Mathematical Model of the Inflationary Process

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Abstract

Presented here is the mathematical model with one commodity describing the dynamics of the inflationary process. This model is also applied to research how the hypothesis of rational expectations could affect the commodity's demand and production after increasing the amount of money on the market.

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1. Introduction

In this paper I applied the Continuos-Time Model of Business Fluctuations developed by me earlier (see Krouglov, 1997a, 1997b, and 1998) for the purpose of describing the dynamics of the inflationary process. I used also the obtained model for looking at the problem how so-called *rational expectations* introduced into economics by Robert E. Lucas, Jr. (see Lucas, 1972) could affect the demand and production of the commodity on market.

2. Model Description

Consider model with one commodity, and denote its volume of production V_p, of savings V_s, and commodity's demand V_d. Designate r_p, r_s, and r_d to be the production, savings, and demand rates respectively.

We assume that until time $t = 0$ economic system was at rest in the equilibrium point i.e.

$$V_d = V_p = V_s = 0 \ ,$$

$$r_d = r_p = r_s = 0 \ ,$$

where we are using the normalized values for the sake of simplicity.

We assume also that commodity's demand rate is directly proportional to the change of amount of money in the economic system, and is inversely proportional to the commodity's price change. Hence if we assume that since time $t = 0$ we begin to pump money into system with some constant rate, the commodity's demand can be expressed by the following formula,

$$\dot{V}_d = \begin{cases} \Delta_r - \lambda_{pr} \cdot \dot{P}, & 0 \le t \le T \\ -\lambda_{pr} \cdot \dot{P} \quad , & T < t < +\infty \end{cases} \qquad (1)$$

where Δ_r is a constant, T is the period of money turnover, and $\lambda_{pr} > 0$ is some constant coefficient.

Other values are expressed respectively,

$$\dot{V}_p = \lambda_p \cdot V_s , \qquad (2)$$

$$\dot{V}_s = \lambda_s \cdot (V_d - V_p) , \qquad (3)$$

$$\dot{P} = \lambda_{df} \cdot (V_d - V_p) , \qquad (4)$$

where $\lambda_p > 0$, $\lambda_s > 0$, and $\lambda_{df} > 0$ are some coefficients (see details in Krouglov, 1997b and 1998).

Note that value $y = V_d - V_p$ is commodity's deficit on the market.

Therefore it is followed from the equations (1) - (4),

$$\ddot{y} + \lambda_{pr} \cdot \lambda_{df} \cdot \dot{y} + \lambda_p \cdot \lambda_s \cdot y = 0 \ , \tag{5}$$

and we have the *damped oscillations* for $0 < \lambda_{pr} \cdot \lambda_{df} < 2 \cdot \sqrt{\lambda_p \cdot \lambda_s}$ (system is *underdamped*), and no oscillations both for $\lambda_{pr} \cdot \lambda_{df} > 2 \cdot \sqrt{\lambda_p \cdot \lambda_s}$ (system is *overdamped*) and for $\lambda_{pr} \cdot \lambda_{df} = 2 \cdot \sqrt{\lambda_p \cdot \lambda_s}$ (*critically damped* case) (see Piskunov, 1965).

3. Solution for the Critically Damped Case

It is known that the step response of a second-order system has the shortest settling time for the critically damped case (see Oppenheim et al., 1997). We show the solution of the equations (1) - (4) for this case. Solutions of the differential equations (1) - (4) for other two cases can be found similarly.

We have from equation (5),

$$y = \Delta_r \cdot t \cdot e^{-\sqrt{\lambda} \cdot t} \tag{6}$$

for $0 \le t < +\infty$ where $\lambda = \lambda_p \cdot \lambda_s$.

Hence

$$V_s = -\frac{\lambda_s \cdot \Delta_r}{\sqrt{\lambda}} \cdot \left(t + \frac{1}{\sqrt{\lambda}}\right) \cdot e^{-\sqrt{\lambda} \cdot t} + \frac{\lambda_s \cdot \Delta_r}{\lambda} \tag{7}$$

and

$$P = -\frac{\lambda_{df} \cdot \Delta_r}{\sqrt{\lambda}} \cdot (t + \frac{1}{\sqrt{\lambda}}) \cdot e^{-\sqrt{\lambda} \cdot t} + (P^0 + \frac{\lambda_{df}}{\lambda}) \cdot \Delta_r \qquad (8)$$

where P^0 is the commodity's price at the time $t = 0$.

Therefore

$$V_d = \begin{cases} \Delta_r \cdot t + 2 \cdot \Delta_r \cdot (t + \frac{1}{\sqrt{\lambda}}) \cdot e^{-\sqrt{\lambda} \cdot t} - \frac{2 \cdot \Delta_r}{\sqrt{\lambda}}, & 0 \le t \le T \\ 2 \cdot \Delta_r \cdot (t + \frac{1}{\sqrt{\lambda}}) \cdot e^{-\sqrt{\lambda} \cdot t} + \Delta_r \cdot (T - \frac{2}{\sqrt{\lambda}}), & T < t < +\infty \end{cases} \qquad (9)$$

and

$$V_p = \begin{cases} \Delta_r \cdot t + \Delta_r \cdot (t + \frac{2}{\sqrt{\lambda}}) \cdot e^{-\sqrt{\lambda} \cdot t} - \frac{2 \cdot \Delta_r}{\sqrt{\lambda}}, & 0 \le t \le T \\ \Delta_r \cdot (t + \frac{2}{\sqrt{\lambda}}) \cdot e^{-\sqrt{\lambda} \cdot t} + \Delta_r \cdot (T - \frac{2}{\sqrt{\lambda}}), & T < t < +\infty \end{cases} \qquad (10)$$

So, we see that commodity's production increases for $T > \frac{2}{\sqrt{\lambda}} = \frac{2}{\sqrt{\lambda_p \cdot \lambda_s}}$,

and decreases otherwise.

To explain this phenomenon I would say that if $T > \frac{2}{\sqrt{\lambda}}$ then the amount of energy introduced into economic system during the period T exceeds the work of a damping force (which decreases the commodity's demand proportionally to the commodity's price increase) during the time between the instant when system left the first equilibrium point and the moment when system arrived to the second equilibrium point.

4. Hypothesis of Rational Expectations

We assume that if the market knows the details of monetary policy it can react respectively trying get all indicators intact.

Therefore

$$0 = \dot{V}_d = \begin{cases} \Delta_r - \lambda_{pr} \cdot \dot{P}, & 0 \le t \le T \\ -\lambda_{pr} \cdot \dot{P} \quad , & T < t < +\infty \end{cases} \qquad (11)$$

and if commodity's price is described by the following function,

$$P = \begin{cases} P^0 + \dfrac{\Delta_r}{\lambda_{pr}} \cdot t \ , & 0 \le t \le T \\ P^0 + \dfrac{\Delta_r}{\lambda_{pr}} \cdot T \ , & T < t < +\infty \end{cases} \qquad (12)$$

the economic system won't leave the point of equilibrium.

However if *rational expectations* duration exceeds the point T, the commodity's demand and hence production will decrease i.e.

$$\dot{V}_d = -\Delta_r, \quad t > T \qquad (13)$$

and

$$V_d = -\Delta_r \cdot (t - T), \quad t > T \ , \qquad (14)$$

$$V_p = \frac{\Delta_r}{\sqrt{\lambda}} \cdot \sin(\sqrt{\lambda} \cdot (t - T)) - \Delta_r \cdot (t - T), \quad t > T \ . \qquad (15)$$

This fact shows in my opinion that *rational expectations* are rather speculative than real economic phenomenon.

5. Conclusion

I received recently a proposal from the publisher to write a book on the mathematical aspects of business fluctuations. Hope that I will find time and energy to cope with it.

4. References

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