On Endogenously Staggered Prices

V. Bhaskar*
University of St. Andrews

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Abstract

Taylor's model of staggered contracts is an influential explanation for nominal inertia and the persistent real effects of nominal shocks. However, in standard imperfect competition models, if agents are allowed to choose the timing of pricing decisions, they will typically choose to synchronize. This paper provides a simple model of imperfect competition which produces stable staggering. Our argument relies on strategic interaction at two levels — between firms within an industries, and across industries — and produces a continuum of stable staggered price equilibria.

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*Dept. of Economics, University of St. Andrews, St. Andrews KY16 9AL, UK. Email: vb1@st-and.ac.uk. I thank Ramon Caminal, Partha Sen, Ted To and seminar audiences at the London School of Economics, St. Andrews, the European Economic Association (Istanbul), and the Econometric Society South and South-East Asia Meeting (Delhi), for helpful comments.

1 INTRODUCTION

An enduring problem for business cycle theory is an explanation for the persistent effects of nominal shocks. Explaining persistence is particularly problematic for theories based on nominal shocks, as opposed to real shocks. This is because rational expectations implies that nominal shocks themselves cannot be serially correlated, since nominal shocks are the deviation of realized nominal magnitudes from expected values. In two important papers, Taylor (1979,1980) provided an argument as to why a nominal shock can have persistent effects.[1] Taylor's argument required two important assumptions. First, he assumed that firms could adjust prices [2] only in alternate periods, an assumption which may be justified by menu or information costs. Taylor's second assumption was that price–setting was *staggered* — half the firms were constrained to adjust prices in odd periods, while the other half were allowed to adjust prices in even periods. These two assumptions combined to provide dramatic results on nominal inertia — the adjustment of prices to a nominal shock is slow, and hence output effects are long lasting. Taylor's results on persistence arise due to strategic complementarity between price-setting decisions. This implies that each firm's optimal price is an increasing function of the aggregate price level. If a shock arrives in an even period, firms in the even cohort will not adjust fully to the nominal shock since firms in the odd cohort have fixed prices in the current period. Furthermore, with rational expectations, the even cohort also takes into account the fact that its own failure to adjust fully will also make for partial adjustment in future periods. This elegant story has made Taylor's model possibly the most influential of "New-Keynesian" models, by providing a dynamic setting within which small nominal rigidities may result in persistent nominal inertia. Indeed, it has also been argued (West (1988); Phaneuf (1990)) that Taylor's model is capable of generating a near random walk in output.[3]

[1] This argument has been developed further by Blanchard (1983, 1986).

[2] Taylor's assumptions pertained to wage setting rather than price setting; however, since similar results follow in either case, we shall follow the subsequent literature, and discuss his model in terms of price setting.

[3] This claim is more controversial. See Chari et. al. (1996) and Ascari (1997) for a contrary view.

Taylor's assumption, of exogenous staggering, plays a critical role in this result — with synchronized price setting, prices take at most one period to adjust fully. This raises the question, can staggering be an equilibrium outcome when we endogenize timing and allow firms to choose between the options of adjusting prices in odd and adjusting price in even periods? The most plausible answer has unfortunately been negative, as Blanchard and Fischer (1988, p400) point out. They conclude that "staggering is unlikely to be an (stable) equilibrium". The reason for this is quite straightforward, and follows from precisely the same strategic complementarity between the firms which plays an important part in Taylor's results. Strategic complementarity implies that each firm would like to adjust its price in the period when most firms are also adjusting price. Let the proportion of firms are adjusting price in even periods be π. If π is greater than one-half, all firms would like to adjust price in even periods, while if π is less than one-half, all firms would like to adjust price in odd periods. This implies that synchronization ($\pi = 0$ or $\pi = 1$) is always a stable equilibrium. Staggering can only be an equilibrium if $\pi = 1/2$, but this equilibrium is unstable, since any small perturbation of π will result in synchronization.

Nor can the addition of idiosyncratic real shocks (i.e. firm-specific real shocks which can arrive in any period) justify staggering. Idiosyncratic shocks imply that any firm has no apriori reason to prefer one period to another, and will hence prefer to adjust in the period when most other firms are also adjusting (see Blanchard and Fischer (1988, p401) for a discussion). To generate stable staggering one needs to assume that some firms receive real shocks mostly in odd periods while other firms receive real shocks mainly in even periods. This assumption, of *asymmetric seasonality* of shocks, is adopted by Ball and Romer (1989). However, as Blanchard and Fischer (1988) point out, the empirical importance of such shocks seems limited.

This paper proposes a model which generates endogenous staggering. Our argument relies on adding one additional level of interaction to the model of imperfectly competitive price-setters which has become standard in this literature (e.g. Blanchard and Fischer). In the standard model the aggregate economy is composed of a large number of individual producers whose products are equally substitutable with each other. Our model introduces the notion of an *industry*, and assumes that products of firms within the same industry are more substitutable with each other than products produced by firms in different industries. This suffices to produce a richer pattern of strategic interaction between producers in this economy, and can generate stable staggering.

The basic intuition for our argument is simple, and relies on the fact that the strategic complementarities are stronger within the industry than across industries. Consider the situation where all firms in any industry adjust price in the same period, and let π now represent the proportion of industries which adjust price in even periods. Let π be close to but less than one-half. A firm in an industry which adjusts in even periods will be more synchronized with the aggregate economy if it switches to adjusting in odd periods; however, it becomes de-synchronized with its own industry by doing so. Since demand is more elastic with respect to the industry price than with respect to the aggregate price, the loss from being de-synchronized with its own industry is greater than the gain from being more synchronized with the aggregate economy, if π is sufficiently close to one-half. Hence this configuration, with synchronization within the industry, but staggering in the aggregate economy, is an strict Nash equilibrium. We show that there exist a continuum of values of π, in an interval $[\underline{\pi}, \bar{\pi}]$, such that if π proportion of industries adjust in even periods and $(1-\pi)$ in odd periods, no firm will have an incentive to change. Furthermore, if π is in the interior of this interval, the equilibrium is stable — if a small number of industries switch timing, this does not influence the timing decisions of other firms in the economy.

The organization of the rest of this paper is as follows. Section 2 sets out the basic model, section 3 analyzes the conditions under which staggering is a stable equilibrium and section 4 the conditions under which synchronization is an equilibrium. The final section concludes.

2 THE MODEL

Our modelling strategy is to use the standard model of an imperfectly competitive macroeconomy, with the modification that we distinguish between individual producers and the industry. Hence our model set out in this section can be considered to be a generalization of the basic macroeconomic model of imperfect competition which has been developed by a number of papers. [4] Assume that there is a continuum of industries distributed uniformly on the unit interval. In each industry, there is a continuum of producer-consumers also distributed uniformly on the unit interval. Index industries by upper case letters, and producers within an industry by lower case letters, so that producer r in industry S has index rS. The utility of the typical producer-consumer is given by:

$$U_{rS} = C_{rS} - \left(\frac{\gamma - 1}{\gamma\beta}\right)(Y_{rS})^{\beta} \tag{1}$$

where C_{rS} is an index of consumption by this producer, and Y_{rS} is the producer's output. The index of consumption is given by

$$C_{rS} = \left\{ \int_{J\in[0,1]} \left(C_{rS}^{J}\right)^{(\theta-1)/\theta} dJ \right\}^{\theta/(\theta-1)} \tag{2}$$

where C_{rS}^{J} is an index of the total consumption of goods from industry J, by producer rS, and is defined by

$$C_{rS}^{J} = \left\{ \int_{i\in[0,1]} \left(C_{rS}^{iJ}\right)^{(\gamma-1)/\gamma} di \right\}^{\gamma/(\gamma-1)} \tag{3}$$

where C_{rS}^{iJ} is the consumption of the good produced by producer i in industry J by consumer rS. The key parameters of our model are β, γ and θ. $(\beta-1)$ is the elasticity of the marginal disutility of labour, and is assumed to be greater than zero. γ is the elasticity of substitution between two products which are produced by the same industry, while

[4] These papers include Akerlof and Yellen (1985) and Blanchard and Kiyotaki (1987), and allow for distinct labor and product markets. We follow Blanchard and Fischer (1988) and Ball and Romer (1989) in dispensing with the labor market, since this interaction does not provide any different insights for our specific purpose.

θ is the elasticity of substitution between two products which are produced in different industries. Our key assumption is that the latter is greater than the former, so that $\gamma > \theta$. Note that if $\gamma = \theta$, our model is very similar to the standard model of Blanchard and Fischer (1988) so that in this sense their model is a special case of ours. We assume that $\gamma > 1$, which is necessary in order to ensure the existence of a solution to the producer's maximization problem. Having assumed this, we need not make any assumption about θ.

The utility function implies that the appropriate industry price index, P^J, and output index, Y^J,are given by

$$P^J = \left\{ \int_{i\in[0,1]} \left(P^{iJ}\right)^{(1-\gamma)} di \right\}^{1/(1-\gamma)} \tag{4}$$

$$Y^J = \frac{1}{P^J} \int_{i\in[0,1]} \left(P^{iJ} Y^{iJ}\right) di \tag{5}$$

The aggregate price level, P, and aggregate output index, Y, are given by

$$P = \left\{ \int_{J\in[0,1]} \left(P^J\right)^{(1-\theta)} dJ \right\}^{1/(1-\theta)} \tag{6}$$

$$Y = \frac{1}{P} \int_{J\in[0,1]} \left(P^J Y^J\right) dJ \tag{7}$$

We follow Ball and Romer (1989), and assume that the relation between money balances and spending on goods is given by

$$Y = \frac{M}{P} \tag{8}$$

An alternative approach, which yields equivalent results is to follow Blanchard and Kiyotaki (1987) and introduce money as an argument in the utility function. We have chosen to adopt the Ball and Romer approach since this allows us to use their welfare results. The above equations imply that the demand faced by producer iJ is given by

$$Y^{iJ} = \left(\frac{P^{iJ}}{P^J}\right)^{-\gamma} \left(\frac{P^J}{P}\right)^{-\theta} \left(\frac{M}{P}\right) \tag{9}$$

We solve first for the static optimal price. The individual producer, with index iJ, takes the industry price P^J, the aggregate price P, and the money supply M as given and chooses price P^{iJ} to maximize utility. We shall use the corresponding lower case letters to denote the natural logarithms of variables (e.g. p is $\ln P$). The optimal price is given by

$$p^*_{iJ} = \alpha_1 p^J + \alpha_2 p + (1 - \alpha_1 - \alpha_2)m \tag{10}$$

where α_1 and α_2 are given by

$$\alpha_1 = \frac{(\gamma - \theta)(\beta - 1)}{1 + \gamma(\beta - 1)} \tag{11}$$

$$\alpha_2 = \frac{1 + (\theta - 1)(\beta - 1)}{1 + \gamma(\beta - 1)} \tag{12}$$

Note that if $\gamma > \theta$, $\alpha_1 > 0$, so that the firm's optimal price is an increasing function of the industry price. In other words, *strategic complementarity at industry level* follows directly from the assumption that the elasticity of substitution is greater for products within the industry than across the industry. On the other hand, α_2 can be either positive or negative, since we do not assume that the industry elasticity of demand θ exceeds one. If $\alpha_2 > 1$, we have *aggregate strategic complementarity* — this is the interesting case, since it gives rise to a stable staggered price equilibrium which also produces nominal inertia.

3 Equilibrium Staggering

We now consider optimal pricing in a dynamic context. Assume that the logarithm of the money supply at date t, m_t, follows a driftless random walk, so that its first difference, Δm_t, has mean zero and constant variance σ^2. Our focus is on *time-dependent* pricing rules. A time dependent pricing rule is optimal if the firm does not know the state (i.e. m_t and other prices), and must incur a fixed information cost each time it finds out about the state. On the other hand, if the costs of changing price are menu costs rather than information costs, state-dependent pricing will be optimal — see Blanchard and Fischer (1988) for a clear discussion of the relative merits of the two types of rule. The microeconomic analysis of the optimal frequency of adjustment in a stochastic environment has been analyzed by Caminal (1991), who shows that this frequency is an increasing function of σ^2 — if the variance of the state variable increases, more frequent adjustment becomes optimal. In addition, the firm's optimal frequency also depends upon the behaviour of other firms, and this may give rise to multiple equilibria in the frequency of adjustment — for a given process for the money supply, there may be equilibria where firms adjust more frequently as well as less frequently.

Since our purpose is to discuss staggering, we shall henceforth assume that the frequency of adjustment is fixed. Specifically, assume that firms find it optimal to adjust every alternate period. The question is, will all firms adjust in the same period or will we have staggering. To verify that a particular configuration is an equilibrium, we compute the average expected loss to a typical firm from conforming to the equilibrium, and compare it with its average expected loss from deviating from this equilibrium. We assume that firms do not discount the future, i.e. formally we are using the limit of means criterion in evaluating alternative policies.[5] Finally, in order to compute the loss of the firm in any period, we use a quadratic approximation to its utility, as is standard in this literature.

We now proceed to construct an equilibrium with staggering. We look for an equilibrium where industries behave differently, but all firms in any industry behave in the same way. Let π be the proportion of industries which adjust price in even periods, so that a proportion $(1-\pi)$ adjust price in odd periods. Since all firms and all industries

[5]Since we have fixed the frequency of adjustment, any firm need compare only two alternative policies — adjusting in odd periods and adjusting in even periods. The limit of means criterion implies that payoffs in any finite number of periods are irrelevant, and hence one need not consider issues such as the optimal timing of transition from one policy to the other.

are symmetrically placed, all firms which adjust price in an even period will set the same price, which we denote by x_t^E . If t is even, $x_t^E = x_{t+1}^E$. Let x_t^O denote the price set by firms which adjust price in odd periods.

We solve now for the firm's optimal price, x_{iJt}. Since the firm does not discount the future, it chooses its price in order to minimize its average loss over the two periods, t and $t+1$. We approximate the loss in each period to the second order, by a quadratic function of the difference between the chosen price and the (static) optimal price. Let p^*_{iJt} be the firm's optimal price in period t for firm iJ. A firm which adjusts price in period t knows the realization of m_t, and can also predict the aggregate price level in period t. Hence its optimal price p^*_{iJt} is known. However, the firm's optimal price for period $t+1$ is a random variable whose realization is not known at date t, since this optimal price depends upon the realization of m_{t+1} , which is not known at date t. Let $E_t p^*_{iJt+1}$ denote the firm's expectation of the optimal price at date $t+1$ given its information at date t. We assume that if firm iJ is able to adjust price in period t, the firm chooses this variable (x_{iJt}) to minimize[6]

$$Z = \frac{(x_{iJt} - p^*_{iJt})^2 + (x_{iJt} - E_t p^*_{iJt+1})^2}{2} \tag{13}$$

This implies that the firm's price is given by

$$x_{iJt} = \frac{(p^*_{iJt} + E_t p^*_{iJt+1})}{2} \tag{14}$$

Let π be the fraction of industries which adjust price in even periods. We approximate

[6]This approximation may be justified as follows. Let $\tilde{x}_{iJt}$ denote the true optimal price for the firm. A quadratic approximation to the firm's utility implies that the firm minimizes the true loss function given by

$$\begin{aligned} \tilde{Z} &= \frac{(\tilde{x}_{iJt} - p^*_{iJt})^2 + (\tilde{x}_{iJt} - E_t p^*_{iJt+1})^2}{2} \\ &= Z + (x_{iJt} - \tilde{x}_{iJt})^2 \end{aligned}$$

Since x_{iJt} differs from $\tilde{x}_{iJt}$ in any regime (staggering or synchronization) by a term of order σ^2 (see Ball and Romer (1987)), the true loss differs from the one used in the text only by terms in σ^4 and higher powers of σ.

the aggregate price level as follows [7]

$$p_t = \pi x_t^E + (1-\pi)x_{t-1}^O \text{ if } t \text{ is even} \tag{15}$$

$$p_t = (1-\pi)x_t^O + (1-\pi)x_{t-1}^E \text{ if } t \text{ is odd} \tag{16}$$

We now solve for the evolution of x_t^E and x_{t+1}^O when t is even. Assume that these are given by

$$x_t^E = \lambda^E x_{t-1}^O + (1-\lambda^E)m_t \tag{17}$$

$$x_{t+1}^O = \lambda^O x_t^E + (1-\lambda^O)m_{t+1} \tag{18}$$

By using the method of undetermined coefficients, we solve for λ^E and λ^O, choosing the stable solutions (i.e. when $\left|\lambda^E\right| < 1$ and $\left|\lambda^O\right| < 1$). These can be written as

$$\lambda^E = K_O - \sqrt{K_O^2 - K_O/K_E} \tag{19}$$

$$\lambda^O = K_E - \sqrt{K_E^2 - K_E/K_O} \tag{20}$$

where K_O and K_E are given by

$$K_E = \frac{1-\alpha_1-\alpha_2\pi}{\alpha_2(1-\pi)} \tag{21}$$

$$K_O = \frac{1-\alpha_1-\alpha_2(1-\pi)}{\alpha_2\pi} \tag{22}$$

Note that λ^E and $\lambda^O$are continuous functions of π on $(0,1)$, and are equal to each other if $\pi = 1/2$. Denote this common value by λ, which is given by

[7] As in footnote 6, this approximation is valid if one wants to approximate the agent's loss to a second order, since the true price level differs from the one we use only by terms in σ^2 and higher powers of σ.

$$\lambda = K - \sqrt{K^2 - 1} \tag{23}$$
$$K = \frac{1 - \alpha_1 - \alpha_2/2}{\alpha_2/2}$$

Let J be an industry where all firms are adjusting price in even periods. Consider firm i in this industry, and compare the loss of this firm from conforming to this policy (i.e. from adjusting in even periods also), with the loss from not conforming, (i.e. switching to a policy of adjusting in odd periods). If firm i is a conformist, its optimal price for period t, where t is even is given by

$$(p^*_{iJt})_c = \alpha_1 x^E_t + \alpha_2[\pi x^E_t + (1-\pi)x^O_{t-1}] + (1 - \alpha_1 - \alpha_2)m_t \tag{24}$$

The conformist's expected optimal price for period $t+1$, given information at date t is

$$(E_t p^*_{iJt})_c = \alpha_1 x^E_t + \alpha_2[\pi x^E_t + (1-\pi)E_t x^O_{t+1}] + (1 - \alpha_1 - \alpha_2)m_t \tag{25}$$

We can now compute the expected per-period loss to a conformist firm, in periods t and $t+1$ where t is even. This loss depends upon the expected change in price of odd industries, $x^O_{t-1} - E_t x^O_{t+1}$,and is given by

$$\mathcal{L}_c(\pi, x^O_{t-1} - E_t x^O_{t+1}) = [(1-\pi)\alpha_2/2]^2 (x^O_{t-1} - E_t x^O_{t+1})^2 + \frac{1}{2}[(1-\alpha_1-\alpha_2) + \alpha_2(1-\pi)(1-\lambda^O)]^2 (\Delta m_{t+1})^2 \tag{26}$$

The first term in the above expression is the anticipated loss that the firm suffers, due to the fact that it expects x^O_{t-1} to differ from $E_t x^O_{t+1}$, given its information at date t. The second term is the unanticipated loss it suffers due to the surprise in the money supply at date $t+1$, which affects its optimal price for $t+1$ directly as well as indirectly, by changing the price of odd industries. Note that in both these terms, a fraction of the loss suffered by the firm arises from the change in price of odd firms, and the coefficient multiplying this is $(1-\pi)\alpha_2$.

We now compute the average expected loss from a *policy* of conformism, over the infinite horizon. This is given by

$$\mathcal{L}_c^{\infty}(\pi) = \left\{[(1-\pi)\alpha_2/2]^2(\frac{1-\lambda^O\lambda^E}{1+\lambda^O\lambda^E}[1+(\lambda^O)^2] + \frac{1}{2}[(1-\alpha_1-\alpha_2)+\alpha_2(1-\pi)(1-\lambda^O)]^2\right\}\sigma^2 \tag{27}$$

Finally, we evaluate this loss function at the point $\pi = 1/2$,

$$\mathcal{L}_c^{\infty}(\pi)|_{\pi=1/2} = \left\{\left(\frac{\alpha_2}{4}\right)^2(1-\lambda^2) + \frac{1}{2}\left[(1-\alpha_1-\alpha_2)+\alpha_2(1-\lambda)\right]^2\right\}\sigma^2 \tag{28}$$

Consider now the optimal price of firm i, given that it is a non-conformist, i.e. given that it is adjusts price in odd periods while its industry (J) adjusts in even periods. Let t be odd.

$$(p^*_{iJt})_{nc} = (1-\alpha_2)(1-\pi)x_t^O + (\alpha_1+\alpha_2\pi)x_{t-1}^E + (1-\alpha_1-\alpha_2)m_t \tag{29}$$

The non-conformist's expected optimal price for period $t+1$, given information at date t is

$$(E_t p^*_{iJt})_{nc} = (1-\pi)\alpha_2 x_t^O + (\alpha_1+\alpha_2\pi)E_t x_{t+1}^E] + (1-\alpha_1-\alpha_2)m_t \tag{30}$$

The loss of a non-conformist is given by

$$\mathcal{L}_{nc}(\pi, x_{t-1}^E - E_t x_{t+1}^E) = [(\alpha_1+\pi\alpha_2)/2]^2(x_{t-1}^E - E_t x_{t+1}^E)^2 + \frac{1}{2}[(1-\alpha_1-\alpha_2)+(\alpha_1+\alpha_2\pi)(1-\lambda^E)]^2(\Delta m_{t+1})^2 \tag{31}$$

Observe that this loss has the same form as (27), and consists of an anticipated loss, due to the change in the price of even firms, and the unanticipated component arising from the surprise in money at date $t+1$, which also affects the price of even firms at date

$t+1$. Note that in both these terms, a fraction of the loss suffered by the firm arises from the change in price of even firms, and the coefficient multiplying this is $\pi\alpha_2 + \alpha_1$. In other words, in the current expression, the loss arising from the price changes of firms in the other cohort includes a term in α_1 in addition, and will hence tend to be larger as compared to (27).

The average expected loss from a non-conformist *policy*, over the infinite horizon, is given by

$$\mathcal{L}_{nc}^{\infty}(\pi) = \left\{ \left(\frac{\alpha_1 + \alpha_2\pi}{2}\right)^2 \frac{1-\lambda^O\lambda^E}{1+\lambda^O\lambda^E}[1+(\lambda^E)^2] + \frac{1}{2}\left[(1-\alpha_1-\alpha_2) + (\alpha_1+\alpha_2\pi)(1-\lambda^O)\right]^2 \right\} \sigma^2 \tag{32}$$

Evaluating the average expected value loss of a non-conformist at $\pi = 1/2$,we have

$$\mathcal{L}_{nc}^{\infty}(\pi)|_{\pi=1/2} = \left[\left(\frac{\alpha_1 + \alpha_2/2}{2}\right)^2 (1-\lambda^2) + \frac{1}{2}\left[(1-\alpha_1-\alpha_2) + (\alpha_1 + \frac{\alpha_2}{2})(1-\lambda)\right]^2 \right\} \sigma^2 \tag{33}$$

An examination of the above expressions shows that the expected loss of a non-conformist ($\mathcal{L}_{nc}^{\infty}(\pi)|_{\pi=1/2}$) exceeds the expected loss of a conformist ($\mathcal{L}_{c}^{\infty}(\pi)|_{p=1/2}$) if $\alpha_1 > 0$. From equation (11) $\alpha_1 > 0$ if $\gamma > \theta$, i.e. the elasticity of substitution between products from the same industry exceeds the elasticity of substitution between products from different industries. This verifies that a situation where all firms within any industry adjust in the same period, and where half the industries adjust in odd periods while the other half of the industries adjust in even periods is a *strict Nash equilibrium* – any firm which deviates from this policy will be strictly worse off.

We now show that there is continuum of strict Nash equilibria — i.e. there exists a non-degenerate interval of values of π, centred around 1/2, such that if π fraction of the industries adjust in even periods and $1-\pi$ fraction of the industries adjust in odd periods, each firm will find it strictly optimal to behave as a conformist. To see this, observe that the loss functions, for a conformist (see equation 27), as well as for a non-conformist (equation 32) are each continuous functions of π. More precisely, each loss function is polynomial in π and λ^E or λ^O, and both λ^E and λ^O are continuous functions of π as long as π is in the interior of the unit interval. Hence $\mathcal{L}_{nc}^{\infty}(\pi)$ and $\mathcal{L}_{c}^{\infty}(\pi)$ are both continuous functions of π. Since $\mathcal{L}_{c}^{\infty} > \mathcal{L}_{nc}^{\infty}$ when $\pi = 1/2$, it follows that this strict inequality also holds when π is sufficiently close to 1/2. Since odd and even periods are indistinguishable, this interval must also be symmetric around 1/2.

Since staggered price setting is a strict Nash equilibrium, it is also stable under any adaptive adjustment or learning rule that firms may adopt. To see this, let $[\underline{\pi}, \bar{\pi}]$ be the interval of values of π such that staggering is a Nash equilibrium, and let π belong to the interior of this interval. Suppose that a small fraction of firms, chosen at random from industries which belong to the even cohort, shift to the policy of adjusting price in odd periods. Since each firm's policy was strictly optimal before this change, this will not induce a change in policy for any other firm in the economy. Furthermore, the firms who have changed policy will find it optimal to shift back to the even cohort. Consider now a second experiment, where all firms in a small fraction of the even industries change policy, and adopt a policy of adjusting in odd periods. It will be optimal for firms in these industries to continue to adjust price in odd periods (if they expect other firms in their industry to also continue), and hence the change will not be reversed. However, this will not affect the incentives of firms in other industries which adjust in even periods, and hence staggering will continue at the aggregate level.

Our explanation for staggering is similar to macro-economic models of coordination failure — both staggering and synchronization (see the following section) are stable equilibria. Firms which adjust price in even periods continue to do so because they expect their industry to adjust in even periods, and because a significant fraction of the economy adjusts in even periods. This raises the question, is the equilibrium with staggering robust to a large monetary shock which hits the economy in a particular period t? Specifically, suppose that t is odd, and suppose also that all firms in the economy are publicly informed about this shock.[8] Would all even firms shift to a policy of adjusting in odd periods, thus

[8]This caveat, that all firms, odd and even are publicly informed about this shock in necessary — as

resulting in synchronization? Not necessarily; even firms might make a one-time price adjustment in period t, but stick to a policy of adjusting in even periods if they expect other even firms to stick to a policy of adjusting in even periods. In concluding section we also discuss informally some modifications of our model which might make staggering even more stable.

We summarize the results of this section in the following proposition.

Proposition 1 *If $\gamma > \theta$, there exists an interval $[\underline{\pi}, \bar{\pi}]$ centred on one- half, such for any π in this interval, there exists a stable equilibrium where all firms in a fraction π of the industries adjust price in even periods, and all firms in a fraction $(1-\pi)$ of the industries adjust price in odd periods.*

4 Equilibrium Synchronization

We now turn to the conditions under which complete synchronization is an equilibrium. Suppose that all firms in all industries adjust prices in even periods. In this case it is clear that if t is even, $p^*_{iJt} = E_t p^*_{iJt+1} = m_t$. Hence $p_t = m_t$ if t is even, and $p_t = m_{t-1}$ if t is odd. The firm's loss from conforming with this policy is given by

$$\mathcal{F}_c^{\infty} = \frac{1}{2}(1 - \alpha_1 - \alpha_2)^2 \sigma^2 \tag{34}$$

If the firm chooses a non-conformist policy of adjusting in odd periods, its optimal prices in periods t and $t+1$, where t is odd are given by

$$p^*_{iJt} = (1 - \alpha_1 - \alpha_2) m_t + (\alpha_1 + \alpha_2) m_{t-1} \tag{35}$$

$$E_t p^*_{iJt+1} = m_t \tag{36}$$

we have discussed, the rationale for time dependent pricing arises from costs of acquiring information about the state rather than menu costs. If all firms are not informed about this shock, it will have no effect on timing decisions.

Hence its price in period t is given by

$$x_{iJt} = m_t - \frac{\alpha_1 + \alpha_2}{2}\Delta m_t \tag{37}$$

The firm's loss from a non-conformist policy is given by

$$\mathcal{F}_{nc}^{\infty} = \frac{1 + [(\alpha_1 + \alpha_2)/2]^2}{2}\sigma^2 \tag{38}$$

Comparing (34) and (38), we see that the latter is always larger, since $\alpha_1 + \alpha_2 > 0$. To verify this note that

$$\alpha_1 + \alpha_2 = \frac{1 + (\gamma - 1)(\beta - 1)}{1 + \gamma(\beta - 1)} \tag{39}$$

which is strictly positive since $\gamma > 1$ and $\beta > 1$.Hence synchronization is always a strict Nash equilibrium, and hence stable, as the following proposition states:

Proposition 2 *Synchronization is a stable equilibrium, for all parameter values.*

Observe that synchronization is stable even if $\alpha_2 < 0$, so that there is no aggregate strategic complementarity. In the absence of aggregate strategic complementarity, it is possible that each *industry* has an incentive not to synchronize — it may be better off by switching to moving in odd periods. However, an individual firm cannot break the synchronized equilibrium, since by doing so, it becomes de-synchronized with its own industry. Indeed, synchronization at the industry level can maintain inefficient aggregate synchronization just as it can maintain inefficient staggering.

5 Discussion

Propositions 1 and 2 show that our model has two types of stable equilibria. While industries may stagger or synchronize price changes, there is complete synchronization within each industry in all these equilibria. It is easy to see that there are no other stable equilibria. Consider any equilibrium where a fraction ξ of firms in industry J adjust price in even periods while a fraction $1-\xi$ adjust price in odd periods. Since this is an equilibrium, it must be optimal for the even firms to adjust in even periods, and for the odd firms to adjust in odd periods. Since all firms in industry J are identical, this establishes that the average expected loss of even firms equals the average expected loss of odd firms. Consider a perturbation of this configuration so that the fraction of even firms increases to $\xi+\epsilon$. Since $\gamma>\theta$, $\alpha_1>0$, and hence the loss of even firms in industry J must now be less than the loss of odd firms in industry J. Hence all odd firms will find it optimal to switch and the equilibrium is not stable.

What are the relative welfare properties of the two types of equilibria identified in the above propositions? In order to calculate welfare, it is not sufficient to compare the average expected losses under the two regimes, i.e. the expressions for $\mathcal{L}_c^{\infty}(\pi)_{|\pi=1/2}$ in (27) and $\mathcal{F}_c^{\infty}$ in (34). Recall that these loss functions are calculated using a linear approximation to the firm's optimal price setting rule, i.e. (14). The true optimal price for the firm differs from this expression by a term which is regime specific, and is of order σ^2 in each of the regimes, staggering and synchronization. This is immaterial for approximating the loss of the agent from any policy to a second order, as footnote 6 demonstrates — the error due to the linear approximation affects the loss only by terms in σ^4 and higher powers of σ. However, in a distorted economy, this has first order effects on welfare — the aggregate price level and mean of real money (as well as its variance) differ from our approximations by a term of order σ^2, and each of these has a first order effect on welfare. A second order approximation to welfare requires that we use a second order approximation to the firm's optimal price-setting rule.[9] These calculations are lengthy as well as complex — indeed, so much so that the published version of Ball and Romer (1989) omits these calculations. Fortunately, one may use the results of Ball and Romer (the discussion paper version of their paper (1987) provides full calculations of their welfare results), and extend these to our model. Our model, with $\gamma=\theta$ coincides exactly with the Ball and Romer (1987,1989) model in the absence of any firm specific seasonal shocks. Ball and Romer show that in this case welfare is strictly greater under

[9] Similarly, one must also use a second order approximation to the aggregate price level.

synchronization than under staggering.[10] Since welfare in each regime is a continuous function of the parameters of the model (in particular γ), this implies that if γ is greater than but sufficiently close to θ, synchronization will continue to be welfare superior to staggering.

We now provide a brief discussion of alternative explanations for staggering. The current paper is most closely related to the work of Ball and Romer (1989), whose welfare comparison of staggered and synchronization has been very useful. However, their explanation for staggering as an equilibrium phenomenon is less compelling, since it hinges upon the assumption of *asymmetric seasonality* of shocks — half the firms receive firm-specific shocks in even periods while the other half receive such shocks in odd periods. Observe that this argument does not work if there is a small fraction of firms which receive asymmetrically seasonal shocks while most firms receive idiosyncratic real shocks in any period. In this case, the firms receiving idiosyncratic shocks would bunch together, and indeed, this may also result in complete synchronization being optimal. However, a small amount of asymmetric seasonality in *conjunction* with our assumption that elasticities of substitution vary between groups of products, may make staggering very stable — indeed, it could even be the unique equilibrium. A rough sketch of this argument is as follows. Suppose that a small group of "even" firms receive real shocks only in even periods, while a small group of "odd" firms receive real shocks only in odd periods, with most firms belonging to the "neutral" category and receiving idiosyncratic shocks in every period. However, some of the neutral firms produce goods which are closer substitutes for the products of odd firms, while other neutral firms supply goods which are more substitutable with the products of even firms. The former group would prefer to adjust in odd periods while the latter prefer even periods, thus giving rise to stable staggering. If the preferences of the non-neutral firms to move in their preferred period are sufficiently strong, this may even rule out synchronization as an equilibrium.[11]

[10] If $\gamma = \theta$, one necessarily has aggregate strategic complementarity. Hence Ball and Romer's results cannot be used to compare welfare in the two regimes when there is no aggregate strategic complementarity.

[11] Ball and Romer also briefly discuss a continuous time version of their model, where idiosyncratic real shocks are given by a Poisson process, and show that there is an equilibrium where firms adjust when the idiosyncratic shock arrives. However, this equilibrium seems an example of state-dependent pricing

Several other explanations rely on individual agents having a strategic influence on the behavior of others, and are hence more microeconomic in spirit. [12] Fethke and Policano (1986) show that staggering is possible if the economy consists of a small number of large firms. Maskin and Tirole (1988) focus on Markov perfect equilibria in a repeated homogeneous duopoly, where firms must keep prices fixed for two periods. With synchronized moves, firms earn zero payoffs in the Bertrand outcome, while with staggered moves, firms earn positive profits in any Markov perfect equilibrium. This ensures that when timing is endogenized, firms will choose to stagger. De Fraja (1993) and Lau (1996) also present models which emphasize similar strategic considerations.

Ball and Cecchetti (1988) provide an argument for staggering which is based on firms being imperfectly informed about demand conditions, and using the behavior of other firms in order to make inferences about demand. Firms are clustered into neighborhoods, and the demand facing the firm depends upon an aggregate shock and a neighborhood specific shock. The firm can observe the sum of these shocks, but is unable to directly observe each component separately. Under staggering, the firm can observe the pricing decisions of its neighbors, and use this information in order to estimate the two shocks separately. The firm's information is maximal if it moves in a period when fewer firms in its neighborhood move, i.e. there is strategic substitutability in the timing decision between firms in the same neighborhood. This ensures that a configuration where half the firms in each neighborhood move in odd periods, with the other half moving in even periods, is a stable equilibrium. Ball and Cecchetti interpret the neighborhood as an industry or geographical area, and on the former interpretation, their model has exactly the opposite empirical predictions as the model of this paper. Whereas our model predicts synchronization at the industry level, their model predicts staggering at the industry level. It is also worth noting that on this interpretation, products will be more substitutable within the neighborhood, and this provides an incentive to synchronize which may offset

rather than time dependent pricing. Under state-dependent pricing, the fact that price changes are not synchronized does not necessarily give rise to aggregate nominal inertia (see Caplin and Spulber (1987) and also Tsiddon (1993)).

[12] In addition to the work we discuss, see also Matsukawa (1986) and Parkin (1986).

the informational incentive to stagger.

The merits of our explanation for staggering at the macroeconomic level are its plausibility and simplicity as well as the fact individual agents have no strategic influence in our model. Indeed, since the individual agent is of measure zero within the industry, the behavior of a single agent has no influence on any other agent in this economy. In our view, the assumption of no strategic influence is an appropriate one to make in the context of a macroeconomic model. Our model also makes strong empirical predictions about the pattern of price setting behavior, since it predicts that price changes will be more synchronized if we consider firms which produce similar products, but less synchronized if firms produce distinct products. This prediction is easily tested, using disaggregated data of the sort which as been used in several studies on the timing of pricing decisions (e.g. Lach and Tsiddon (1992); Domberger and Fiebig (1993)). For example, Domberger and Fiebig (1993) use firm level data from a variety of industries to investigate intra-industry pricing decisions.They do not explicitly analyze inter-industry variation in the timing of pricing decisions, but this would seem easy to do.

Finally, we note that the basic model set out in section 2 of this paper may be useful in analysing other questions in macro-economics. Our innovation is the explicit modelling of intra-industry interaction, in combination with the modelling of inter-industry interaction which is usual in the literature. The insight that this model offers is that coordination failures may be multi-level. Hence the structure of equilibria may be richer than suggested in simple models. We propose to use this model to analyze other issues such as indexation and the dynamics of state-dependent pricing

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