Growth and Risk-Sharing with Private Information

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Abstract

We examine the impact of incomplete risk-sharing on growth and welfare. The source of market incompleteness in our economy is private information: a household's idiosyncratic productivity shock is not observable by others. Risk-sharing between households occurs through long-term contracts with intermediaries. We …nd that incomplete risk-sharing tends to reduce the rate of growth relative to the complete risk-sharing benchmark. Numerical examples indicate the contracts are relatively e¢cient and that the growth e¤ects of private information are small.

1. Introduction

Recent research has found evidence that is inconsistent with the full insurance predictions of the complete markets model. For example, Cochrane (1991), Mace (1991) and Hayashi et al. (1996) provide evidence against complete risk-sharing within the US at the individual level; Townsend (1994) and Maitra (1997) reject full insurance across households within Indian villages; and Backus et al. (1992), Baxter and Crucini (1994) and Athanasoulis and van Wincoop (1997) provide evidence against cross-country consumption risk-sharing. Motivated by this finding, we consider the effect of risk on growth and welfare. We develop an environment where household production is subject to idiosyncratic shocks which are private information, and growth is endogenous. The assumption of private information provides a basis for market incompleteness; the resulting problem of incentive compatibility eliminates the possibility of complete risk-sharing. Households share risk through long-term contracts with competitive intermediaries.[1] The enduring relationship allows intermediaries to exploit intertemporal trade-offs, thereby providing (partial) insurance.

Previous work on risk and growth typically has contrasted complete risk-sharing with autarchy. Since the extent of market incompleteness is endogenous in our environment, we are able to examine an intermediate case. We find that the presence of uninsurable risk reduces the rate of growth relative to the complete risk-sharing benchmark. This, for example, differs from the result in Devereux and Smith (1994). Comparing autarchy with complete risk-sharing, in a model of capital risk which essentially shares our technology and preferences, they find

[1] Thus, the extent of market incompleteness is endogenous in our economy.

that the e¤ect of risk on savings, and hence growth, is ambiguous.[2] Our results indicate that the impact of risk on growth and welfare is likely to be sensitive to the origin of market incompleteness and the types of insurance arrangements allowed.

In related work, Marcet and Marimon (1992) examine a two-agent model with capital accumulation where a risk-neutral investor with unlimited resources invests in the technology of a risk-averse producer whose output is subject to productivity shocks which are private information.[3] Our work extends their analysis to a market-clearing economy with endogenous growth. In contrast to Marcet and Marimon, we …nd that investment, as well as consumption, is a¤ected by shocks to production. As a result, there are growth e¤ects of private information. However, numerical examples indicate that, on average, the growth and welfare e¤ects of incomplete risk-sharing are likely to be small.

In section 2 we describe technology, preferences and the contract. Section 3 solves the contract assuming logarithmic utility, while section B of the appendix examines the contract when utility is iso-elastic. Numerical examples are presented in section 4; these provide quantitative measures of the size of the growth and welfare e¤ects resulting from private information. Section 5 discusses additional applications of our model. In particular, there are several interesting di¤erences between our long term contracting economy with production and the

[2] The ambiguous e¤ect of risk on savings was noted by Levhari and Srinivasan (1969). Specifically, when the elasticity of intertemporal substitution is low (high), risk tends to raise (reduce) savings. See Weil (1990). Obstfeld (1994) shows that when risk sharing leads to a portfolio shift into riskier, more productive assets, it may be growth promoting.

[3] See also Aiyagari and Williamson (1997) for a model of credit in which only the social planner has access to capital.

more standard model of contracts with risky endowments. These may be of independent interest.

2. The Environment

In each period, there is a large number of households each of which operates a technology of the form $Y_t = z_t K_t$ where Y_t is output, K_t is capital, and z_t is the level of productivity at time $t = 0; 1; \ldots$. Productivity, which is independently and identically distributed across households at any time, and over time for any household, takes on one of two possible values: it is z_i with probability $\mu_i > 0$, $i = 1; 2$, where $0 < z_1 < z_2$ and $\mu_1 + \mu_2 = 1$. We define the expected value of productivity as $\xi = \sum_{i=1}^{2} \mu_i z_i$ and assume that capital completely depreciates after production.

Households are infinitely lived, and possess time separable preferences over sequences of consumption with period utility from current consumption, C, of the form

$$v(C) = \begin{cases} (1 - \beta)\frac{C^{1-\sigma}}{1-\sigma} & \text{for } \sigma > 0 \text{ and } \sigma \neq 1, \\ (1 - \beta)\log C & \text{for } \sigma = 1. \end{cases}$$

Note that, for convenience, we normalize the utility function by $(1 - \beta) \in (0; 1)$ where future utility is discounted by β.

Each household participates in a permanent contract with a risk-neutral, competitive intermediary. In any period, only the household observes its own productivity, thus there is private information with respect to output. At the beginning of the period the household has a predetermined capital stock, K. Given the household's capital, the intermediary announces a set of potential transfers, B_i, and investments, K_i^0, as functions of the impending productivity report. Upon observing its output, $z_i K$, the household determines a report for the intermediary. Subsequently, the intermediary executes the transfer, and implements the investment for the household, which determines its capital stock at the onset of the next period. By definition, if the contract is incentive-compatible, then, at every point in time, the household will truthfully report the level of productivity. Hence consumption in state i will be $C_i = z_i K + B_i$.

Our approach in solving the contract adapts the methods used to characterize the risky endowment model of long-term contracts.[4] An important assumption in extending these existing results to our analysis is that the household has no ability to invest in an unobservable manner. The value of misreporting productivity lies in being able to consume hidden output. In order to ensure truth-telling (incentive-compatibility), we constrain the contract so there are no gains from one-period temporary deviations from truth-telling. That is, the contract is temporarily incentive compatible (t.i.c.) in the sense of Green (1987). Provided certain boundary conditions, satisfied by our problem, hold, temporary incentive compatibility is equivalent to incentive compatibility. Next, since the t.i.c. constraints introduce future expected lifetime utility as a state variable, we follow Green in characterizing the contract by solving a dual, expenditure minimization problem for the intermediary. Standard duality theorems ensure that this solution also solves the utility maximization problem faced by household.[5] Finally,

[4] See for example Green (1987), Taub (1990), Phelan and Townsend (1991), Atkeson and Lucas (1992) and the related analysis of Spear and Srivastava (1987) and Thomas and Worrall (1990).

[5] A formal proof of these results, which are standard but require considerable additional

we impose an aggregate resource constraint upon our economy: the sum of consumption and investment cannot exceed output. This is related to the approach taken by Atkeson and Lucas (1992) in the context of an endowment economy.

Let U_i' represent expected lifetime utility, starting next period, for the household, assuming that it will accurately report productivity from that date onward, given a current productivity report of z_i. When the state is z_1, temporary incentive compatibility is ensured by the following constraint.

$$\text{if } z_1K + B_2 > 0 \text{ then } v(z_1K + B_1) + \beta U_1' \geq v(z_1K + B_2) + \beta U_2' \tag{2.1}$$

The left hand side of (2.1) represents the value to the household with actual output z_1K of truthfully reporting its productivity. Provided that misreporting the level of productivity generates a feasible level of consumption, then the right hand side of the constraint represents the value of following this strategy. The t.i.c. constraint when productivity is z_2 is given below.

$$v(z_2K + B_2) + \beta U_2' \geq v(z_2K + B_1) + \beta U_1' \tag{2.2}$$

Note that, as $z_2 > z_1$, non-negativity of C_1 ensures that $z_2K + B_1 \geq 0$, eliminating the need for a conditional constraint. As discussed in Oh and Green (1992), concavity of v implies that if both (2.1) and (2.2) are to hold, then $B_1 \geq B_2$ and $U_1' \leq U_2'$. Furthermore, if (2.2) binds and $B_1 \geq B_2$ ($B_1 > B_2$) then (2.1) is satisfied (holds with inequality). These results will prove useful below.

As indicated earlier, we obtain equilibrium allocations for the contracting economy using a dual approach. Given an initial utility entitlement, U, and capital

notation, may be found in Khan and Ravikumar (1996).

stock, K, for the household, the intermediary solves an expenditure minimization problem. Hereafter, we will refer to the solution of the expenditure minimization problem as the contract. In this formulation, we must impose a promise-keeping constraint upon the contract which ensures that the household's expected lifetime utility satisfies its initial entitlement.

$$U \leq \sum_{i=1}^{2} \pi_i \left(v(z_i K + B_i) + \beta U_i' \right) \tag{2.3}$$

The intermediary can borrow from, or lend to, other intermediaries at the constant discount factor, $q \in (0,1)$. Let the expected present value of expenditure be $E(U;K)$. The contract, which minimizes the intermediary's net expenditure, by choice of $\left(B_i; K_i'; U_i' \right)_{i=1}^{2}$ subject to the incentive compatibility constraints (2.1) - (2.2) and the promise keeping constraint (2.3) satisfies the following Bellman equation.

$$E(U;K) = \min \sum_{i=1}^{2} \pi_i \left(B_i + K_i' - z_i K + qE(U_i'; K_i') \right) \tag{2.4}$$

The expected present value of expenditure, at the optimum, will equal the sum of expected current expenditure and the discounted expected present value of expenditures incurred from the next period onwards. A competitive intermediary must maximize the household's expected lifetime utility; in equilibrium, this implies a zero profit condition $E(U;K) = 0$ which determines U given K.

3. Analysis

In this section we provide a complete characterization of the private information economy for the case of logarithmic preferences. The case of general iso-elastic utility is similar, and is summarized in section B of the appendix.

3.1. The Contract

In order to solve the Bellman equation, we deflate the value function by capital. This allows us to reformulate the contract into an intensive form which, by exploiting a homogeneity property of the problem, reduces the dimension of the state vector. Let $b_i K = B_i$ and $\gamma_i K = K_i'$ for $i = 1, 2$. Rewrite the objective as

$$E(U, K)/K = \min \sum_{i=1}^{2} \pi_i \left(b_i + \gamma_i - z_i + q\gamma_i E(U_i', K_i')/K_i' \right). \tag{3.1}$$

Now define a composite state variable, $u \equiv U - \log K$. Consistency requires that the future state, conditional on i, is given by $u_i' = U_i' - \log k_i'$. Since this implies that $U_i' = u_i' + \log \gamma_i + \log k$, the t.i.c. constraint at z_1 may be revised as

$$\text{if } z_1 + b_2 \geq 0 \text{ then} \tag{3.2}$$

$$(1-\beta)\log(z_1 + b_1) + \beta\left(u_1' + \log \gamma_1\right) \geq (1-\beta)\log(z_1 + b_2) + \beta\left(u_2' + \log \gamma_2\right),$$

while the t.i.c. constraint at z_2 is equivalent to

$$\begin{aligned} &(1-\beta)\log(z_2 + b_2) + \beta\left(u_2' + \log \gamma_2\right) \\ \geq\ &(1-\beta)\log(z_2 + b_1) + \beta\left(u_1' + \log \gamma_1\right). \end{aligned} \tag{3.3}$$

Subtracting $\log K$ from both sides of (2.3) the promise-keeping constraint becomes

$$u \leq \sum_{i=1}^{2} \pi_i \left[(1-\beta)\log(z_i + b_i) + \beta\left(u_i' + \log \gamma_i\right) \right]. \tag{3.4}$$

Since the constraints above depend only upon the composite state variable, we are able to define $W(u) = E(U, K)/K$. The intensive form problem, which describes expenditures per unit capital, satisfies the following Bellman equation.

$$W(u) = \min \sum_{i=1}^{2} \pi_i \left(b_i + \gamma_i - z_i + q\gamma_i W(u_i') \right) \tag{3.5}$$

where the minimization is with respect to $(b_i, \gamma_i, u_i')_{i=1}^{2}$.

We now analyze the intensive form contract. Let λ and θ be the multipliers for the constraints (3.3) and (3.4). We suppress (3.2) which never binds, as is shown below in proposition 3.1. The first order conditions, with respect to $(b_i, \gamma_i, u_i')_{i=1}^{2}$, are listed below.[6]

$$\pi_1 + \frac{\lambda(1-\beta)}{z_2 + b_1} - \frac{\theta\pi_1(1-\beta)}{z_1 + b_1} = 0 \tag{3.6}$$

$$\pi_2 - \frac{\lambda(1-\beta)}{z_2 + b_2} - \frac{\theta\pi_2(1-\beta)}{z_2 + b_2} = 0 \tag{3.7}$$

$$\pi_1 \left(1 + qW\left(u_1'\right)\right) + \lambda\frac{\beta}{\gamma_1} - \theta\frac{\pi_1\beta}{\gamma_1} = 0 \tag{3.8}$$

$$\pi_2 \left(1 + qW\left(u_2'\right)\right) - \lambda\frac{\beta}{\gamma_2} - \theta\frac{\pi_2\beta}{\gamma_2} = 0 \tag{3.9}$$

$$\pi_1 q\gamma_1 W'(u_1') + \lambda\beta - \theta\pi_1\beta = 0 \tag{3.10}$$

$$\pi_2 q\gamma_2 W'(u_2') - \lambda\beta - \theta\pi_2\beta = 0 \tag{3.11}$$

The Benveniste-Scheinkman theorem implies $W'(u) = \theta$.

The efficiency conditions allow a strong characterization of the risk-sharing contract. Firstly, the introduction of productive capital offers a channel for adjust-

[6] See Khan and Ravikumar (1996) where we establish, for a more general problem, that W is strictly increasing, convex and differentiable and that the equilibrium described here is unique.

ing utility entitlements absent in the endowment model. As a result, the continuation value of the state variable, u_i^0, is independent of productivity and the initial state, u. The linear production structure implies that utility entitlements, U, are linear functions of the logarithm of the capital stock, K. Changes in expected lifetime utility which occur in response to productivity reports are implemented through changes in the household's stock of capital. Secondly, risk-aversion on part of the household implies that the contract insures current consumption: when the household reports low productivity the net transfer is higher than when it reports high productivity ($b_1 > b_2$). Alternatively, repayment is lower. However, the presence of private information limits the extent of risk-sharing. Households must be prevented from under-reporting income during periods when income is relatively high. As a result, reports of low productivity reduce lifetime consumption. Given diminishing marginal utility, the cost minimizing intermediary will spread this fall in lifetime consumption over time. Consequently, our third result is that low productivity results in both lower current consumption and reduced investment ($c_1 < c_2$ and $°_1 < °_2$). These qualitative characteristics of the contract are summarized in the following proposition. (All proofs are in appendix A.)

Proposition 3.1. In the log case, $u_1^0 = u_2^0$, $°_1 < °_2$, $b_1 > b_2$ and $c_1 < c_2$.

The higher transfer when $z = z_1$, given the binding incentive constraint at z_2, implies that the t.i.c. constraint at z_1 does not bind, as assumed above.

3.2. Equilibrium

As noted earlier, since E is strictly increasing in U given K, the zero pro…t condition $E(U; K) = 0$ will determine the highest level of expected lifetime utility feasible for the household given its initial stock of capital. Since $E(U; K) = KW(U - \log K)$, this zero profit condition implies, given strict monotonicity of W, that U is proportional to $\log K$. Hence u, and thus the contract $\left(b_i; \gamma_i; u_i'\right)_{i=1}^{2}$, is the same for all households. As a result, any household with capital stock K and productivity z_i will be allocated current consumption $(z_i + b_i)K$ and investment $\gamma_i K$. Average output for all households with K units of capital will be ξK, assuming a positive measure of such households; average consumption for this group will be $\sum_{i=1}^{2} \mu_i (z_i + b_i) K$ and average investment will be $\sum_{i=1}^{2} \mu_i \gamma_i K$.

Economy-wide market clearing requires that aggregate output equal the sum of aggregate consumption and investment. This equilibrium restriction on aggregate allocations implies an equivalent restriction on the expected or average current expenditure within the contract which determines q. Let $\psi(K)$ represent the distribution of capital across households over the space of current capital holdings, K. Equilibrium requires that

$$\int_K \xi K \psi(K) = \int_K \left(\sum_{i=1}^{2} \mu_i (z_i + b_i + \gamma_i) \right) K \psi(K).$$

This market-clearing condition requires that $\sum_{i=1}^{2} \mu_i (b_i + \gamma_i) = 0$. Next, using equation 3.5, we have the necessary condition, $\sum_{i=1}^{2} \mu_i q \gamma_i W\left(u_i'\right) = 0$. Recalling $u_1' = u_2'$, this implies $u_i' = u$, $i = 1; 2$, since $W(u) = 0$. Finally, (3.8) and (3.10) yield the equilibrium condition $q\theta = 1$. Note that the recursive equilibrium is stationary in the sense that $(b_i; \gamma_i)_{i=1}^{2}$, q and u are time-invariant. This verifies our earlier conjecture that q is constant.

We now contrast growth between our incomplete risk-sharing economy and the complete risk-sharing benchmark. The latter, a well-known problem, may be retrieved by suppressing (3.3) (setting $\lambda = 0$ everywhere) and repeating the above analysis. The solution, denoted by superscript f, is characterized by $c_i^f = (1-\beta)\lambda$ and $\gamma_i^f = \beta\lambda$ for $i = 1, 2$. Furthermore, under complete risk-sharing $q^f\lambda = 1$ and $u^f = \log(1-\beta)\left(\beta^{\beta}\lambda\right)^{\frac{1}{1-\beta}}$.

The introduction of private information reduces the mean rate of growth, $\sum_{i=1}^{2}\pi_i\gamma_i$, relative to the complete risk-sharing value of $\beta\lambda$. As a result, the intermediary's discount rate, $q^{-1} - 1$ falls. We suggest the following explanation. If, upon observing z_2, the household truthfully reports the productivity then it consumes C_2, while misrepresentation yields consumption equal to $(z_2 - z_1)K + C_1$. All else being equal, higher levels of capital tend to increase the current gains to deviations from truth-telling. The contract then requires larger variations in both C_i and U_i' in order to ensure incentive-compatibility. Given convexity of preferences, this tends to reduce welfare for any given level of resources. This welfare reducing aspect of additional capital makes investment less attractive in the private information economy relative to the complete risk-sharing economy. Hence the overall rate of capital accumulation is lower under private information.

Proposition 3.2. In equilibrium, $q > \lambda^{-1}$ and $\sum_{i=1}^{2}\pi_i\gamma_i < \beta\lambda$.

We calculate the expected increase in lifetime utility as $\sum_{i=1}^{2}\pi_i\left(U_i' - U\right) = \sum_{i=1}^{2}\pi_i\log\gamma_i$. Proposition (3.2) and Jensen's Inequality jointly imply that the expected increase in welfare is lower under private information. However, this does not imply that welfare has a negative trend leading to the immiserization of almost all households. This result, due to the possibility of economic growth, is in sharp contrast to the endowment model.

4. Numerical examples

We examine several numerical examples. These allow us to describe the risk sharing arrangement in more detail and obtain preliminary evidence on the magnitude of the growth and welfare effects of the incomplete risk-sharing environment. The baseline parameter values we use are in table 1. The average level of productivity is set equal to the long run return on equity in the U.S., $\xi = 1.065$, as indicated in Mehra and Prescott (1985). We allow productivity to vary symmetrically around its mean. Thus we assume that $\pi_1 = 0.5$ and $x = \xi - z_1 = z_2 - \xi$. The parameter x is difficult to calibrate. In our baseline case we set its value to imply that the coefficient of variation of z is 0.1. This value implies a standard deviation of consumption growth of 0.0468, which is close to 0.044 predicted by the base case of Heaton and Lucas (1996, table 4, p.458).[7] Finally we choose β so that $\beta\xi = 1.02$. The aggregate rate of growth for the complete risk-sharing economy, when household preferences are logarithmic, matches the long run growth data, as documented in Parente and Prescott (1993). This is also the average rate of growth under autarchy, and, as we shall see, not significantly different from the rate of growth under incomplete risk-sharing.

We first examine the case of logarithmic preferences. Across the three different allocations, autarchy (A), incomplete risk-sharing (I) and complete risk-sharing (C), $u + \log K$ represents the level of expected lifetime utility for a household with capital K. Thus u is the expected lifetime utility for a household with one unit of capital. Each entry in the rows of tables 2 through 5 marked **loss** represents the percentage decrease necessary in the level of consumption under complete risk-

[7] Below, we will examine examples involving different values of x.

sharing, at every point in time, to match the level of welfare associated with the other economies. We deflate all quantity variables by the level of capital. Thus, given a shock z_i, the household's savings is $-b_i$ and c_i is consumption, per unit capital. Investment per unit capital is denoted γ_i, which is also the gross rate of growth of capital. The average rate of growth is denoted $E(\gamma)$, while r is the percentage discount rate ($q = \frac{1}{1+r}$). Finally, ΔU represents the expected increase in lifetime utility.

In table 2 we see that, in the complete risk-sharing allocation, consumption and investment are unresponsive to the productivity shock. The household's savings varies with productivity so as to completely smooth the consumption profile. The incomplete risk-sharing economy induces fluctuations in current consumption, but this variability in consumption is low relative to that under autarchy. There is a net transfer of resources from households with high current productivity to those with low current productivity: $c_1 + \gamma_1 > z_1$ while $c_2 + \gamma_2 < z_2$. For those experiencing below average productivity, this reduces savings, while boosting both consumption and investment, relative to autarchy. The residual variability in consumption, and the reduced average growth rate, causes expected welfare to increase more slowly than under complete risk-sharing, $\Delta U = 0.0184 < 0.0198$. The inability to smooth consumption under autarchy implies high variability in both consumption and investment rates. Consequently, welfare increases yet more slowly, $\Delta U = 0.0148$.

In figure 1, we illustrate initial lifetime expected utility for the complete risk-sharing, incomplete risk-sharing and autarchy economies. As indicated by the loss measures in table 2, the move from complete to incomplete insurance is equivalent to a 1.6% decrease in the level of consumption, while autarchy implies an 11.2%

decrease. In this example, we see that incentive compatible arrangements are relatively successful in smoothing consumption. The switch from such an economy to autarchy results in a significant loss in expected utility, measured in units of full insurance consumption, for the typical household.

In figures 2 and 3 we graph the evolution of the distribution of capital (K), deflated by the compounded growth factor, and expected lifetime utility (U) for the incomplete risk-sharing economy. All households are initially identical. Recall that the intermediary enforces truth-telling by offering relatively higher lifetime utility entitlements for high productivity reports than for low productivity reports at each point in time. As a result, both distributions of wealth and utility, are characterized by increasing dispersion over time. For this example, the distribution of utility entitlements within each period is symmetric. Convexity of preferences then implies a skewed distribution of capital. In proposition 3.1 we showed that, in the private information economy, changes in welfare are implemented through changes in capital. This log-linear mapping is also, of course, present in the autarchic model. The greater variability in investment present in autarchy implies that the private information economy dampens dispersion over time relative to autarchy.

Next, in table 3, maintaining our other baseline parameters, we allow the coefficient of relative risk aversion, σ, to vary between $1/2$ and 4. We find that the rate of growth under private information is consistently below the complete risk-sharing equivalent. This result, which we have found to be robust, indicates that the growth reducing effect of incomplete risk-sharing, found for logarithmic preferences, extends to the case of iso-elastic utility. Interestingly, both the growth and welfare effects of private information fall as σ rises. Recall that higher values of σ

are associated with increased reluctance to substitute consumption across time. As shown in proposition 3.1, potential deviations from truth-telling raise current consumption at the expense of future consumption. As σ increases, the attractiveness of such behaviour is reduced. This reduces the costs of private information and shifts the incomplete risk-sharing allocation closer to full insurance. For all σ, the contract is relatively efficient. Even when the intertemporal elasticity of substitution is high ($\sigma = 0.5$), the loss is only 2.57%. Note that, for the same σ, the loss under autarchy is more than three times as large, 9.77%. For higher values of σ, autarchy yields larger welfare losses relative to incomplete risk-sharing. Furthermore, under autarchy, income uncertainty generates a strong motive to self-insure through savings when risk aversion is large. This drives the high rates of growth relative to complete risk-sharing. As is well known, the sign of the risk effect on savings changes when σ crosses one. In contrast, the growth rate under incomplete risk-sharing is always below that under complete risk-sharing, and the growth effects are small.

In table 4, we vary the coefficient of variation of z. This implies changes in z_1 and z_2. All other parameters are maintained at the values listed in table 1. Higher variability of productivity implies higher risk and tends to reduce the efficiency of the contract in terms of both growth and welfare. However, the differences in rates of growth never rise above one-tenth of one per cent and the associated welfare effect is small relative to autarchy. Table 5 considers changes in the discount factor, β, while maintaining all other parameters at the table 1 values. The three discount factors we consider, $\beta = 0.9390$, 0.9577 and 0.9765 imply 0, 2 and 4 per cent average growth, respectively. Note that higher values of β imply an increased emphasis on future consumption. As indicated by the negative trend in loss, this increases the efficiency of the contract for the same reason as in table 3.

These numerical examples indicate that, across a range of parameter values, (1) the growth effects of incomplete risk-sharing are small and (2) the incomplete markets economy achieves levels of welfare close to the levels attained under complete risk-sharing. The relative efficiency of the private information economy arises from the ability to adjust capital, and hence output, in response to the changes necessary in lifetime utility entitlement over time. This implies that changes in a household's utility entitlement are matched by proportionate movements in the gain from understating productivity, $(z_2 - z_1)K$.

5. Concluding remarks

We have examined the impact of incomplete risk-sharing, in an environment with private information, on growth and welfare. In our economy, households share risk by entering into enduring relationships with competitive intermediaries. We have found that the aggregate growth rate is lower under private information than under full insurance. Furthermore, the risk-sharing arrangement, while incomplete, is relatively efficient and the growth effects of private information are generally small.

Our work adapts the methods used to study long term contracting with risky, unobservable endowments to an economy with production and capital accumulation. The contract with capital exhibits several properties which contrast with the standard model. First, expected lifetime utility, while growing more slowly than under complete risk-sharing, does not necessarily contain a negative trend. Second, the contract exhibits the property that all changes in welfare are implemented through changes in the household's stock of capital. Consequently, welfare always exceeds the autarchy value of capital. Finally, while both the endowment and production economies share the property that the distribution of wealth or utility entitlements is characterized by increasing dispersion, in the production economy this rising inequality is larger under autarchy.[8]

The contract implements risk-sharing by conditioning the household's future lifetime utility, or wealth, on the current report of productivity. Thus we emphasize the problem of unobservable returns to investment, the common emphasis of the literature on private information in development economics. If investment were itself unobservable, then our risk-sharing arrangement would be infeasible.[9] In particular, the intermediary cannot exploit di¤erences in the rates of intertemporal substitution across households. It is, however, unclear what types of risk-sharing arrangements are then feasible. We view this as an area for future research. An implication of our …ndings is that if resources may be devoted towards either (1) reducing the e¤ects of informational asymmetries and thereby implementing improved insurance services (allowing for observable returns to investment) or (2) developing the legal basis for implementing state-contingent enforceable contracts (allowing for observable investment), such as those we have assumed, then expenditures on the latter may be far more important for welfare gains.

Our framework may also contribute to explanations of several empirical phenomena that are apparently at odds with the complete markets model of capital

[8] See Aiyagari and Alvarez (1996) for an interesting example of an endowment economy where lower bounds on the consumption possibilities set ensure that the economy is characterized by an invariant distribution of wealth.

[9] See Cole and Kocherlakota (1997) for an economy with risky endowments and unobservable storage, where the rate of return to storage is exogenous.

accumulation. For instance, consider the cross-country evidence on savings, investment and consumption. Feldstein and Horioka (1980), Backus et al. (1992) and Baxter and Crucini (1993), among others, have presented evidence that the savings to investment correlation, within several economies, is positive. Backus et al. and Baxter and Crucini (1994) have found that cross-country consumption correlations are lower than the corresponding output correlations. Both empirical regularities have been interpreted as inconsistent with frictionless international borrowing and lending. However, in a two-country model with complete markets, productivity spill-overs and capital adjustment costs, Baxter and Crucini (1993) have reproduced the positive savings-investment correlation. Their result emphasizes country size. In our model, the presence of private information yields positive correlation of savings and investment. We are able to generate this result even though locations are small and productivity is independently distributed. With respect to the consumption correlation anomaly, Baxter and Crucini (1994) have developed explanations which rely, in part, upon exogenous restrictions on financial arrangements. Our economy provides a basis for such departures from the complete markets assumption.

Appendix

A. Proofs

Proof of Proposition 3.1:

We divide the proof into 5 parts.

1) $u_1^0 = u_2^0 = u^0$: Equations (3.8) and (3.10) jointly imply that $1 + qW\left(u_1^0\right) =$

$qW'\left(u_1'\right)$ while (3.9) and (3.11) together yield $1 + qW'\left(u_2'\right) = qW'\left(u_2'\right)$. Thus we see that $u_1' = u_2'$ and this common value, labeled u' is independent of u and thus common to all contracts.

2) $\lambda > 0$: (By contradiction) Given part (1), assume that $\lambda = 0$. Next, from (3.8) and (3.9) we have $\gamma_1 = \gamma_2$ while (3.6) and (3.7) yield $z_1 + b_1 = z_2 + b_2$. Since this implies that $z_2 + b_1 > z_2 + b_2$ we have violated (3.3).

3) $\gamma_1 < \gamma_2$: Given parts (1) and (2), we may solve (3.10) and (3.11) to obtain $\gamma_1 = -\frac{\theta - \frac{\lambda}{\mu_1}}{qW'(u')} < -\frac{\theta + \frac{\lambda}{\mu_2}}{qW'(u')} = \gamma_2$.

4) $b_1 < b_2$: Given parts (1) - (3), we know that $\beta \log \gamma_2 - \beta \log \gamma_1 > 0$ which requires that $(1-\beta)(\log(z_2 + b_2) - \log(z_2 + b_1)) < 0$ for (3.3) to hold with equality.

5) $c_1 = z_1 + b_1 < z_2 + b_2 = c_2$: Given part (2), rearranging (3.6) and 3.7) we have $\mu_1\left(1 - \frac{\theta(1-\beta)}{z_1+b_1}\right) = -\frac{\lambda(1-\beta)}{z_2+b_1} < 0$ while $\mu_2\left(1 - \frac{\theta(1-\beta)}{z_2+b_2}\right) = \frac{\lambda(1-\beta)}{z_2+b_2} > 0$. This requires that $z_1 + b_1 < z_2 + b_2$. ■

Proof of Proposition 3.2

As $W'\left(u_i'\right) = \theta$ and $q\theta = 1$, (3.10) and (3.9) may be solved as $\gamma_1 = \theta\beta - \frac{\lambda}{\mu_1}\beta$ and $\gamma_2 = \theta\beta + \frac{\lambda}{\mu_2}\beta$. Next (3.6) and (3.7) may be rearranged as

$$
\begin{aligned}
\mu_1(z_1 + b_1) + \lambda(1-\beta)\frac{z_1 + b_1}{z_2 + b_1} &= \mu_1\theta(1-\beta) \\
\mu_2(z_2 + b_2) - \lambda(1-\beta) &= \mu_2\theta(1-\beta).
\end{aligned}
$$

It then follows that

$$
\sum_{i=1} \mu_i(b_i + \gamma_i) - \lambda(1-\beta)\left[1 - \frac{z_1 + b_1}{z_2 + b_1}\right] = \theta - \xi.
$$

Given proposition 3.1, we know that $\lambda > 0$, so that, as $z_1 < z_2$, we know that

$\lambda(1-\beta)\left[1-\frac{z_1+b_1}{z_2+b_1}\right] > 0$. Recalling the equilibrium condition $\sum_{i=1}^{2}\mu_i(b_i+\gamma_i) = 0$, we have proven $\theta < \xi$. Therefore $q\xi > 1$ and $\sum_{i=1}^{2}\mu_i\gamma_i = \beta\theta < \beta\xi$. ■

B. Iso-elastic preferences

We solve the iso-elastic case, drawing heavily on the analysis of section 3. The intensive form composite state variable, for this case, is given by $u = \frac{U}{k^{1-\sigma}}$. The contract is determined by solving (3.5) subject to (B.1) - (B.3).

$$\text{if } z_1+b_2 \geq 0 \text{ then } (1-\beta)\frac{(z_1+b_1)^{1-\sigma}}{1-\sigma}+\beta\gamma_1^{1-\sigma}u_1' \geq (1-\beta)\frac{(z_1+b_2)^{1-\sigma}}{1-\sigma}+\beta\gamma_2^{1-\sigma}u_2' \tag{B.1}$$

$$(1-\beta)\frac{(z_2+b_2)^{1-\sigma}}{1-\sigma}+\beta\gamma_2^{1-\sigma}u_2' \geq (1-\beta)\frac{(z_2+b_1)^{1-\sigma}}{1-\sigma}+\beta\gamma_1^{1-\sigma}u_1' \tag{B.2}$$

$$u \leq \sum_{i=1}\mu_i\left[(1-\beta)\frac{(z_i+b_i)^{1-\sigma}}{1-\sigma}+\beta\gamma_i^{1-\sigma}u_i'\right] \tag{B.3}$$

Suppressing (B.1) which, as before, does not bind, defining λ to be the multiplier for (B.2) and θ the multiplier for (B.3), we derive the following efficiency conditions with respect to $\left(b_i;\gamma_i;u_i'\right)_{i=1}^{2}$.

$$\mu_1+\lambda_1(1-\beta)(z_2+b_1)^{-\sigma}-\mu_1\theta(1-\beta)(z_1+b_1)^{-\sigma} = 0 \tag{B.4}$$

$$\mu_2-\lambda_1(1-\beta)(z_2+b_2)^{-\sigma}-\mu_2\theta(1-\beta)(z_2+b_2)^{-\sigma} = 0 \tag{B.5}$$

$$\mu_1\left(1+qW\left(u_1'\right)\right)+\lambda_1\beta\gamma_1^{-\sigma}(1-\sigma)u_1'-\mu_1\theta\beta\gamma_1^{-\sigma}(1-\sigma)u_1' = 0 \tag{B.6}$$

$$\rho_2 \left(1 + qW\left(u_2'\right)\right) - \lambda_1 \beta \gamma_2^{-\sigma} (1 - \sigma) u_2' - \rho_2 \mu \beta \gamma_2^{-\sigma} (1 - \sigma) u_2' = 0 \quad (B.7)$$

$$\rho_1 q \gamma_1 W'(u_1') + \lambda_1 \beta \gamma_1^{1-\sigma} - \rho_1 \mu \beta \gamma_1^{1-\sigma} = 0 \quad (B.8)$$

$$\rho_2 q \gamma_2 W'(u_2') - \lambda_1 \beta \gamma_2^{1-\sigma} - \rho_2 \mu \beta \gamma_2^{1-\sigma} = 0 \quad (B.9)$$

It is straightforward to show that proposition 3.1 holds for the general iso-elastic case. Furthermore, equilibrium in the economy with iso-elastic preferences may be calculated using the method described in section 3.2. An examination of the growth effects of private information given iso-elastic utility, which requires numerical methods, is contained in table 3 and discussed in section 4.

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Table 1: Baseline Parameters

z_1	z_2	μ_1	β	σ
0.9585	1.1715	0.5	0.9577	1.0

Table 2: The contract

	A	I	C
u	-2.7712	-2.6687	-2.6522
Loss (%)	11.2000	1.6000	0.0000
b_1	-0.9180	-0.9138	-0.9135
b_2	-1.1220	-1.1248	-1.1265
c_1	0.0405	0.0447	0.0450
c_2	0.0495	0.0467	0.0450
γ_1	0.9180	0.9809	1.0200
γ_2	1.1220	1.0577	1.0200
$\mathbf{E}(\gamma)$	1.0200	1.0193	1.0200
ΔU	0.0148	0.0184	0.0198

Table 3: Varying the elasticity of substitution

	σ:	0.5	0.75	1	2	3	4
	C	4.03	2.67	2.00	0.99	0.66	0.50
Growth (%)	I	3.93	2.58	1.93	0.96	0.64	0.48
	A	3.77	2.54	2.00	1.50	1.67	2.01
Loss (%)	I	2.57	1.96	1.63	0.83	0.48	0.31
	A	9.77	9.79	11.21	18.45	25.99	33.40

Table 4: Varying the coefficient of variation

	Coeff. var.:	0.05	0.10	0.20
Growth (%)	C	2.00	2.00	2.00
	I	1.96	1.93	1.90
	A	2.00	2.00	2.00
Loss (%)	I	0.79	1.63	2.93
	A	2.92	11.21	38.31

Table 5: Varying the discount factor

	β:	0.9390	0.9577	0.9765
Growth (%)	C	0.000	2.000	4.000
	I	-0.001	1.930	3.970
	A	0.000	2.000	4.000
Loss (%)	I	1.65	1.63	1.50
	A	7.90	11.21	19.25

Figure 1: The primal value functions

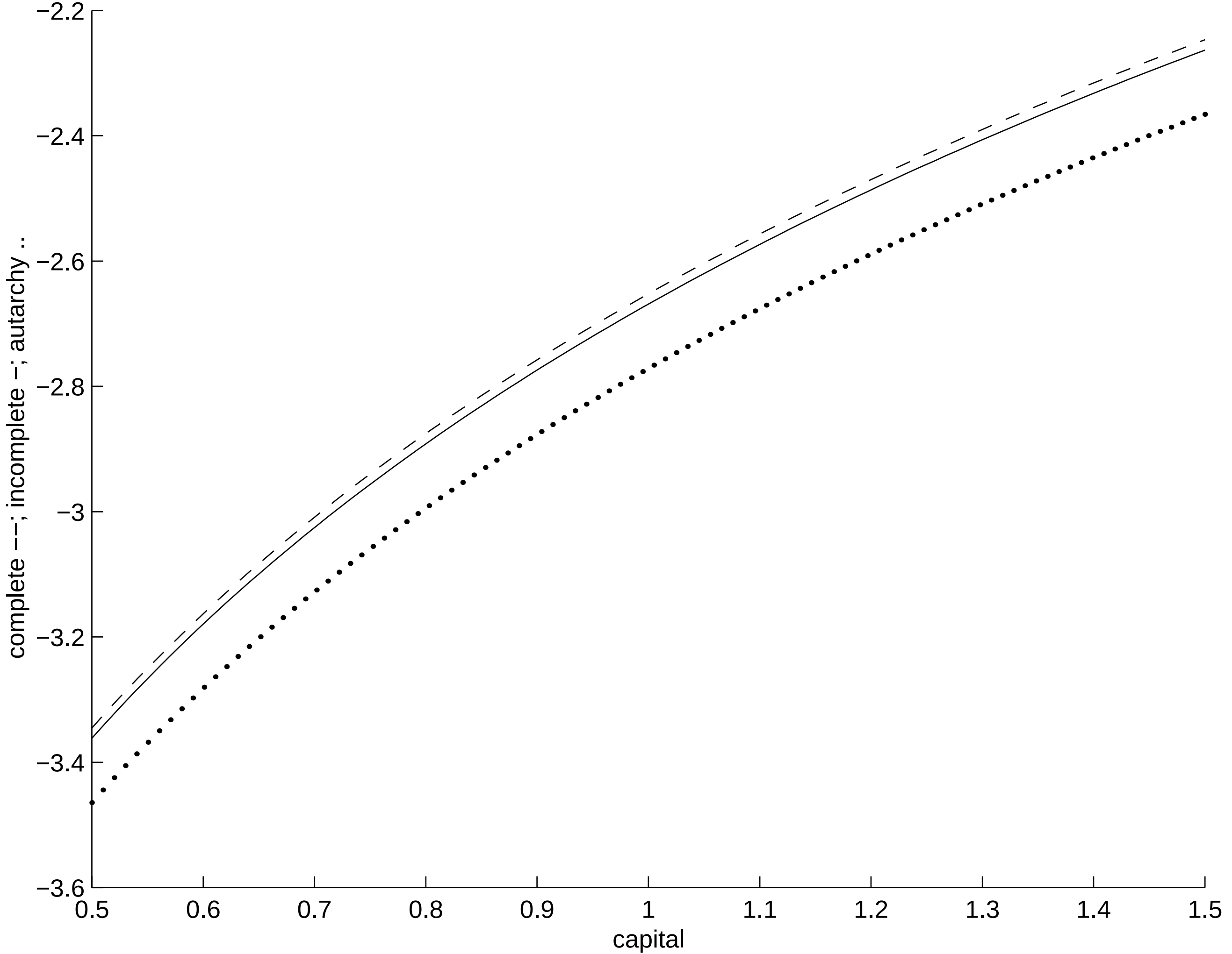

Figure 2: The distribution of capital

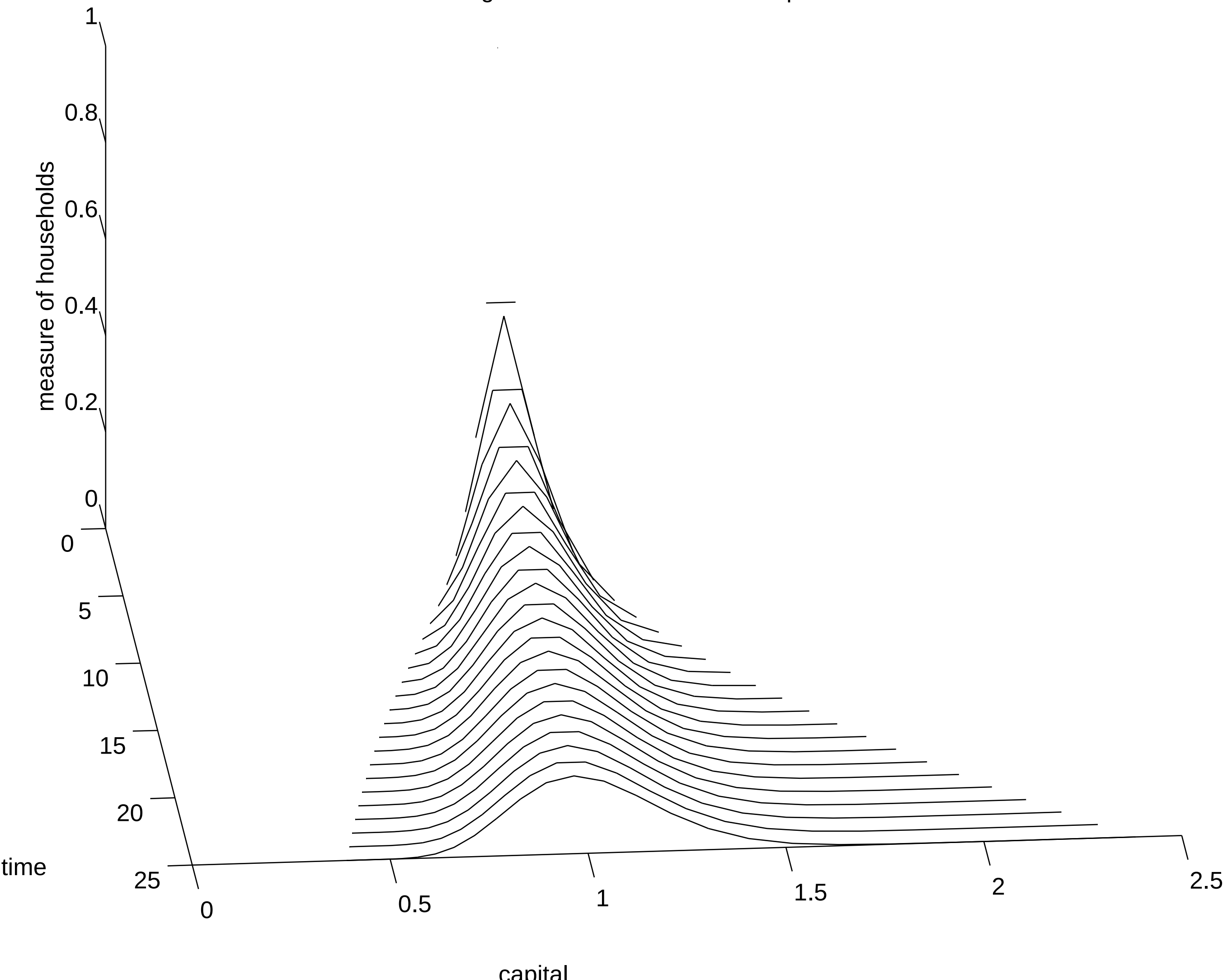

Figure 3: The distribution of utility entitlements