs, then we find evidence of bubbles 40-80% of the time. Even the most stringent tests, which require that all the coefficients are both statistically significant and that they jointly not be equal, find significant evidence of bubbles as much as 43% of the time. However, it should be noted that this power again drops off considerably as π approaches 0 or 1

TABLE 4. Percentage of Draws Passing Multiple Tests

	Regime 1 Explosive and Regime 2 Stable or Collapsing		Regime 1 Explosive Regime 2 Stable or Collapsing and Regimes Not Equal	
π	$\beta > 0$ $\beta < 0$	t-stat > 2 t-stat < –2	$\beta > 0$ $\beta < 0$ Wald Test	t-stat > 2 t-stat < –2 Wald Test
0.25	70.98	34.78	35.72	17.26
0.50	83.36	57.97	54.63	37.79

TABLE 4. Percentage of Draws Passing Multiple Tests

	Regime 1 Explosive and Regime 2 Stable or Collapsing		Regime 1 Explosive Regime 2 Stable or Collapsing and Regimes Not Equal	
π	$\beta > 0$ $\beta < 0$	t-stat > 2 t-stat < –2	$\beta > 0$ $\beta < 0$ Wald Test	t-stat > 2 t-stat < –2 Wald Test
0.75	83.54	63.79	55.94	42.51
0.85	78.49	64.37	52.48	42.82
0.95	67.70	50.16	36.44	27.03
0.99	39.30	24.91	15.96	10.10

4.2 van Norden Bubble Test

Table 5 shows the results of the likelihood ratio tests for bubbles, which compare the fit of the regime-switching model (SW) (EQ 15) to that of the two simpler models NM (EQ 17) and EC (EQ 18). For both high and low values of π, both nulls are almost always rejected in favor of the switching model. (Remember that those draws where the restricted models had likelihood function values greater than the unrestricted models were included as non-rejections of the null! Including these draws did not have a great effect on the rejection rates except for $\pi = 0.5$.) However, the case where $\pi = 0.5$ stands out as an important exception. Here, there were very few rejections of the null, reflecting the very high (over 85%) frequency with which the restricted models gave higher values of the likelihood function than the unrestricted model.

TABLE 5. Adjusted LR Tests.% Rejections

Restriction	**NM**		**EC**	
π	**5%**	**1%**	**5%**	**1%**
.999	98.03	97.45	91.47	90.62
.99	99.24	98.90	94.97	93.81
.95	99.96	99.96	98.45	97.66
.85	99.73	99.73	98.02	97.44
.75	91.00	89.26	87.44	84.92
.5	11.46	11.40	12.01	11.96
.25	97.49	97.49	97.73	97.73

We next examined the parameters of our estimated model (Table 6.) To avoid the identification problem noted in (EQ 16), the parameters are normalized by setting regime 2to be the regime with the greater slope coefficient.The bubble model therefore implies that one should find [$\psi_1 < 0$, $\psi_2 > 0$, $\eta > 0$].

When $\pi = 0.999$ or $\pi = 0.99$ we may never observe a bubble collapse in our relatively small sample of 100 observations. This could cause the estimates to miss the behaviour of regime C and instead make a classification based upon the slight difference between Regimes E and G. This is consistant with the Monte Carlo results. Using the median of the distribution, we find $\psi_1 < 0$ and $\psi_2 > 0$ except for $\pi = \{0.999, 0.99\}$.

TABLE 6. Median Value of Parameters

π Parameter	**.999**	**.99**	**.95**	**.85**	**.75**	**.5**	**.25**
γ_1	-0.0023	0.126	0.552	0.468	-0.541	-0.338	1.02
ψ_1	0.363	0.221	-0.667	-0.941	-1.09	-0.937	-1.03
γ_2	0.0971	0.101	0.136	0.174	0.205	0.242	0.0897
ψ_2	0.749	0.628	0.221	0.339	0.470	0.854	0.0438
λ	0.211	-1.47	-2.69	-3.11	-3.23	-2.31	-185.
η	2.54	2.01	1.76	1.45	1.18	1.10	351.
σ_1	1.05	1.24	2.23	2.21	0.0987	0.0567	2.25
σ_2	0.154	0.0947	0.0643	0.0726	0.107	0.212	0.0914

As shown in Table 7, the t-statistics also provide evidence of bubbles. Independent of the level of π, regime two usually has significantly positive slope and intercept terms, while the slope term in the equation for the probability of being in regime one is usually (correctly) negative for all values of π. However the actual percentage varies greatly. The change in π greatly affects regime one's slope. At high levels of π the slope is often found to be significantly greater than zero. The lack of actual collapses at high levels of π may be responsible for this failure to detect a collapsing regime. As π decreases the slope is found to be significantly less than zero. This corresponds well with the presence of a bubble.

TABLE 7. Percentage of Draws with Parameters Significantly Different From Zero.[a]

π Parameter	**.999**	**.99**	**.95**	**.85**	**.75**	**.5**	**.25**
γ_1	26.17	16.19	7.98	23.1	53.9	81.58	10.28
ψ_1	55.54	47.50	16.9	51.2	84.20	96.00	95.98
γ_2	36.02	56.98	86.3	96.98	99.56	98.89	98.8
ψ_2	95.8	94.54	96.7	98.33	97.97	93.35	48.83
λ	16.58	44.1	76.9	90.52	94.3	52.35	8.02
η	47.7	60.4	83.6	91.88	90.7	29.24	6.76
σ_1	100	99.95	100	99.9	99.9	99.5	100
σ_2	99.8	99.97	100	100	100	99.9	100

a. Shaded are those that are significantly less than zero and Unshaded are significantly greater than zero. Being significantly different was to have a t-statistic greater than two or less than negative two. Both percentages were calculated for each parameter. The greater percentage was reported.

As mentioned earlier, the van Norden test for bubble consists of testing for three coefficients being the correct signs: $\psi_1 < 0$, $\psi_2 > 0$, and $\eta < 0$. When all three coefficients are the correct sign, the series is classified as having a bubble. Requiring that the coefficients only be the correct signs implies that the van Norden test classifies almost all the series as being bubbles for values of π less than 0.99 (Table 8). Requiring statistical significance causes the level of detection to drop. The series for which π equal to 0.25 sees the greatest decrease. This large drop is due to the transition equation's slope coefficient η being found statistically significant only six percent of the time (Table 7).

TABLE 8. Joint Bubble Tests $\psi_1 < 0$ $\psi_2 > 0$ and $\eta < 0$

π	**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
correct signs	8.294	39.51	82.45	95.59	98.87	89.30	94.93
t-test	0.7478	4.883	16.01	48.40	76.78	28.47	2.824

To better understand the behaviour of the test, we also examined the ex post probability that an observation was generated by regime 1 (i.e. the probability conditional on ΔB_{t+1}.)We would expect to find many periods classified as growing and few classified as collapsing. If we identify the collapsing regime with regime one then this is supported by Figure 3. The smaller the value on the horizontal axis, the smaller the number of draws that are classified as regime one. For π between 0.95 and 0.75 regime one is found to be somewhat infrequent, and at π equal to 0.50 there are a number of cases where regime one is very frequent and regime two is very infrequent. The majority of draws lie on the 45 degree line implying that all the periods are clearly distinguished between regimes one and two. Each period has either a very low or very high probability of being in regime one. The exception is when π equals 0.50. Some draws contain a large number of periods whose probability of being in regime one is neither very high, (greater then 0.9), nor very low, (less than 0.1). This may be due to the misspecification of not having a separate regime for State G, the growing state.

FIGURE 3. Place Around Here

4.3 Comparing van Norden and Hall and Sola.

Having already examined the two tests individual performance, we now use the two tests together. First, we examine how often the tests both agree that a bubble is present. Second, we examine how often one test confirms the presence of a bubble already detected by the other test.

In Table 9 using the least stringent conditions that the coefficients are of the correct sign, bubbles are found over 50 percent of the time for values of π equal to 0.85 and 0.75. Even the most stringent test that all the coefficients are statistically significant gives a positive find over one third of the time for the case where $\pi = 0.75$.

TABLE 9. Percentage of draws that agree with both Tests

Tests		π					
Hall and Sola	**van Norden**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
correct signs and Wald test	correct signs	14.02	33.14	50.74	55.38	49.04	34.18
	t-test	1.545	7.172	26.27	43.36	15.05	0.9432
correct t-tests and Wald test	correct signs	10.20	24.51	41.58	42.16	33.80	16.59
	t-test	1.318	5.661	22.01	33.56	10.34	0.5291

Table 10 gives the marginal benefit of running the second test: the percentage of bubbles found by test B given that test A has already found a bubble.

TABLE 10. Percentage of Times Test B agrees with a finding of a bubble by Test A[a]

A	**B**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
Hall and Sola	van Norden	10.94	21.00	51.61	79.00	27.36	3.071
van Norden	Hall and Sola	30.85	36.08	45.49	43.80	36.15	17.97

a. Agrees and finding of a bubble implies that for the van Norden test all three slope coefficients are statistically significant and that for the Hall and Sola test that the t-tests and the Wald test are statistically significant.

For a positive finding by the van Norden test, the Hall and Sola test is more likely to disagree rather than agree for all values of π. The van Norden is more likely to agree rather than disagree for a finding of bubble by the Hall and Sola test only for the cases where π is equal to 0.85 or 0.75.

5.0 Conclusions:

6.0 Bibliography

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