Optimal Financing by Money and Taxes of Productive and Unproductive Government Spending: Effects on Economic Growth, Inflation, and Welfare

by

David Alan Aschauer*

Working Paper No. 241

July 1998

*Elmer W. Campbell Professor of Economics, Bates College and Research Associate, The Jerome Levy Economics Institute

I. Introduction

This paper contains an investigation of the effects of different means of financing government spending on economic growth, inflation, and welfare. In this setting, two different types of government spending are considered: *productive* expenditures which provide services to the private sector in its production activities; and *unproductive* expenditures which have no direct influence on the private economy. In turn, two different forms of finance are considered: proportional income taxation; and money creation.

The primary result of the paper is, perhaps, striking in its simplicity. Specifically, we find that optimal public finance requires productive government expenditure to be financed by money creation and unproductive government expenditure by income taxation. Within the model structure--a representative agent, endogenous growth model with money introduced *via* a cash-in-advance constraint--the basic result is robust to changes in the values of all underlying model parameters such as the intertemporal elasticity of substitution in consumption, the rate of time preference, and the output elasticity of public services.

The paper proceeds as follows. Section II contains a brief description of the model. Section III presents the economic equilibrium, while Sections IV and V compare the effects of financing productive and unproductive expenditures, respectively, by taxation and money creation. Section VI brings together the previous sections and considers the joint financing of government expenditures by taxation and money creation. Section VII concludes the paper and points to directions for future research.

II. Model Description

A. Modeling the Economy: Endogenous Growth

We are primarily concerned with the impact of government policies on the economy in the long run. Consequently, the analysis must revolve around a growth model in which such policies are capable of affecting either the long run level of output *or* the long run growth rate of output. One candidate framework for analysis is the neoclassical growth model of Solow (1956), in which growth is driven by exogenous factors such as population growth and technological change; another is the new growth model of Romer (1986), Lucas (1988), and Barro (1990), in which growth is determined endogenously. In the former model, a permanent change in government policy is likely to have a permanent effect on the long run level of output, while in the latter model such policy is likely to have a permanent effect on the long run rate of economic growth.

While the choice of the appropriate model is best determined by the available empirical evidence, such evidence is fairly inconclusive. The empirical work in this area--which has burgeoned in recent years--tends to reject a simple form of the neoclassical growth model which focuses on *physical* capital accumulation, but is not incompatible with an "augmented" neoclassical growth model which, in addition, includes *human* capital (as in Mankiw, Romer, and Weil (1992)). Still, there is a strand of literature regarding the long run effects of government policy actions--particularly fiscal policies--which favors the endogenous growth model. Specifically, Kocherlakota and Yi (1996) show that permanent changes in certain forms of government expenditures (*e.g.*, education spending and physical capital spending) have an impact on long run economic growth. Using a different approach, Aschauer (1997) presents evidence that permanent changes in the stock of public capital as well as changes in flow government spending affect the long run growth rate. We follow this empirical literature and construct a endogenous growth model with government spending financed by taxation and money creation.

B. Modeling the Public Sector: Financing Productive and Unproductive Spending by Money Creation and Taxes

In recent research on the impact of public sector behavior on the economy, a number of authors have made a crucial distinction between productive and unproductive government expenditures. In particular, productive expenditures, such as education, research and development, job training, and physical infrastructure, are taken to positively affect the efficiency of private sector production. On the theoretical front, Barro (1990) considers the role of government spending in a simple endogenous growth model. His basic result is that productive government spending, financed by a proportional income tax, will maximize economic growth and welfare when the output share of productive government spending, γ, is set equal to the output elasticity of such spending, α. In this context, either a lower ($\gamma < \alpha$) or higher ($\gamma > \alpha$) level of government spending acts as a drag on economic growth; in the former case because of insufficient government services, and in the latter case because of an excessive level of taxation. Turnovsky and Fisher (1995) investigate the composition of government expenditures and its consequences for macroeconomic performance in a neoclassical growth model. One of their main findings is that, to a first approximation, the marginal resource cost of productive government spending (*i.e.*, public infrastructure) should be set equal to unity (or its resource cost)--another way of stating the Barro rule of $\gamma = \alpha$.

There is a large and expanding literature on the empirical front as well. Aschauer (1989, 1990) initially brought attention to the potential importance of public capital in private production, with estimates of the output elasticity of public capital in the range of .35 to .40 in the United States over the post-World War II period. Later work has yielded mixed results. Lynde and Richmond (1993), Fernald (1993), Finn (1993), and Kocherlakota and Yi (1996) obtain results which are generally supportive of Aschauer's original estimates, while others, such as Evans and Karras (1994), present findings which leave little or no room for a direct positive impact of public expenditures on private sector production.

In work which is particularly relevant to the analysis contained in the current paper, Aschauer (1997) uses data for the U.S. states over the period from 1970 to 1990 and estimates the output elasticity of flow government expenditures in a non-linear, endogenous growth model to be in the range of .04 and the output elasticity of the stock of public capital in the range of .33. We will follow these empirical results and assume that a significant portion of total government expenditures is of the productive form.

The total level of government expenditures needs to be financed in one fashion or another. In the above-mentioned papers, when the method of financing is made explicit, government spending is typically supported by use of lump-sum or income taxation. The focus of attention in the present paper, however, is on the relative importance of taxation and money creation in financing public expenditure. In a recent paper, Palivos and Yip (1995) carefully work out the effects of money versus tax finance of public expenditure in an endogenous growth model, but limit themselves to the case of unproductive government spending.[1] We, instead, allow for both productive and unproductive government spending.

C. Modeling Money: The Cash-in-Advance Constraint

The main obstacle to rationalizing the holding of money by economic agents is that it is a rate of return dominated asset; other assets, such as bonds, yield a positive return and, therefore, are superior to money in acting as a store of value. Economists have devised a number of ways to overcome this obstacle in order to introduce money into an economic model. For instance, in Sidrauski (1967), money is allowed to yield a direct flow of utility services. In McCallum (1989) money is taken to reduce "shopping time" and, as a consequence, to increase leisure time. In the present model, we follow the cash-in-advance approach initiated by Clower (1967) which requires that the purchase of goods takes place through the use of money.

Once the cash-in-advance approach has been adopted as the method for introducing money into the analysis, a basic question arises: to what set of purchases should the constraint apply? In Lucas (1980), the set of purchases is comprised of consumption goods; in Stockman (1981), the set of purchases includes consumption and investment goods. The distinction between these two approaches is hardly without significance. In the former approach, the economy essentially dichotomizes into a real and monetary sector and money is superneutral--that is, changes in the growth rate of the money supply *per se* have no impact on the rate of economic growth. In the latter approach, however, the real and monetary sectors become intertwined and changes in the pace of money creation have an important effect on capital accumulation and economic growth.

One way of choosing between the two approaches is to look to the empirical evidence pertaining to the impact of money growth on long run economic growth. In an exhaustive analysis of 62 countries over the period from 1979 to 1984, Dwyer and Hafer (1988) estimate a very small, though negative, relationship between real output growth and money growth; specifically, in a univariate regression of real output growth on money growth, the coefficient on the latter variable is estimated as -0.018 (s.e. .009). Thus, a 10 percentage point increase in the money growth rate is estimated to modestly reduce annual economic growth from a sample average of .026 to .024. In a more recent analysis, Barro (1997) makes use of data for 109 countries over the period from 1960 to 1990 and similarly finds a small effect of money growth and inflation on real output growth; in particular, a 10 percentage point increase in the inflation rate is estimated to reduce the annual growth rate of per capita real output from a sample average of .022 to .020. Again, on purely statistical grounds, this relationship is rather weak.

In light of this evidence, it would seem that the appropriate modeling strategy for this paper--which is pointed toward an analysis of alternative government financing schemes in the long run and for a country such as the United States which, historically, has experienced a relatively low rate of inflation--is one in which money is essentially superneutral. As a consequence, we will assume that the cash-in-advance constraint applies to consumption purchases and that, by implication, the financing of private investment purchases is accomplished through the use of alternative credit instruments.

D. Model

We now describe, in equation form, the explicit model used in this paper. We assume an infinitely-lived representative agent with preferences over current and future consumption given by

$$u = \frac{1}{(1-\sigma)} \cdot \int_0^\infty (c^{1-\sigma} - 1) \cdot e^{-\rho \cdot t} dt \tag{1}$$

where u = utility, c = consumption, ρ = a constant rate of time preference, and σ^{-1} = a constant intertemporal elasticity of consumption.[2] The agent seeks to maximize utility subject to a flow budget constraint as given by

$$c + \dot{k} + \dot{m} + \pi \cdot m = (1 - \tau) \cdot y \tag{2}$$

where k = private capital, m = holdings of real money balances, y = output, τ = a proportional tax rate on income, π = the rate of price inflation, and a dot refers to a time derivative. Evidently, the agent may dispose of after-tax income by consuming, investing in capital, or by acquiring real money balances. In the present setting output is produced via the private sector production technology

$$y = A \cdot k^{1-\alpha} \cdot g_p^{\alpha} \tag{3}$$

where A = a productivity index and g_p = productive government expenditures such as education, worker training, research and development, and infrastructure. Labor input is taken to be inelastically supplied and is normalized to unity--thereby equating output and output per worker. We follow Barro (1990) by assuming that the production function exhibits constant returns to scale in capital and productive public expenditures.

In addition to the budget constraint in (2) and the production technology in (3), the choices of the representative agent are limited by a cash-in-advance constraint

$$c \leq m \tag{4}$$

so that, in effect, the agent must have accumulated a sufficient level of real cash balances in order to consume at a particular level at any particular point in time.

The total level of government spending, g, is given by

$$g = g_p + g_u \tag{5}$$

where g_u = unproductive government spending such as defense spending. Finally, the government budget constraint, which equates total government spending to the sum of the revenue from money creation and from income taxation, is

$$g = \mu \cdot m + \tau \cdot y \tag{6}$$

where μ = rate of monetary expansion and where we have combined the (potentially) separate budget constraints of the central bank and government treasury.

III. Equilibrium

The model equilibrium is attained through: solving the agent's problem of maximizing utility (1) subject to the budget constraint (2) and the cash-in-advance constraint (3); manipulating the resulting first-order conditions; and, finally, imposing various market-clearing and balanced growth conditions. After these steps are accomplished, the equilibrium is described by the following set of equations:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\cdot((1-\tau)\cdot(1-\alpha)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho) \tag{7}$$

$$c + \dot{k} = (1-\gamma)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}\cdot k \tag{8}$$

$$\theta = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{m}}{m} = \frac{\dot{y}}{y} = \frac{\dot{g}}{g} = \frac{\dot{g}_p}{g_p} = \frac{\dot{g}_u}{g_u} \tag{9}$$

$$\frac{\dot{m}}{m} = \mu - \pi \tag{10}$$

$$\frac{m}{k} = \frac{c}{k} \tag{11}$$

$$u = \frac{(c(0))^{1-\sigma}}{(1-\sigma)\cdot(\rho-(1-\sigma)\cdot\theta)}. \tag{12}$$

Equation (7) determines the growth rate of consumption as proportional to the difference between the after-tax marginal product of private capital and the rate of time preference, with the factor of proportionality given by the intertemporal elasticity of consumption. In this expression, γ_p = productive government expenditure expressed as a ratio to private sector output. We note that, as in Barro (1990), the after-tax marginal product of private capital, given by

$$(1-\tau)\cdot(1-\alpha)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}$$

is increasing (at a decreasing rate) in the ratio of productive government spending to output, γ_p, but decreasing in the income tax rate, τ. Importantly, the after-tax marginal product of private capital and the growth rate of consumption are independent of the money growth rate, reflecting the superneutral nature of money in the model.

Equation (8) is an equilibrium condition for the goods market; consumption and private capital accumulation are constrained by output net of government spending, where γ = total government spending as a ratio to output. Equation (9) is a balanced growth condition which implies constant output ratios of consumption, private capital, money balances, and total, productive, and unproductive government expenditure. Equation (10) is an equilibrium condition in the money market; since, in addition, the growth rate of real money balances equals the growth rate of output, this equation determines the rate of inflation as $\pi = \mu - \theta$. Equation (11) is the cash-in-advance condition--which is assumed to be binding in the subsequent analysis. Finally, equation (12) is the representative agent's utility function (apart from a constant term) in the balanced growth equilibrium which is obtained by substituting the constant growth rate of consumption as expressed in equation (7) into the utility function as given by (1) and integrating with respect to time.[3]

IV. Productive Government Spending

In this section, we assume that all government spending is productive, so $\gamma = \gamma_p$, and investigate the relative impacts of money and tax finance.

A. Results with Money Finance [$\tau = 0$]

The financing of government spending by money creation yields the following expression for the rate of economic growth:

$$\theta_M = \frac{1}{\sigma}\cdot((1-\alpha)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho). \tag{13}$$

Clearly, growth is increasing in γ_p. The initial level of consumption, in turn, is given by

$$c_M(0) = (1-\gamma_p)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \theta_M \tag{14}$$

which, depending on the values taken on by various parameters, can be verified to rise or fall in response to an increase in γ_p. We note that in this expression we have imposed, without any loss of generality, the condition that the initial stock of private capital is unity, or $k(0) = 1$. We will maintain this condition for the remainder of the paper.

The corresponding rate of inflation is given by

$$\pi_M = \mu - \theta_M \tag{15}$$

where the rate of money growth is determined as

$$\mu = \frac{\gamma_p \cdot (\frac{y}{k})}{\frac{m}{k}} = \frac{\gamma_p \cdot A^{\frac{1}{1-\alpha}} \cdot \gamma_p^{\frac{\alpha}{1-\alpha}}}{((1-\gamma_p) \cdot A^{\frac{1}{1-\alpha}} \cdot \gamma_p^{\frac{\alpha}{1-\alpha}} - \theta_M)}. \tag{16}$$

Evidently, there are a variety of effects of an increase in γ_p on the rate of inflation. Note, in particular, that an increase in government spending raises economic growth which, in turn, has an ambiguous effect on inflation. *Given* the money growth rate, a rise in output growth directly lowers the inflation rate. At the same time, however, the rise in output growth requires a shift in resources from consumption to investment which, in turn, has the effect of lowering the level of money balances and thereby necessitating a *rise* in the rate of money growth to finance government spending. This latter effect raises the inflation rate.

Despite the fact that economic growth is increasing in γ_p, it is easily verified that the utility-maximizing value of γ_p occurs where $\gamma_p = \alpha$. To verify this point, we substitute $c_M(0)$, as given by equation (14), into the representative agent's utility function, as given by equation (12), and differentiate with respect to θ_M to obtain

$$\frac{du}{d\theta_M} = \frac{(c_M(0))^{-\sigma} \cdot (\alpha - \gamma_p) \cdot A^{\frac{1}{1-\alpha}} \cdot \gamma_p^{\frac{\alpha}{1-\alpha}}}{\sigma \cdot (\rho - (1-\sigma) \cdot \theta_M)}. \tag{17}$$

In this expression, $c_M(0)$ and $(\rho - (1\text{-}\sigma) \cdot \theta_M)$ are assumed legitimately to be positive in order to ensure an economically meaningful equilibrium in which utility is positive (which rationalizes the former assumption) yet bounded (which rationalizes the latter assumption). Thus, since economic growth is increasing in γ_p, the maximization of utility requires γ_p be set equal to α.

B. Results with Tax Finance [$\mu = 0$]

The financing of government spending by taxes requires $\tau = \gamma_p$ and implies that economic growth is given by

$$\theta_T = \frac{1}{\sigma} \cdot ((1-\gamma_p) \cdot (1-\alpha) \cdot A^{\frac{1}{1-\alpha}} \cdot \gamma_p^{\frac{\alpha}{1-\alpha}} - \rho). \tag{18}$$

Here, economic growth is increasing in γ_p for $\gamma_p < \alpha$, achieves a maximum at $\gamma_p = \alpha$, and is decreasing in γ_p for $\gamma_p > \alpha$. This finding, which extends that of Barro (1990) from a non-monetary to a monetary setting, primarily arises as a result of the superneutral nature of money in the present model.

The initial level of consumption is given by

$$c_T(0) = (1-\gamma_p)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \theta_T \tag{19}$$

which, depending on the values of the various parameters of the model, may be an increasing or decreasing function of γ_p. The rate of inflation is given by

$$\pi_T = -\theta_T \tag{20}$$

which is decreasing in γ_p for $\gamma_p < \alpha$, achieves a minimum at $\gamma_p = \alpha$, and is increasing in γ_p for $\gamma_p > \alpha$.

It can be verified that the utility-maximizing value of γ_p is equal to α. Following the same procedure as in the case of money finance, we find

$$\frac{du}{d\theta_T} = \frac{(c_T(0))^{-\sigma}\cdot\alpha\cdot(1-\gamma_p)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}}{\sigma\cdot(\rho-(1-\sigma)\cdot\theta_T)} \tag{21}$$

which is positive for $0 < \gamma_p < 1$. Thus, since economic growth is maximized at $\gamma_p = \alpha$, so too is the utility of the representative agent.

C. A Comparison of Results

In this subsection, we compare the impacts of money and tax finance of productive government spending on economic growth, initial consumption, inflation, and welfare.

1. Economic Growth

A comparison of equations (13) and (18) indicates that the difference between economic growth rates in the cases of money and tax finance is given by

$$\theta_M - \theta_T = \frac{(1-\alpha)\cdot\gamma_p\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}}{\sigma} \tag{22}$$

and so the growth rate is unambiguously higher under money finance than under tax finance. The reason, of course, is straightforward. The financing of a higher level of productive government spending requires either a higher rate of money growth or a higher tax rate. While a higher rate of money growth has no effect on the economic growth rate (as in equation (13)), a higher tax rate has an adverse effect on the economic growth rate (as in equation (18)).

Figure 1 shows a comparison of economic growth rates under money and tax finance where we adopt the following parameter values: $A = .11$, $\alpha = .05$, $\rho = .04$, and $\sigma = 1$. These particular parameter values are chosen in order to generate a reasonable range of economic growth rates.[4] Evidently, the annual economic growth rate rises sharply from initial values of $-\rho = -.04$ for $\gamma_p = 0$ to .039 (under money finance) and .035 (under tax finance) for $\gamma_p = \alpha$. The difference between the two growth rates, which continues to rise to a high of nearly .10, is primarily due to a significant drop in economic growth under tax finance.

2. Initial Consumption

The difference between the initial levels of consumption under money and tax finance is obtained through a comparison of equations (14) and (19), and is expressed as

$$c_M(0) - c_T(0) = \theta_T - \theta_M. \tag{23}$$

The negative differential between levels of initial consumption is a direct result of the positive differential between rates of economic growth. Thus, we see that the choice between money and tax finance is ultimately a choice between current and future consumption; specifically, money finance is neutral with respect to the current/future consumption mix, while tax finance favors current consumption by reducing the net return to capital accumulation and the rate of economic growth.

3. Inflation

The difference between the inflation rates under money and tax finance can be shown to equal

$$\pi_M - \pi_T = \frac{(\theta_M - \theta_T)}{c_M(0)} \cdot ((\frac{\sigma}{1-\alpha}) - c_M(0)) \tag{24}$$

which, in general, takes on an ambiguous sign. For example, the inflation differential tends to be higher the lower the initial level of consumption or the lower the intertemporal elasticity of consumption. This is because a higher level of productive government spending, under money finance, is then associated with a relatively low level of real money balances and requires a relatively high rate of money creation to meet the requirements of the government budget constraint. However,

Figure 1:
Economic Growth under Money and Tax Finance

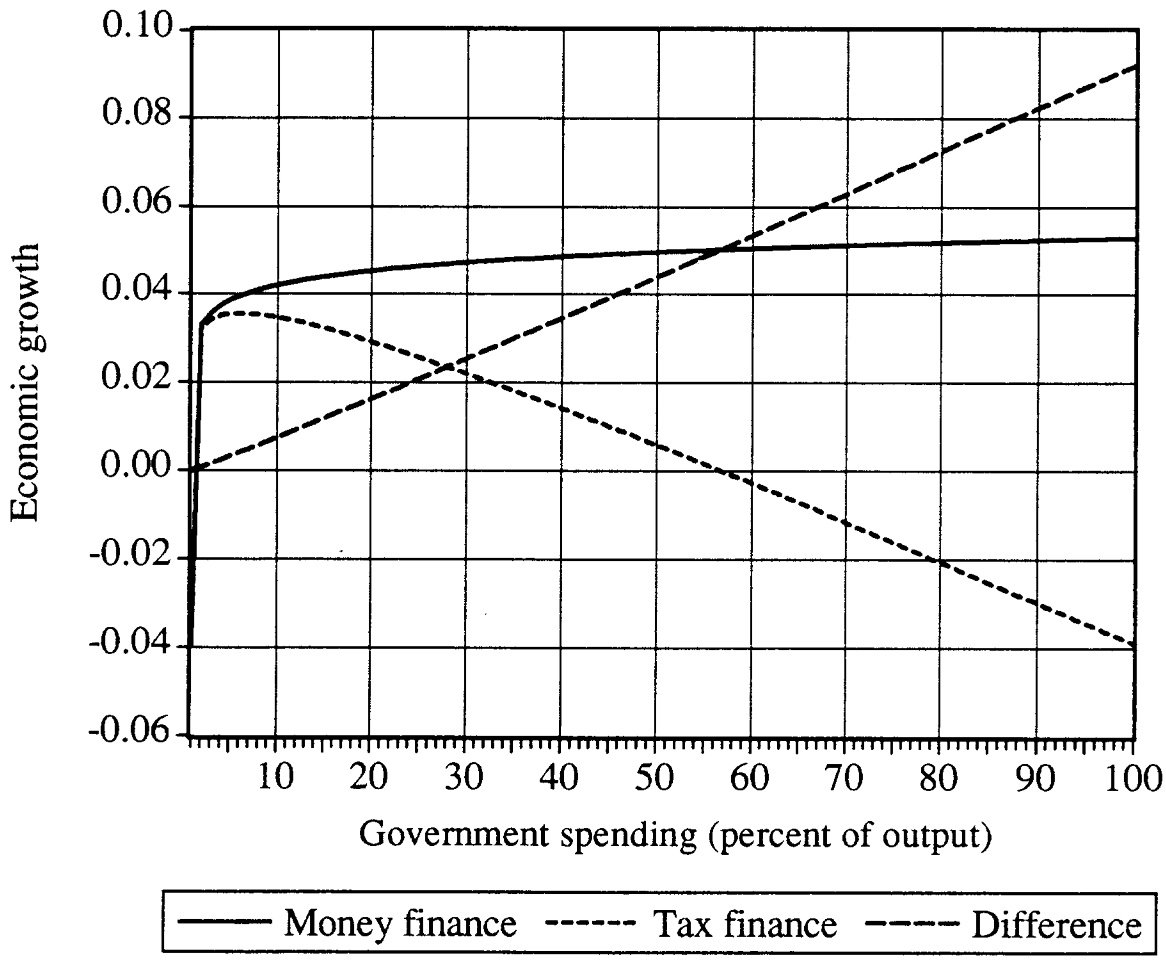
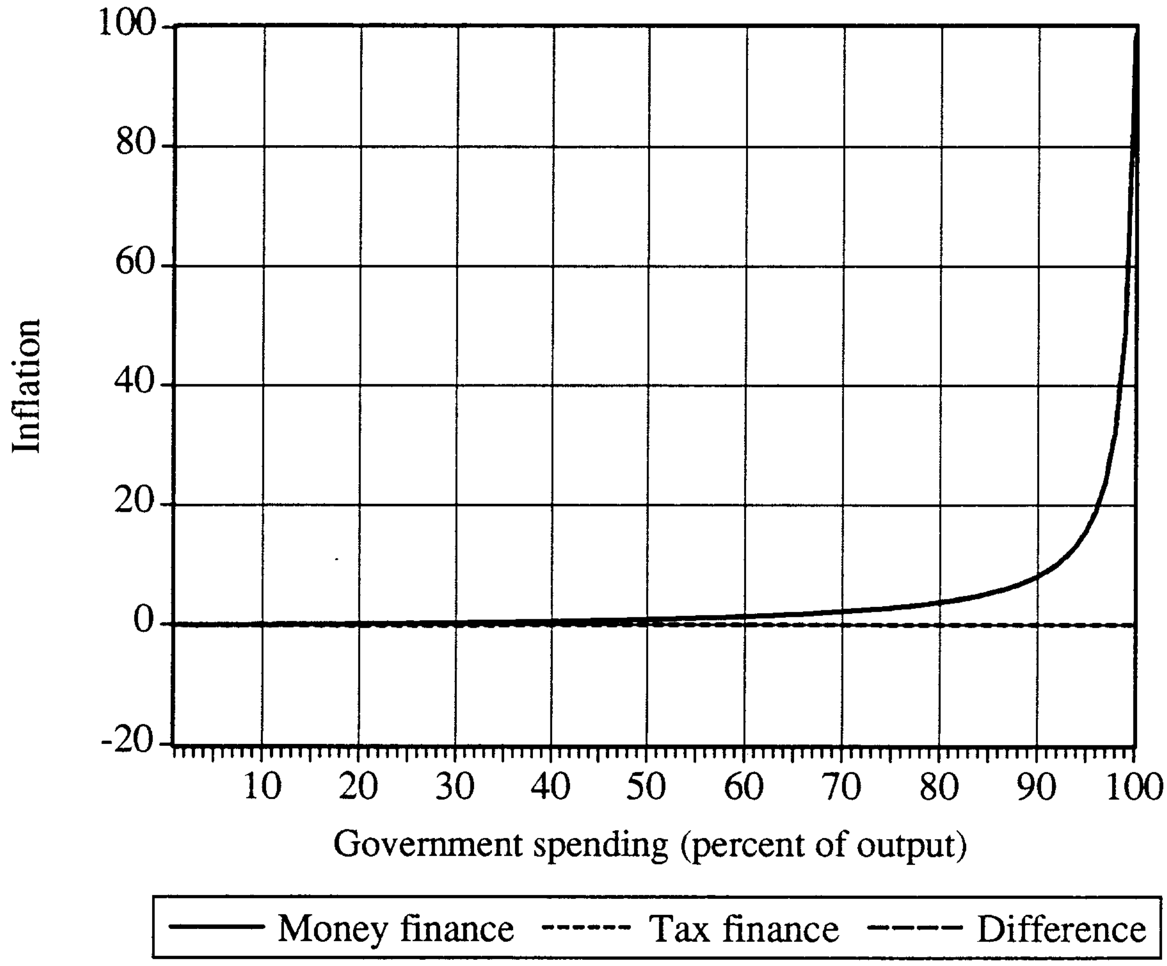


Figure 2:
Inflation under Money and Tax Finance



for reasonable parameter values, the inflation differential is positive and increasing in γ_p. Figure 2 indicates that an attempt to finance government spending through money creation leads to a hyper-inflationary situation (defined here as an annual inflation rate in excess of 1.00) for levels of government spending in excess of 50 percent of output.

4. Welfare

The discussion surrounding equations (17) and (21) established that the utility of the representative agent is maximized at $\gamma_p = \alpha$ under each financing scheme. We now demonstrate that the level of attained utility at $\gamma_p = \alpha$ is higher under money finance than under tax finance, or

$$u_M - u_T > 0 \qquad @\ \gamma_p = \alpha. \tag{25}$$

Substitution of the initial level of consumption, expressed as a function of the economic growth rate, into the utility function (12) and differentiating with respect to θ yields

$$\frac{du}{d\theta} = \left(\frac{(c(0))^{-\sigma}}{(\rho-(1-\sigma)\cdot\theta)^2}\right)\cdot\left((1-\gamma_p)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho - \sigma\cdot\theta\right) \tag{26}$$

which implies that utility is maximized when economic growth equals

$$\theta^* = \frac{1}{\sigma}\cdot\left((1-\gamma_p)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho\right). \tag{27}$$

Inspection of equations (13), (18), and (27) then indicates that the utility-maximizing economic growth rate can be supported under money finance with $\gamma_p = \alpha$ but cannot be supported under tax finance.

V. Unproductive Government Spending

In this section, we assume that a portion of government spending is exogenously determined and unproductive, so $\gamma = \gamma_p + \gamma_u$, and, as before, consider the relative impacts of money and tax finance.

A. Results with Money Finance [$\tau = 0$]

The financing of government spending by money creation yields the following expression for the rate of economic growth:

$$\theta_M = \frac{1}{\sigma}\cdot((1-\alpha)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho). \tag{28}$$

Clearly, growth is increasing in γ_p and is unaffected by γ_u. The initial level of consumption, in turn, is given by

$$c_M(0) = (1-\gamma)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \theta_M \tag{29}$$

which can be shown to rise or fall in response to an increase in γ_p and to fall in response to a rise in γ_u. The corresponding rate of inflation is given by

$$\pi_M = \mu - \theta_M \tag{30}$$

where the rate of money growth is determined as

$$\mu = = (\frac{\gamma\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}}{((1-\gamma)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \theta_M)}). \tag{31}$$

As in the prior case, where all government spending was taken to be productive, there are a variety of effects of an increase in γ_p on the rate of inflation. However, an increase in γ_u unambiguously raises the money growth rate in equation (31) and, by implication, raises the inflation rate in equation (30).

It is straightforward to show that the utility-maximizing value of γ_p now occurs where $\gamma_p = \alpha - \gamma_u$. To verify this point, we substitute $c_M(0)$, as given by equation (29), into the representative agent's utility function, as given by equation (12), and differentiate with respect to θ_M to find:

$$\frac{du}{d\theta_M} = \frac{(c_M(0))^{-\sigma}\cdot(\alpha-\gamma)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}}{\sigma\cdot(\rho-(1-\sigma)\cdot\theta_M)}. \tag{32}$$

For a given value of γ_u, the maximization of utility requires γ be set equal to α, from which it follows that γ_p be set equal to $\alpha - \gamma_u$.

B. Results with Tax Finance [$\mu = 0$]

The financing of government spending by taxes requires $\tau = \gamma$ and implies that economic growth is given by

$$\theta_T = \frac{1}{\sigma}\cdot((1-\gamma)\cdot(1-\alpha)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho). \tag{33}$$

Here, economic growth is increasing in γ_p for $\gamma_p < \alpha\cdot(1 - \gamma_u)$, achieves a maximum at $\gamma_p = \alpha\cdot(1 - \gamma_u)$, and is decreasing in γ_p for $\gamma_p > \alpha\cdot(1 - \gamma_u)$. Further, the economic growth rate is decreasing in γ_u.

The initial level of consumption is given by

$$c_T(0) = (1-\gamma)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \theta_T \tag{34}$$

which, depending on the values of the various parameters of the model, may be an increasing or decreasing function of γ_p and γ_u. The rate of inflation is given by

$$\pi_T = -\theta_T \tag{35}$$

which is decreasing in γ_p for $\gamma_p < \alpha\cdot(1 - \gamma_u)$, achieves a minimum at $\gamma_p = \alpha\cdot(1 - \gamma_u)$, and is increasing in γ_p for $\gamma_p > \alpha\cdot(1 - \gamma_u)$.

Using the previous procedure, it can be shown that the utility-maximizing value of γ_p is equal to $\alpha\cdot(1 - \gamma_u)$. Specifically, we find

$$\frac{du}{d\theta_T} = \frac{(c_T(0))^{-\sigma}\cdot\alpha\cdot(1-\gamma)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}}{\sigma\cdot(\rho-(1-\sigma)\cdot\theta_T)} \tag{36}$$

which is positive for $0 < \gamma < 1$. As the economic growth rate is maximized at $\gamma_p = \alpha\cdot(1 - \gamma_u)$, the same can be said for utility.

C. A Comparison of Results

In this subsection, we compare the impacts of money and tax finance of government spending on economic growth, initial consumption, inflation, and welfare.

1. Economic Growth

A comparison of equations (28) and (33) indicates that the difference between economic growth rates in the cases of money and tax finance is given by

$$\theta_M - \theta_T = \frac{(1-\alpha)\cdot\gamma\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}}}{\sigma} \tag{37}$$

and so the economic growth rate remains unambiguously higher under money finance than under tax finance.

2. Initial Consumption

The difference between the initial levels of consumption under money and tax finance is obtained through a comparison of equations (29) and (34), and is expressed as

$$c_M(0) - c_T(0) = \theta_T - \theta_M. \tag{38}$$

As in the previous section, the negative differential between levels of initial consumption is a direct result of the positive differential between rates of economic growth. We see, again, that the choice between money and tax finance can be viewed as fundamentally a choice between current and future consumption.

3. Inflation

The inflation rate differential under money and tax finance can be shown to equal

$$\pi_M - \pi_T = \frac{(\theta_M - \theta_T)}{c_M(0)}\cdot\left(\left(\frac{\sigma}{1-\alpha}\right) - c_M(0)\right) \tag{39}$$

which duplicates equation (24), where government spending was limited to productive spending. Consequently, the inflation differential continues to take on an ambiguous sign without additional restrictions being placed on the various model parameters.

4. Welfare

In general, the attained level of utility may be higher or lower under money than under tax finance of government expenditure. However, clear results can be established for two specific cases where we set the total level of government expenditure at a certain level of output, $\gamma = \omega$.

(a) all government spending is productive, $\gamma_u = 0$, $\gamma_p = \alpha = \omega$. This case is merely a restatement of the results of the previous section. Money finance dominates tax finance in this case, and

$$u_M - u_T > 0 \qquad @\ \gamma_u = 0,\ \gamma_p = \alpha = \omega. \tag{40}$$

(b) all government spending is unproductive, $\gamma_u = \omega$, $\gamma_p = \alpha = 0$. Tax finance dominates money finance in this case, and

$$u_M - u_T < 0 \qquad @\ \gamma_u = \omega,\ \gamma_p = \alpha = 0. \tag{41}$$

To establish this result, we use the same procedure as in the previous section and differentiate the utility function, expressed as a function of θ, to obtain

$$\frac{du}{d\theta} = \left(\frac{(c(0))^{-\sigma}}{(\rho-(1-\sigma)\cdot\theta)^2}\right)\cdot((1-\omega)\cdot A - \rho - \sigma\cdot\theta) \tag{42}$$

which implies that utility is maximized when economic growth equals

$$\theta^{**} = \frac{1}{\sigma}\cdot((1-\omega)\cdot A - \rho) \tag{43}$$

which can be supported under tax finance with $\tau = \omega$ but not under money finance.

The reasoning behind the contrasting results in these two cases has been discussed by Barro (1990) and Palivos and Yip (1995). Government spending, whether productive (as in Barro (1990) in a model without money) or unproductive (as in Palivos and Yip (1995) in a model with money) involves an externality which, depending on the situation, may or may not need to be internalized by the use of taxes. The decision by the representative agent to increase (or decrease) private output by a unit carries with it the public sector response of an increase (or decrease) in government spending by ω units. In the case of productive spending, with the government setting the level of spending in an optimal fashion at $\gamma_p = \alpha = \omega$, money finance yields the proper incentive, while tax finance yields too small an incentive, for capital accumulation and economic growth. In the case of unproductive spending, however, with $\gamma_u = \omega$, tax finance provides the appropriate incentive, while money finance provides too large an incentive, for investment and growth.

In between these two cases, there typically exists a range of values of productive and unproductive spending for which money finance dominates tax finance and a range for which tax finance dominates money finance. But this leads to the likelihood that some *combination* of money and tax finance-- rather than each alone-- will allow a higher level of welfare for the representative agent. We now investigate this possibility by considering the joint determination of money and tax financing of public expenditure.

VI. Joint Money and Tax Financing of Productive and Unproductive Spending

In this section, we determine the optimal structure of public sector spending and finance. In general, the rate of economic growth and initial consumption are given by

$$\theta = \frac{1}{\sigma}\cdot\left((1-\tau)\cdot(1-\alpha)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \rho\right) \tag{44}$$

$$c(0) = (1-\gamma_p-\gamma_u)\cdot A^{\frac{1}{1-\alpha}}\cdot\gamma_p^{\frac{\alpha}{1-\alpha}} - \theta. \tag{45}$$

We now assume that a certain fraction, ϕ, of total government expenditure is financed by income taxation (with the remainder financed by money creation) so that

$$\tau = \phi\cdot\gamma = \phi\cdot(\gamma_p+\gamma_u). \tag{46}$$

Direct substitution of equation (46) into equations (44) and (45) and subsequent substitution of the resulting expressions into the utility function (12) gives

$$u = u(\phi,\ \gamma_p;\ \gamma_u). \tag{47}$$

Differentiation of equation (47) with respect to ϕ and γ_p and equating the results to zero yields two first order conditions for utility maximization. The solution of these two equations then gives

$$\gamma_p^* = \alpha\cdot(1-\gamma_u) \tag{48}$$

so that the chosen level of productive government spending is proportional to the output elasticity of public services, and

$$\phi^* = \frac{\gamma_u}{\gamma} \tag{49}$$

so that the fraction of income taxation in total government revenue equals the fraction of unproductive government spending in total government spending.

Making use of the government budget constraint, we then have

$$\mu\cdot\frac{m}{y} = \gamma_p = \alpha\cdot(1-\gamma_u) \tag{50}$$

and

$$\tau = \gamma_u \tag{51}$$

which is the central result of the paper: specifically, we find that *optimal public finance requires productive government spending to be financed by money creation and unproductive government spending by income taxation.*

Figure 3 shows contours of the utility function of the representative agent, equation (47), for the following parameterization of the model: $A = .11$, $\alpha = .05$, and $\gamma_u = .05$ as well as for various values of ρ and σ. The baseline case in panel (a) assumes $\rho = .04$ and $\sigma = 1$ and displays utility contours that are relatively symmetric around the optimal values of $\phi = .5128$ and $\gamma_p = .0475$. Panels (b) and (c) show the effect of varying the intertemporal elasticity of consumption, with respective values of σ equal to .5 and 2. Evidently, a decrease in the intertemporal elasticity of consumption (*i.e.*, an increase in σ) tends to increase the importance of choosing the correct level of productive government spending relative to the correct fraction of income taxation in total revenue. That is, the utility function becomes relatively steeper in the γ_p dimension (and relatively flatter in the ϕ dimension) as σ increases. Panels (d) and (e) show the effect of varying the rate of time preference, with ρ respectively equal to .01 and .10. As with the (inverse) intertemporal elasticity of consumption, an increase in the rate of time preference tends to increase the relative importance of choosing the correct level of productive government spending.

In order to interpret the results obtained above, it is useful to focus on the partial problem of the

Figure 3:
Utility Contours under Money and Tax Finance

a) $\rho = .04, \sigma = 1$

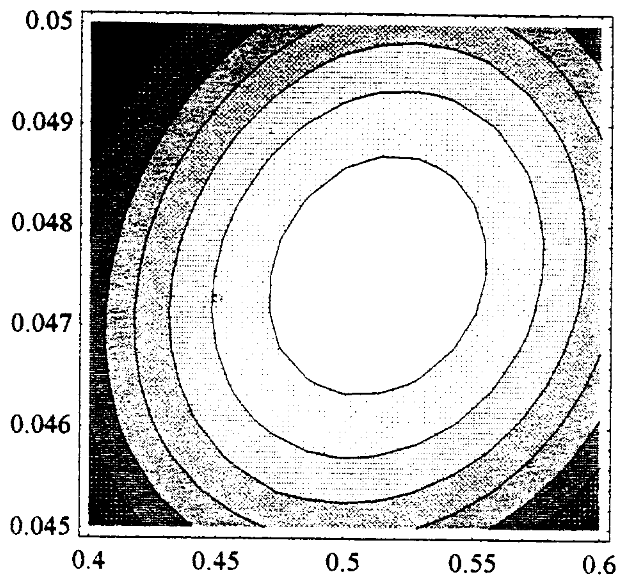
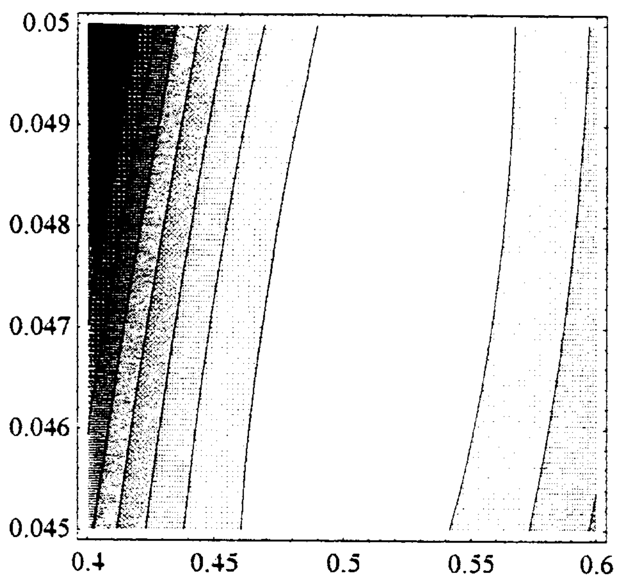


Figure 3 (con't):
Utility Contours under Money and Tax Finance

b) $\rho = .04, \sigma = .5$



c) $\rho = .04, \sigma = 2$

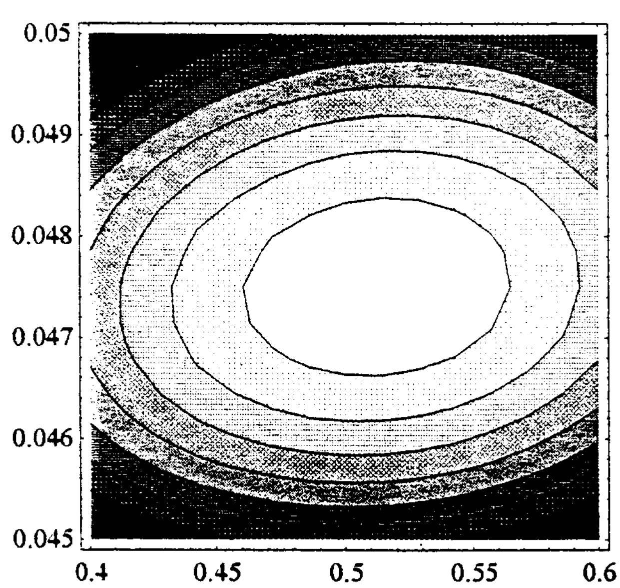
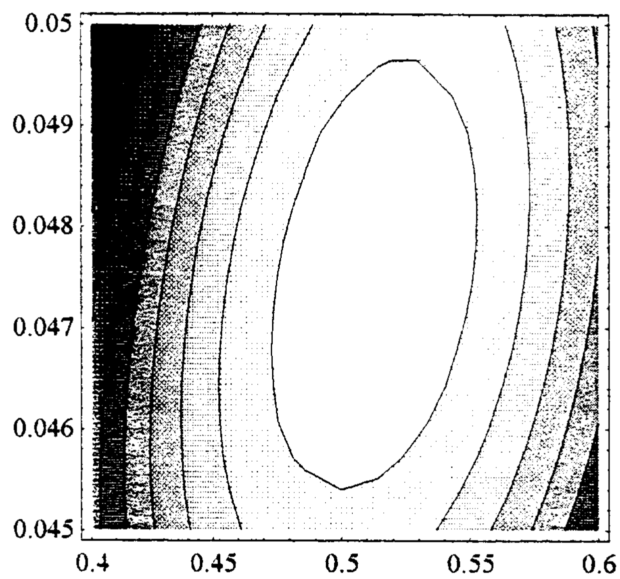
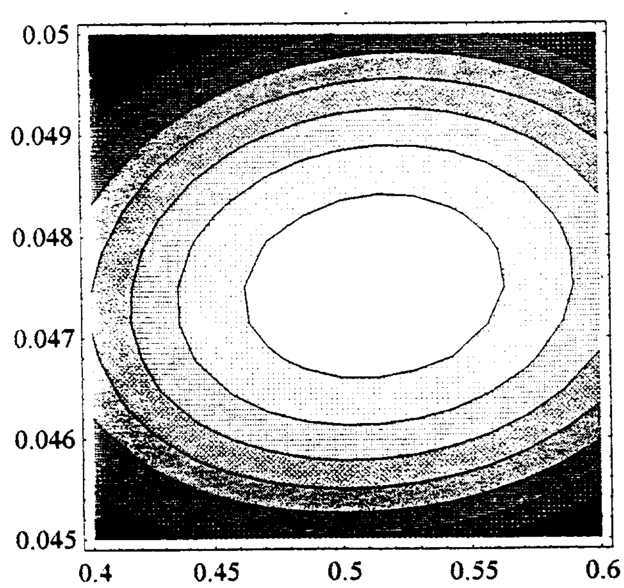


Figure 3 (con't):
Utility Contours under Money and Tax Finance

d) $\rho = .01$, $\sigma = 1$



e) $\rho = .10$, $\sigma = 1$



optimal choice of money and tax finance, *given* the optimal choice of the level of productive government spending. Accordingly, we set $\gamma_p = \alpha\cdot(1 - \gamma_u)$ and $\tau = \phi\cdot(\gamma_p + \gamma_u)$ in equations (44) and (45) to obtain

$$\theta = h_\theta(\phi;\ \gamma_p = \alpha\cdot(1-\gamma_u)) \tag{52}$$

and

$$c(0) = h_c(\phi;\ \gamma_p = \alpha\cdot(1-\gamma_u)). \tag{53}$$

It is then straightforward to verify that economic growth is decreasing in ϕ and initial consumption is increasing in ϕ. In particular,

$$\frac{d\theta}{d\phi} = -\frac{dc(0)}{d\phi} < 0.$$

Figure 4 graphs $h_\theta(\cdot)$ and $h_c(\cdot)$ for the baseline parameter values of the model. An increase in ϕ raises the level of consumption (relative to $k(0) = 1$) and lowers the economic growth rate and the associated level of private investment. Once again, we note that the choice of finance is, in essence, a choice of present versus future consumption.

Substituting equations (52) and (53) into the utility function (12) now gives

$$u = u(\phi;\ \gamma_p = \alpha\cdot(1-\gamma_u)) \tag{55}$$

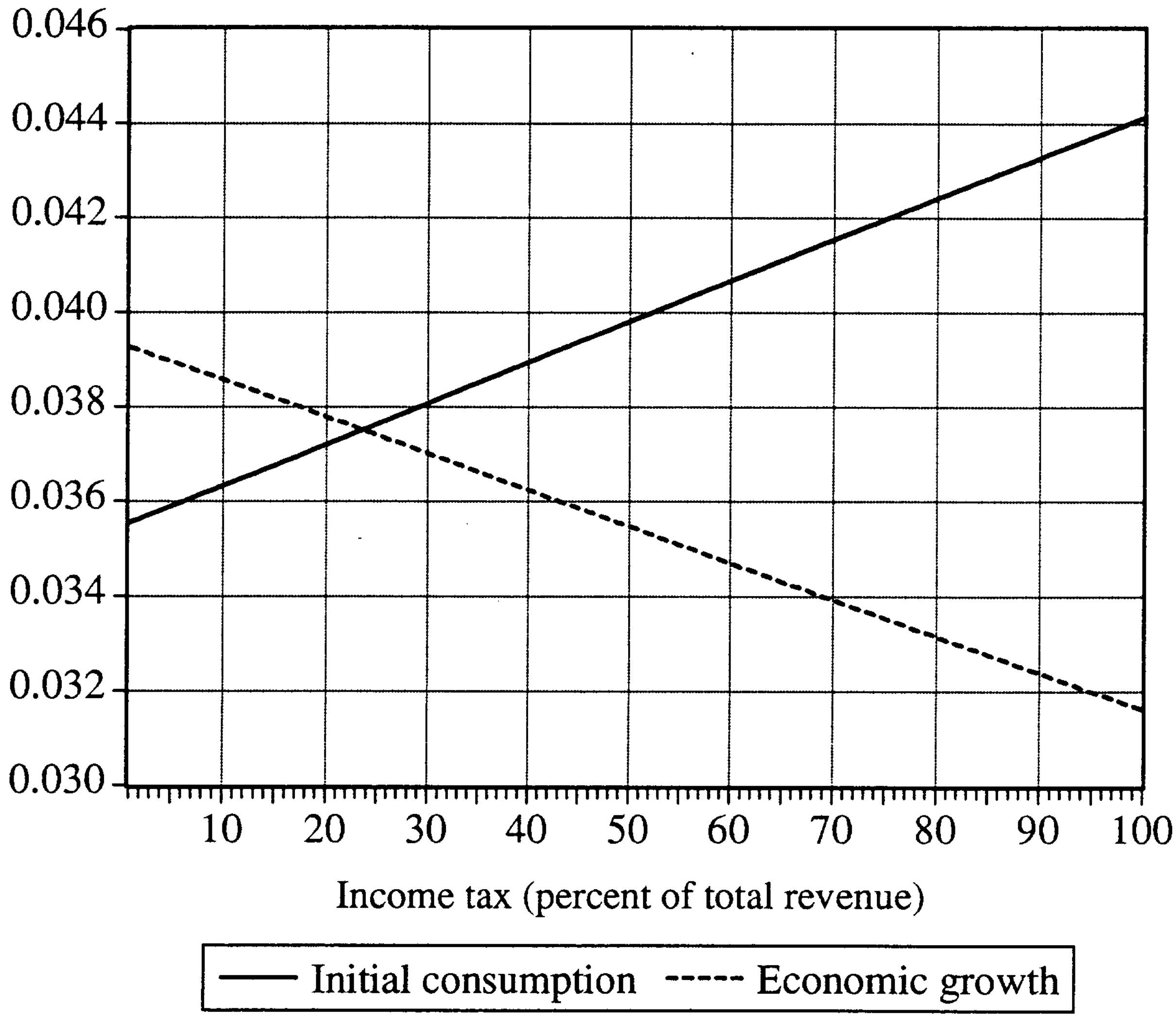
which, upon differentiation, yields

$$\frac{du}{d\phi} = \left(\frac{(c(0))^{-\sigma}}{(\rho-(1-\sigma)\cdot\theta)}\right)\cdot\left(\frac{dc(0)}{d\phi}\right)\cdot(\rho-(1-\sigma)\cdot\theta-c(0)) \tag{56}$$

which implies that utility is maximized when $c(0) = \rho - (1\text{-}\sigma)\cdot\theta$. This, then, gives the attained level of utility, under optimal public finance, as

$$u = \frac{(c(0))^{-\sigma}}{(1-\sigma)} \tag{57}$$

Figure 4:
Initial Consumption and Economic Growth
under Money and Tax Finance



which is shown in Figure 5 for the baseline model parameters. Thus, under optimal public spending and revenue creation policies, the level of attained utility is a simple function of the level of initial consumption.

VII. Conclusion

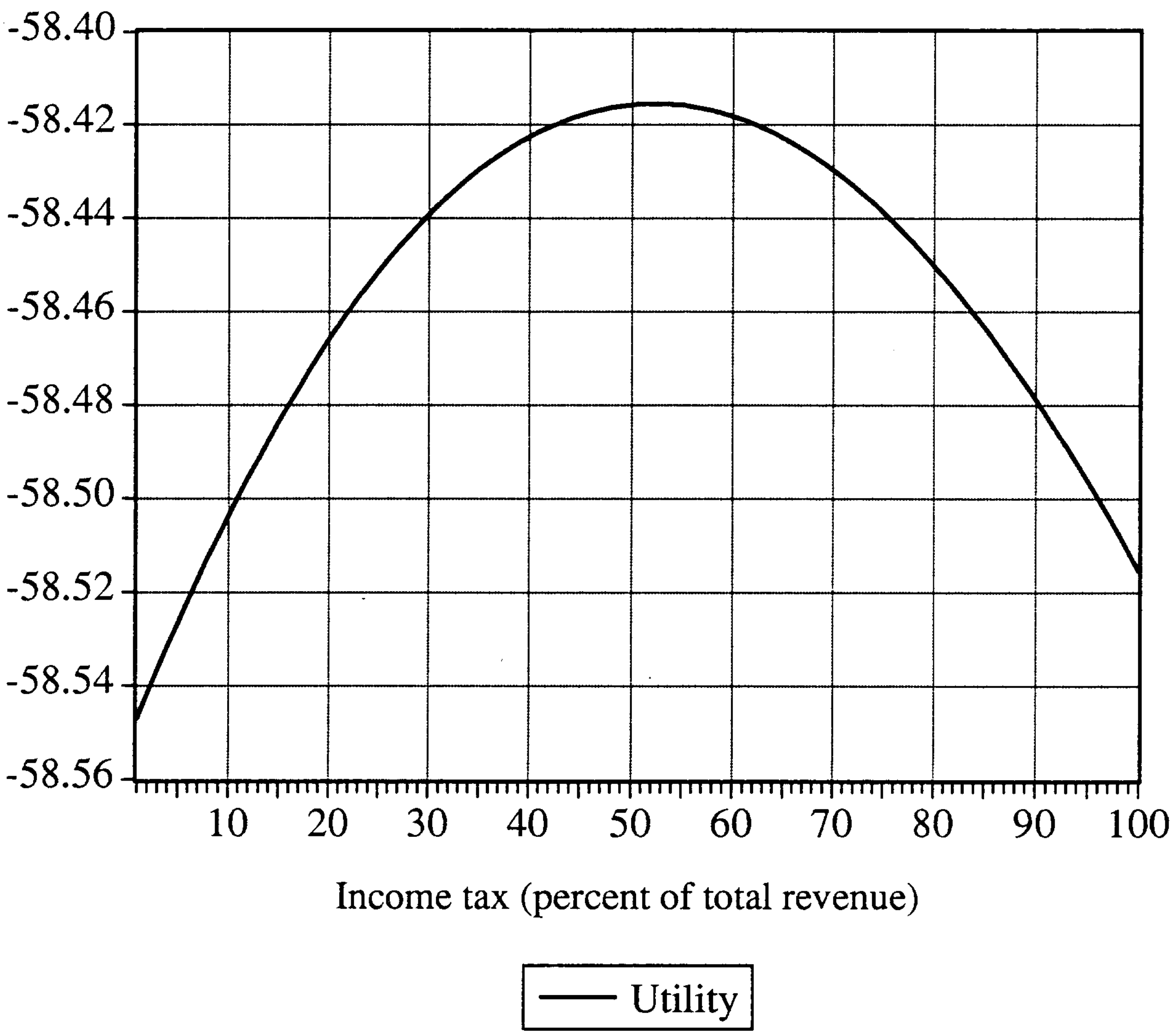
This paper has investigated the relative impact on economic growth, inflation, and welfare, of the use of money creation versus income taxation for the purpose of financing productive and unproductive public expenditures. On empirical grounds, we have rationalized the use of an endogenous growth model, supplemented with a cash-in-advance constraint, as an appropriate framework of analysis. Within this model structure, a clear-cut theoretical result emerges--specifically, optimal public finance involves the use of money creation to finance productive government spending and income taxation to finance unproductive spending. This result is robust to various parameterizations of the underlying model.

The primary finding of this paper is of practical relevance to the current economic situation in the United States as well as many other developed and developing countries. A host of empirical results indicate that certain forms of productive government expenditure--*e.g.*, on physical infrastructure--may be significantly below the levels which, according to the criteria of this paper, would maximize social welfare. Furthermore, the conduct of monetary policy is almost exclusively focused on the goal of eliminating inflation, resulting in measured annual inflation rates in the range of .03 and actual annual inflation rates (taking appropriate account of quality improvements, etc.) in the range of .01 (or even lower). The analysis contained in this paper suggests an alternative focus for the conduct of monetary policy, that it be pointed toward increased funding of productive government expenditures. Such a reorientation of monetary policy promises to raise economic growth, future standards of living, and the overall level of welfare of the average person in society.

As is almost always true with theoretical models which generate clear-cut results and carry unambiguous policy implications, several qualifications are in order. First, the model structure is one in which money is superneutral; adopting a different model, in which money is not superneutral, no doubt will alter the fundamental result of the paper. However, we point out that higher money growth (and associated inflation) has been found to dampen economic growth in some economic models but to enhance economic growth in other economic models. Consequently, it is not clear how the introduction of the non-superneutrality of money will affect--either qualitatively or quantitatively-- the results of this paper.

Second, it is arguable that the treatment of unproductive government expenditures as exogenously determined is unsatisfactory. A richer model structure would allow for other forms of government spending--*e.g.*, on government consumption services--which may widen the scope of optimizing behavior of the government and carry interesting implications for the choice of money versus tax finance of public expenditures. Indeed, a broader analysis such as this may well generate an empirically useful framework for the proper classification of government spending and associated financing schemes and, by extension, lead to a valid quantitative judgement as to whether the current

Figure 5:
Utility under Money and Tax Finance



rate of money growth and inflation may be too low to support an optimal structure of government activities.

Notes

1. It should be noted, however, that the model constructed by Palivos and Yip (1995) allows for the non-superneutrality of money, while the model in the present paper is one in which the superneutrality of money holds. We make an empirical argument in favor of the present approach in the next section of the paper. However, we stress that the model in the present paper, when compared to Pavilos and Yip's model, is both more general (along the dimension of the composition of government spending) and more specific (along the dimension of the superneutrality of money).

2. In the special case of $\sigma = 1$, the utility function takes on the logarithmic form

$$u = \int_0^\infty log(c) \cdot e^{-\rho \cdot t} dt.$$

3. For the logarithmic case when $\sigma = 1$, we have

$$u = \frac{log(c(0))}{\rho} + \frac{\theta}{\rho^2}.$$

4. We note that these particular parameter values also yield reasonable values for the average product of private capital and initial consumption. For instance, when evaluated at $\gamma_p = \alpha$, the average product of capital equals .084 while the level of initial consumption (relative to the initial capital stock of $k(0) = 1$) equals .040 under money finance and .044 under tax finance.

References

Aschauer, D.A. 1989. "Is Public Expenditure Productive?" *Journal of Monetary Economics* 23: 177-200.

Aschauer, D.A. 1990. "Why is Infrastructure Important?" In A.H. Munnell, ed,. *Is There a Shortfall in Public Capital Investment.* Boston, MA: Federal Reserve Bank of Boston.

Aschauer, D.A. 1997. "Do States Optimize? Public Capital and Economic Growth." *Jerome Levy Economics Institute Working Paper No. 189*, April.

Barro, R.J. 1990. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98: S103-S125.

Barro, R.J. 1997. *Determinants of Economic Growth: A Cross-Country Empirical Study.* Cambridge, MA: MIT Press.

Clower, R.W. 1967. "A Reconsideration of the Microfoundations of Monetary Theory." *Western Economic Journal* 6: 1-8.

Dwyer, G.P. and R.W. Hafer. 1988. "Is Money Irrelevant?" *Economic Review.* Federal Reserve Bank of St. Louis, May/June: 3-14.

Evans, P. and G. Karras. 1994. "Are Government Activities Productive? Evidence from a Panel of U.S. States. *Review of Economics and Statistics* 76: 1-11.

Fernald, J. 1993. "How Productive Is Infrastructure? Distinguishing Reality and Illusion With a Panel of Industries." Unpublished manuscript, Harvard University.

Finn, M. 1993. "Is All Government Capital Productive?" Unpublished manuscript, Federal Reserve Bank of Richmond.

Kocherlakota, N.R., and K. Yi. 1996. "A Simple Time Series Test of Endogenous Vs. Exogenous Growth Models: An Application to the United States." *Review of Economics and Statistics* 78: 126-134.

Lucas, R.E. 1980. "Equilibrium in a Pure Currency Economy." *Economic Inquiry* 18: 203-220.

Lucas, R.E. 1988. "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22: 3-42.

Lynde, C. and J. Richmond. 1993. "Public Capital and Total Factor Productivity." *International Economic Review* 34: 401-414.

Mankiw, N.G., D. Romer, and P. Weil. 1992. "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics* 108: 407-437.

McCallum, B.T. 1989. *Monetary Economics: Theory and Policy.* New York, NY: Macmillan Publishing Company.

Palivos, T. and C.K. Yip. 1995. "Government Expenditure Financing in an Endogenous Growth Model: A Comparison." *Journal of Money, Credit, and Banking* 27: 1159-1178.

Romer, P.M. 1986. "Increasing Returns and Long-Run Growth." *Journal of Political Economy* 94: 1002-1037.

Sidrauski, M. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review* 57:534-544.

Solow, R.M. 1956. "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics* 70: 65-94.

Stockman, A.C. 1981. "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics* 8: 387-393.

Turnovsky, S.J. and W.H. Fisher. 1995. "The Composition of Government Expenditure and Its Consequences for Macroeconomic Performance." *Journal of Economic Dynamics and Control* 19: 747-786.