Cyclical Effects of the Composition of Government Purchases

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Abstract

This paper constructs a general equilibrium model with monopolistically competitive firms and endogenous markups where government spending consists of both consumption and investment goods. It is shown that when markups are countercyclical, an increase in the share of investment goods in total public expenditure, raises output, employment, and capital stock in the long-run leading to increases in welfare and productivity. However, this also raises the short run cyclical variability of the economy. In particular, variance of output and employment arising from technological and aggregate demand shocks increase as the long run share of government investment goes up implying a trade-off between greater long-run efficiency and higher short-run volatility.

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*The views expressed are those of the authors and do not necessarily represent those of the Fund. An earlier version of the paper was issued as a *Working Paper of the International Monetary Fund No. WP/97/19*. It is available at www.imf.org.

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1. Introduction

Since the mid-1980s, many of the OECD countries have undertaken significant budgetary changes. Much of the subsequent analyses on the implications of these changes have focussed primarily on the effects of reducing the levels of overall expenditure fiscal deficits. However, along with reductions in overall spending and fiscal deficits, in many of these countries the composition of government spending has also undergone substantial changes. In particular, over the last ten years or so, except in Japan, governments have generally shifted away from investment spending. While the shift has been most dramatic in the smaller OECD countries such as Australia, Ireland, and New Zealand, among many of the larger countries, such as France, Germany, and the United States, the compositional change has also been quite pronounced (see Figure 1). This paper focuses on the effects of compositional changes in government expenditure and shows how changes in the relative shares of public consumption and investment goods can affect both the long run efficiency as well as short run fluctuations of an economy even when public spending is assumed to be unproductive.

Using a model with horizontally differentiated goods and monopolistic competition we find that when markups are countercyclical, a reduction in the share of investment goods in total public expenditure, lowers output, employment, and capital stock in the steady state. While this leads to declines in welfare and productivity in the long run, it also lowers the short run cyclical variability of the economy. In particular, variance of output and employment arising from technological and aggregate demand shocks fall as the long run share of government investment goes down. Quantitative estimates using the model calibrated, to the postwar U.S. and other selected OECD countries, show that the effects of changing the composition of public spending can be quite significant. These effects are, however, reversed if markups are procyclical.

The link between the composition of public expenditure and growth has been the focus of endogenous growth models such as those of Barro (1990) and Devarajan et. al (1996), where a part or all of government expenditure is assumed to be directly productive. Some authors have also explored the impact of aggregate government purchases on short run output and employment when public spending is not directly productive. In perfectly competitive economies, an increase in government spending raises households' willingness to supply more labor in response to the corresponding fall in wealth arising from financing additional government spending and through an increase in real interest rate induced by the higher government outlay (see, for example, Aiyagari et. al (1989) and the references therein). In a model of oligopolistic price setting, Rotemberg and Woodford (1992) showed that higher government spending can result in increased output and employment without any shift in households' consumption-leisure choices. However, these studies have concentrated on the effect of aggregate public spending rather than on its composition.

The economy we consider is one where final goods have two different end uses — consumption and investment. Under the assumption that firms cannot discriminate between the two sets of buyers, the price elasticity of aggregate demand is given by the average of the elasticities of consumption and investment, weighted by the shares of the two end uses in total demand. This implies that the composition of aggregate demand will determine the final price elasticity faced by the producers of the commodity. In a world where firms have some market power, the markup charged by a firm over its marginal cost is then determined by this average elasticity. Consequently, whenever the composition of the demand changes between the two end uses so does the elasticity and in turn the markup. Since decisions regarding the use of different inputs by firms depend on the markup, i.e., in equilibrium $MPK_t = \mu_t r_t$, where MPK_t is the marginal product of capital, r_t its rental price, and μ_t the markup, and $MPL_t = \mu_t w_t$, where MPL_t is the marginal product of labor and w_t the real wage rate, changes in the composition of aggregate output affect both investment and employment in the economy.

The model is based on a framework originally developed by Gali (1994). Goods are horizontally differentiated with each good being produced by a single firm in a monopolistic market The price elasticity of demand for each good is different for households — who use it for consumption, and firms — who use it for investment. In this environment, we show that the government can affect the efficiency and cyclical behavior of the economy by changing composition of aggregate demand by altering the composition of its own purchases. In order to isolate the effect of compositional changes in public expenditure on the economy, we assume throughout the analysis that government purchases neither enter households' utility nor the production process.

When markups are countercyclical, which is a largely accepted feature of the U.S. and OECD economies (Woodford and Rotemberg (1990), Bils (1987), Martins et. al (1996)), decreasing the share of public investment while keeping government expenditure unchanged, reduces output, employment, capital stock, and welfare in the steady state. In this model, for markups to be countercyclical, the price elasticity of investment needs to be larger than that of consumption. When this happens, an decrease in the share of investment lower the average elasticity, raises markups charged by profit maximizing firms, leading to lower output, employment, investment, and finally welfare. Moreover, the rise in the markup pushes the economy farther away from the perfectly competitive equilibrium such that there are efficiency losses, in terms of both labor and total factor productivity. However, if the elasticity of investment is lower than that of consumption so that markups become procyclical, decreasing the share of public investment has the opposite effect.

Although, when the price elasticity of investment is larger than that of consumption, decreases in steady state public investment lowers steady state output and welfare by increasing the markup, the reduction in elasticity which leads to the rise in the markup, also makes the aggregate demand less elastic. Consequently, both exogenous demand and supply shocks lead to a lower adjustment in quantity rather than prices, so that output and employment variability are also reduced. Consequently, as the economy is moved farther away from the perfectly competitive allocation, the cyclical variability is lowered. With lower cyclical variations, depending on the level of risk aversion, the increase in welfare can overcompensate the losses in welfare arising from the lower efficiency so that decreases in public investment can increase overall welfare. Thus, changes in the long run composition of public expenditure entail a trade-off between lower long run efficiency and lesser short run volatility.

The mechanism by which shocks affect markups in this model differs significantly from the processes relied upon in Basu (1995) and Rotemberg and Woodford (1992). In Basu, each producer faces a state dependent "menu cost" for changing prices. This generates price rigidity over some range of aggregate shocks such that markups become countercyclical. Rotemberg and Woodford, uses a structure based on Rotemberg and Saloner (1986) where firms within an oligopoly collude to keep prices above marginal cost with the collusion being supported by the threat to revert to the competitive price in future in response to any deviation. An exogenous increase in demand raises the gains from undercutting the industry set price. To prevent a breakdown of the collusion, the industry reverts to a lower price and hence to a lower markup. In contrast to these mechanisms, in our model, all prices are flexible and there is no collusion. The importance of markup variations in understanding business cycle fluctuations dates back to Kalecki (1939) and Keynes (1938). And in this paper, we add to this literature by showing how permanent changes in the composition of public expenditure can affect the cyclical response of the economy by changing the elasticity of aggregate demand. Although, Bils (1989) provides a setup where the price elasticity changes in response to demand shocks, the paper does not make a persuasive case as to why this can occur. In contrast, in our model this link is made explicit.

Among others, Hall (1988) and Evans (1992) have shown that empirically productivity shocks can be correlated with variables, such as changes to government expenditure and monetary impulses, although in principle these innovations should be orthogonal to the technology. We show that, government expenditure shocks can be positively correlated to productivity (as in Basu 1995), measured as the standard Solow residual, when markups are countercyclical. As a result, the standard measures of the Solow residual will tend to overestimate the contribution of technological shocks. It is also shown that when markups are countercyclical if the average elasticity is sufficiently large, then increases in public investment *crowds in* private investment. As public investment is not used in production, the crowding in of private investment occurs because of gains in efficiency, which increases the marginal revenue product of capital despite a fall in the markup.

The original Gali (1994) framework was constructed to show how in the absence of exogenous shocks, self-fulfilling revisions of expectations or sunspots can generate cyclical fluctuations that are similar to those in real economies. Although this property of the model is retained in this paper, we find that in light of the more recent evidence on econometric estimates of markups it is difficult to justify sunspot equilibria to be reasonable mechanisms by which cyclical fluctuations occur. In particular, when markups are procyclical, the existence of sunspots requires that, at the steady state, the markup has to be larger than 2. Recent studies such as those by Domowitz et. al (1988), Morrison (1990), and Martins et. al (1996) typically estimate markups to be less than 1.7.

The rest of the paper is organized as follows. Section II presents the environment including the assumptions made with respect to the behavior of government expenditure. Section III characterizes the equilibrium while Section IV discusses the properties of the steady state. In Section V the long run effects of changing the composition of public spending is discussed. Issues involving the plausibility of sunspot equilibria are discussed in section VI. Section VII shows the effects of changing the composition of government expenditure on the economy when exogenous demand and technological shocks are present. Section VIII concludes the paper.

2. The Model

2.1. Households

We assume that there is a large number of identical consumers denoted by $i = 1, 2, ...N$ whose preferences are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t^i) - v(l_t^i) \right]$$

where E_0 is the expectation operator at time $t = 0$ and $\beta \in [0, 1]$ is the discount factor. The utility $u(c_t^i)$ derived by consumer i at time t is given by

$$u(c_t^i) = M^{\frac{1}{1-\sigma}} \left[\sum_{s=1}^{M} \left(c_{st}^i \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 0$ and c_{st}^{i} is the quantity of good s consumed by i at t . This is a constant elasticity of substitution utility function used previously in the literature on monopolistic competition (Spence 1976, Dixit and Stiglitz 1977). The disutility from working l_t^i hours is given by

$$v(l_t^i) = \frac{\zeta}{1+\zeta}\left(l_t^i\right)^{\frac{1+\zeta}{\zeta}}.$$

where $\zeta > 0$. Normalizing each period wage to unity, denoting the total income of consumer i prior to paying taxes as a_t^i and the lump-sum tax as T_t^i, letting p_t^s denote the price paid by consumers for a unit of good s, and using P_t as the consumer price index corresponding to the composite consumption good purchased by the consumers which is defined as

$$P_t = \left[\frac{1}{M}\left(\sum_{s=1}^{M}(p_t^s)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\right]$$

the following are the consumption and labor supply functions (derived in the appendix) of the ith household:

$$c_{st}^{i} = \left(\frac{p_t^s}{P_t}\right)^{-\sigma}\left(\frac{a_t^i - T_t^i}{P_t M}\right),\ s = 1,2,...M, \forall\ i = 1,2,...N$$
$$l_t^i = P_t^{-\zeta},\ \forall\ i = 1,2,...N \tag{2.1}$$

2.2. Firms

There are M firms, each producing a horizontally differentiated commodity indexed by $j = 1,2,...M$. Without loss of generality assume that $N = M$. The technology to produce these goods is identical across firms and requires both labor and capital with a Cobb-Douglas production function given by

$$y_t^j = z_t\left(k_t^j\right)^{\alpha}\left(l_t^j\right)^{1-\alpha} \tag{2.2}$$

with $0 < \alpha < 1$ and where y_t^j, k_t^j, and l_t^j, denote the jth firm's output, capital stock and labor input respectively. z_t denotes the productivity shock and follows

$$z_{t+1} = \phi^z z_t + \varepsilon_{t+1}^z$$

where $\phi^z < 1$, measures the persistence in the shock and ε_t^z is an $i.i.d.$ disturbance with mean $\overline{\varepsilon}$ and a finite positive support $[\varepsilon^-, \varepsilon^+]$. Capital accumulation requires the use of all M commodities and the law of motion of capital is given by the following equation

$$k_{t+1}^j = (1-\delta)k_t^j + f(i_t^j), \tag{2.3}$$

where

$$f(i_t^j) = \left(\frac{1}{M}\right)^{\frac{1}{\eta-1}}\left[\sum_{h=1}^{M}\left(i_{ht}^j\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.4}$$

$\eta > 0$ and where $\delta \in [0,1]$ represents the rate of depreciation. Thus the capital stock of each firm is increased by purchasing goods from the other firms that are mixed together (through the same function for all firms) to produce a composite investment good $f(i_t^j)$. Basu (1995) uses a technology that is similar in some respects. There output is produced using a constant returns to scale technology in labor and intermediate goods, $Q_t^j = \left(L_t^j\right)^\alpha \left(I_t^j\right)^{1-\alpha}$, where $I_t^j = \left(\int_0^1 I_{ki}^{1-\phi} dk\right)^{\frac{1}{1-\phi}}$. However, ϕ which measures the elasticity of substitution among different goods in production is also the same among the goods in consumption so that the price elasticities of firms and households are identical. Consequently, in Basu the transmission mechanism by which shocks filter through to firms is different from the process relied upon in this model.

If we denote, as above, by p_t^h, $h = 1,2,...M$, the price of good h, the cost of the composite investment good purchased at each period (i.e. the total investment expenditure of the firm) is given by $\nu_t^j = \sum_{h=1}^M p_t^h i_{ht}^j$. Implicitly, this framework requires that firms are not able to discriminate between types of buyers. When a buyer purchases one unit of the good sold by a firm, that buyer can be a consumer purchasing a part of his consumption composite, a firm purchasing a part of its investment composite, or the government buying either. We assume here that in none of these cases can the firm discriminate and extract a different margin.

As generally followed in this literature, the firm's decision problem is solved in two steps. In the first step, for a given level of capital accumulated the firm solves the *static* allocation problem of choosing the optimal mix among M commodities to produce the intermediate capital goods and the number of labor hours and then in the second step, using this solution, the *intertemporal* problem of accumulating capital is addressed. This results in the following decision rules for the firm:

$$\begin{gathered} i_{ht}^j = \left(\frac{p_t^h}{\Pi_t}\right)^{-\eta} \left(\frac{\nu_t^j}{\Pi_t M}\right),\ h = 1,2,...M \\ f(i_t^j) = \frac{\nu_t^j}{\Pi_t} \end{gathered} \tag{2.5}$$

determine the optimal choice among the M investment goods where

$$\Pi_t = \left[\left(\frac{1}{M}\right) \sum_{h=1}^{M} \left(p_t^h\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.6}$$

is the price index of the composite investment good.

$$p_t^j = \mu_t^j \omega_t^j \tag{2.7}$$

where $\mu_t^j = \frac{\xi_t^j}{\xi_t^j - 1}$ is the mark-up, ξ_t^j is the price elasticity of the jth firm's final product, and

$$\omega_t^j = \frac{1}{1-\alpha} z_t^{-\frac{1}{1-\alpha}} \left(\frac{y_t^j}{k_t^j}\right)^{\frac{\alpha}{1-\alpha}}$$

is the marginal cost. The optimal capital accumulation policy function, obtained by taking the first order condition in k_{t+1} is given by

$$\frac{\Pi_t}{P_t} = \beta E_t \frac{1}{P_{t+1}} \left[\frac{\alpha}{1-\alpha} z_{t+1}^{-\frac{1}{1-\alpha}} \left(\frac{y_{t+1}^j}{k_{t+1}^j}\right)^{\frac{1}{1-\alpha}} + (1-\delta)\Pi_{t+1}\right] \tag{2.8}$$

2.3. The Government

As discussed previously, we assume that the government collects lump sum taxes from consumers for a total of T_t, with $T_t = \sum_{i=1}^{N} T_t^i$. The government spends the entire amount on purchasing consumption and investment goods i.e., $G_t = T_t$. Once T_t is determined, the budget allocation is carried out in a two step process. First, the overall sectoral expenditure limits are determined. We denote them by $\theta_t G_t$ for public investment and $(1-\theta_t)G_t$ for public consumption, with $\theta_t \in [0,1]$. Alternatively, the budget constraint of the government can be written as

$$\begin{aligned} \sum p_t^s c_{gt}^s &\leq (1-\theta_t)G_t \\ \sum p_t^h i_{gt}^h &\leq \theta_t G_t \end{aligned} \tag{2.9}$$

Two assumptions regarding the behavior of government purchases are made next. First, $\theta_t = \theta$ for all t, and second we assume that for any fixed amount earmarked for public consumption (public investment), the government allocates the money among different goods exactly in the same way as it would be done by the private sector. This would in effect occur if the aggregator function for the government is a linear combination of $u(c_t^i)$ and $f(i_t^j)$ i.e., the government's allocation problem is

$$\max\ \omega M^{\frac{1}{1-\sigma}} \left[\sum_{s=1}^{M} \left(c_{st}^i\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + (1-\omega) \left(\frac{1}{M}\right)^{\frac{1}{\eta-1}} \left[\sum_{h=1}^{M} \left(i_{ht}^j\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

subject to

$$\sum p_t^s c_{gt}^s \leq (1-\theta_t)G_t$$

$$\sum p_t^h i_{gt}^h \leq \theta_t G_t$$

where θ_t and G_t is given and the weight $\omega \in [0,1]$[1]. The following two equations are obtained as the solution to the spending patterns of the government[2].

$$\begin{aligned} c_{gt}^s &= \left(\frac{p_t^s}{P_t}\right)^{-\sigma} \left(\frac{(1-\theta)G_t}{P_t}\right),\ s = 1,2,\ldots M \\ i_{gt}^h &= \left(\frac{p_t^h}{\Pi_t}\right)^{-\eta} \left(\frac{\theta G_t}{\Pi_t}\right),\ h = 1,2,\ldots M \end{aligned} \tag{2.10}$$

Finally, assume that government expenditure is stochastic in the sense that $g_t \equiv \frac{G_t}{Y_t} = (1-\phi^g)\bar{g} + \phi^g g_{t-1} + \varepsilon_t^g$, where ε_t^g has a finite support $[g^-, g^+]$ containing $\bar{g}$ and has an $i.i.d.$ distribution. This constitutes the demand shock to the system as opposed to z_t which is the supply shock.

3. Symmetric Market Equilibrium

The optimal decision rules derived in the previous two sections and the overall feasibility conditions, and the price indices describe the market equilibrium. However, before proceeding further

[1] In particular, note that if an overall budget constraint is only imposed, i.e., $\sum p_t^s c_{gt}^s + \sum p_t^h i_{gt}^h \leq G_t$, then $\theta_t = \omega$.

[2] Note that the elasticities of government's consumption goods and investment goods expenditures are the same as for individual agents i.e., the elasticities are σ and η.

with the characterization of the equilibrium it is useful to discuss the aggregate demand faced by firms. Note that the demand faced by the firm j comprising of the demand by consumers, other firms and the government is given by

$$y_t^j = \left(\frac{p_t^s}{P_t}\right)^{-\sigma}\left(\frac{a_t - G_t}{P_t}\right) + \left(\frac{p_t^h}{\Pi_t}\right)^{-\eta}\left(\frac{\nu_t}{\Pi_t}\right) + \left(\frac{p_t^s}{P_t}\right)^{-\sigma}\left(\frac{(1-\theta)\Upsilon_t}{P_t}\right) + \left(\frac{p_t^h}{\Pi_t}\right)^{-\eta}\left(\frac{\theta\Upsilon_t}{\Pi_t}\right)$$

$$y_t^j = \left(\frac{p_t^s}{P_t}\right)^{-\sigma}\left(\frac{a_t - \theta G_t}{P_t}\right) + \left(\frac{p_t^h}{\Pi_t}\right)^{-\eta}\left(\frac{\nu_t + \theta G_t}{\Pi_t}\right) \tag{3.1}$$

As is evident from (14), if $\theta = 0$, then aggregate demand is not affected at all by government expenditure.

Now let λ_t^j denote the share of investment demand. Then the price elasticity of the jth good is given by

$$\xi_t^j = (1-\lambda_t^j)\sigma + \lambda_t^j\eta \tag{3.2}$$

The price elasticity ξ_t^j is a weighted sum of the consumers and investors elasticities. And therefore, the markup will depend on the weight, λ_t^j. Furthermore, note that if the two elasticities are the same then $\mu_t = \sigma$, a constant.

Focusing our attention to only symmetric equilibria where all firms produce the same quantity and use the same amount of inputs. Moreover, all consumers consume and save the same amount. Furthermore, since firms cannot discriminate between consumers and investors price charged for any particular commodity will be the same, regardless of its end use. As a result $P_t = \Pi_t$. With the assumption of symmetry, all firm and household specific indices are dropped.

The demand for labor by firms can be written as $l_t = \left(\frac{x_t}{z_t}\right)^{\frac{1}{1-\alpha}} k_t$ where, $x_t \equiv \frac{y_t}{k_t}$, the output-capital ratio. Using this and equation (11) one derives the P_t as

$$P_t = \left(\frac{x_t}{z_t}\right)^{-\frac{1}{\zeta(1-\alpha)}} k_t^{-\frac{1}{\zeta}} \tag{3.3}$$

The real wage in the economy w_t is then given by

$$w_t = \frac{1}{P_t}$$

Replacing (16) in (8) provides a closed form expression for the output-capital ratio

$$x_t = A_x z_t^{\frac{1+\zeta}{1+\alpha\zeta}} \mu_t^{-\frac{\zeta(1-\alpha)}{1+\alpha\zeta}} k_t^{-\frac{(1-\alpha)}{1+\alpha\zeta}},\ A_x = (1-\alpha)^{\frac{\zeta(1-\alpha)}{1+\alpha\zeta}} \tag{3.4}$$

Using (17) one can subsequently derive

$$y_t = A_x z_t^{\frac{1+\zeta}{1+\alpha\zeta}} \mu_t^{\frac{-\zeta(1-\alpha)}{1+\alpha\zeta}} k_t^{\frac{\alpha(1+\zeta)}{1+\alpha\zeta}} \tag{3.5}$$

$$l_t = A_l z_t^{\frac{\zeta}{1+\alpha\zeta}} \mu_t^{\frac{-\zeta}{1+\alpha\zeta}} k_t^{\frac{\alpha\zeta}{1+\alpha\zeta}},\ A_l = (1-\alpha)^{\frac{\zeta}{1+\alpha\zeta}} \tag{3.6}$$

Note that both output and labor are functions of capital stock k_t and λ_t. One can use (18) to find solutions for consumption, c_t, and investment, i_t. However, λ_t is an endogenous variable and needs to be solved. We next turn out attention to solving for λ_t. To do so, consider equation (11). Under the assumption that $P_t = \Pi_t$, (10.6) becomes

$$\rho+\delta = E_t \frac{1}{P_t}\left[\frac{\alpha}{1-\alpha} z_{t+1}^{-\frac{1}{1-\alpha}} \left(\frac{y_{t+1}}{k_{t+1}}\right)^{\frac{1}{1-\alpha}}\right] \qquad \rho = \frac{1}{\beta}-1 \tag{3.7}$$

Using the fact that at period t, k_{t+1} is known the above can be written as

$$k_{t+1} = A_k \left(E_t\left[\left(\frac{z_{t+1}}{\mu_{t+1}}\right)^{\frac{1+\zeta}{1+\alpha\zeta}}\right]\right)^{\frac{1+\alpha\zeta}{1-\alpha}} \qquad A_k = \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1+\alpha\zeta}{1-\alpha}} (1-\alpha)^{\zeta} \tag{3.8}$$

Note that μ_{t+1} is a function of λ_{t+1}. Therefore, if λ_{t+1} is perceived to increase and $\mu^{'}(.) \neq 0$, then private investors will increase or decrease k_{t+1}. If, for example, $\mu^{'}(.) < 0$, then k_{t+1} will increase. However, this will imply that λ_t will also rise. The law of motion of capital is given by (2.3)

$$k_{t+1} = \left[(1-\delta)+(\lambda_t-\theta g_t)x_t\right]k_t \tag{3.9}$$

Rearranging the above yields,

$$(\lambda_t-\theta g_t)x_t = \frac{k_{t+1}}{k_t}-(1-\delta)$$

equivalently,

$$(\lambda_t-\theta g_t)x_t = \frac{A_k}{k_t}\left(E_t\left[\left(\frac{z_{t+1}}{\mu_{t+1}}\right)^{\frac{1+\zeta}{1+\alpha\zeta}}\right]\right)^{\frac{1+\alpha\zeta}{1-\alpha}} - (1-\delta) \tag{3.10}$$

Using the derived first order conditions a symmetric market equilibrium for this economy can be defined as:

Given $\{k_0, z_0\}$*, a sequence of* $\{z_t, g_t\}$*, and government policy described by* $\{\theta, \tau_t\}$*, asymmetric equilibrium for this economy is a sequence* $\{y_t, c_t, l_t, k_{t+1}, \lambda_t, \mu_t, P_t\}$ *such that:*

(1) $x_t = A_x z_t^{\frac{1+\varsigma}{1+\alpha\varsigma}} \mu_t^{-\frac{\varsigma(1-\alpha)}{1+\alpha\varsigma}} k_t^{-\frac{(1-\alpha)}{1+\alpha\varsigma}}$

(2) $y_t = A_x z_t^{\frac{1+\varsigma}{1+\alpha\varsigma}} \mu_t^{-\frac{\varsigma(1-\alpha)}{1+\alpha\varsigma}} k_t^{\frac{\alpha(1+\varsigma)}{1+\alpha\varsigma}}$

(3) $l_t = A_l z_t^{\frac{\varsigma}{1+\alpha\varsigma}} \mu_t^{-\frac{\varsigma}{1+\alpha\varsigma}} k_t^{-\frac{\alpha\varsigma}{1+\alpha\varsigma}}$

(4) $P_t = \left(\frac{x_t}{z_t}\right)^{-\frac{1}{\varsigma(1-\alpha)}} k_t^{-\frac{1}{\varsigma}}$

(5) $\varpi_t = \frac{1}{P_t}$

(6) $\mu_t = \frac{\varepsilon_t}{\varepsilon_t - 1}$

(7) $\xi_t = (1-\lambda_t)\sigma + \lambda_t \eta$

(8) $\tau_t = \frac{\Upsilon_t}{y_t}$

(9) $(\lambda_t - \theta g_t)x_t = \frac{k_{t+1}}{k_t} - (1-\delta)$

(10) $k_{t+1} = A_k \left(E_t \left[\left(\frac{z_{t+1}}{\mu_{t+1}} \right)^{\frac{1+\varsigma}{1+\alpha\varsigma}} \right] \right)^{\frac{1+\alpha\varsigma}{1-\alpha}}$

(11) $g_t = \tau_t$

4. Steady State and Long Run Policy

Conditions $(1)-(11)$ completely characterize the competitive equilibrium. The only intertemporal equations are (9)-(11). Assuming that $\tau_t = \tau$ and $\overline{z}$ and $\overline{g}$ are the mean values of z_t and g_t and letting $\psi \equiv \theta\overline{g}$, in a *deterministic steady state* i.e., $z_t = \overline{z}$ and $g_t = \overline{g}$, the steady state values of capital, share of investment, and mark-up are given by

$$(\overline{\lambda} - \psi)\overline{\mu} = \frac{\alpha\delta}{\rho+\delta} \tag{4.1}$$

$$\overline{k} = A_k \overline{z}^{\frac{1+\zeta}{1-\alpha}} \left(\frac{1}{\overline{\mu}} \right)^{\frac{1+\zeta}{1-\alpha}} \tag{4.2}$$

$$\overline{\mu} = \left(\frac{(\eta-\sigma)\overline{\lambda} + \sigma}{(\eta-\sigma)\lambda - (1-\sigma)} \right) \tag{4.3}$$

Note that from (4.3) it follows that when $\frac{\partial\mu}{\partial\lambda} > 0$ as long as $\sigma > \rho$. When $\lambda = \psi$, the *RHS* of (4.1) is 0 and monotonically increasing if $\sigma > \rho$. Therefore, as long as $\frac{\alpha\delta}{\rho+\delta} < (1-\psi)\left(\frac{\eta}{\eta-1}\right)$ there will exist some $0 \leq \lambda \leq 1$ for which (4.1) is satisfied and a steady state exists. In particular, given the monotonicity of $(\lambda - \psi)\overline{\mu}$ the steady state will also be unique. However, if $\sigma < \rho$ then $\frac{\partial\mu}{\partial\lambda} < 0$ and in which case multiple steady states can exist (Gali, 1994).

From (4.2) it follows that when $\overline{\mu}$ is high, $\overline{k}$ is low. Since steady state output is given by

$$\overline{y} = A_x \overline{z}^{\frac{1+\zeta}{1+\alpha\zeta}} \overline{\mu}^{\frac{-\zeta(1-\alpha)}{1+\alpha\zeta}} \overline{k}^{\frac{\alpha(1+\zeta)}{1+\alpha\zeta}}$$

a fall in the steady state markup unambiguously increases steady state output. Similarly, when the markup falls both employment,

$$\overline{l} = A_l \overline{z}^{\frac{\zeta}{1+\alpha\zeta}} \overline{\mu}^{\frac{-\zeta}{1+\alpha\zeta}} \overline{k}^{\frac{\alpha\zeta}{1+\alpha\zeta}}$$

and labor productivity,

$$\frac{\overline{y}}{\overline{l}} = const \times \overline{z}^{\frac{1}{1-\alpha}} \ \overline{\mu}^{-\frac{\alpha}{1-\alpha}}$$

also increase.

Recall that $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$ or equivalently, $\ln y_t = \ln z_t + \alpha \ln k_t + (1-\alpha)\ln l_t$. One method of computing total factor productivity, TFP, is to define $TFP_t = \ln y_t - \alpha \ln k_t - (1-\alpha)\ln l_t$. In particular, this formulation has been used to derive the contribution of productivity shock z_t in growth and cyclical variability. In computing the total factor productivity, following Solow (1957), researchers have used the share of labor income in national income to approximate $(1-\alpha)$. In the presence of perfect competition the share of labor income approximates $(1-\alpha)$ quite well. However, under imperfect competition i.e., $\mu > 1$, the share of labor income is $\frac{1-\alpha}{\mu}$ and that of capital $\frac{\alpha}{\mu}$. If these shares are used to compute productivity then

$$\frac{\partial TFP}{\partial \mu} = \frac{\partial \ln y}{\partial \mu} - \left(\frac{\alpha}{\mu}\right)\frac{\partial \ln k}{\partial \mu} - \left(\frac{1-\alpha}{\mu}\right)\frac{\partial \ln l}{\partial \mu}$$

It turns out that

$$\frac{\partial TFP}{\partial \mu} = -\left(\zeta(1-\alpha)^2 + \alpha(1+\zeta)^2\right)\left(1-\frac{1}{\mu}\right) \tag{4.4}$$

For $\mu > 1$, $\frac{\partial TFP}{\partial \mu} < 0$ rather than 0 if α, the true labor elasticity, was used. Consequently, if the steady state markup increases, not only does output increase as shown before but also total factor productivity. This, however, should not be surprising. The presence of the markup causes the equilibrium to be suboptimal in the first place so that when the markup falls there are efficiency gains reflected in the increase in total factor productivity. But, more importantly, it also implies that factors that can change the markup, such as changes in government expenditure (of a permanent kind for the steady state analysis) will be correlated with the Solow residual, i.e., the measured factor productivity. In particular, since $\mu = \left(1-\frac{1}{\xi}\right)^{-1}$, where $\xi = (\eta-\sigma)\lambda + \sigma$, if λ increases and $\eta - \sigma > 0$, then μ falls as a higher elasticity implies a lower markup as predicted by standard microeconomic theory. If $\eta - \sigma < 0$ the opposite holds true. We summarize these findings in the following proposition.

Proposition 4.1. *A decrease in the steady state markup is associated with higher steady state levels of capital stock, employment and output and also higher labor and total factor productivity. Moreover, if $(\eta-\sigma)$ is positive(negative) then the share of investment in output is positively(negatively) correlated with total factor productivity in the long run.*

4.1. Public Expenditure Policy — The Long Run Effects

Proposition 1 makes it clear that changes in the equilibrium markup can have important consequences for the economy in general. In this section we look at a particular way of changing the markup, namely by altering the share of public investment, ψ. To see how government policies reflected in the composition of public expenditure can affect the markup of producers consider the expression for the weighted elasticity $\xi_t = (1-\lambda_t)\sigma + \lambda_t\eta$. Denote by λ_t^p, the share of investment in total private spending, then $\lambda_t = (1-g_t)\lambda_t^p + \theta_t g_t$ and

$$\xi_t = (\eta-\sigma)\left[(1-g_t)\lambda_t^p + \theta_t g_t)\right] + \sigma$$

Government expenditure, g_t, by itself can only have a second-order effect on the elasticity since if $\theta_t = 0$ then $\xi_t = (\eta - \sigma)(1 - g_t)\lambda_t^p + \sigma$ and g_t can have an effect only if λ_t^p changes, i.e., if changes in government spending affects the way private agents allocate their income between consumption and investment. In this model since investment is determined by firms, who care only for the overall breakdown between investment and consumption, and not who the end-users are (by assumption all end-users for the same functional purposes have the same elasticity) changes in g_t will be completely offset by changes in λ_t^p such that $(1 - g_t)\lambda_t^p$ remains constant. If in contrast, households decided on the level of investment then λ_t^p would probably not move enough to completely offset the change in g_t such that $(1 - g_t)\lambda_t^p$ would change leading to a change in the elasticity. In particular it would depend on how government expenditure changes both disposable income of households and the interest rate in the economy and the reaction of private investment to such changes. However, if $\theta_t \neq 0$ then changes in both θ_t and g_t matter in equilibrium. When g_t or θ_t is altered, the overall composition of output changes causing the elasticity to change which in turn leads to a change in the markup.

In the steady state, $(\bar{\lambda} - \psi)\bar{\mu} = \frac{\alpha\delta}{\rho+\delta}$ and $\bar{\mu} = \left(\frac{(\eta-\sigma)\bar{\lambda}+\sigma}{(\eta-\sigma)\bar{\lambda}-(1-\sigma)}\right)$. Substituting the second equation in the first we have $(\lambda - \psi)\left(\frac{(\eta-\sigma)\lambda+\sigma}{(\eta-\sigma)\lambda-(1-\sigma)}\right) - \frac{\alpha\delta}{\rho+\delta} = 0$. Since $\frac{(\eta-\sigma)\bar{\lambda}+\sigma}{(\eta-\sigma)\bar{\lambda}-(1-\sigma)} \neq 0$ for $0 \leq \lambda \leq 1$ and $\eta \geq 0$, the steady state relationship between λ and ψ can be expressed as $\lambda = f(\psi)$.

Lemma 4.2. *If $\eta < \sigma$ then $0 < \frac{df}{d\psi} < 1$. Otherwise, $\frac{df}{d\psi} > 1$ if $\xi(\xi - 1) > (\lambda - \psi)(\eta - \sigma)$ and $\frac{df}{d\psi} < 0$ if $\xi(\xi - 1) < (\lambda - \psi)(\eta - \sigma)$.*

Proof. Follows directly from differentiating $(f(\psi) - \psi)\left(\frac{(\eta-\sigma)f(\psi)+\sigma}{(\eta-\sigma)f(\psi)-(1-\sigma)}\right) - \frac{\alpha\delta}{\rho+\delta} = 0$. ■
From the above lemma it follows that when $\eta < \sigma$, if public investment increases, although share of total investment goes up — $\frac{d\lambda}{d\psi} > 0$ — the increase is less than the amount by which public investment is raised — $\frac{d\lambda}{d\psi} < 1$. This implies that the share of private investment must fall. Since λ increases when ψ is raised and $\eta - \sigma < 0$, it follows from $\xi = (\eta - \sigma)\lambda + \sigma$ and $\mu = \left(1 - \frac{1}{\xi}\right)^{-1}$ that the markup also increases. With μ being raised steady state output falls (Proposition 1) so that not only does the share of private investment decrease but also its level. Permanent increases in public investment, in this case, *crowds out* private investment. However, if $\eta - \sigma > 0$ and the price elasticity of aggregate demand is sufficiently high, i.e., $\xi(\xi - 1) > (\lambda - \psi)(\eta - \sigma)$, not only does the share of total investment increase but also induces an increase in the share of private investment — $\frac{d\lambda}{d\psi} > 1$. Given that $\eta - \sigma > 0$, the steady state markup falls and output also increases (again due to Proposition 1) so that the level of private investment also increases. In contrast to the previous case, public investment *crowds in* private investment. Since increasing θ or g while keeping the other unchanged in equivalent to increasing ψ we have the following proposition.

Proposition 4.3. *If θ is increased keeping g constant or g is increased keeping θ constant and if $\eta - \sigma > 0$ and $\xi(\xi - 1) > (\lambda - \psi)(\eta - \sigma)$ then the steady state levels of output, private investment, and employment are increased along with labor productivity and total factor productivity. The opposite occurs if $\eta - \sigma < 0$.*

Proof. Follows from proposition 1 and lemma 1. ■

The long run effects of changing government expenditure and its composition depend on whether investor elasticity is larger than consumer elasticity and the weighted average of the two elasticities — the elasticity of aggregate demand. As we argue later in the paper, it is more likely that $\eta - \sigma > 0$ for the post war U.S. and other OECD economies. In which case, increasing the share of investment in government expenditure does in fact increase output, employment and productivity. The same effects are also obtained if the share of government spending is increased keeping its composition unchanged. To ascertain the short run effects we need to compute the temporal equilibria of the economy. However, given the nature of the environment closed form solutions cannot be obtained. To get around the problem, we construct a linear approximation to the equilibria and study the short run impact of government spending using the approximate solution.

5. Sunspot Equilibria

In a recent paper, Gali (1994) used this environment to show how equilibria that are driven by self-fulfilling revisions of expectations can generate time series with properties that are similar to those observed in U.S. postwar business cycles. The belief that sunspot equilibria can generate empirically reasonable time series properties have been shared by others such as Farmer and Guo (1994) and Woodford (1991) among others. However, unlike in Gali, the other models rely on some form of increasing returns or externalities to deliver the sunspot equilibria. Although Gali's approach is appealing since it eliminates the need to rely on exogenous shocks to generate cycles, we find that for the U.S. postwar economy it is not a plausible mechanism to use in studying cyclical fluctuations.

5.1. Existence of sunspots

To see how sunspot equilibria may be generated in this economy and why they are not plausible we turn our attention to studying the model's cyclical behavior. First, equilibrium conditions $(1), (6), (9)$, and (10) are linearized around the steady state, implicitly given by (4.1) and (4.2) to obtain approximate closed form solutions. Let $\overline{x} = \{\overline{\lambda}, \overline{k}, \overline{z}\}$ denote the steady state of the system. The linearized equations are

$$\hat{x}_t = a_1 \hat{z}_t - a_2 \hat{\mu}_t - a_3 \hat{k}_t \tag{5.1}$$

$$\hat{\mu}_t = b_1 \hat{\lambda}_t \tag{5.2}$$

$$\hat{\lambda}_t = \frac{1}{c_1} \hat{k}_{t+1} - \frac{1}{c_1} \hat{k}_t - \frac{\delta}{c_1} \hat{x}_t + c_2 \hat{g}_t \tag{5.3}$$

$$\hat{k}_{t+1} = d_1 E_t \left[\hat{z}_{t+1} - \hat{\mu}_{t+1} \right] \tag{5.4}$$

where the coefficients are the partial derivatives evaluated at their steady state values (*see the appendix for details*). By using (5.1) , (5.2) , (5.3), (5.4) becomes

$$\hat{k}_{t+1} = E_t \left[e_1 \hat{z}_{t+1} + e_2 \hat{k}_{t+2} + e_3 \hat{g}_{t+1} \right] \tag{5.5}$$

Now suppose $| e_2 | > 1$. Consider first the perfect foresight equilibrium i.e., $\hat{z}_{t+1} = 0$ and $\hat{g}_t = 0$ $\forall t = 0, \ldots \infty$. The first equation in this system is $\hat{k}_1 = e_2 \hat{k}_2$. Consequently, there is no restriction on k_1 imposed by the initial capital stock k_0 i.e., the system of equations, $\hat{k}_{t+1} = e_2 \hat{k}_{t+2}$ $\forall t = 0, \ldots \infty$, are unrestricted by the initial stock of capital. There is an unique steady state given by $\hat{k}_{t+1} = 0$ $\forall t = 0, \ldots \infty$. Moreover, the perfect foresight equilibrium is stable. Any sequence of $\left\{ \hat{k}_{t+1} \right\}$

starting from an arbitrary k_1 will converge to the steady state $\hat{k}_{t+1} = 0$. Since, k_0 does not restrict the choice of k_1, there is an indeterminate number of perfect foresight equilibria. However, of these equilibria there is one that is stationary namely, $\left\{\hat{k}_{t+1}\right\} = 0\ \forall t = 0,\ldots\infty$ which is the steady state equilibrium. But this is not the only stationary equilibrium. Around a sufficiently small neighborhood of the steady state there may be more than one such equilibria (Woodford 1991, Gali 1994). To see why the steady state is not the only stationary equilibrium note that $E_t\left[\hat{k}_{t+1} - e_2\hat{k}_{t+2}\right] = 0$ is the version of (5.5) when there is no intrinsic uncertainty ($\varepsilon_t^z = 0$ and $\varepsilon_t^g = 0$) which is the optimal capital accumulation rule followed by investors. Lagged one period, the rule is given by $E_{t-1}\left(\hat{k}_{t+1} - \frac{1}{e_2}\hat{k}_t\right) = 0$. Now suppose investors' believe that investment follows $\hat{k}_{t+1} = \frac{1}{e_2}\hat{k}_t + s_t$, where the random variable s_t is a realization of an $i.i.d.$ probability distribution with $E_{t-1}s_t = 0$ and described over a closed support $[a,b]$ containing the steady state. Although this rule still implies that $E_{t-1}\left(\hat{k}_{t+1} - \frac{1}{e_2}\hat{k}_t\right) = 0$ which is consistent with the optimal policy (5.5), the extrinsic uncertainty s_t can generate investment paths that depend solely on it. If $\mid e_2 \mid < 1$ then the solution to $\hat{k}_{t+1} = \frac{1}{e_2}\hat{k}_t + s_t$ is given by $\hat{k}_{t+1} = E_t\sum_{j=0}(e_2)^j s_{t+j}\ \forall t = 1,\ldots\infty$. As $E_t s_{t+j} = 0$, $\hat{k}_{t+1} = 0\ \forall t = 1,\ldots\infty$ is the unique stationary solution. But if $\mid e_2 \mid > 1$, then the solution is

$$\hat{k}_{t+1} = \sum_{j=0}\left(\frac{1}{e_2}\right)^j s_{t-j}\ \forall t = 0,\ldots\infty \tag{5.6}$$

with $\hat{k}_1 = e_2 s_0$ and the equilibrium path of capital stock is determined entirely by current and past realizations of the sunspot variable s_t[3]. However, if agents do not believe that the sunspot matters for the capital stock i.e., $\hat{k}_{t+1} \neq \frac{1}{e_2}\hat{k}_t + s_t$ but $\hat{k}_{t+1} = \frac{1}{e_2}\hat{k}_t$, then the only stationary equilibrium possible is the one where $\{\hat{k}_{t+1}\} = 0\ \forall t = 0,\ldots\infty$.

Since the existence of sunspots depend on the parameter e_2 we next provide its characterization.

Lemma 5.1. $\mid e_2 \mid < 1$ *if and only if* $1 - \frac{\delta(1-\alpha)}{1+\zeta}\left[1 + \left(\frac{\bar{\lambda}}{\bar{\lambda}-\bar{\psi}}\right)\frac{1}{\varepsilon_\mu}\right] > 1$[4].

Proof. See appendix. ■

While the theoretical possibility of sunspots existing have been established it still remains to be shown whether empirically such equilibria should be considered seriously. To this end, in the next section we use a suitably parameterized economy to provide the answer.

5.2. The Likelihood of Sunspots

In this section, parameters of the model are calibrated to be compatible with the post-war U.S. economy using long-run and micro-level data and econometric studies. Following, Kydland

[3]The reason why such sunspot equilibria can exist is identical to that in Woodford (1991). Without loss of generality assume that in period t the realization of the sunspot is a positive s_t. Furthermore, suppose all investors believe that the choice of $\hat{k}_{t+2}$ will follow the rule $\hat{k}_{t+2} = \sum_{j=0}\left(\frac{1}{e_2}\right)^j s_{t-j+1}$. Then the positive realization of s_t increases the expected value of $E_t\hat{k}_{t+2}$. However, this also implies that investment in $t+1$ will need to be higher such that $E_t\hat{y}_{t+1}$ will be increased. In order to support a higher equilibrium output in $t+1$ investors will then, as an optimal response, increase their desired capital stock in period t i.e., $\hat{k}_{t+1}$. Consequently, the seemingly inconsistent behavior of $\hat{k}_{t+1} = \frac{1}{e_2}\hat{k}_t + s_t$ instead of $\hat{k}_{t+1} = \frac{1}{e_2}\hat{k}_t$ – as determined by the fundamentals of the environment – is justified.

[4]This lemma is identical to the result in Gali (1994) except that it is modified to include ψ.

and Prescott (1990) set $\bar{\lambda} = 0.21$, the share of investment in aggregate output. Depending on whether investment in defense equipment is included $\bar{\psi} = 0.023$ or 0.03. The depreciation rate is set at $\delta = 0.016$ per quarter such that the steady state capital-output ratio is around 10. This depreciation rate is different from 0.025 which is normally used approximating an annual rate of 10 percent. Using the average risk-free interest rate per quarter in the postwar U.S. economy $\rho = 0.01$ or $\beta = 0.99$. Typically, equilibrium business cycle models assume extremely large labor supply elasticities. However, estimated elasticities have been in general very low (close to zero for adult white males) and have rarely been over 2 (Killingsworth and Heckman 1986). In this model $\varsigma = 1$. This leaves behind unresolved the values for μ, α, η and σ. Note that the steady state condition $(\bar{\lambda} - \psi)\mu = \frac{\alpha\delta}{\rho+\delta}$ implies that for any value of μ there is an unique α consistent with it.

Using industry data, Hall (1987, 1989) estimated μ of over 1.8 for all the one-digit industries. Subsequent work by Domowitz, Hubbard and Petersen (1988) and Morrison (1990) have shown that estimates of markups that range from 1.2 to 1.7 are more reasonable. This range of values for μ seem more reasonable also on the ground that for the postwar U.S. economy profit margins have been rather small. More recently, Martins, Scarpetta and Pilat (1996) estimated the average markup for the U.S. economy to be around 1.15. On sectoral basis, they found that markups for most industries were between 1.05 and 1.54. However, Gali (1994) assumes a steady state value of $\mu \in [2.0, 3.03]$ using evidence from Hall (1988). These two ranges i.e., $[2.0, 3.03]$ and $[1.15, 1.7]$ have significant behavioral differences. To see these differences in a more transparent manner consider the data in Table 1. It describes the values of $\{\eta, \sigma\}$ consistent with a given μ. Table 1 also lists the corresponding sign of e_2 and whether it is greater or less than 1. Recall that if e_2 is greater than 1, then sunspots are possible.

Table 1. Sunspot Zones and Markups

μ	η	σ	α	e_2	*sunspot*
1.2	0.00 – 5.15	7.50 – 6.21	0.32	$+$	*yes*
1.2	5.18 – 5.60	6.20 – 6.10	0.32	$-$	*yes*
1.2	5.61 – 5.98	6.09 – 6.00	0.32	$-$	*no*
1.2	6.00 – 30.0	5.99 – 0.00	0.32	$+$	*no*
1.6	0.00 – 2.58	3.33 – 2.7	0.47	$+$	*yes*
1.6	2.59 – 2.61	2.69 – 2.68	0.47	$-$	*yes*
1.6	2.62 – 2.65	2.68 – 2.67	0.47	$-$	*no*
1.6	2.66 – 13.33	2.66 – 0.00	0.47	$+$	*no*
2.8	0.00 – 1.53	1.94 – 1.56	0.74	$+$	*yes*
2.8	1.53 – 155	$\simeq 1.55$	0.74	$-$	*yes*
2.8	1.56 – 5.05	1.54 – 0.68	0.74	$+$	*no*
2.8	5.06 – 7.78	0.67 – 0.00	0.74	$+$	*yes*

For every value of μ considered, for some region of $\{\eta, \sigma\}$, $| e_2 | > 1$ such that it is possible that sunspot equilibria may exist for a wide range of markups. Moreover, as can be seen in Table 1, the intervals of $\{\eta, \sigma\}$ for which sunspots exist are also quite large. Consequently, the existence of such equilibria in the environment discussed in this paper does not depend on very specific values of parameters. Note, that for values of $\{\eta,\sigma\}$ close to one another $|e_2| < 1$ and for sufficiently apart values $|e_2| > 1$. This implies, as described by Gali, for sunspots to exist the two elasticities need to be sufficiently far apart. If η and σ are close to one another $\bar{\varepsilon}_\mu$ is extremely small such that $\frac{1}{\bar{\varepsilon}_\mu}$ is very large which in turn implies that $|e_2| = \left\{1 - \frac{\delta(1-\alpha)}{1+\zeta}\left[1 + \left(\frac{\bar{\lambda}}{\lambda-\psi}\right)\frac{1}{\bar{\varepsilon}_\mu}\right]\right\}^{-1}$ will be small and sunspot equilibria cannot exist.

However, when μ is small e.g., 1.2 or 1.5, sunspots exist only in the region $\eta < \sigma$. For larger values of the markup e.g., for values considered by Gali i.e., $\mu \in [2, 3.03]$, $|e_2| > 1$ in regions where $\eta < \sigma$ as well as $\eta > \sigma$. The difference in the behavior of the economy for larger values of the steady state μ turns out to be quite significant. In the region $\eta > \sigma$, the elasticity of the markup to λ, $\bar{\varepsilon}_\mu = \frac{(\sigma-\rho)\bar{\lambda}}{\xi(\xi-1)}$ is negative. At the empirical level, the U.S. economy displays two strong regularities

1. 1. a strongly *procyclical* investment share
   2. a strongly *countercyclical* markup[5].

If these two features are to hold simultaneously then when output increases, λ should increase and μ should fall. This implies that $\varepsilon_\mu < 0$. Therefore, if the model is to be consistent with the regularities displayed by the U.S. economy it follows that $\eta > \sigma$ since $\bar{\varepsilon}_\mu = \frac{(\sigma-\eta)\bar{\lambda}}{\xi(\xi-1)}$. While for parameter values consistent with a steady state markup $\mu > 2.0$, sunspots can exist ($|e_2| > 1$) when $\eta > \sigma$, in the range $\mu \in [1, 2)$ this is not true. In fact for $\mu \leq 2$, sunspots fail to exist for $\eta > \sigma$.

Proposition 5.2. *If $\eta > 0$, $\sigma > 0$, and $\eta > \sigma$, then at the steady state $\mu > 2$ for sunspot equilibria to exist.*

Proof. See appendix. ■

Note that in the proof of Proposition 3, we did not use any specific parameter values except for restricting the parameters to be positive and the markup to be greater than 1. Consequently, the proposition holds quite generally and for this class of economies if the share of investment is to be procyclical and the optimal markup countercyclical, at the steady state the markup has to be over 100 percent. Given the more recent studies discussed previously, it is unlikely that the long-run markup for the U.S. economy can be that large. In fact, the Martins, Scarpetta and Pilat (1996) study shows that for most OECD countries markups are substantially lower than 2.

Simplifying the parameters e_1 and e_3, (5.5) becomes

$$\hat{k}_{t+1} = E_t\left[d_1(1-e_2)\hat{z}_{t+1} + e_2\hat{k}_{t+2} + \left(\frac{\rho+\delta}{\alpha}\right)\psi^2 e_2\hat{g}_{t+1}\right]$$

This implies that if $e_2 > 1$ then $\hat{k}_{t+1}$ is negatively correlated with technological shocks as $d_1(1-e_2) < 0$. In turn, $e_2 > 1$ also implies a negative correlation for investment which we find to be counter intuitive. Furthermore, if $\mu \in [2, 3.03]$, the values used by Gali, $\alpha \in [0.53, 0.8]$ and the share of wages in national income lies between $[0.24, 0.07]$. This value is significantly different

[5]See for example, Rotemberg and Woodford 1990, Bils, 1987.

from the observed share of around 0.64 (Prescott, 1986). Consequently, given the preponderance of both empirical evidence and the implied inconsistencies with empirical regularities it seems unlikely that the version of this model with sunspots will be a reasonable approximation of the real economy.

6. Equilibrium with Exogenous Fluctuations

In keeping with the restrictions derived in the previous section we follow studies that have used a similar framework such as Rotemberg and Woodford (1992) and Basu (1995) to restrict steady state markups to the interval $[1.2, 1.7]$ that matches the more recent econometric estimates. As shown in proposition 3, this restriction forces $|e_2| < 1$, so that sunspot equilibria are ruled out. Moreover, in the rest of this section unless otherwise noted it will be assumed that $\eta > \sigma$ so that $\varepsilon_\mu < 0$. Therefore, in order to generate cyclical fluctuations the model will depend exclusively on the two exogenous shocks : (i) the technological shock $\hat{z}_t$ and (ii) the demand shock $\hat{g}_t$. Consider first the perfect foresight equilibrium i.e., $\hat{z}_t = 0$ and $\hat{g}_t = 0$, $\forall t = 0, \ldots \infty$. The first equation in this system is $\hat{k}_1 = e_2 \hat{k}_2$. Consequently, there is no restriction on k_1 imposed by the initial capital stock k_0. The unique steady state of the system is $\hat{k} = 0$ and since $\mid e_2 \mid < 1$ it is unstable, such that the only rational expectations equilibrium is the one where $\{\hat{k}_{t+1}\} = 0$ $\forall t = 0, \ldots \infty$. Therefore, given any initial capital stock k_0, there is an unique perfect foresight equilibrium which is also stationary. The instability of the steady state allows the stochastic version of the difference equation (5.5) to be forward stable such that there is an unique rational expectations equilibrium given by

$$\hat{k}_{t+1} = e_1 E_t \left[\sum_{j=0} e_2^j \hat{z}_{t+j+1} \right] + e_3 E_t \left[\sum_{j=0} e_2^j \hat{g}_{t+j+1} \right] \tag{6.1}$$

Since $\hat{z}_{t+1} = \phi^z \hat{z}_t + \hat{\varepsilon}^z_{t+1}$ and $E_t \left[\hat{\varepsilon}_{t+1}\right] = 0$ and $\hat{g}_{t+1} = \phi^g \hat{g}_t + \hat{\varepsilon}^g_{t+1}$ and $E_t \left[\hat{\varepsilon}^g_{t+1}\right] = 0$ we have,

$$\hat{k}_{t+1} = \phi^z e_1 E_t \left[\sum_{j=0} (\phi^z e_2)^j \hat{z}_t \right] + \phi^g e_3 E_t \left[\sum_{j=0} (\phi^g e_2)^j \hat{g}_t \right] \tag{6.2}$$

which can be expressed as

$$\hat{k}_{t+1} = \frac{\phi^z e_1}{1 - \phi^z e_2} \hat{z}_t + \frac{\phi^g e_3}{1 - \phi^g e_2} \hat{g}_t \tag{6.3}$$

This is the capital accumulation law of motion generated by the equilibrium conditions of the economy and using the linearized system of equations $(5.1) - (10.10)$ all other variables can be expressed in terms of $\hat{z}_t$ and $\hat{g}_t$. In particular[6],

$$\hat{\lambda}_t = \lambda^z \hat{z}_t - \lambda^k \hat{k}_t + \lambda^g \hat{g}_t \tag{6.4}$$

$$\hat{\mu}_t = b_1 \lambda^z \hat{z}_t - b_1 \lambda^k \hat{k}_t + b_1 \lambda^g \hat{g}_t \tag{6.5}$$

$$\hat{y}_t = y^z \hat{z}_t + y^k \hat{k}_t + y^g \hat{g}_t \tag{6.6}$$

[6]The appendix lists the complete expressions for the different coefficients.

$$\hat{l}_t = l^z \hat{z}_t + l^g \hat{g}_t + l^k \hat{k}_t \tag{6.7}$$

We investigate the effects of changing the long run share of investment goods in overall government purchases, θ, on the cyclical behavior of aggregate output y_t, investment i_t, consumption c_t and labor l_t keeping the long run level of overall government expenditure $\bar{g}$ constant. Using the expression for equilibrium output (6.6)derived in the appendix and ignoring the variations in the stock of capital which are generally small,

$$var(\hat{y}) = (y_1^a)^2 \sigma_z^2 + (y_3^a)^2 \sigma_g^2$$

since $Cov(\hat{z}, \hat{k}) = 0$, $Cov(\hat{z}, \hat{g}) = 0$, $Cov(\hat{g}, \hat{k}) = 0$. Consequently, the way output responds to the exogenous shocks, $\hat{z}_t$ and $\hat{g}_t$, depends on the parameters y^z and y^g. These two parameters are affected by θ. As a result, when the long-run composition of government outlay changes, it affects the way the economy reacts to both supply and demand shocks. The cyclical variability of output can therefore, be accentuated or reduced by the choice of θ.

As noted in the discussion on the long run effects of changes in the share of public investment, when θ changes the price elasticity of the commodity also changes at the steady state also changes since

$$\bar{\xi} = (\eta - \sigma)\left[(1-\bar{g})\bar{\lambda}^p + \theta\bar{g}\right] + \sigma$$

In particular, when $(\eta - \sigma) > 0$, then an increase in θ will increase the elasticity as long as $\xi(\xi - 1) > (\lambda - \theta\bar{g})(\eta - \sigma)$ (see lemma 1). This will lead to a reduction in the markup. Apart from increasing welfare by reducing the markup and increasing productivity (Proposition 1) an increase in θ also impinges on the way the variability of output and variables such as employment, consumption and investment. This occurs since an increase in the elasticity results in a "flattening" of the aggregate demand function. With a flatter demand function, exogenous shocks have a larger effect on quantity than prices (see Figure 3). Consequently, the variability in aggregate output for the same exogenous shocks will be larger resulting in a larger short-run volatility leading to the following situation. On one hand, increased government investment enhances productivity and welfare while on the other it also raises the volatility in income. Depending on the risk aversion of households, the loss in welfare due to increased volatility could, in principle, overshadow the welfare gains due to increased productivity leading a to fall in the total welfare.

7. Growth and Cyclical Effects of Changing Public Investment

Although it is possible to derive closed form solutions expressing all the variables in terms of the exogenous shocks (see the appendix) evaluating the effects of increased θ analytically is still too cumbersome. Instead, we use the parametric restrictions derived in the previous section to simulate the effects first for the U.S. economy and then for other OECD countries.

7.1. Simulation Results for the U.S. Economy

For the U.S. economy, in the experiments we keep g constant at the long run average for government purchases which is around 0.21. The steady state value of θ is around 0.15 which includes investment goods for defense. This implies that government investment is about 3.1 percent of GDP.

Figure 6.1:

Table 2. Parameters used in Simulations

Parameter description	Value
β– household's discount factor	0.99
ς– wage elasticity of labor	1
α– output elasticity of capital	0.467
$\underline{\delta}$– rate of depreciation of capital stock	0.016
$\bar{\lambda}$– steady state share of total investment in GDP	0.21
$\underline{\bar{\mu}}$– steady state markup	1.6
θ– share of investment in government expenditure	0.15
$\bar{g}$– steady state share of government expenditure in GDP	0.21
ϕ^z– serial correlation of technology shock	0.95
ϕ^g– serial correlation of government expenditure shock	0.82
σ_z– standard deviation of technology shock	0.00763
σ_g– standard deviation of government expenditure shock	0.0241

We take as the benchmark the steady state implied by the parameter values enumerated in Table 2 . The unresolved parameters are η and σ. However, given the procyclicality of markups as discussed in the previous section we know that $\eta > \sigma$. In this region we consider two sets of values for these variables $\{\eta = 2.75, \sigma = 2.65\}$ and $\{\eta = 3.5, \sigma = 2.46\}$. In the first set the difference between the two elasticities is small and in the second relatively large. For each set of parameters we vary the θ keeping g constant at 0.21. A five percentage point increase (decrease) in θ implies a one percentage point increase(decrease) in share of government investment in GDP. The steady state share of public investment, ψ, is around 0.03. Tables 3a and 3b summarizes the effects of changing this share by *one* percentage point or equivalently changing θ from its steady state value of 0.15 by $five$ percentage points.

Table 3a. Steady State Effects of a One Percent Decrease in Public Investment-to-GDP ratio

	$\{\eta = 2.75, \sigma = 2.65\}$	$\{\eta = 3.5, \sigma = 2.46\}$
Percentage change in $\overline{\lambda}$	-1.054	-1.096
Percentage change in $\overline{\mu}$	0.038	0.41
Percent change in $\overline{k}$	-0.089	-0.95
Percent change in $\overline{l}$	-0.044	-0.476
Percent change in $\overline{c}$	-0.058	-0.625
Percent change in $\overline{y}$	-0.065	-0.702
Percent change in welfare	-0.82	-0.502

When ψ is lowered by 1 percentage point, the share of total investment λ, falls by more than 1 percentage for both cases i.e., η and σ are close to one another (Table 3a, column A) and η and σ relatively far apart (Table 3a, column B). However, the multiplier effect is somewhat larger when the two elasticities are further apart. On the other hand, when η and σ are close to one another, the effect on the steady state markup is very small. The markup, μ remains almost unchanged. This occurs because the average elasticity, which determines the level of the markup, given by $(\eta - \sigma)\lambda + \sigma$ is approximately constant at σ when $\eta - \sigma$ is small. When the difference between η and σ is large the change in the markup is by almost half a percentage point. In a similar vein, capital stock, labor, output and private consumption rise in both cases although the increase is sharper when the two elasticities are further apart.

When $\eta = 3.5$ and $\sigma = 2.46$, the effect of percentage decrease in public investment leads to almost half a percent decrease in employment. If θ was held constant at 0.15, to achieve the same effect government expenditure as a percentage of output would have had to increase by 5 percentage points, i.e., from 0.21 to about 0.26. Thus keeping the composition of public expenditure constant, permanent decreases in government spending permanently decreases employment and output. In models with constant markup, although such effects are present, the causality works through the labor supply decisions of households and depends almost entirely on the income effect on leisure. A decrease in government expenditure leads to an increase in private wealth and lowers real interest rates. Both these have the effect of lowering the marginal utility of wealth which, in turn, causes labor supply to fall (Barro 1981). However, in this environment the causality works from the demand side. With a decrease in the share of investment, the price elasticity of a firm's output increases causing the markup to rise and consequently leading to a decrease in labor demand (for a related analysis see Rotemberg and Woodford 1992).

Table 3b. Cyclical Effects of a One Percentage Decrease in Public Investment-to-GDP ratio

	$\{\eta = 2.75, \sigma = 2.65\}$	$\{\eta = 3.5, \sigma = 2.46\}$
Percent change in σ_y due to $\hat{z}$	-0.927	-3.52
Percent change in σ_y due to $\hat{g}$	-85.16	-100.41
Percent change in σ_l due to $\hat{z}$	-2.26	-4.72
Percent change in σ_l due to $\hat{g}$	-85.16	-100.41
Percent change in welfare	0.03	0.05

The short run effects of permanently changing θ are quite significant, even when the difference between η and σ is small (column A, Table 3b). While the percent decrease in the variance of output from the technology shock is by almost 1 percent, for labor it is by over 2 percent. The effect is more dramatic for the variance caused by demand shocks. For both employment and output the variance falls by over 85 percent and by over 100 percent when the elasticities are further apart. This reduction in the variance of employment and output leads to a rise in households' welfare. Welfare increases by around 0.03 percent. As discussed previously, a decrease in the public investment reduces the elasticity and increases the markup. While this leads to an decline in welfare due to efficiency losses, it also makes aggregate demand less elastic. With a more inelastic demand, adjustments in quantity (output and therefore employment) are smaller than in prices. This fall in volatility acts to increase the level of welfare.

7.1.1. Correlation of Total Factor Productivity with Demand Shocks

Assume in this section that $\hat{z}_t = 0$, so that the only exogenous shock to the system is $\hat{g}_t$. Letting $\hat{x}_t \equiv \frac{x_t - \bar{x}}{\bar{x}} \simeq \ln\left(\frac{x_t}{\bar{x}}\right)$, total factor productivity is given by

$$\hat{q}_t = y^k\left(1 - \frac{1}{\bar{\mu}}\right)\hat{k}_t + y^g\left(1 - \frac{1}{\bar{\mu}}\right)\hat{g}_t$$

where $y^g = -\frac{\varsigma(1-\alpha)\xi_\mu\lambda^g}{1+\alpha\zeta}$. As long as the elasticity of λ with respect to markup is negative, a condition satisfied if $\eta > \sigma$ — which is the same requirement to ensure that the markup is procyclical — the Solow residual, $\hat{q}_t$, will be positively correlated to $\hat{g}_t$, namely the aggregate demand shock. Consequently, in this economy if Solow residuals are measured in the standard way i.e., the difference between actual output and the share weighted contribution of capital and labor, it will be biased upwards as shocks such as $\hat{g}_t$ that are exogenous to the true technological shock, will be picked up. While, Hall (1988) and Evans (1992) have empirically shown that productivity is correlated with variables such as government expenditure shocks, Basu (1995) provides an argument why this may take place in a model where final good production requires intermediate goods. In this environment however, the driving force behind the result is the wedge between the elasticities of firms and households for the same goods.

7.2. Simulation Results for Other OECD Economies

This section is based largely on a study by Royen (1998), who uses a similar environment to estimate the effects of changing the composition of government purchases on output, employment, capital stock and private consumption in OECD member countries

7.2.1. Calibration Issues

The depreciation rate and the elasticity of labor supply are assumed to be common across the 14 OECD member countries used in this study. The specific values for the two parameters $\delta = 0.10$ and $\varsigma = 1$. Estimates of mark-ups for OECD countries are available from Martins, Scarpetta, and Pilat, who calculated markup ratios for 14 countries for the period 1970 – 1992. Markups, μ, are available for 36 manufacturing sectors. The average markup ratios range from 1.13 in Belgium and Finland to 1.26 for Japan. In this paper, for the US, the markup has been estimated to 1.18

for the 1970's period and to 1.15 for the 1980's - substantially lower than the 1.6 used in our simulations. Furthermore, Martins, Scarpetta, and Pilat, provide evidence of the countercyclical markups in these countries. As before, for the model economy, this implies that investment elasticity is higher than consumption elasticity i.e., $\eta > \sigma$. Royen uses two sets of specific values for these elasticities which are consistent with the estimated OECD markups. In the first set, the distance between the two elasticities is set approximately at 1.5, while in the other it is 3. Given the countercyclical mark-ups and their low levels, this results in high values for η and σ. The elasticity of capital in production, α, is given for each country by the steady state relation $(\overline{\lambda} - \psi)\overline{\mu} = \frac{\alpha\delta}{\rho+\delta}$, where ρ is given by the average risk-free interest rate in each country[7]. Country-specific values for the long-run share of investment in government expenditure, the share of government expenditure in GDP, and share of investment in GDP were used.

7.2.2. Steady State Effects

In the 1970s, the impact of an increase in $\overline{\theta}$ by five percentage points, with $\overline{g}$ remaining constant, on the long-run share of investment in GDP is the largest in the UK and Denmark of around 1.7 percentage points, while it has a relatively low impact Japan and Germany of about 0.7 percentage. The OECD average is around 1.3 percentage point change. In the 1980's, the impact on share of investment increases for almost all countries. The average increase for OECD countries is about 1.6 percentage points, with the highest impact in France, Sweden, and Denmark of over 2 percentage points. This result does not alter when the difference between the two elasticities is altered. The effect on the markup ratios, however, increases, as one would expect, when the gap between investment and consumption elasticity increases. When the difference in the two elasticities is around 1.5, markups fall by about 0.1 percent in these countries, while the decline is about 0.25 percent when the difference is doubled. The 1970's, with the exception of Japan and Norway, the effect of a 5 percentage change in $\overline{\theta}$ has little impact on output, ranging from 0.07 percent increase in Belgium to 0.5 percent in the Netherlands. In Norway, however, a significantly different pattern emerges. Changing $\overline{\theta}$ by five percentage point increases output by more than 1.6 percent when the distance between the two elasticities is about 3. For the 1980's, the differences among countries are less pronounced. Changing θ has a relatively larger impact in France, Italy, Australia, Norway and Finland, where it increase output by almost 0.7 percent.

8. Concluding Remarks

That aggregate demand affects the cyclical behavior of an economy by changing the markup is well established in the literature with markups behaving in a countercyclical manner as increases in demand raise the elasticity of demand faced by monopolistically competitive firms such that profit maximization leads to a reduction in their markups. Building on the framework developed by Gali (1994), who showed that the composition of aggregate demand can have similar impact, we found that changes in government purchases can have efficiency effects as well as affect the short run volatility of macroeconomic variables like output and employment. In particular, when the price elasticities of investment is larger than that of consumption, markups behave countercyclically. In such a situation if the share of investment in total public expenditure is increased aggregate demand becomes more elastic and consequently, optimal markup is reduced. With a fall in the markup the economy is pushed towards the competitive equilibrium with the

[7]The average three month treasury bill rate for the period considered is used.

resulting increase in productivity raising the marginal revenue products of both capital and labor, such that in equilibrium, firms hire more workers and accumulate more capital, which in turn increases the steady output, employment and capital stock. However, the increase in the elasticity of aggregate demand implies that in response to exogenous shifts in demand or supply quantity adjustment will be more than price adjustment thereby increasing the cyclical variability of output and employment. This trade-off between welfare gains due to increased efficiency and welfare losses due to greater variability is the critical decision that policy makers need to make when changing the composition of public expenditure.

For the postwar U.S. economy, a one percentage increase in the share of public investment, keeping the share of total government spending constant, raises steady state output by almost 0.07 percent and welfare by 0.8 even when the elasticities of consumption and investment are close to one another. For larger differences in the two elasticities while the increase in output is larger the welfare gains are less since there is an offsetting increase in disutility from increased hours of work. However, this also increases the short run variability of the economy. The variance of output from the same technology shock increases by almost one percent while the variance of employment is raised by about 2 percent. The resulting loss in welfare from this increased uncertainty is about 0.03 percent.

We conclude the paper by discussing briefly a particular shortcoming of the model. Typically government expenditure is made of consumption and investment goods and labor, i.e., public sector employees. If a government changes the number of its employees, the economy can also be affected. However, this effect will be through the supply side. By changing its level of employment, the government can affect the amount of labor input available to the private sector. i.e., the effective labor supply faced by firms in the private sector. The effect of such changes could be significant both quantitatively as well as in terms of its direction. One cannot draw a strict analogy between changes in the composition of government purchases and public sector employment since the latter will affect the economy primarily by changing the composition of available inputs and their relative prices. In this paper, however, we did not consider the effects of changes in the public sector employment.

9. References

1. Aiyagari, S. Rao, Lawrence Christiano, and Martin Eichenbaum, “The Output and Employment Effects of Government Purchases,” *Manuscript, Federal Reserve Bank of Minneapolis*, (February 1989).

2. Basu, Susanto, “Intermediate Goods and Business Cycles: Implications for Productivity and Welfare,” American Economic Review, Vol. 85, No. 3, (1995), pp. 512-531.

3. Barro, Robert J., “Government Spending in a Simple Model of Endogenous Growth,” *Journal of Political Economy* (Chicago), Vol. 98, (1990), pp. S103-S125.

4. Barro, Robert J., “Output Effects of Government Purchases,” *Journal of Political Economy* (Chicago), Vol. 89, (December 1981), pp. 191-225.

5. Bils, Mark, “Pricing in a Customer Market,” *Quarterly Journal of Economics*, Vol. 104, (November 1989), pp. 699-718.

6. Bils, Mark, “The Cyclical Behavior of Marginal Cost and Price,” *American Economic Review*, Vol. 77, (December 1987), pp. 838-57.

7. Devarajan, Shantayanan, Vinaya Swaroop, and Heng-fu Zou, “The Composition of Public Expenditure and Economic Growth,” *Journal of Monetary Economics*, Vol. 37, (1996), pp. 313-344.

8. Domowitz Ian, R. Glenn Hubbard, and Bruce C. Petersen, “Market Structure and Cyclical Fluctuations in U.S. Manufacturing,” *Review of Economics and Statistics*, Vol. 70 (February 1988), pp. 55-66.

9. Evans, Charles L., “Productivity Shocks and Real Business Cycles,” *Journal of Monetary Economics*, Vol. 29, (April 1992), pp. 191-208.

10. Farmer, Roger E.A., and Jang-Ting Guo, “Real Business Cycles and the Animal Spirits Hypothesis,” *Journal of Economic Theory*, Vol. 63 (June 1994), pp. 42-72.

11. Galí, Jordi, “Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand,” *Journal of Economic Theory*, Vol. 63 (June 1994), pp. 73-96.

12. Hall, Robert E., “The Relation Between Price and Marginal Cost in U.S. Industry,” *Journal of Political Economy* (Chicago), Vol. 96, (1988), pp. 921-948.

13. Kalecki, Michael, “The Determinants of Distribution of the National Income,” *Econometrica*, Vol. 6, (April 1938), pp. 97-112.

14. Keynes, John Maynard, “Relative Movements of Real Wages and Output,” *Economic Journal*, Vol. 49, (March 1939), pp. 34-51.

15. Killingsworth, Mark R., and James J. Heckman, “Female Labor Supply: A Survey,” in *Handbook of Labor Economics* (Orley Ashenfelter and Richard Layard eds.,), Vol. 1, Amsterdam: North-Holland, (1986).

16. Kydland, Finn E. and Edward C. Prescott, " Business Cycles: Real Facts and a Monetary Myth," *Quarterly Review* (Federal Reserve Bank of Minneapolis), (Winter 1990), pp. 3-18.

17. Martins, Joaquim O., Stefano Scarpetta, and Dirk Pilat, "Mark-up Ratios in Manufacturing Industries: Estimates for 14 OECD Countries," *OECD Working Paper No. 162*, Paris, (1996).

18. Morrison, Catherine J., "Market Power, Economic Profitability and Productivity Growth Measurement: An Integrated Structural Approach," *NBER Working Paper No. 3355*, Cambridge, Mass., (May 1990).

19. Rotemberg, Julio J., and Garth Saloner, "A Supergame-Theoritic Model of Price Wars During Booms," *American Economic Review*, Vol. 76, (June 1986), pp. 390-407.

20. Rotemberg, Julio J., and Michael Woodford, "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity," *Journal of Political Economy* (Chicago), Vol. 100, No. 6, (1992), pp. 1153-1207.

21. Rotemberg, Julio J., and Michael Woodford, "Cyclical Markups: Theories and Evidence," *NBER Working Paper No. 3534*, Cambridge, Mass., (December 1990).

22. Royen, Marie-Christine, "Impact of a Change in Public Spending Policies on Margins and Business Cycles in OECD Countries," GEMME Working Paper, University of Liege, (October 1998).

23. Solow, Robert M., "Technical Change and the Aggregate Production," *Review of Economic Studies* (Edinburgh), Vol. 39 (August 1957), pp. 312-20.

24. Woodford, Michael, "Self-fulfilling Expectations and Fluctuations in Aggregate Demand," in *New Keynesian Economics* (N.Gregory Mankiw and David Romer, Eds.), Vol. 2, MIT Press, Cambridge, MA, (1991).

10. Appendix

10.1. Solution to the Household's Problem

The objective function of the household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t^i) - v(l_t^i)\right]$$

where

$$u(c_t^i) = M^{\frac{1}{1-\sigma}} \left[\sum_{s=1}^{M} \left(c_{st}^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \ \sigma > 0$$

and

$$v(l_t^i) = \frac{\zeta}{1+\zeta} \left(l_t^i\right)^{\frac{1+\zeta}{\zeta}}, \ \zeta > 0.$$

Denoting by q_t^h the price of the shares in firm h, we can write the budget constraint of consumer i as a function of his income, the return on the shares that he owns and the taxes he pays to the government. Thus, if s_{ht}^i is the number of shares he owns in firm h at time t and d_t^h the dividend he receives from these shares, the budget constraint is given by

$$\sum_{s=1}^{M} p_t^s c_{st}^i \leq l_t^i + \sum_{h=1}^{M} \left(d_t^h + q_t^h\right) s_{ht}^i - \sum_{h=1}^{M} q_t^h s_{h,t+1}^i - T_t^i$$

where, T_t^i is the tax paid by the ith household.

If we define the income of consumer i net of taxes as a_t^i, with

$$a_t^i \equiv l_t^i + \sum_{h=1}^{M} \left(d_t^h + q_t^h\right) s_{ht}^i - \sum_{h=1}^{M} q_t^h s_{h,t+1}^i$$

and assume that all shares are distributed equally between consumers at time $t = 0$, the consumer's problem can be written as

$$\begin{gathered} \max \ E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \left[u(c_t^i) - v(l_t^i)\right] \\ \text{s.t. } \textstyle\sum_{s=1}^{M} p_t^s c_{st}^i \leq a_t^i - T_t^i \\ s_{h0}^i = \tfrac{1}{N} \end{gathered} \tag{10.1}$$

As discussed in the text let P_t be the consumer price index defined as

$$P_t = \left[\frac{1}{M} \left(\sum_{s=1}^{M} (p_t^s)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\right]$$

then the solution to the consumer's problem is

$$c^i_{st} = \left(\frac{p^s_t}{P_t}\right)^{-\sigma}\left(\frac{a^i_t - T^i_t}{P_t M}\right),\ s = 1,2,...M, \forall\ i = 1,2,...N$$

$$l^i_t = P_t^{-\zeta},\ \forall\ i = 1,2,...N$$

$$\frac{q^h_t}{P_t} = \beta E_t \frac{1}{P_{t+1}}\left(d^h_{t+1} + q^h_{t+1}\right),\ h = 1,2,...M,\ \forall\ i = 1,2,...N \tag{10.2}$$

$$\lim\inf_{T\to\infty} E\beta^T \sum_{h=1}^{M}\left(\frac{q^h_T}{P_T}\right) s^i_{hT} = 0,\ h = 1,2,...M,\ \forall\ i = 1,2,...N$$

The impact of the taxes clearly appears in the first equation characterizing the demand for the consumption goods. The second equation is the labor supply as a function of the composite price index with an elasticity of ζ as defined above. The third is the Euler condition and corresponds to the familiar capital asset pricing equation. The fourth equation is a limit condition that guarantees that the value function is well defined.

10.2. Solution to the Firm's Problem

Using the solutions to the static allocation problem faced by the firm from equation (2.5), the *intertemporal* problem of the jth firm is given by

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \frac{d^j_t}{P_t}$$

$$\text{s.t.}\qquad d^j_t = p^j_t y^j_t - l^j_t - \nu^j_t$$

$$y^j_t = z_t \left(k^j_t\right)^{\alpha}\left(l^j_t\right)^{1-\alpha} \tag{10.3}$$

$$k^j_{t+1} = (1-\delta)k^j_t + \frac{\nu^j_t}{\Pi_t}$$

where β is the discount factor, d^j_t is the dividend paid by firm j at time t, P_t the price index for the consumption goods defined similarly to Π_t, and E_0 is the expectation operator at time $t = 0$. Since the intertemporal problem is also identical across firms, we can drop the subscript j.

One can thus summarize the problem in (10.3) by

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{P_t}\left[p_t y_t - z_t^{-\frac{1}{1-\alpha}} y_t^{\frac{1}{1-\alpha}} k_t^{-\frac{\alpha}{1-\alpha}} - \Pi_t\left(k_{t+1} - (1-\delta)k_t\right)\right] \tag{10.4}$$

where $z_t^{-\frac{1}{1-\alpha}} y_t^{\frac{1}{1-\alpha}} k_t^{-\frac{\alpha}{1-\alpha}}$ is the variable cost at time t. Solving (10.4) by choosing $\{y_t, k_{t+1}\}$ one gets

$$p^{'}_t y_t + p_t = \frac{1}{1-\alpha} z_t^{-\frac{1}{1-\alpha}} \left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}$$

where $p^{'}_t$ represents the derivative of p_t with respect to y_t. The above equation can be rearranged as

$$p_t\left(\frac{p_t^{'}y_t}{p_t}+1\right)=\frac{1}{1-\alpha}z_t^{-\frac{1}{1-\alpha}}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}$$

or,

$$p_t\left(1-\frac{1}{\xi_t}\right)=\frac{1}{1-\alpha}z_t^{-\frac{1}{1-\alpha}}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}$$

where ξ_t, is the price elasticity of the commodity and $\frac{1}{1-\alpha}z_t^{-\frac{1}{1-\alpha}}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}$ is the marginal cost. This gives us the standard price setting rule in the case of monopolistic competition namely, that the price is set equal to a mark-up over the marginal cost, i.e.,

$$p_t=\mu_t\omega_t \tag{10.5}$$

where $\mu_t=\frac{\xi_t}{\xi_t-1}$ is the mark-up and

$$\omega_t=\frac{1}{1-\alpha}z_t^{-\frac{1}{1-\alpha}}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}$$

is the marginal cost. After rearranging, the optimal capital accumulation policy function, obtained by taking the first order condition in k_{t+1} is given by

$$\frac{\Pi_t}{P_t}=\beta E_t\frac{1}{P_{t+1}}\left[\frac{\alpha}{1-\alpha}z_{t+1}^{-\frac{1}{1-\alpha}}\left(\frac{y_{t+1}}{k_{t+1}}\right)^{\frac{1}{1-\alpha}}+(1-\delta)\Pi_{t+1}\right] \tag{10.6}$$

where $\frac{\alpha}{1-\alpha}z_{t+1}^{-\frac{1}{1-\alpha}}\left(\frac{y_{t+1}}{k_{t+1}}\right)^{\frac{1}{1-\alpha}}$ corresponds to the decline in the variable or labor cost due to a unit increase in the capital stock. Equations (**??**), (10.5), and (10.6) characterize the optimal policy functions of the firms in terms of output at time t and capital stock at time $t+1$, thus fully characterizing the solution to the firm's problem.

10.3. The Linear Approximations

The linearization of the following equations

$$x_t-A_xz_t^{\frac{1+\zeta}{1+\alpha\zeta}}\mu_t^{-\frac{\zeta(1-\alpha)}{1+\alpha\zeta}}k_t^{-\frac{(1-\alpha)}{1+\alpha\zeta}}=0 \tag{10.7}$$

$$\mu_t-\frac{(\eta-\sigma)\lambda_t+\sigma}{(\eta-\sigma)\lambda_t-(1-\sigma)}=0 \tag{10.8}$$

$$(\lambda_t-\theta g_t)x_t-\frac{k_{t+1}}{k_t}+(1-\delta)=0 \tag{10.9}$$

$$k_{t+1}-A_k\left\{E_t\left[\left(\frac{z_{t+1}}{\mu_{t+1}}\right)^{\frac{1+\zeta}{1+\alpha\zeta}}\right]\right\}^{\frac{1+\alpha\zeta}{1-\alpha}}=0 \tag{10.10}$$

resulted in

$$\hat{x}_t = a_1\hat{z}_t - a_2\hat{\mu}_t - a_3\hat{k}_t \tag{10.11}$$

$$\hat{\mu}_t = b_1\hat{\lambda}_t \tag{10.12}$$

$$\hat{\lambda}_t = \frac{1}{c_1}\hat{k}_{t+1} - \frac{1}{c_1}\hat{k}_t - \frac{\delta}{c_1}\hat{x}_t + c_2\hat{g}_t \tag{10.13}$$

$$\hat{k}_{t+1} = d_1 E_t\left[\hat{z}_{t+1} - \hat{\mu}_{t+1}\right] \tag{10.14}$$

where,
$a_1 = \left(\frac{1+\zeta}{1+\alpha\zeta}\right)$, $a_2 = \frac{\zeta(1-\alpha)}{1+\alpha\zeta}$, $a_3 = \frac{1-\alpha}{1+\alpha\zeta}$, $b_1 = \left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\rho)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)\frac{\bar{\lambda}}{(\bar{\lambda}-\psi)}$,
$c_1 = \delta\frac{\bar{\lambda}}{(\bar{\lambda}-\psi)}$, $c_2 = \left(\frac{\psi}{\bar{\lambda}}\right)$, $d_1 = \left(\frac{1+\zeta}{1-\alpha}\right)$, and $\hat{\omega}_t = \frac{\omega_t-\bar{\omega}}{\bar{\omega}}$ for $\omega = x, \lambda, k, z, v, g$. Note that at the steady state $(\bar{\lambda}-\psi)\left(\frac{(\eta-\sigma)\bar{\lambda}+\sigma}{(\eta-\sigma)\bar{\lambda}-(1-\sigma)}\right) = \frac{\alpha\delta}{\rho+\delta}$. Consequently,

$$\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right) = (\bar{\lambda}-\psi)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}-(1-\sigma)\right)\left((\eta-\sigma)\bar{\lambda}+\sigma\right)}\right)$$

Since $\bar{\xi} = (\eta-\sigma)\bar{\lambda}+\sigma$, the b_1 becomes $\bar{\lambda}\left(\frac{(\sigma-\rho)}{\bar{\xi}(\bar{\xi}-1)}\right)$. From the definition of the elasticity of the markup with respect to λ, we know that $\bar{\varepsilon}_\mu = \dfrac{(\sigma-\eta)\bar{\lambda}}{\bar{\xi}\left(\bar{\xi}-1\right)}$. Consequently, $b_1 = \bar{\varepsilon}_\mu$. Replacing (10.11) and (10.12) in (10.13) we have under the assumption that $c_1 - \delta a_2 b_1 \neq 0$

$$\hat{\lambda}_t = \frac{1}{c_1-\delta a_2 b_1}\hat{k}_{t+1} - \frac{(1-\delta a_3)}{c_1-\delta a_2 b_1}\hat{k}_t - \frac{\delta a_1}{c_1-\delta a_2 b_1}\hat{z}_t + \frac{c_1 c_2}{c_1-\delta a_2 b_1}\hat{g}_t \tag{10.15}$$

Using the above and leading (10.12) by one period

$$\hat{\mu}_{t+1} = b_1\left(\frac{1}{c_1-\delta a_2 b_1}\hat{k}_{t+2} - \frac{(1-\delta a_3)}{c_1-\delta a_2 b_1}\hat{k}_{t+1} - \frac{\delta a_1}{c_1-\delta a_2 b_1}\hat{z}_{t+1} + \frac{c_1 c_2}{c_1-\delta a_2 b_1}\hat{g}_{t+1}\right)$$

Note that $c_1 c_2 = \frac{\delta\psi}{\bar{\lambda}-\psi}$. Replacing the above in (10.14) we have, provided $c_1 - b_1(d_1-\delta) \neq 0$,

$$\hat{k}_{t+1} = E_t\left[e_1\hat{z}_{t+1} + e_2\hat{k}_{t+2} + e_3\hat{g}_{t+1}\right] \tag{10.16}$$

where,

$e_1 = \frac{d_1[c_1+b_1\delta(a_1-a_2)]}{c_1-b_1(d_1-\delta)}$

$e_2 = -\frac{d_1 b_1}{c_1-b_1(d_1-\delta)} = -\frac{d_1\bar{\varepsilon}_\mu}{c_1-\bar{\varepsilon}_\mu(d_1-\delta)}$

$e_3 = -\frac{d_1 c_1 c_2 b_1}{c_1-b_1(d_1-\delta)} = -\frac{d_1 c_1 c_2\bar{\varepsilon}_\mu}{c_1-\bar{\varepsilon}_\mu(d_1-\delta)}$

Next, we list below some of the other coefficients of the linearized system of equations:

$$k^z = \frac{\phi^z e_1}{1-\phi^z e_2}, k^g = \frac{\phi^g e_3}{1-\phi^g e_2}, \lambda^z = \frac{k^z-\delta a_1}{c_1-\delta a_2 b_1}, \lambda^k = \frac{(1-\delta a_3)}{c_1-\delta a_2 b_1}, \lambda^g = \frac{k^g+c_1 c_2}{c_1-\delta a_2 b_1}$$

$$y^z = \left(\frac{(1+\zeta)-\zeta(1-\alpha)b_1\lambda^z}{1+\alpha\zeta}\right), y^k = \left(\frac{\alpha(1+\zeta)+\zeta(1-\alpha)b_1\lambda^k}{1+\alpha\zeta}\right), y^g = -\frac{\varsigma(1-\alpha)b_1\lambda^g}{1+\alpha\zeta}$$

$$l^z = \frac{\zeta}{1+\alpha\zeta}(1-b_1\lambda^z), l^g = -\frac{\zeta b_1\lambda^g}{1+\alpha\zeta}, l^k = \frac{\zeta}{1+\alpha\zeta}(\alpha+b_1\lambda^k)$$

10.4. Proof of Lemma 2

Using the expression for d_1, $c_1$and b_1 one gets

$$e_2 = -\left\{\frac{\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)}{\delta\left(\frac{1-\alpha}{1+\zeta}\right)\left[1-\left(\frac{1+\zeta}{\delta(1-\alpha)}-1\right)\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)\right]}\right\}$$

$$\frac{1}{e_2} = -\left\{\frac{\delta\left(\frac{1-\alpha}{1+\zeta}\right)\left[1-\left(\frac{1+\zeta}{\delta(1-\alpha)}-1\right)\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)\right]}{\left(\frac{1+\zeta}{1-\alpha}\right)\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\rho)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)}\right\}$$

$$= -\delta\left(\frac{1-\alpha}{1+\zeta}\right)\left[\frac{1}{\left(\frac{1+\zeta}{1-\alpha}\right)\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)}+1-\left(\frac{1+\zeta}{\delta(1-\alpha)}\right)\right]$$

$$= -\left\{\frac{\delta(1-\alpha)}{1+\zeta}\left[1+\frac{1}{\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right)}\right]-1\right\}$$

Using that fact that the steady state $\left(\frac{\alpha\delta}{\rho+\delta}\right)\left(\frac{(\sigma-\eta)}{\left((\eta-\sigma)\bar{\lambda}+\sigma\right)^2}\right) = \left(\frac{\bar{\lambda}-\psi}{\lambda}\right)\bar{\varepsilon}_\mu$, we have

$$\frac{1}{e_2} = -\left\{\frac{\delta(1-\alpha)}{1+\zeta}\left[1+\left(\frac{\bar{\lambda}}{\lambda-\psi}\right)\frac{1}{\bar{\varepsilon}_\mu}\right]-1\right\}$$

Taking modulus, $\frac{1}{|e_2|} = 1-\frac{\delta(1-\alpha)}{1+\zeta}\left[1+\left(\frac{\bar{\lambda}}{\lambda-\psi}\right)\frac{1}{\varepsilon_\mu}\right]$. If $|e_2| < 1$, then $|\frac{1}{e_2}| > 1$ i.e., $1-\frac{\delta(1-\alpha)}{1+\zeta}\left[1+\left(\frac{\bar{\lambda}}{\lambda-\psi}\right)\frac{1}{\varepsilon_\mu}\right] > 1$.

10.5. Proof of Proposition 3

Consider the expression for $e_2 = \left\{1 - \frac{\delta(1-\alpha)}{1+\zeta}\left[1+\left(\frac{\bar{\lambda}}{\lambda-\psi}\right)\frac{1}{\bar{\varepsilon}_\mu}\right]\right\}^{-1}$. Since $\bar{\varepsilon}_\mu = \frac{(\sigma-\eta)\bar{\lambda}}{\bar{\xi}(\bar{\xi}-1)}$ and $\bar{\xi} = (\eta-\sigma)\bar{\lambda}+\sigma$, using the steady state condition $(\bar{\lambda}-\psi)\mu = \frac{\alpha\delta}{\rho+\delta}$, we get $e_2 = C_2\left[\frac{C_1^2}{\eta-\sigma}+C_3\right]^{-1}$ where $C_1 = (\eta-\sigma)\lambda+\sigma > 0$, $C_2 = \frac{\alpha(1+\zeta)}{(\rho+\delta)(1-\alpha)} > 0$ and $C_3 = C_2 - \frac{\alpha\delta}{(\rho+\delta)} > 0$. Now if $\mid e_2 \mid > 1$ when $\eta-\sigma > 0$ then $C_2 > \left[\frac{C_1^2}{\eta-\sigma}+C_3\right]$. This implies that $C_2 - C_3 > \frac{C_1^2}{\eta-\sigma}$, i.e. $\frac{\alpha\delta}{(\rho+\delta)} > \frac{C_1^2}{\eta-\sigma}$ which in turn results in $\eta-\sigma > C_1^2\left(\frac{\alpha\delta}{\rho+\delta}\right)^{-1}$. Since, $\eta-\sigma = \frac{C_1}{\lambda}-\sigma$, the inequality can be rearranged as $\frac{C_1}{\lambda}-\sigma > C_1^2\left(\frac{\alpha\delta}{\rho+\delta}\right)^{-1}$. Again using the steady state condition $(\bar{\lambda}-\psi)\mu = \frac{\alpha\delta}{\rho+\delta}$ and rearranging the expression for optimal markup such that $C_1 = (1-\frac{1}{\mu})^{-1}$, the inequality becomes $\mu C_1 - C_1^2\left(\frac{\lambda}{\lambda-\psi}\right) > \mu\sigma$. Now if $\sigma > 0$ then $\mu - C_1\left(\frac{\lambda}{\lambda-\psi}\right) > 0$ i.e., $\mu - \left(\frac{\mu}{\mu-1}\right)\left(\frac{\lambda}{\lambda-\psi}\right) > 0$. This implies that $\mu > 1+\frac{\lambda}{\lambda-\psi}$. Since $\frac{\lambda}{\lambda-\psi} > 1$ it implies that $\mu > 2$.