Long-Run Neutrality in a Long-Memory Model

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Abstract. In this paper we use a bivariate, fractionally integrated, autoregressive, moving average model of money and real output to extend Fisher and Seater (1993) long-run neutrality requirements to long-memory processes. We derive new restrictions on the order of the nominal and real variable and discuss their implications when long-run neutrality is tested with a reduced form econometric model. These new restrictions show how finding money to be nonstationary is not sufficient for testing long-run neutrality. A long-memory process can be both nonstationary and mean reverting, meaning an exogenous monetary shock has no long-run effect on money if it is such a process. We also use the fractionally integrated, autoregressive, moving average model to estimate and test the order of integration of money and real output. Since unit root tests have low power against long-memory processes, directly estimating the differencing parameter is more robust to the presence of short and long-memory. Lastly, we apply our long-memory framework to a century worth of annual money supply and real output data for Argentina, Australia, Canada, Italy, Sweden, the UK, and the US and discover that long-run neutrality is testable in six of the seven countries and holds in five of the six countries.

Keywords: Long-Memory, Long-Run Neutrality
JEL Classification: C2; E5

1 Introduction

Recent investigations of the long-run neutrality hypothesis of money (LRN) testing for constrained coefficients in a reduced-form model of money and output have shown that meaningful tests can only be conducted if both the monetary and real variable are nonstationary with a non-zero impulse response function (see King and Watson (1992), Fisher and Seater (1993), Boschen and Mills (1995), Bullard and Keating (1995), Serletis and Krause (1996), and Serletis and Koustas (1998)). This requirement has been expressed by the order of integration of the processes, i.e., in the notation of Engle and Granger (1987) money and output must be $I(1)$ processes. Testing the long-run neutrality of money hypothesis with a zero-restriction coefficient approach thus amounts to the researcher correctly characterizing the long-run properties of the data as that of a unit root process.

Most empirical work testing for LRN in a reduced-form model have used unit root and/or mean/trend stationarity tests to determine the nominal and real variables' order of integration. The theoretical consistency of these unit root and stationarity tests have been proven by assuming that the alternative to the null hypothesis be a short-memory process. There is, however, a larger class of models in which the order of integration is a continuum of real numbers. Aptly called fractionally integrated models, these models not only include the unit root and trend stationary processes, but also include processes exhibiting more complex long-run dynamics. Because of this ability to model long-run behavior series which are fractionally integrated are referred to as long-memory processes.

Sowell (1990) has conjectured that conventional unit root tests have low power against long-memory processes. This has been confirmed by Diebold and Rudebusch (1991) in Monte Carlo simulations with the Dickey-Fuller (1981) unit root test. Concluding that a process is $I(1)$ when it is actually a fractionally integrated series can be very misleading when testing for long-run neutrality. For instance, certain values of integration cause the fractionally integrated model to be nonstationary, but with an impulse response function that decays hyperbolically to zero. If money were such a long-memory process, the real effect of an exogenous monetary disturbance would diminish to zero at such a slow rate one might mistaken the shock as having a permanent effect and incorrectly conclude that tests of long-run neutrality are possible.

In this paper we extend Fisher and Seater (1993) order of integration restrictions necessary for testing LRN within a bivariate autoregressive, moving average (ARMA) framework to orders of integration associated with fractionally integrated models. We then apply this framework of testing long-run neutrality to seven countries who have at least a century worth of annual money supply and real output data.

2 Fractionally Integrated ARMA Models

Let $x(t)$ be the fractionally integrated, autoregressive, moving average [ARFIMA(p,d,q)] process defined by

$$\Phi(L)(1-L)^d(x(t)-\mu)=\Theta(L)\epsilon(t) \tag{1}$$

where

$$\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$$
$$\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$$

are polynomials of degrees p and q, respectively, and whose roots lie outside the unit circle. L is the lag operator, i.e., $L^j x(t) \equiv x(t-j)$, $j \in \{0, 1, 2, \ldots\}$, $\epsilon(t) \sim i.i.d.\mathcal{N}(0, \sigma_\epsilon^2)$, and μ is the unknown mean.

For non-integer values of d, the differencing operator, $(1-L)^d$, is defined by the binomial expansion

$$(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} L^j$$

where $\Gamma(\cdot)$ is the gamma function. When $-1 < d < 0.5$, $x(t)$ is stationary with finite variance and invertible in the sense that the mean square error of the one-step-ahead predictor from a finite-ordered autoregressive model of $x(t)$ converges to σ_ϵ^2. If $-1 < d < 0$, $x(t)$ is said to have intermediate-memory, whereas if $0 < d < 0.5$, $x(t)$ is said to possess long-memory.

If $d \geq 0.5$, one can difference the data until it is stationary, i.e., continue to apply the linear operator $(1-L)$ to $x(t)$ until the transformed process's d is less than 0.5. In this manner any nonstationary ARFIMA series can be made to be stationary just as in the ARIMA case.

It is well known that $x(t)$'s spectrum equals

$$f(\omega) \sim 1/|\omega|^{2d}, \qquad \text{as } \omega \to 0,$$

which approaches infinity when $0 < d$ and has a slow hyperbolically decaying autocorrelation function

$$\gamma(\tau) = |\tau|^{2d-1}$$

whose sum diverges [Granger and Joyeux (1980), Hosking (1981), and Brockwell and Davis (1993)]. Because of this slow decay in $x(t)$'s autocorrelation function it is often mistaken for a nonstationary series and differenced. The transformed series will have a spectrum equal to zero at the origin. Thus, the series has been over-differenced, causing long-run analysis of the transformed series with an ARMA model to be incorrect and misleading.

As pointed out by Sowell (1992b), modeling the long-run dynamics of a series with a short-memory ARMA model is subject to criticism. An ARMA model can only capture the long-run behavior of a series with parameter values close to the boundary of its parameter space, where statistical inference is weak. If the series's long-run dynamics are not captured, then the model's short-run properties will suffer. There is no way for the researcher to direct the short-run parameters to capture the series long-run characteristics even though these are the characteristics she is interested in.

As can be seen by the above power spectrum and autocovariance function for $x(t)$, the ARFIMA model's differencing parameter, d, characterizes the long-run behavior of $x(t)$, while its autoregressive and moving average polynomials, $\Phi(L)$ and $\Theta(L)$, capture short-run properties. Together, they enable the ARFIMA model to provide a parsimonious representation of an economic time series's long and short-run dynamics not possible with a ARMA model.

The ARFIMA model's flexibility enables the Wold representation of $x(t)$,

$$(1-L)(x(t)-\mu) = A(L)\epsilon(t) \tag{2}$$

where

$$A(L) = (1-L)^{1-d}\Phi(L)^{-1}\Theta(L)$$

to be more accurate in accessing the long-run impact of an exogenous shock to $x(t)$. The long-run effect of a unit shock on $x(t)$ equals $A(1)$. When the order of integration is restricted to an integer value, $x(t)$ is incorrectly fit as either a mean/trend stationary model ($d=0$), where $A(1)=0$ ($f(0)<\infty$), or a unit root process ($d=1$), where $A(1)\neq 0$ ($f(\omega)\to\infty$, as $\omega\to 0$).

Since $A(1)$ represents the total effect an innovation has on $x(t)$, in other words, the value of the spectrum at frequency zero, the flexibility of the ARFIMA model's spectrum to take on a wide variety of shapes expands the possible values of $A(1)$. When d is a non-integer, the binomial series portion of the moving average polynomial, $(1-L)^{1-d}$, can be written in terms of the hypergeometric function, giving

$$A(L) = F(d-1, 1; 1; L)\Phi(L)^{-1}\Theta(L),$$

where the hypergeometric function is defined as

$$F(\alpha, \beta; \gamma; L) = \sum_{j=0}^{\infty} \frac{\Gamma(\alpha+j)\Gamma(\beta+j)\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma+j)\Gamma(j+1)} L^j.$$

It is well known (Gradshteyn and Ryzhik (1994), p. 1066) that $F(\cdot)$:

1. Converges for $L = 1$, when $\alpha + \beta - \gamma = d - 1 < 0$,

2. Diverges for $L = 1$, when $\alpha + \beta - \gamma = d - 1 \geq 0$.

Applying these two conditions to $A(L)$, an innovation will have a permanent effect on $x(t)$ if $1 \leq d$, whereas the effect of the innovation will dissipate to zero when $d < 1$. Thus, although $x(t)$ is a nonstationary process when $0.5 \leq d < 1$, its cumulative impulse response to a exogenous shock will still be zero.

Even though the long-run effect of a unit shock to a fractionally integrated series of order less than one is zero, the energy found in its spectrum near the origin suggests that the finite persistence could last for years. Contrasting this with the long-run neutrality requirement that an exogenous shock to the money supply must have an infinitely long effect, inferences made with an ARFIMA model concerning the long-run are more likely to be correct than those made with a unit root model. A unit root process's response to a stochastic shock is never ending. Such dynamics requires an infinite number of observations to accurately model and since data is finite, a researcher will never have enough observations to make a meaningful inference about the series's infinite behavior with the unit root model.

3 Fractional Integration Long-Run Neutrality Restrictions

For the most part we follow the notation found in Fisher and Seater's (1993) bivariate ARMA system where m is the log of the nominal money supply and y is the log of real output. However, given our use of the ARFIMA model, our notation for the orders of integration differs with $d_m, d_y \in \Re$ denoting respectively the orders of integration for m and y.

We assume m and y constitute a bivariate ARFIMA system whose autoregressive representation is given by

$$\begin{aligned} a(L)(1-L)^{d_m} m(t) &= b(L)(1-L)^{d_y} y(t) + u(t) \\ d(L)(1-L)^{d_y} y(t) &= c(L)(1-L)^{d_m} m(t) + w(t) \end{aligned} \tag{3}$$

where $a(L)$ and $d(L)$ are p-order polynomials, $b(L)$ and $c(L)$ are q-order polynomials, and the vector of innovations, $(u(t), w(t))' \sim iid(\mathbf{0}, \mathbf{\Sigma})$, where $\mathbf{0} = (0,0)'$ and

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{uu} & \sigma_{uw} \\ \sigma_{uw} & \sigma_{ww} \end{bmatrix}.$$

As in Fisher and Seater (1993), Eq. (3) is econometrically unidentified. We adopt Fisher and Seater's identifying restriction and assume the long-run exogeneity of money and the independence between the exogenous shocks to m and y, i.e., $b(1) = \sigma_{uw} = 0$. Under these restrictions and the long-run neutrality restrictions to be shown below, the frequency-zero regression coefficient from regressing $(1-L)^{d_y} y$ on $(1-L)^{d_m} m$ measures the long-run response of y to a permanent unit change in money. Testing the hypothesis that changes in the money-supply have no affect on real output amounts to testing if the frequency-zero regressions coefficient equals zero.

Fisher and Seater (1993) summarize the above test of long-run neutrality by computing the long-run derivative of y with respect to m,

$$LRD_{y,m} \equiv \lim_{k \to \infty} \frac{\partial y(t+k)/\partial u(t)}{\partial m(t+k)/\partial u(t)},$$

where the order of integration for y and m are restricted to non-negative integers. In the more general context of fractional orders of integrations, Fisher and Seater (1993) $LRD_{y,m}$ formula continues to convey the total impact an exogenous shock to money will have on output.

Both the numerator and denominator of $LRD_{y,m}$ can be evaluated separately using the moving average representations of m and y

$$m(t) = (1-L)^{-d_m} \left[\alpha(L) u(t) + \beta(L) w(t)\right]$$

$$y(t) = (1-L)^{-d_y}\left[\gamma(L)u(t) + \lambda(L)w(t)\right] \tag{4}$$

where α, β, γ, and λ are functions of the lag polynomials found in Eq. (3). The impact of a change in $u(t)$ on m and y is computed with the Wold representation

$$\begin{aligned}(1-L)m(t) &= \Theta(L)u(t)\\ (1-L)y(t) &= \Xi(L)u(t)\end{aligned} \tag{5}$$

where $\Theta(L) = (1-L)^{1-d_m}\alpha(L)$ and $\Xi(L) = (1-L)^{1-d_y}\gamma(L)$. Thus, the long-run effect of an exogenous money supply disturbance on m and y is $\Theta(1)$ and $\Xi(1)$, respectively. If $d_m \geq 1$, the hypergeometric function, $F(d_m - 1, 1; 1; 1) = (1-L)^{1-d_m}|_{L=1}$, diverges causing $\Theta(1)$ to be nonzero and $LRD_{y,m}$ to equal

$$\begin{aligned}LRD_{y,m} &= \frac{\Xi(L)|_{L=1}}{\Theta(1)}\\ &= \frac{(1-L)^{d_m-d_y}\gamma(L)\Big|_{L=1}}{\alpha(1)}.\end{aligned} \tag{6}$$

Fisher and Seater (1993) define money to be long-run neutral if $LRD_{y,m} = 0$. Using the properties of the hypergeometric function found in Section 2 and the possible fractional nature of m and y's order of integration, we rewrite Fisher and Seater's four cases, (*Case (i)-(iv)*), of $LRD_{y,m} = 0$ and add three new restrictions.

3.1 Restrictions

Case(i). — When $d_m \in (0,1)$, $LRD_{y,m}$ is not defined since,

$$\begin{aligned}\lim_{k\to\infty}\frac{\partial m_{t+k}}{\partial u_k} &= (1-L)^{1-d_m}\alpha(L)\Big|_{L=1}\\ &= F(d_m - 1, 1; 1; 1)\alpha(1)\\ &= 0\cdot\alpha(1)\end{aligned}$$

when $d_m - 1 < 0$.[1]

Since m is a long-memory process permanent changes to the money supply can not occur. The effects of an exogenous shock to m will eventually decay to zero. This case is

[1]If $d_m \in (0,1)$, $d_y \geq 1$ and $\gamma(1) \neq 0$, then $LRD_{y,m} \to \infty$ since $F(d_y - d_m, 1; 1; 1) \to \infty$.

plagued by the criticisms of Lucas (1972) and Sargent (1971); testing for long-run neutrality is impossible with a reduced form model when there is no sustained change to money.

Case(ii). — When $d_m \geq 1$ and $d_y \in (0,1)$, $LRD_{y,m} \equiv 0$ since,

$$\begin{aligned}
\lim_{k\to\infty} \frac{\partial y_{t+k}}{\partial u_t} &= (1-L)^{1-d_y}\gamma(L)\Big|_{L=1} \\
&= F(d_y - 1, 1; 1; 1)\gamma(1) \\
&= 0 \cdot \gamma(1)
\end{aligned}$$

when $d_y - 1 < 0$. It directly follows that LRN holds.

Although m is permanently affected by an exogenous shock, this change in the monetary level will not permanently affect y. Since y is fractionally integrated of order less than one, it is a mean reverting process whose dynamics slowly converge to those observed before the shock. Thus, the quantity theory of money holds, output will be unaffected in the long-run by a change in monetary policy.

Case(iii). — When $d_m = d_y \geq 1$, $LRD_{y,m}$ implies

$$LRN = \frac{c(1)}{d(1)}.$$

Except for the possibility of fractional orders of integration, this case is the same as Fisher and Seater's (1993) *Case(iii)*. If the orders of integration for m and y are equal and at least equal to one, exogenous shocks will have a permanent affect on m and y. Under these conditions the long-run neutrality hypothesis can be tested by calculating $LRD_{y,m}$ and determining if it is significantly different from zero.

Case(iv). — When $d_m \geq 1$, $d_y \geq 2$ and $d_m = d_y - 1$, LRN implies

$$\begin{aligned}
LRD_{y,m} &= \frac{(1-L)^{-1}c(1)}{d(1)} \\
&= \frac{c^*(1)}{d(1)}
\end{aligned}$$

where $c^*(L) = (1-L)^{-1}c(L)$.

This is a generalization of Fisher and Seater's (1993) *Case(iv)* to fractional orders of integrations. The LRD restriction has the same interpretation as given to it by Fisher and Seater; a permanent change in the monetary stock might have a permanent effect on the growth rate of y when $d_y \geq 2$ and $c^*(1)/d(1)$ does not equal zero.

Case(v). — When $1 \leq d_y < d_m$, $LRD_{y,m} \equiv 0$ since,

$$\begin{aligned} LRD_{y,m} &= \frac{(1-L)^{d_m-d_y}\gamma(L)\Big|_{L=1}}{\alpha(1)} \\ &= F(d_y - d_m, 1; 1; 1)\frac{\gamma(1)}{\alpha(1)} \\ &= 0 \cdot \frac{\gamma(1)}{\alpha(1)} \end{aligned}$$

when $d_y - d_m < 0$. Hence, LRN holds.

The intuition for *Case(v)* can be found by differencing both m and y, $d_m - 1$ times to obtain m^* and y^*. The above formula for $LRD_{y,m}$ continues to hold since differencing m and y is equivalent to multiplying the numerator and denominator of $LRD_{y,m}$ by $(1-L)^{d_m-1}$. Because the differenced series m^* is by definition a unit root process, an exogenous shock will have a permanent affect on m^*. However, the filtered process y^* is a fractionally integrated process with a differencing parameter less than one. Since such processes are mean reverting, permanent changes to y^* are not possible.[2]

Case(vi). — When $1 \leq d_m < d_y$, $LRD_{y,m}$ is not defined since,

$$\begin{aligned} (1-L)^{d_m-d_y}\Big|_{L=1} &= F(d_y - d_m, 1; 1; 1) \\ &\to \infty \end{aligned}$$

when $d_y - d_m > 0$.[3]

The same intuition holds for *Case(vi)* as did for *Case(i)*. Differencing m and y by $(1-L)^{d_y-1}$ we obtain m^* and y^*. Although y^* will be permanently affected by an exogenous shock, m^* is a long-memory process with $d_{m^*} \in (0,1)$. The money supply can never be permanently affected by an exogenous shock, hence, the neutrality of money can not be addressed.

Case(vii). — When $1 \leq d_{\Delta y} < d_m$, where $d_{\Delta y} = d_y - 1$, LRN implies

$$LRD_{y,m} = \frac{(1-L)^{d_m-d_y}\gamma(L)\Big|_{L=1}}{\alpha(1)}$$

[2]Although economic data is rarely $I(3)$ or larger, which is not surprising given that economic theory has little to say about series that are integrated of order three or greater, the above intuition still holds when the order of integration of m and y are both greater than some positive integer.

[3]If $\gamma(1) = 0$ then $LRD_{y,m} \equiv 0$ and LRN holds.

Case	Relative Order of Integration	$LRN \Rightarrow LRD_{y,m}$
i)	$0 < d_m < 1$	*undefined*
ii)	$0 < d_y < 1 \leq d_m$	$\equiv 0$
iii)	$1 \leq d_m = d_y$	$c(1)/d(1)$
iv)	$1 \leq d_m = d_{\Delta y}$	$c^*(1)/d(1)$
v)	$1 \leq d_y < d_m$	$\equiv 0$
vi)	$1 \leq d_m < d_y$	*undefined*
vii)	$1 \leq d_{\Delta y} < d_m$	$\equiv 0$

Table 1: Long-run neutrality restrictions

$$
\begin{aligned}
&= \frac{(1-L)^{d_m - d_{\Delta y} - 1}\gamma(L)\Big|_{L=1}}{\alpha(1)} \\
&= F(d_{\Delta y} - d_m, 1; 1; 1)\frac{\gamma^*(1)}{\alpha(1)} \\
&= 0 \cdot \frac{\gamma^*(1)}{\alpha(1)},
\end{aligned}
$$

where $\gamma^*(L) = (1-L)^{-1}\gamma(L)$, when $d_{\Delta y} - d_m < 0$. In other words, the permanent effect from an exogenous shock to the level of money will not have a permanent effect on the growth rate of y. Each of the LRD conditions are summarized in Table 1.

4 Evidence of Fractional Long-Run Neutrality

4.1 Data

Since the characteristics of long-memory and our tests of long-run neutrality depend on the low frequency dynamics of the monetary and output series, a long time-horizon is needed to test for the presence of long-run neutrality in a long-memory model. We test for long-run neutrality using over a century worth of low frequency data for Argentina [Bae (1998)], Australia, Canada, Italy, Sweden, the United Kingdom, and the United States [Backus and Kehoe (1992)].[4] Each country's data set contains annual observations over the general

[4]The data compiled by Backus and Kehoe (1992) also contains data for other developed countries, e.g., Denmark, Germany, and Japan. However, these data sets suffer from five to ten consecutive years of missing observations, and hence, we exclude them from our study of long-run neutrality.

period 1870 to 1970 for the country's monetary stock, M, and real output, Y.[5] In our discussion below, we denote the logarithmic transformation of M and Y by their lowercase letter and Δm and Δy as their logged first differences.

4.2 Order of Integration

To determine which of the cases in Table 1 the seven countries satisfy, we employ Sowell's (1992a) exact maximum likelihood approach in estimating an array of ARFIMA(p,d,q) models, where $p, q = 0, 1, 2, 3$, for each country's Δm and Δy.[6] Sowell (1990) and Diebold and Rudebusch (1991) both show unit root tests to have low power against long-memory processes, whereas Canova (1994) urges the use of alternative tests to the unit root test in discerning the true long-run characteristics of the data. By estimating an ARFIMA model we replace the restrictive knife-edge approach of the unit root test, where the process's order of integration must be either $I(0)$ or $I(1)$, with a continuum of stationary and non-stationary orders of integration, $I(d)$. This continuum lends itself to standard hypothesis testing.

Since our interest is in how an exogenous shock affects the future behavior of money and output, we use the Schwartz Information Criterion (SIC) to choose the best ordered ARFIMA model out of he sixteen estimated models. Table 2 contains the parameter estimates (standard deviations in parenthesis) and the value of the log-likelihood function, $\mathcal{L}$, for each country's best SIC selected ARFIMA model of monetary and output growth rates. Because the growth rate of a series, x, equals $(1 - L)x$, the order of integration for the level series equals, $d_x = 1 + d_{\Delta x}$. Hence, the order of integration for money and output equal one plus the differencing parameter estimates found in Table 2.

[5]Bae (1998) merges the data collected by Della Paolera (1988) and Domenech (1986) with the International Financial Statistics (IFS) to create an annual series that goes from 1884-1996.

[6]The exact maximum likelihood estimator can lead to incorrect inference if the orders of the ARFIMA model are not correctly specified. A possible alternative to the exact MLE would be one of the semi-parametric estimation methods of Geweke and Porter-Hudak (1983), Jensen (1999a) and Jensen (1999b).

Country	Variable	d	ϕ_1	ϕ_2	θ_1	$\mathcal{L}$
Argentina	Δy	-0.1550	0.5931*			121.6397
		(0.2102)	(0.1836)			
	Δm	0.4504*	0.6411*	-0.3698*		-12.7642
		(0.0607)	(0.1034)	(0.0898)		
Australia	Δy	0.0040				138.5244
		(0.0853)				
	Δm	0.2169*				140.7538
		(0.0822)				
Canada	Δy	-0.4953*	0.6975*			159.8583
		(0.2119)	(0.1700)			
	Δm	0.2588*				150.2857
		(0.0850)				
Italy	Δy	0.0743				132.6292
		(0.0855)				
	Δm	-0.0178	0.8542*			147.9991
		(0.1394)	(0.0874)			
Sweden	Δy	0.0462				217.7343
		(0.0883)				
	Δm	-0.2627	0.7906*			146.9351
		(0.1599)	(0.1155)			
U.K	Δy	-0.3024	0.6169*			214.8545
		(0.1833)	(0.1540)			
	Δm	-0.1800	0.7850*		0.6091*	229.7252
		(0.2145)	(0.1287)		(0.1371)	
U.S	Δy	-0.5225*	0.6219*			151.5620
		(0.2179)	(0.1908)			
	Δm	-1.7157*	1.8081*	-0.8463*	0.4638*	179.9065
		(0.2004)	(0.0857)	(0.0793)	(0.1202)	

Table 2: Exact maximum likelihood estimates of the best SIC selected ARFIMA models with standard deviations contained in parenthesis. $*$ denotes significance at the 1% level.

4.3 Money

4.3.1 Temporary effects

Except for US money, every country's estimated order of integration for the logged monetary stock satisfies Section 3's necessary conditions for testing long-run neutrality. Unlike Fisher and Seater (1993), Serletis and Krause (1996), and Serletis and Koustas (1998), who each find US money to be $I(1)$, our estimate of $d_{\Delta m} = -1.726$ for the US produces a Wald-stat that rejects at the 1% significance level the null hypothesis, $H_0 : d_{\Delta m} = 0$, in favor of a negative order of integration. From this finding we conclude that US money is fractionally integrated of positive order, but significantly less than the $I(1)$ requirement necessary for testing long-run neutrality.

Because US money has a negative order of integration for its growth rate, the monetary stock's is stationary with an order of integration that is less than 0.5 and hence, can be directly estimated. We estimate the sixteen ARFIMA(p,d,q) models, where $p, q = 0, 1, 2, 3$, for the US money supply and find the best SIC model to equal

$$(1 - \underbrace{0.9949}_{0.0065} L)(1 - L)^{\underbrace{0.3624}_{0.0817}} m_t \quad = \quad (1 + \underbrace{0.4587}_{0.0984} L)\epsilon_t \tag{7}$$

with $\hat{\sigma}^2 = 0.0024$, $\mathcal{L} = 165.75$, and the curly braces denoting the parameter's standard deviation.

The order of integration, $d_m = 0.3624$, and the first-order autoregressive parameter, $\phi_1 = 0.9949$, in Eq. (7) are both significantly less than the minimum unit value required for testing long-run neutrality. In particular, the Wald-stat for the $I(1)$ null equals 4.44, and 153.06 for $H_0 : \phi_1 = 1.0$, both significant at the 1% level. Thus, in the words of Fisher and Seater (1993), "there are no permanent stochastic changes in m, so LRN is not addressable," in the United States.

With an order of integration so close to the nonstationary boundary value of 0.5, and an autoregressive parameter so near to one, the estimated model in Eq. (7) gives reason for why Fisher and Seater (1993), Serletis and Krause (1996) and Serletis and Koustas (1998) all fail to reject the presence of a unit root in US money. As stated earlier, unit root tests have very low power against long-memory processes. The estimated ARFIMA model in Eq.

(7) shows US money to follow a long-memory process ($d_m = 0.36$) with a short-memory parameter ($\phi = 0.995$) close to the boundary of the parameter space. Both parameter values contribute to the long-memory behavior of US money. An autoregressive parameter close to one contributes to the long-memory process's slow hyperbolically decaying response to an exogenous shock. This slow often unnoticeable decay leads many to incorrectly infer that a stochastic shock to US money does have a permanent effect. This, however, is not the case. Although the effect of an exogenous shock will be long lasting, it will eventually decay to zero.

4.3.2 Permanent effects

The differencing parameter estimates in Table 2 for the growth rate of money in Argentina, Australia, Canada, Italy, Sweden, and the UK can not reject the hypothesis that an exogenous shock to their money supply will have a permanent effect. The nature of these permanent effects fall into one of two groups:

- Group A: countries whose monetary series is a unit root process, i.e., $d_m = 1$.
- Group B: countries whose monetary series is fractionally integrated, i.e., $d_m > 1$.

4.3.3 Unit roots in money

With the p-values of the Wald-stat testing the hypothesis, $d_{\Delta m} = 0$, contained in parenthesis, the money supply in Italy (0.4493), Sweden (0.0520), and the UK (0.2017) are integrated of order one. The presence of a unit root in these three country's money supply is also supported by the likelihood ratio test (LR-test) between the restricted ARMA ($d_{\Delta m} = 0$) and the unrestricted ARFIMA model. With the p-value of the LR-test in parenthesis, the unrestricted ARFIMA models provide no more explanatory power about the long-run behavior of the growth rate of money in Italy (0.3086), Sweden (0.8278) and the UK (0.5632) than is explained by the restricted ARMA model's estimated parameters found in Table 3. The presence of a unit root in these country's monetary series insures that an exogenous shock will have a permanent affect on their money supply, thus allowing us to test for the presence of long-run neutrality.

Country	Variable	ϕ_1	ϕ_2	θ_1	$\mathcal{L}$
Argentina	Δy	0.5931*			121.6397
		(0.1836)			
	Δm	1.0554*	-0.2715*		-18.3959
		(0.0910)	(0.0910)		
Canada	Δy	0.2701*			157.9077
		(0.0935)			
Italy	Δm	0.8448*			147.9913
		(0.0506)			
Sweden	Δm	0.5957*			146.0033
		(0.0845)			
U.K	Δy	0.3880*			213.5816
		(0.0900)			
	Δm	0.6817*		0.5439*	229.4229
		(0.0845)		(0.1182)	
U.S	Δy	0.1865*			148.8413
		(0.0952)			
	Δm	0.3139	0.0456	0.4274	173.9110
		(0.3919)	(0.2692)	(0.3757)	

Table 3: Maximum likelihood estimates of the restricted ($d = 0$) ARFIMA models with standard deviations contained in parenthesis. $*$ denotes significance at the 1% level. Note: The restricted ARFIMA models for monetary growth in Canada and real output growth in Italy and Sweden can not be identified since their best unrestricted models were ARFIMA$(0, d, 0)$.

Using a number of different unit root and mean/trend stationarity tests, Serletis and Koustas (1998) found Italy's money supply to be an $I(2)$ process, i.e., exogenous shocks were thought to have a permanent affect on the growth rate of Italian money. This finding satistfied the necessary condition for testing long-run superneutrality. Serletis and Koustas, however, could find no evidence in support of superneutrality for Italy.

Serletis and Koustas (1998) rejection of long-run superneutrality can be explained by the ARFIMA model's estimates of Italy's monetary growth rate. The order of integration estimate in Table 2 for the growth rate of the Italian money supply, $d_{\Delta m} = -0.0178$, is significantly less than the necessary unit order of integration required for testing superneutrality. The order of integration is negative with little probability of differing from zero. Hence, it is unlikely that the growth rate of the money supply in Italy will be permanently affected by a stochastic shock. Rather, it is more likely that a shock will have at most a very short-lived affect on the growth rate of money, if any affect at all. Long-run superneutrality of Italian money is not "addressable" under the current methodology of calculating the LRD, and any inferences made concerning the acceptance or rejection of superneutrality is mere speculation.

4.3.4 Fractionally integrated money

As can be seen by the p-values in parenthesis, the Wald-stat for the null of a unit root in the monetary series of Argentina (1.0), Australia (0.9952), and Canada (0.9985), are all significantly greater than commonly accepted critical values. Each test finds in favor of a persistent, fractionally integrated monetary series that will be permanently affected by an exogenous shock.

The growth rate of Argentina's money supply produces the following estimate for the restricted ARFIMA(2,0,0) model,

$$(1 + 1.06L - 0.27L^2)\Delta m(t) = \epsilon(t) \tag{8}$$

where $\hat{\sigma}^2 = 0.0804$, and $\mathcal{L} = -18.40$. This also produces a LR-test statistic of 11.263, which rejects at the 1% level the model of Eq. (8) in favor of the long-memory model.

Because the best fitting ARFIMA models in Table 2 for monetary growth in Australia and Canada have neither an autoregressive or moving average component, or in other words, their monetary growth rate follows a Gaussian fractionally integrated process, the restricted ARFIMA model can not be identified. We, instead, plot in Fig. 1 the theoretical power spectrums using the parameters from the best SIC selected ARFIMA and the parameter estimates from the best SIC selected short-memory ARMA model of the monetary growth rate in Australia

$$\Delta m(t) = (1 + \underbrace{0.3043}_{0.1008} L)\epsilon(t) \tag{9}$$

where $\hat{\sigma}^2 = 0.0035$ and $\mathcal{L} = 140.96$, and in Canada

$$\Delta m(t) = (1 + \underbrace{0.2765}_{0.0857} L)\epsilon(t) \tag{10}$$

where $\hat{\sigma}^2 = 0.0034$ and $\mathcal{L} = 149.92$, against the series empirical power spectrum.

Inspection of the theoretical spectrums in Fig. 1 shows the tracking ability of the ARFIMA model and inability of the ARMA model. The revealing point is the upward movement in the ARFIMA model's power spectrum and the constancy in the ARMA model's spectrum as frequencies approach the origin. The ARMA model's theoretical spectrums fail to capture any of the long-run energy found in the two series'ss periodogram. Because the shape of the power spectrum near the origin is highly dependent on the value of the differencing parameter, the value of $d_{\Delta m}$ produced by the ARFIMA models are better in explaining the long-run behavior of Δm than the short-memory ARMA model.

4.4 Existence of Long-Run Neutrality

4.4.1 Output

To test for long-run neutrality in the six countries whose monetary series satisfies the necessary integration of order one or greater condition, we first compute the Wald-stat under the null that real output in these countries is integrated of order zero, i.e., $d_{\Delta y} = 0$. Using the estimates of $d_{\Delta y}$ found in Table 2, we find the output series for Argentina (0.2315), Australia (0.5187), Canada (0.0107), Italy (0.8066), Sweden (0.6990) and the UK (0.0510) all fail to reject the unit root hypothesis (p-values in parenthesis).

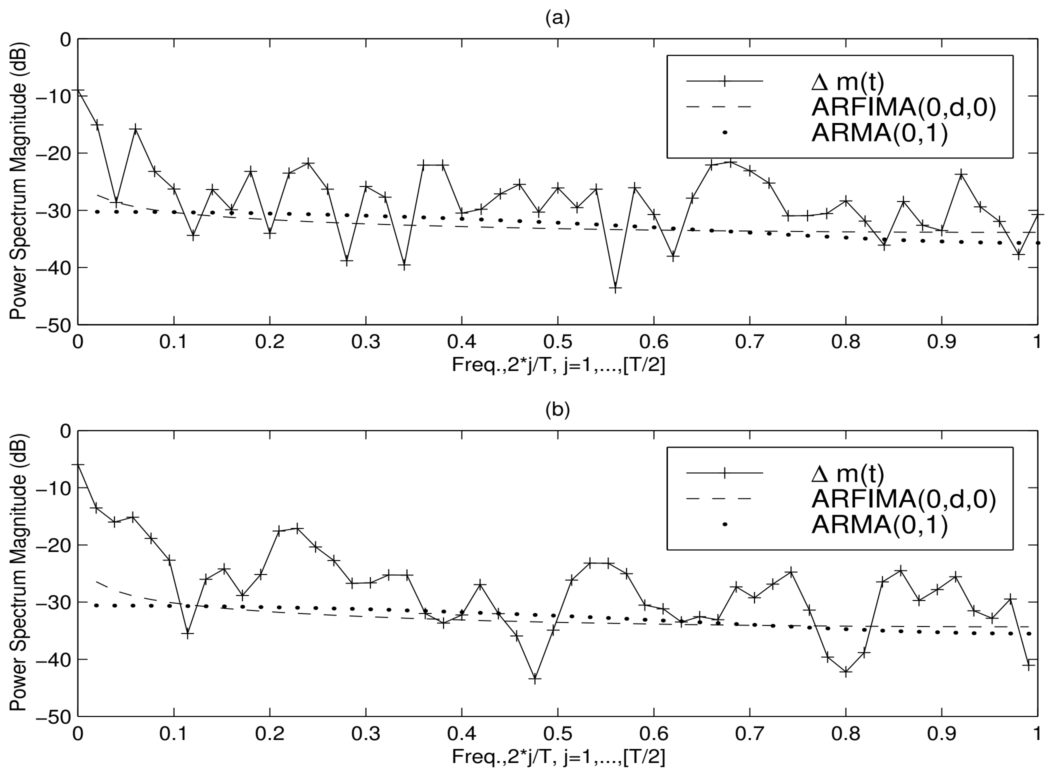

Figure 1: Power spectrum $[10 \log_{10} f(\omega)]$ for the estimated ARFIMA and ARMA models plotted over the periodogram of the first log difference of money-supply for (a) Australia and (b) Canada.

Long-run neutrality test results for Group A

Country	d_y	*Case*	$LRD_{y,m}$	LRN
Italy	$d_y = 1$	*iii*	$c(1)/d(1) \neq 0$	$-$
Sweden	$d_y = 1$	*iii*	$c(1)/d(1) = 0$	$+$
UK	$d_y = 1$	*iii*	$c(1)/d(1) = 0$	$+$
	$d_y < 1$	*ii*	0	$+$

Long-run neutrality test results for Group B

Country	d_y	*Case*	$LRD_{y,m}$	LRN
Argentina	$d_y = 1$	*v*	0	$+$
Australia	$d_y = 1$	*v*	0	$+$
Canada	$d_y = 1$	*v*	0	$+$
	$d_y < 1$	*ii*	0	$+$

Table 4: Long-run neutrality tests for Group A countries (those countries where a unit root in their money supply can not be rejected) and Group B countries (those countries whose money supply are fractionally integrated of order greater than one).

4.4.2 Sweden

Our inferences of the order of integration for Sweden's monetary and output series are no different from those found by Serletis and Krause (1996) and Serletis and Koustas (1998). Both money and output are integrated of order one for Sweden and by *Case (iii)* we infer from Serletis and Krause (1996) result of $LRD_{y,m} = 0.078$ that long-run neutrality holds. This result of LRN in Sweden and the LRN results for the other five countries are summarized in Table 4.

4.4.3 United Kingdom

If we are willing to accept a Type I error of 5% instead of 1%, the unit root hypothesis can be rejected for the UK output series. Assuming that we are willing to bear the possible repercussions, the estimated order of integration for total goods and services in the UK is now significantly less than one. Although not a unit root process, we do not conclude as do Serletis and Krasue (1996) that UK output is integrated of order zero. Instead, we find UK output to follow a nonstationary, non-persistent, fractionally integrated process of order, $I(0.7)$, where any exogenous shock caused by a change in monetary policy, a supply shock, or any other stochastic disturbance to output, will slowly dissipate away. It immediately follows from the conditions found in *Case (ii)* that long-run neutrality is trivially accepted for the UK.

If, however, we are unwilling to accept the increased possibility of rejecting the presence of a unit root in UK output when in fact a unit root does exist, then $d_m = d_y = 1$ and by *Case (iii)* a estimate of $c(1)/d(1)$ is required to test LRN. We apply the Bartlett estimator of the frequency-zero coefficient to the series Δy and Δm to estimate $c(1)/d(1)$. In other words, we compute the ordinary least squares estimator of the equation

$$H(L)(1-L)y(t) = \alpha_k + \beta_k H(L)(1-L)m(t) + e_k(t). \tag{11}$$

where $H(L) = (1 + L + L^2 + \cdots + L^k)$ is a moving average filter, and set $c(1)/d(1) = \hat{\beta}_k$ as $k \to \infty$, where $\hat{\beta}_k$ is the OLS estimator of β_k.

The results from the Bartlett frequency-zero estimator of $c(1)/d(1)$ using UK output and money support the existence of LRN. In Fig. 2 the OLS estimators, $\hat{\beta}_k$, for $k = 1, \ldots, 30$, from Eq. (11) are plotted along with the Newey-West adjusted 95% confidence intervals. For every value of k, $\hat{\beta}_k$ is insignificant and never more than 0.1 away from origin. As k increases, $\hat{\beta}_k$ is nearly zero. Hence, $c(1)/d(1)$ equals zero and LRN is accepted.

4.4.4 Argentina and Australia

Both Argentina and Australia's monetary series are fractionally integrated of order larger than one and have an output series that is an $I(1)$ process. The Wald-stat of the null

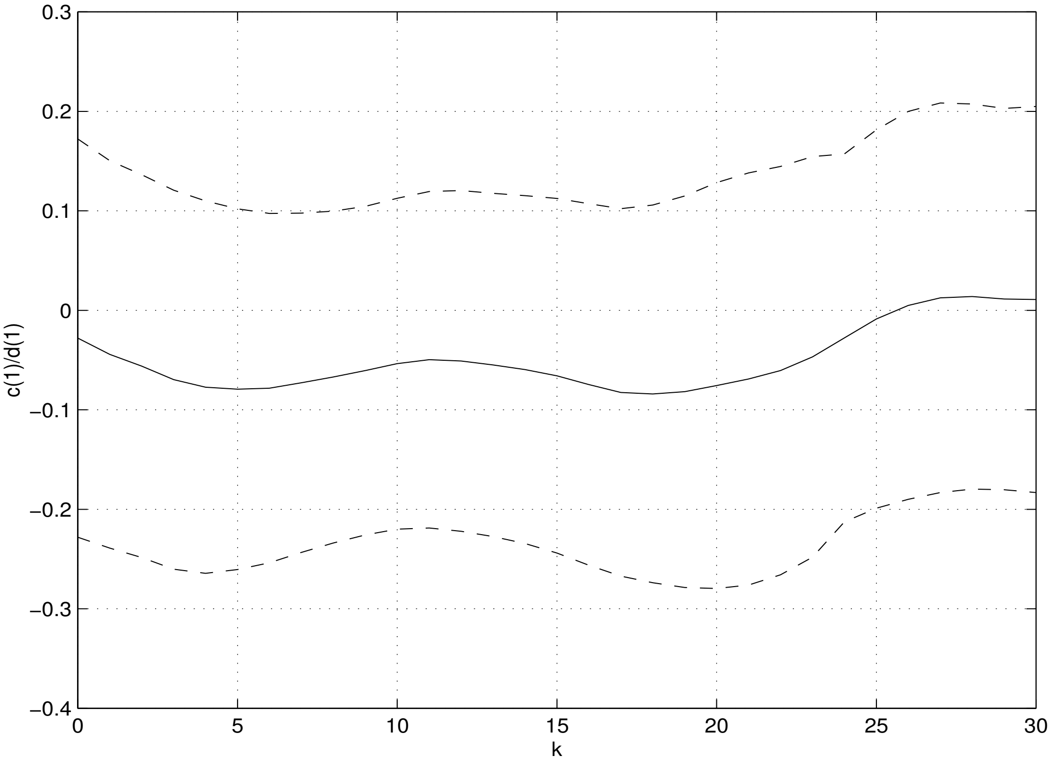

Figure 2: Bartlett zero-frequency estimator of UK's Δy against Δm.

hypothesis, $d_{\Delta y} = d_{\Delta m}$, are significant for both countries and support $d_{\Delta y} < d_{\Delta m}$. By *Case (v)* we can immediately infer that long-run neutrality holds for Argentina and Australia. Both Serletis and Krause (1996) and Serletis and Koustas (1998) also find LRN to hold for Australia.

4.4.5 Canada

For Canadian output the estimated order of integration for logged differenced output equals, $d_{\Delta y} = -0.4953$. This estimate produces a p-value of 0.012 for the unit root Wald-stat. Thus, output in Canada is either a $I(1)$ or $I(d)$ series with $d \in (0, 1)$.

If we conclude that y follows a unit root process, then combined with our finding of Canadian money being fractionally integrated of order greater than one, by *Case (v)* we trivially find long-run neutrality to hold. If, instead, we conclude output in Canada is a fractionally integrated series of order less than one, by the arguments of *Case (ii)* long-run neutrality again holds. Thus, either specification of Canadian output supports the quantity-theory of money.

Serletis and Krause (1996) argue that the second of our two scenarios holds, except instead of output being fractionally integrated, they find output to be an $I(0)$ process. Regardless of output in Canada being a unit root process, a fractionally integrated process, or a stationary $I(0)$ process, since m is a $I(d)$ process where $d > 1$, either of the above processes supports long-run neutrality.

4.4.6 Italy

Of the six countries whose money supply satisfy the necessary conditions for testing LRN, Italy is the only country where LRN can not be accepted. In Table 2 the estimated order of integration equals 0.0743 for the growth rate of Italy's output. This estimate is not significant, and hence, does not reject the presence of a unit root in Italian output. According to *Case (iii)* the LRN hypothesis is testable by estimating $c(1)/d(1)$ with the Bartlett frequency-zero estimator, β_k, as $k \to \infty$, and comparing it with zero.

Fig. 3 plots the OLS estimator $\hat{\beta}_k$ for $k = 1, \ldots, 45$ and the Newey-West 95%-confidence interval. As Fig. 3 shows, the value of $\hat{\beta}_k$ never descends below 0.1 and zero is only contained

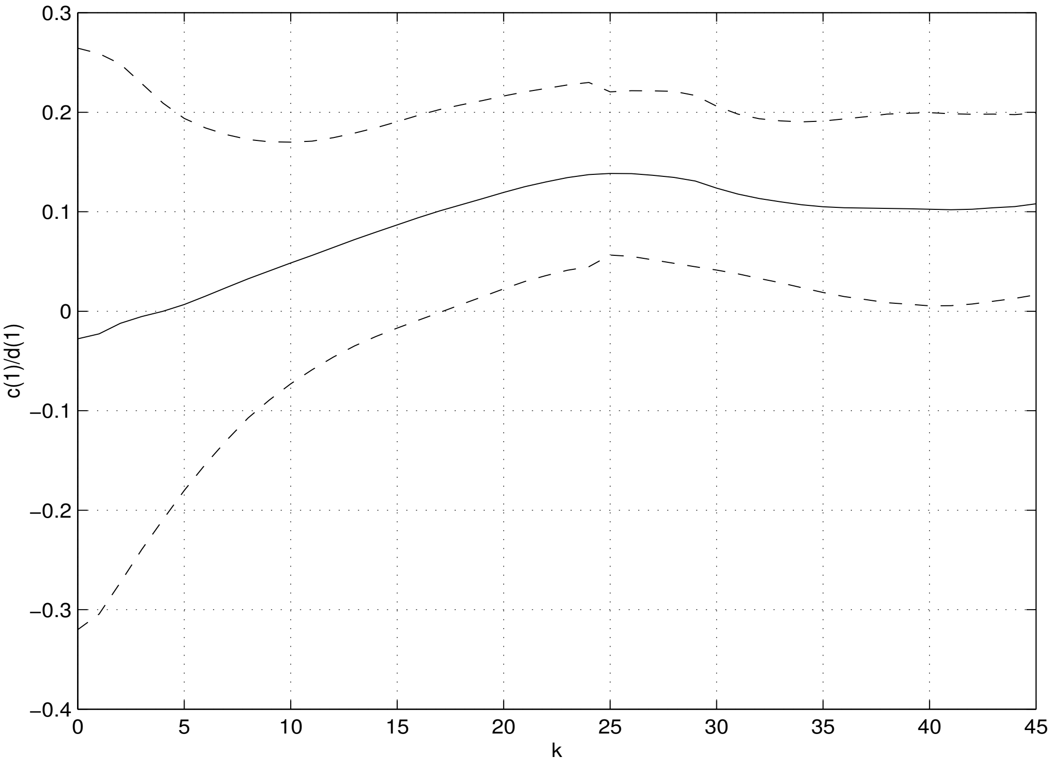

Figure 3: Bartlett zero-frequency estimator of Italy's Δy against Δm.

by the confidence interval when $k < 20$. Thus, we cannot accept the testable long-run neutrality of Italian money hypothesis.

5 Conclusion

In this paper we have extended the reduced form, zero-restriction test of the long-run neutrality of money to the larger class of long-memory bivariate ARFIMA models. The flexibility of the ARFIMA model captures both the short and long-memory dynamics of a series and enables it to fit differenced unit root processes. More importantly, it is the long-memory model's ability to produce a nonstationary process whose long-run dynamics are such that on the surface it appears that the impulse response function does not decay to zero, but in fact decays hyperbolically to zero. This flexibility allowed us to produce three new restrictions to Fisher and Seater (1992) conditions for the long-run derivative,

$LRD_{y,m}$, of the quantity theory of money hypothesis.

We apply the long-memory ARFIMA model to a century worth of annual monetary and output data from seven countries. We find four of the country's money supply follow a long-memory process. The other three country's money can not reject the presence of a unit root. Our findings in general support the long-run neutrality of money. Of the six countries that satisfy the necessary LRN conditions, five of these countries are found to be supportive of the quantity theory of money hypothesis.

The one country whose money supply fails to satisfy the necessary conditions of being integrated of order one or greater is the US. We find the US money supply to follow a fractionally integrated process that is nonstationary with an impulse response function that slowly decays to zero. Under these conditions the effect of a monetary shock to the US money supply will be finite in length. Thus, the zero-restriction approach to testing the quantity theory of money can not be conducted.

In our empirical work we also find the money supply in Italy to be an $I(1)$ process. This discovery refutes the $I(2)$ finding for the Italian money supply by Serletis and Koustas (1998). It also provides a justification for Serletis and Koustas (1998) failure to find evidence of long-run superneutrality. Because the money supply in Italy is an $I(1)$ process, the effect of an exogenous shock to the growth rate of money will dissipate quickly to zero. No inference can be made either positively or negatively towards the existence of superneutrality with a reduced form econometric model of monetary growth and real output. It also happens that among the testable countries, Italy is the one country that fails to accept the long-run neutrality of money hypothesis.

References

Backus, D.K. and P.J. Kehoe (1992) "International Evidence on the Historical Properties of Business Cycles," *American Economic Review,* 82, 864-888.X

Bae, S. (1998) "Essays on Financial System, Inflation and Growh," Dissertation, University of Missouri.

Boschen, J.F. and L.O. Mills, (1995) "Tests for Long-Run Neutrality Using Permanet Monetary and Real Shocks," *Journal of Monetary Economics*, 35, 25-44.

Brockwell, P. and R. Davis (1993) *Time Series: Theory and Methods,* 2nd ed. (Springer-Verlag: New York).

Bullard, J. and J.W. Keating (1995) "The Long-Run Relationship Between Inflation and Output in Postwar Economies," *Journal of Monetary Economics,* 36, 477-496.

Della Paolera, G., (1988) "How the Argentina Economy Performed During the International Gold Standard: A Reexamination," Dissertation, University of Chicago.

Canova, F. (1994) "Testing for Long-Run Neutrality: Empirical Evidence for G7-Countries with Special Emphasis on Germany. A Comment," *Carnegie-Rochester Conference Series on Public Policy,* 41, 119-125.

Dickey, D.A., and W.A. Fuller (1981) "A Likelihood Ratio Tests for Autoregressive Time Series with a Unit Root," *Econometrica,* 49, 1057-1072.

Diebold, F.X., and G.D. Rudebusch (1991) "On the Power of Dickey-Fuller Tests Against Fractional Alternatives," *Economics Letters*, 35, 155-160.

Domenech, R.A. (1986) "Estadisticas de la Evolicion Economica de Argentina 1913-1984," *Estudios*, 9, 103-184.

Engle, R.F. and C.W. Granger (1987) "Cointegration and Error Correction: Representation, Estimation, and Testing," *Econometrica,* 55, 251-276.

Fisher, M.E. and J.J. Seater (1993) "Long-Run Neutrality and Superneutrality in an ARIMA Framework," *American Economic Review,* 83, 402-415.

Geweke, J. and S. Porter-Hudak (1983) "The Estimation and Application of Long Memory Time Series Models," *Journal of Time Series Analysis,* 4, 221-238.

Granger, C. and R. Joyeux (1980) "An Introduction to Long-Memory Time Series Models and Fractional Differencing," *Journal of Time Series Analysis*, 1, 15-29.

Gradshteyn, I.S., and I.M. Ryzhik, (1994) *Table of Integrals, Series, and Products,* 5th ed. (Academic Press: San Diego)

Hosking, J. R. (1981) "Fractional Differencing," *Biometrika*, 68, 165-176.

Jensen, M.J. (1999a) "Using Wavelets to Obtain a Consistent Ordinary Least Squares Estimator of the Fractional Differencing Parameter," *Journal of Forecasting,* 18, 17-32.

Jensen, M.J. (1999b) "An Alternative Maximum Likelihood Estimator of Long-Memory Processes Using Compactly Supported Wavelets," forthcoming in *Journal of Economic Dynamics and Control.*

King, R., and M. Watson (1992) "Testing Long-Run Neutrality," National Bureau of Economic Research, Working Paper No. 4156.

Lucas, R.E. Jr. (1972) "Econometric Testing of the Natural Rate Hypothesis," in *The Econometrics of Price Determination* . O. Eckstein, ed. (Washington D.C.: Board of Governors of the Federal Reserve System).

Newey, W.K., and K.D. West (1987) "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 703-708.

Sargent, T.J. (1971) "A Note on the Accelerationist Controversy," *Journal of Money, Credit, and Banking,* 3, 50-50.

Serletis, K., and D. Krause (1996) "Empirical Evidence on the Long-Run Neutrality Hypothesis Using Low-Frequency International Data," *Economics Letters,* 50, 323-327.

Serletis, K., and Z. Koustas (1998) "International Evidence on the Neutrality of Money," *Journal of Money, Credit, and Banking,* 30, 1-25.

Sowell, F. (1990) "Fractional Unit Root Distribution," *Econometrica,* 58, 495-506.

Sowell, F. (1992a) "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models," *Journal of Econometrics,* 53, 165-188.

Sowell, F. (1992b) "Modeling Long-Run Behavior With the Fractional ARIMA Model," *Journal of Monetary Economics,* 29, 277-302.