

Dynamic Output and Employment Effects of Public Capital

by

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I. Introduction

Some of my previous research investigates the static, or short run impacts of changes in the public capital stock on economic performance. For instance, in Aschauer (1997a) I use state level data for the period 1970 to 1990 and find that the public capital stock is an important determinant of the rate of growth of output per worker. Specifically, a one standard deviation increase in public capital (relative to private capital) induces an increase in the growth rate of output per worker of some 1.4 percent per year.¹ Similarly, in Aschauer (1997b) I determine that the public capital stock is also a key factor lying behind the rates of growth of output *and* employment, with a one standard deviation rise in public capital generating an increase in the growth rate of output and employment, respectively, of about 1.6 and 0.5 percent per year.²

However, these findings leave open the question of the dynamic, or long run effects of public capital on the economy. To answer this question, it is just as important to understand the dynamic interrelationship between productivity, output, and employment as the economy evolves over time as it is to know the effect of public capital on the initial growth rates of these variables. For example, depending on the persistence of the increase in the productivity growth rate, any particular static

increase in productivity growth can translate into a rather small or large increase in the long run level of output per worker. Consider the following formula for the cumulative change in productivity given a series of changes in productivity growth:

$$dy = \sum_{t=0}^{\infty} dDy_t = \sum_{t=0}^{\infty} r_y^t \cdot dDy$$

where y represents (the natural logarithm of) productivity, Dy_t , the growth rate of productivity in period t , Dy the initial growth rate of productivity, and r_y a persistence parameter for productivity growth. Making use of the estimate cited above--that a one standard deviation increase in public capital raises initial productivity growth by 1.6 percent per year--a purely transitory, one period increase in productivity growth (represented by $r_y = 0$) will lift long run output per worker by only 1.6 percent, while a highly persistent increase in productivity growth (represented by, perhaps, $r_y = 0.9$) will boost the long run level of productivity by 14.4 percent. In the extreme case of a permanent increase in productivity growth (where $r_y = 1$), a given increase in initial productivity growth ultimately will generate an infinite increase in output per worker.

This paper explores these persistence concerns by simulating the dynamic, long run effects of public capital on output and employment. Section II lays out a dynamic model relating output and employment growth to public capital, initial output, and initial employment--a minimalist model capable of capturing in a compact fashion the interrelationship between output and employment as

the economy evolves over time. Section III presents empirical estimates of the model based on fixed effects regression analysis of U.S. state level data over the period 1970 to 1990. Section IV employs the estimated model to simulate the long run impact of public capital under two scenarios--where the state unemployment rate and the labor force, respectively, are assumed to be exogenous. Section V concludes the paper with some suggestions regarding future research.

II. Conceptual Framework

The analysis is predicated on a constant returns to scale production function, written in natural logarithms as

$$Y = A + aK + (1-a)E \quad (1)$$

where Y = natural logarithm of output of goods and services, K = natural logarithm of physical capital stock, E = natural logarithm of employment, and A = natural logarithm of total factor productivity. Total factor productivity is a function of the allocation of the total--public and private--capital stock as in

$$A = A\left(\frac{KG}{K}\right) \quad A'' < 0. \quad (2)$$

At low levels of public capital relative to private capital, the marginal product of public capital

exceeds that of private capital and output rises with an increase in public capital; consequently, $A' > 0$. However, at sufficiently high levels of public capital relative to private capital, the marginal product of public capital is exceeded by that of private capital and output falls with an increase in public capital; thus, $A' < 0$. In the empirical analysis to follow, A takes the quadratic form

$$A = 1 - \left(\frac{KG}{K}\right) + \left(\frac{1}{2m}\right) \left(\frac{KG}{K}\right)^2 \quad (3)$$

where

$$\frac{KG}{K} < (>) m \Rightarrow A' > (<) 0$$

and so an estimate of the parameter m represents an estimate of the level of the public capital stock (relative to the private capital stock) which maximizes output.

In this framework, the marginal products of private capital and of employment, written in natural logarithms, are given by

$$mp_K = \ln a + A - (1-a)(K-E) \quad (4)$$

and

$$mp_E = \ln(1-a) + A + a(K-E). \quad (5)$$

Thus, an increase in the public capital stock also increases the marginal products of both factors of

production as long as the public capital stock ratio lies below the output maximizing level of m .

Now, given that there are increasing costs of adjusting the private capital stock and employment, an increase in the public capital stock will cause a persistent differential between the marginal products of private capital and employment and their respective costs--the user cost of capital, r , and the wage, w --and will generate persistent increases in the growth rates of private capital and employment. Analytically, letting DK and DE represent the growth rates of capital and employment, respectively, we have

$$DK = b_K \cdot (mp_K - r) \quad b_K > 0 \quad (6)$$

$$DE = b_E \cdot (mp_E - w) \quad b_E > 0 \quad (7)$$

It is assumed that the user cost of capital is exogenously determined in the national capital market.

It is further assumed that the wage is determined in the state labor market by the employment function

$$E = E_w \cdot w + E_L \cdot L + E_U \cdot U \quad E_w > 0, E_L > 0, E_U < 0$$

where L represents the labor force and U represents the unemployment rate. The labor force and the unemployment rate are taken to depend on the same set of factors which determines the level of output per worker, so that the labor force bears a positive, and the unemployment rate a negative, relationship to output per worker. Inverting the employment function then yields

$$w = w_E \cdot E + w_y \cdot (Y - E) \quad w_E > 0, w_y < 0. \quad (8)$$

Here, the assumption that w_E is positive merely reflects a positively sloped labor supply schedule. The assumption that w_y is negative reflects two possibilities. First, given the unemployment rate, an increase in per capita income can be expected to attract workers to a particular state and, as a consequence, to expand the labor force and reduce the wage. In the subsequent discussion, this will be termed the *endogenous labor force* specification of the labor market. Second, given the labor force, an increase in per capita income can be taken to be consistent with a lower unemployment rate and a larger effective labor force which, in turn, is reflected in a lower wage. This will be termed the *exogenous labor force* specification of the labor market.

The form of the production function implies that the growth rate of output, DY , may be written as the weighted sum of three components--the growth rate of total factor productivity, the growth rate of the private capital stock, and the growth rate of employment. Accordingly,

$$DY = DA + a \cdot DK + (1-a) \cdot DE \quad (9)$$

or

$$DY = DA + a \cdot b_K (mp_K - r) + (1-a) \cdot b_E (mp_E - w). \quad (10)$$

We reduce the model to a two equation system relating the growth rate of output, DY , and the growth rate of employment, DE , to total factor productivity, initial output, and initial employment.

This reduction is accomplished by the following four steps:

(1) assuming that total factor productivity growth, DA , is *potentially* a function of the level of total factor productivity, A , as in

$$DA = g_A \cdot A \quad g_A >, =, < 0$$

(2) solving for the initial capital stock, K , by inverting the production function in equation (1) to get

$$K = \frac{1}{a} [Y - A - (1 - a) \cdot E]$$

(3) using this result to eliminate the capital stock from the expressions for the marginal product of capital (equation (4)) and the marginal product of labor (equation (5)) leaving both as functions of output, employment, and total factor productivity as in

$$mp_K = \ln a + \frac{1}{a} [A + (a - 1) \cdot Y + E]$$

$$mp_E = \ln(1 - a) + Y - E$$

(4) substituting for the wage from equation (8) and for the marginal products of capital and labor (as written directly above) in the expressions for the growth rate of output (equation (9)) and employment (equation (10)).

After these algebraic steps, the following two equation system is obtained:

$$\begin{pmatrix} DY \\ DE \end{pmatrix} = \begin{pmatrix} c_{YY} & c_{YE} \\ c_{EY} & c_{EE} \end{pmatrix} \cdot \begin{pmatrix} Y \\ E \end{pmatrix} + \begin{pmatrix} \overline{c}_Y + g_Y A \\ \overline{c}_E + g_E A \end{pmatrix} \quad (11)$$

where the various parameters are given by:

$$\begin{aligned} c_{YY} &= (1-a)(b_E(1-w_y) - b_K) \\ c_{YE} &= -(1-a)(b_E(1-w_y + w_E) - b_K) \\ c_{EY} &= b_E(1-w_y) \\ c_{EE} &= -b_E(1-w_y + w_E) \\ g_Y &= g_A + ab_K + (1-a)b_E(1-w_y) \\ g_E &= b_E(1-w_y) \end{aligned}$$

and constant terms--of no particular significance for the analysis--are denoted by overlines. Under the assumptions of the model, the last four of these coefficients can be signed-- $c_{EY} > 0$, $c_{EE} < 0$, $g_Y > 0$, and $g_E > 0$. If, in addition, it is assumed that the speed of adjustment for employment sufficiently exceeds the speed of adjustment for the capital stock, then the first two coefficients also may be signed -- $c_{YY} > 0$ and $c_{YE} < 0$.

The long run equilibrium values for output, Y^L , and employment, E^L , are given by

$$Y^L = \frac{(1 - c_{YE}c_{EY})(\bar{c}_Y + g_Y A) - c_{YY}c_{YE}(\bar{c}_E + g_E A)}{|C|} \quad (12)$$

$$E^L = \frac{c_{EY}(\bar{c}_Y + g_Y A) + c_{YY}(\bar{c}_E + g_E A)}{|C|}. \quad (13)$$

The impact of economic shocks on output and employment depends on the stability of the model, which, in turn requires two conditions on the C matrix: that the trace be negative and the determinant be positive. Thus, stability requires

$$tr = c_{YY} + c_{EE} < 0 \quad |C| = \begin{vmatrix} c_{YY} & c_{YE} \\ c_{EY} & c_{EE} \end{vmatrix} = c_{YY}c_{EE} - c_{YE}c_{EY} > 0.$$

Now, in terms of the fundamental parameters of the model, we have

$$c_{YY} + c_{EE} = -(1 - a)b_E w_E < 0$$

$$c_{YY}c_{EE} - c_{YE}c_{EY} = (1 - a)b_E b_K w_E > 0.$$

Thus, as specified, the model is dynamically stable. Intuitively, an increase in (initial) output tends to lower the wage and raise the growth rates of employment and output--a destabilizing force--while

an increase in (initial) employment tends to raise the wage and lower the growth rates of employment and output--a stabilizing force. The strength of the destabilizing force depends on the coefficient w_y , while the strength of the stabilizing force depends, in turn, on the coefficient $w_y - w_E$; as the former coefficient is smaller (in absolute value) than the latter coefficient (also in absolute value), the stabilizing force dominates and the model is dynamically stable.

Figure 1 illustrates the long run equilibrium of the model. Under the (more restrictive) assumption stated previously, both the $DY = 0$ and $DE = 0$ schedules take on positive slopes, specifically

$$\left. \frac{dY}{dE} \right|_{DY=0} = - \frac{c_{YE}}{c_{YY}}; \quad \left. \frac{dY}{dE} \right|_{DE=0} = - \frac{c_{EE}}{c_{EY}}.$$

The dynamic structure of the model is depicted by the arrows in the diagram. For instance, in the region above (below) both the $DY = 0$ and $DE = 0$ loci, the growth rates of output and employment are positive (negative). In the region above (below) the $DY = 0$ locus but below (above) the $DE = 0$ locus, the growth rate of output is positive (negative) and the growth rate of employment is negative (positive).

Depending on the magnitude of the various parameters, the structure of the model admits a variety of possibilities for dynamic paths for output and employment. Two such paths, beginning from an arbitrary beginning position for the economy labeled B, are shown in Figure 2. The first path,

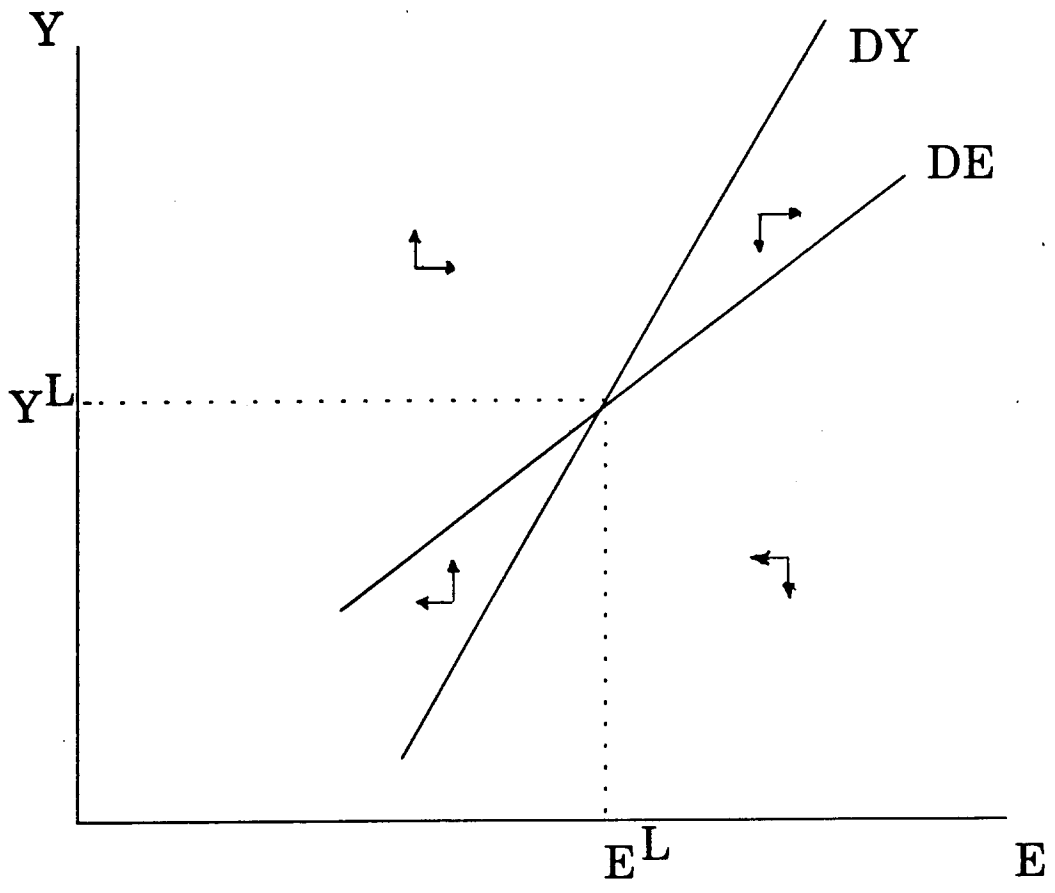


Figure 1

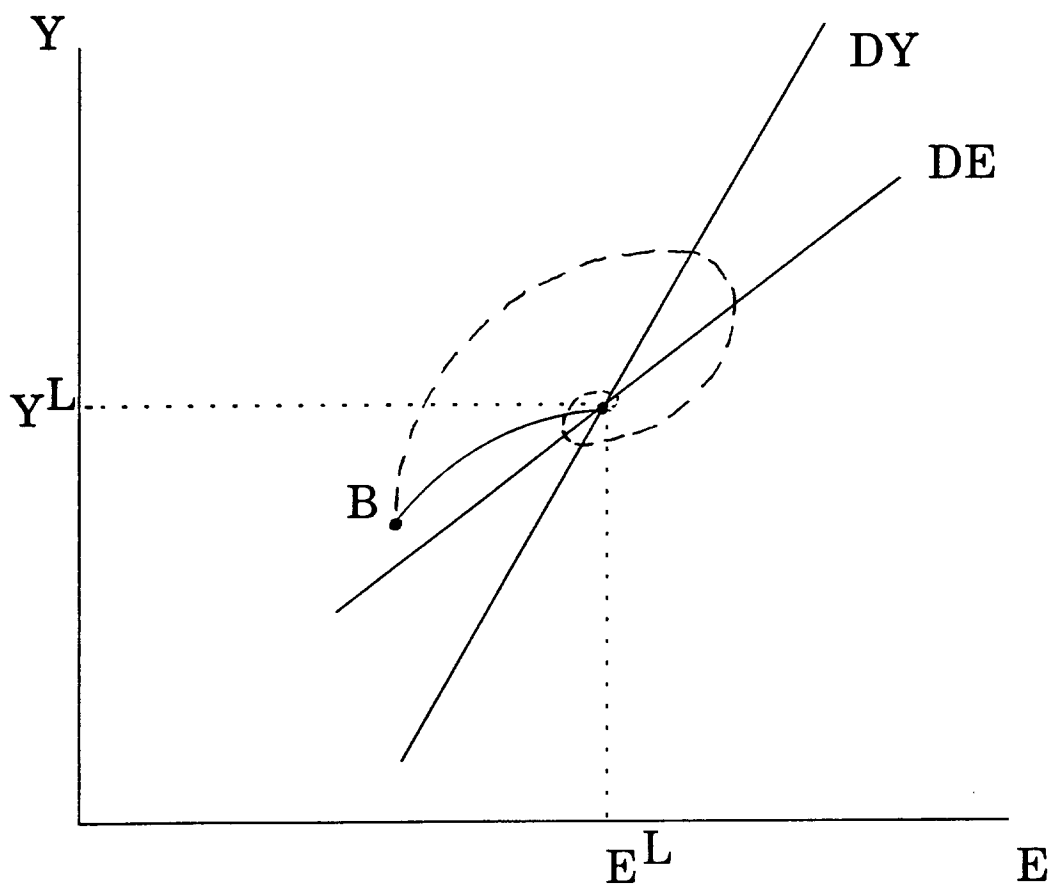


Figure 2

denoted by the solid line, depicts a smooth transition from B, with output and employment gradually rising to higher levels.³ The second path, represented by the dashed line, reveals an oscillatory transition from B, with output and employment overshooting, and then undershooting, the long run equilibrium.⁴

Consider, now, an increase in total factor productivity arising as a consequence of an increase in public capital (provided that the public capital ratio lies below the output maximizing value of m). In Figure 3, the increase in public capital induces rightward shifts in the $DY = 0$ and $DE = 0$ schedules, since

$$\frac{dE}{dA} \Big|_{DY=0} = -\frac{g_Y}{c_{YE}}; \quad \frac{dE}{dA} \Big|_{DE=0} = -\frac{g_E}{c_{EE}}.$$

Clearly, the long run level of employment rises while, apparently, the long run level of output rises or falls. However, differentiation of equations (12) and (13) yields

$$\frac{dY}{dA} = \frac{(1 - c_{YE}c_{EY})g_Y - c_{YY}c_{YE}g_E}{|C|} > 0 \tag{14}$$

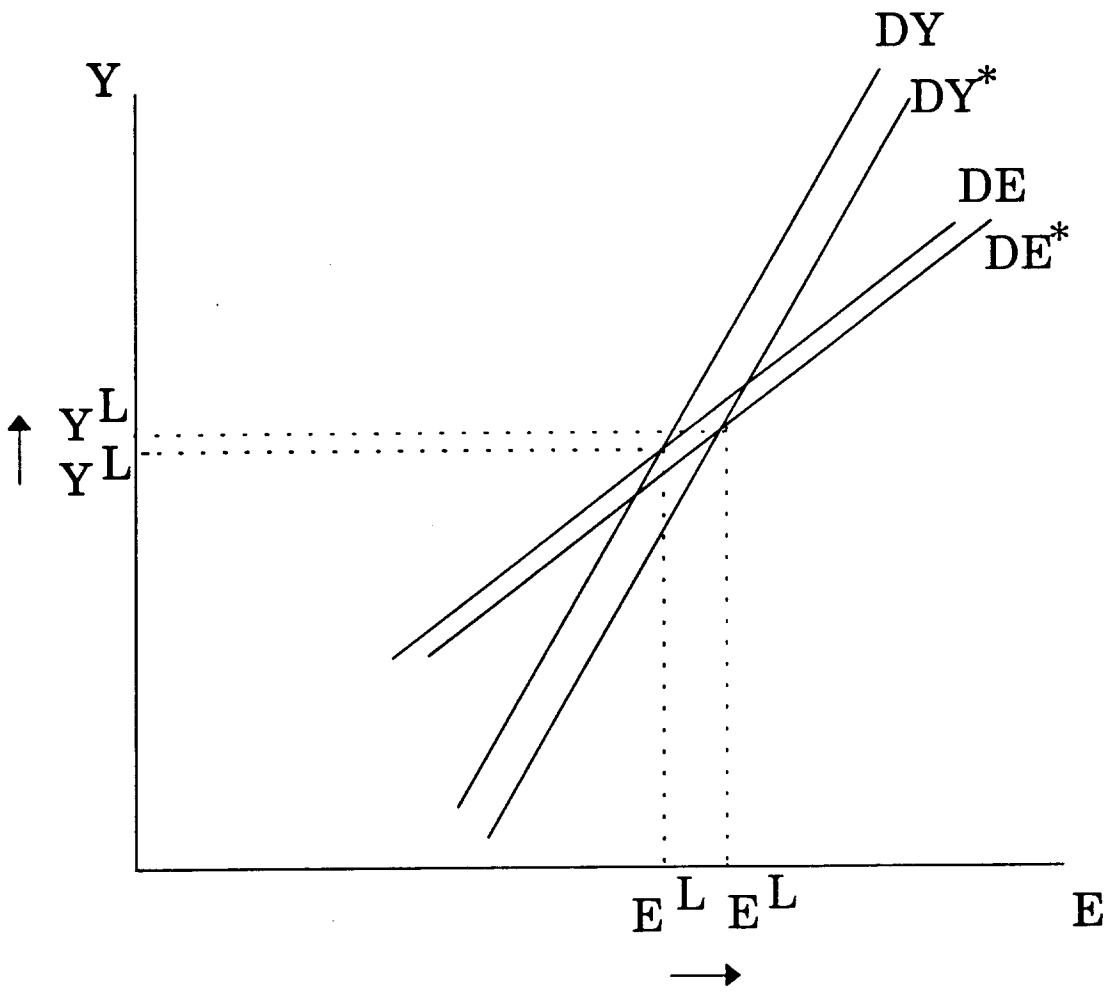


Figure 3

$$\frac{dE}{dA} = \frac{c_{EY}g_Y + c_{YY}g_E}{|C|} > 0 \quad (15)$$

so that both output and employment rise in the long run as a result of an increase in the public capital stock. Diagrammatically, the rightward shift in the DY locus--given by $-(g_Y/c_{YE})dA$ --exceeds the rightward shift in the DE locus--given by $-(g_E/c_{EE})dA$.

Figures 4 and 5 depict the effects of a 10 percent increase in public capital on output and employment under the assumption that the C matrix is given by

$$(C) = \begin{pmatrix} .07 & -.42 \\ .09 & -.36 \end{pmatrix}$$

which, it can be verified, has a negative trace and positive determinant and so fulfills the dynamic stability conditions.⁵ As is revealed by the figures, these coefficient values imply that the levels of output and employment rise to their new, higher levels--by roughly 1.2 percent and 0.4 percent, respectively--at a positive, but diminishing rate. As a direct consequence of the built-in stability of the model, both the growth rate of output and of employment eventually fall to zero.

Figures 6 and 7 show the impact of a similar 10 percent increase in public capital under the alternative assumption that the C matrix is given by

GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

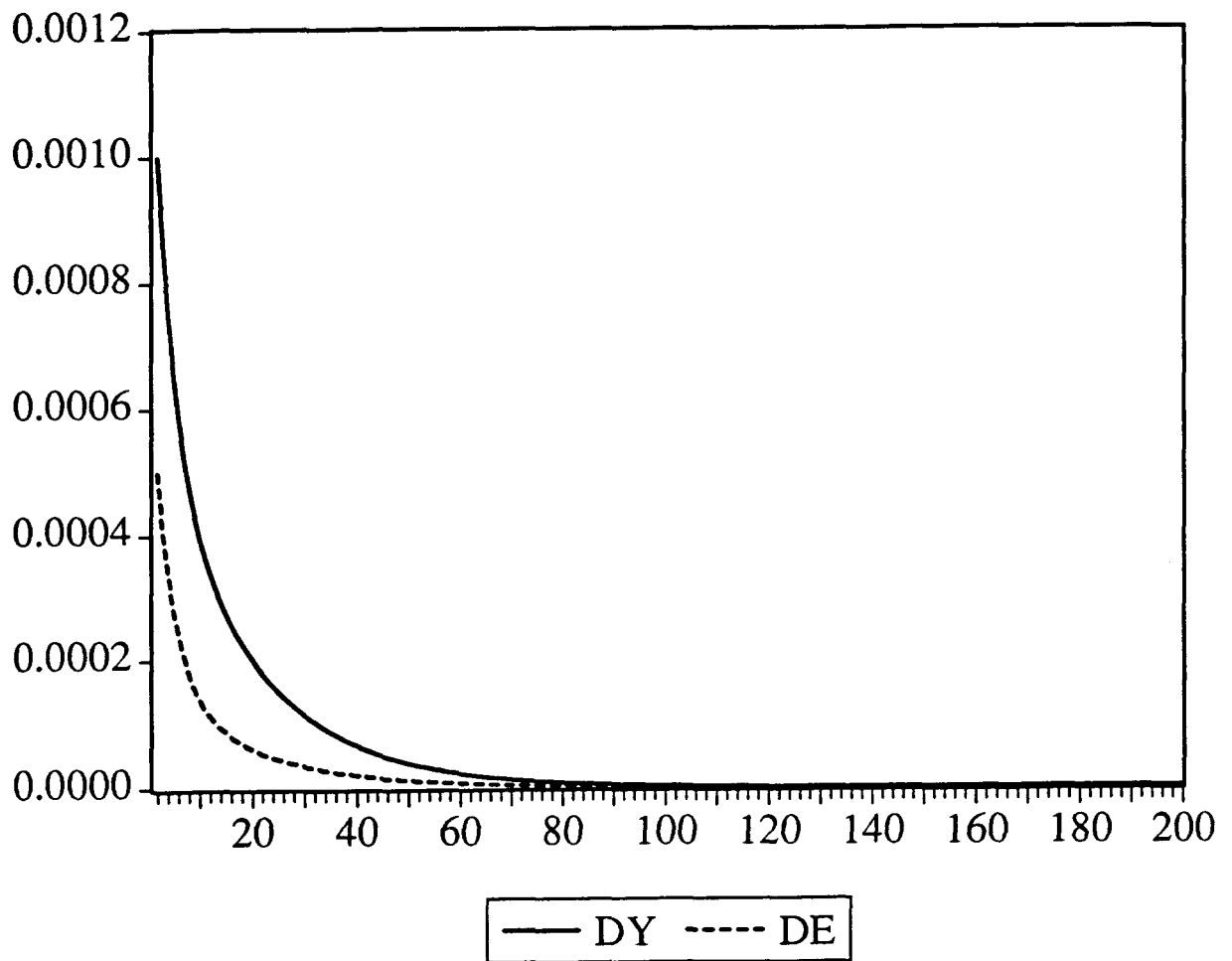


Figure 4

CUMULATIVE GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

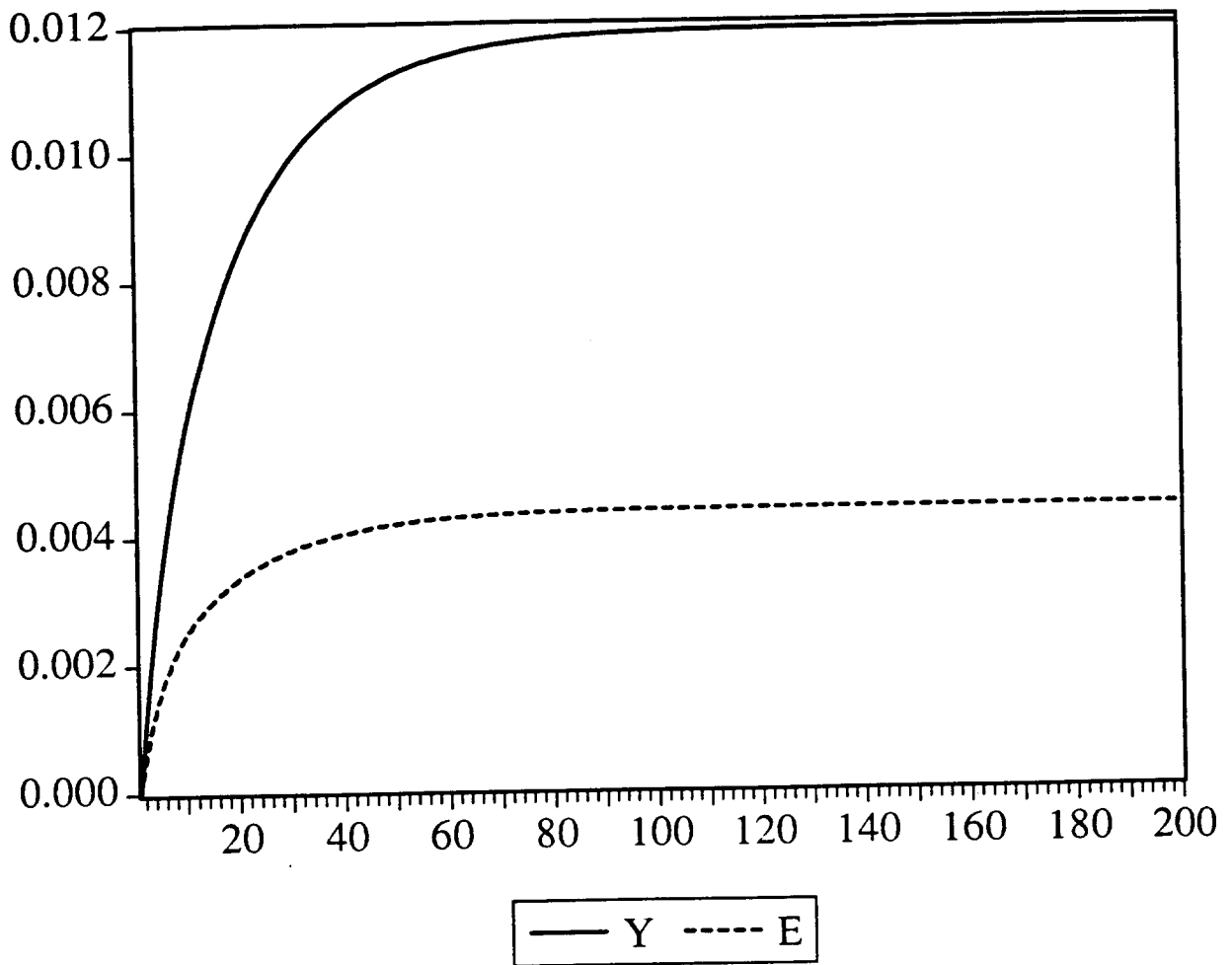


Figure 5

GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

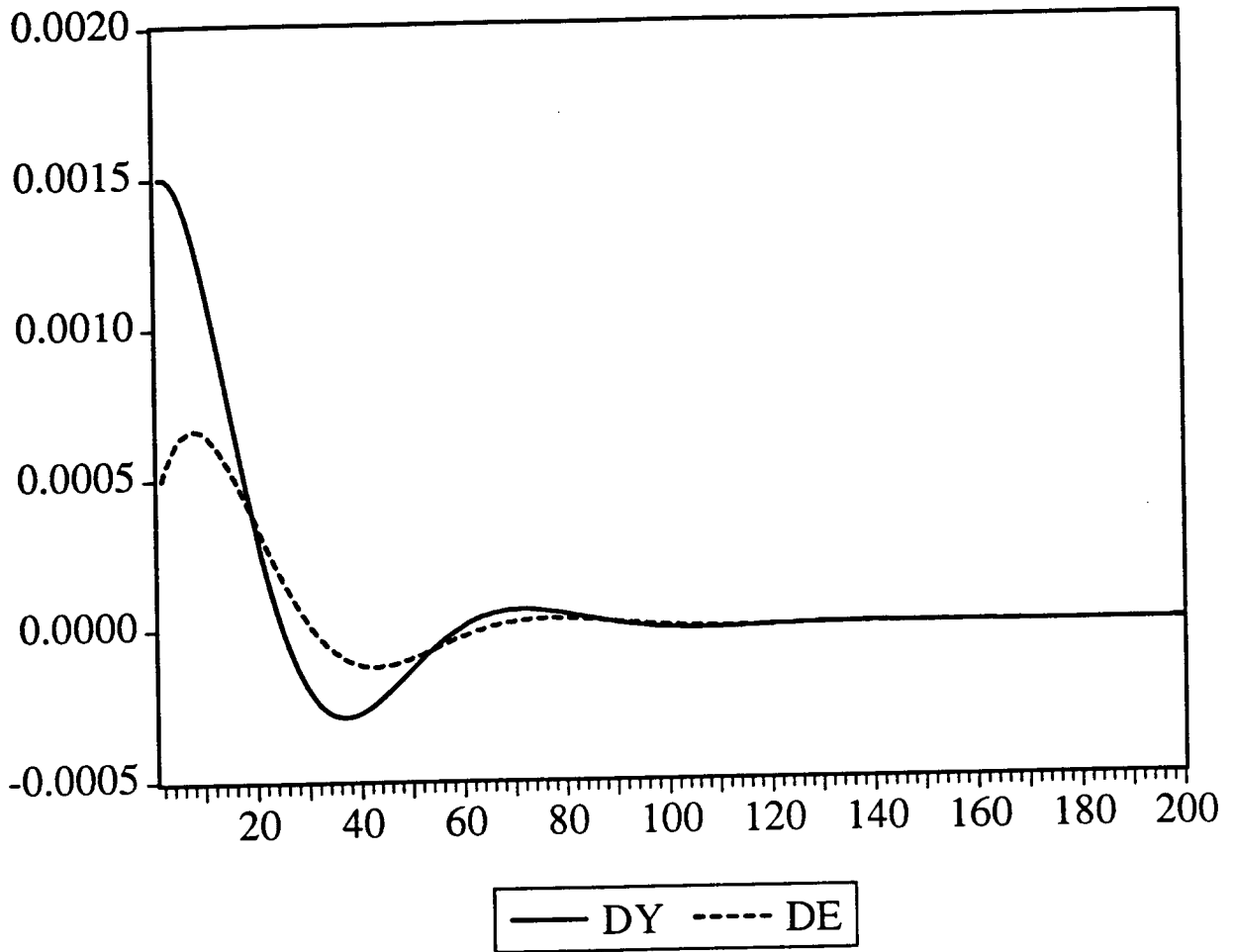


Figure 6

CUMULATIVE GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

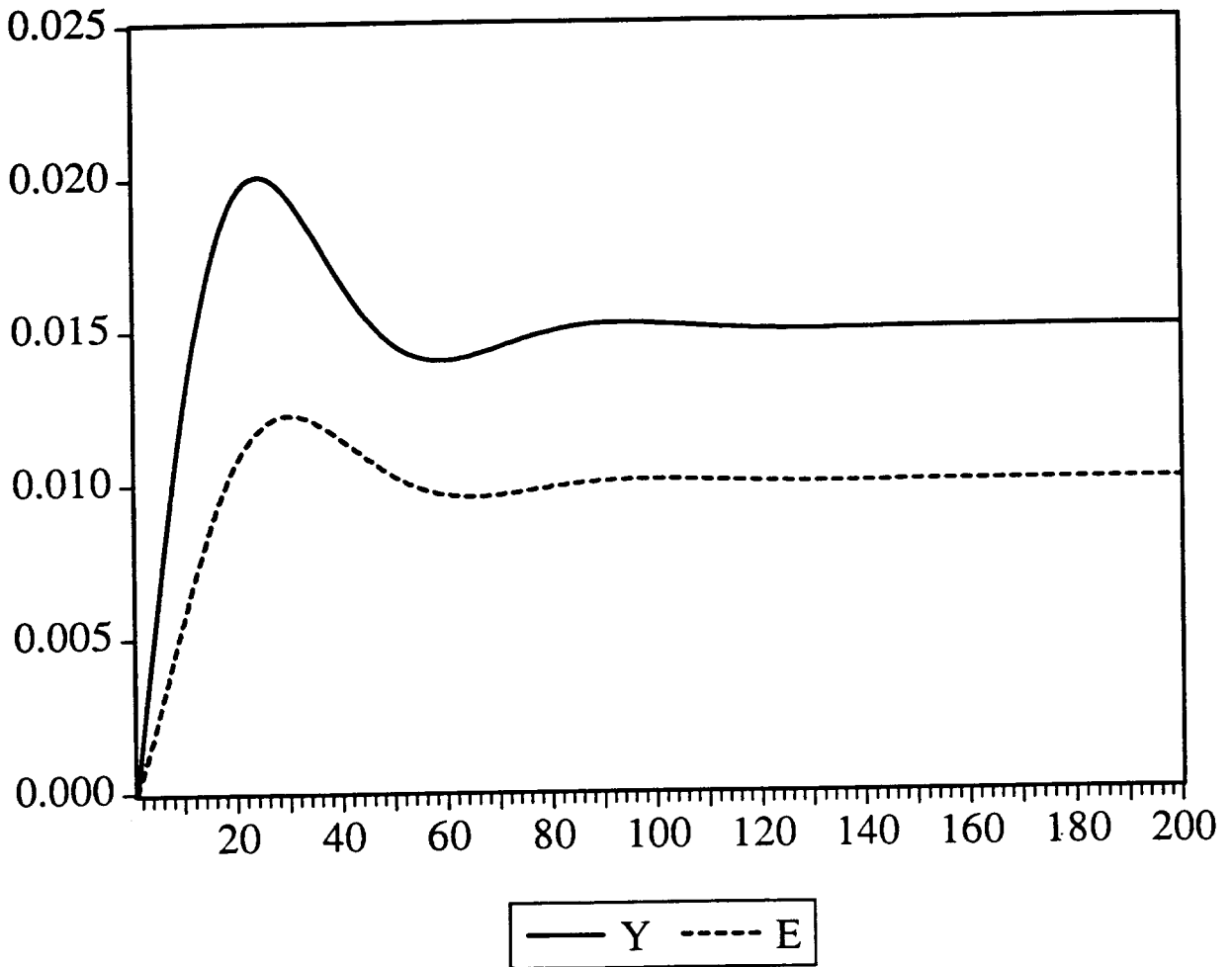


Figure 7

$$(C) = \begin{pmatrix} .10 & -.30 \\ .10 & -.20 \end{pmatrix}$$

which, too, can be shown to fulfill the dynamic stability conditions.⁶ Now, however, it can be seen that output and employment rise in an oscillating fashion to their new levels--about 1.5 percent higher for output and 1.0 percent higher for employment. Again, because of the assumed stability of the model, the growth rates of output and employment converge to zero over time.

III. Empirical Estimates

This section presents empirical estimates of the model contained represented by equation (11). The data and estimation procedure are only sketched out here; for a more complete description, the reader is referred to Aschauer (1997b). The data are for the 48 contiguous US states over the two decades of the 1970s and 1980s. Output is measured as gross state product; the published current dollar values for gross state product have been converted to constant (1982) dollar values using the deflator for gross national product. Employment is measured as private non-agricultural employment. The public and private capital stocks, expressed in constant (1982) dollars, are calculated from annual investment flows using a perpetual inventory method. The fixed effects estimation procedure assumes that there are idiosyncratic influences on output and employment growth at the state level and in each decade.

Table III.1 presents results for the basic model. Here, the coefficients l and m capture the importance

Table III.1
Output and Employment Growth Effects of Public Capital

$$DX = l_x \left(\frac{KG}{K} \right) \cdot \left(1 - \frac{1}{2 \cdot m_x} \left(\frac{KG}{K} \right) \right) + a_x \cdot z + e_x$$

$$X = Y, E$$

	OLS		WLS [√y]		WLS [√Y]		WLS [√E]		SUR	
	DY	DE	DY	DE	DY	DE	DY	DE	DY	DE
l_x	.779 (.202)	.358 (.095)	.830 (.189)	.394 (.100)	.731 (.157)	.319 (.073)	.710 (.163)	.311 (.072)	.779 (.121)	.358 (.059)
m_x	.603 (.044)	.568 (.035)	.597 (.040)	.571 (.033)	.634 (.023)	.613 (.021)	.639 (.024)	.612 (.022)	.603 (.022)	.568 (.022)
Y	.069 (.033)	.090 (.019)	.075 (.031)	.098 (.018)	.084 (.026)	.096 (.014)	.074 (.031)	.089 (.016)	.069 (.027)	.090 (.013)
E	-.130 (.040)	-.132 (.021)	-.134 (.038)	-.140 (.020)	-.154 (.033)	-.137 (.016)	-.144 (.040)	-.129 (.019)	-.130 (.028)	-.132 (.014)
U	.258 (.226)	.237 (.086)	.305 (.227)	.253 (.090)	.070 (.203)	.195 (.074)	.044 (.205)	.194 (.071)	.195 (.074)	.237 (.071)
R ²	.598	.863	.637	.888	.835	.931	.826	.923	.598	.863
SER x10(-3)	10.672	5.249	10.634	5.406	8.488	4.289	8.510	4.251	10.672	5.249
SSR x10(-3)	4.783	1.157	4.749	1.228	3.026	.773	3.042	.759	4.783	1.157

Notes: standard errors are in parentheses; all equations include individual state and decade fixed effects.

of the public capital ratio to *initial* output and employment growth. As a concrete example, suppose that $l = 1.0$, $m = 0.6$, and that $KG/K = 0.5$. Then the marginal impact on growth of a 1 percentage point (0.01) increase in the public capital stock (relative to the private capital stock) can be obtained by differentiation of the growth expression (11) and is given by

$$g_l = l \cdot [1 - (\frac{l}{m}) \cdot (\frac{KG}{K})] = 1 \cdot [1 - (\frac{5}{.6})] = .167$$

times the increase in the public capital stock--0.00167--or 0.167 percent per year. Note that for larger increases in the public capital ratio it becomes important to take into consideration the non-linear nature of the relationship between public capital and growth; for instance, if the increase in the public capital ratio were 10 percentage points (0.1) then the public capital ratio would reach the output maximizing ratio of $m = .6$ and it would be appropriate to use the average of the original (0.5) and new (0.6) public capital ratios, or 0.55, to determine that the effect on the growth rate would be 0.083, or 0.83 percent per year.

The estimates of the coefficients l and m are positive and statistically significant for both output growth and employment growth for all estimation methods--ordinary least squares [OLS], weighted least squares [WLS] with output per worker [\sqrt{y}], output [\sqrt{Y}], and employment [\sqrt{E}] as weights, and seemingly unrelated regression [SUR]. The point estimates obtained by the various methods are quite similar: for l , the point estimates range between 0.710 and .830 for output and between 0.311 and 0.394 for employment; for m the point estimates lie between 0.597 and 0.613 for output and between 0.568 and 0.613 for employment. Furthermore, the estimates of the output maximizing

ratio of public capital to private capital obtained from the output growth expression and the employment growth expression, respectively, are close to one another; the estimate of m obtained from the output growth equation always exceeds that obtained from the employment growth equation, but never by more than 3 or 4 percentage points. As a result of this similarity of point estimates, and because the SUR method allows for more efficient estimates than the OLS method, the SUR results will be employed for descriptive purposes in much of the subsequent discussion.

The effect of a 1 percentage point increase in the public capital ratio on initial output and employment growth--relative to the sample average public capital ratio of 0.446--is equal to 0.203 percent per year for output and 0.077 percent per year for employment. These effects are meaningful--amounting to a 0.137 of 1 standard deviation increase in output growth and a 0.054 of 1 standard deviation increase in employment growth--and yet seem reasonable. For instance, assume momentarily that the C matrix is diagonal and that the model is stable.⁷ If the initial increases in output and employment growth were to decline exponentially at the (common) rate $r_Y = r_E = 0.5$, then the cumulative rise in output would equal 0.406 percent while the rise in employment would equal 0.154 percent. Even if the growth rate increases were more persistent, declining at a rate of perhaps $r_Y = r_E = 0.2$, the cumulative rise in output and employment would remain reasonable, at 1.015 percent and 0.385 percent, respectively.

However, the conceptual model allows for (even demands) a richer dynamic structure for output and employment; specifically, both output and employment growth are seen to depend on the initial levels

of output and employment. Thus, it is necessary to look at the estimated coefficients associated with the initial levels of output and employment in order to fully judge the impact of public capital on the long run levels of output and employment. As can be seen in Table III.1, both output growth and employment growth depend positively (and significantly) on the initial level of output, with point estimates between 0.069 and .084 for output growth and between 0.089 and 0.096 for employment growth. Conversely, both output growth and employment growth depend negatively (and, again, significantly) on the initial level of employment, with point estimates between -0.130 and -0.154 for output growth and between -0.129 and -0.140 for employment growth. As indicated by Table III.2, the model turns out to be dynamically stable for each estimation method, with $tr < 0$ and $|C| > 0$. Finally, for each set of equations the roots of the characteristic equation are complex, by the condition $tr^2 - 4|C| < 0$. Consequently, given a particular disturbance to the economy, the paths for output growth and employment growth will be oscillatory.

IV. Dynamic Output and Employment Effects of Public Capital

The estimates of the dynamic model contained in the previous section imply that the paths for output and employment, subject to a shock to the economy, will be stable yet oscillatory. Figure 8 shows the impact of a permanent increase in the public capital ratio from 0.45 (near its sample average of 0.446) to 0.50 on the growth rates of output, employment, and productivity. As is evident from the figure, the growth rate of output initially rises by 0.8 of a percent per year while the growth rate of employment climbs by a smaller 0.3 percent per year; as a consequence, productivity growth is lifted by some 0.5 percent per year. Interestingly, output and employment growth continue to expand for

Table III.2
Stability: Public Capital and Economic Growth

	OLS	WLS [\sqrt{y}]	WLS [\sqrt{Y}]	WLS [\sqrt{E}]	SUR
tr	-0.063	-0.065	-0.053	-0.055	-0.063
$ C $	0.003	0.002	0.003	0.003	0.003
$tr^2 - 4 \cdot C $	-0.008	-0.004	-0.004	-0.009	-0.008

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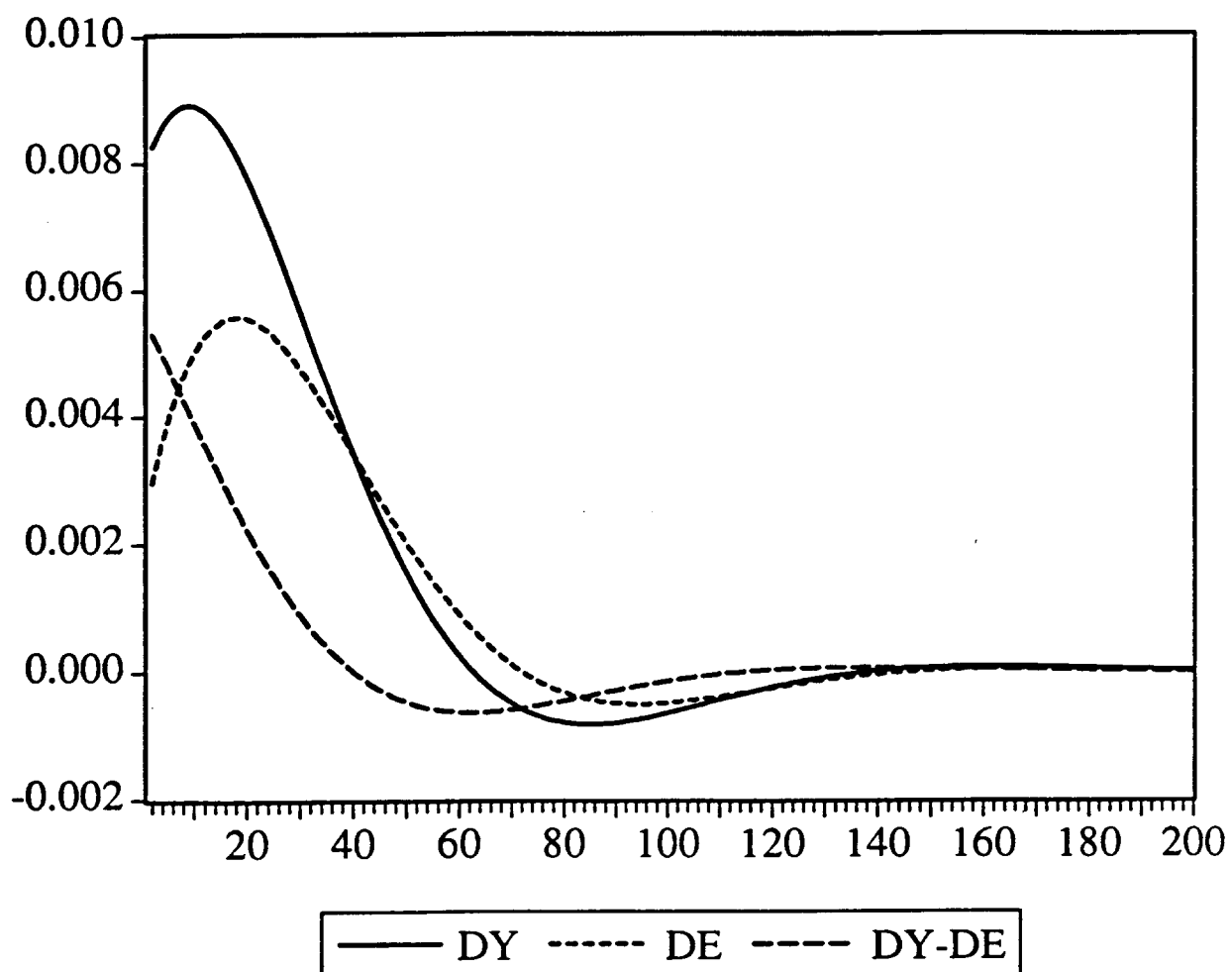


Figure 8

some period of time. Output growth peaks at 0.9 percent per year after 9 years while employment growth peaks at 0.5 percent per year after 18 years. Still, output growth remains above employment growth--generating persistent productivity gains--for forty years. The impact of the change in public capital on economic growth are quite persistent. Output growth remains above 0.4 percent per year (one half of the initial impact) for nearly 40 years. In similar fashion, employment growth stays above 0.15 percent per year for 55 years. Indeed, it takes a full 100 years--perhaps 150 years--before the growth effects of the increase in public capital have essentially disappeared.

Figure 9 illustrates the cumulative effect of the change in public capital on the level of output, employment, and productivity. The roughly 10 percent (or 0.368 of 1 standard deviation) rise in the public capital ratio can be seen to generate very substantial increases in each of these variables over a 200 year horizon. Output climbs by 27.2 percent, nearly three times the percentage increase in the public capital ratio. Over three-quarters of the increase in long run output can be seen to come from gains in employment (which increases by 20.8 percent), with less than one-quarter arising from capital accumulation and productivity improvements (equalling 6.4 percent).

The magnitude of these effects may strike some readers as implausibly large. In response, three points may be made. First, the fundamental reason for the substantial cumulative effects is not to be found in the *initial* increase in output and employment growth--at a fairly modest 0.8 and 0.3 percent per year--as much as in the *persistence* of the increase in growth rates. The calculation of persistence rates is rendered difficult by the interaction between output and employment during the transition to

CUMULATIVE GROWTH EFFECTS OF PUBLIC CAPITAL
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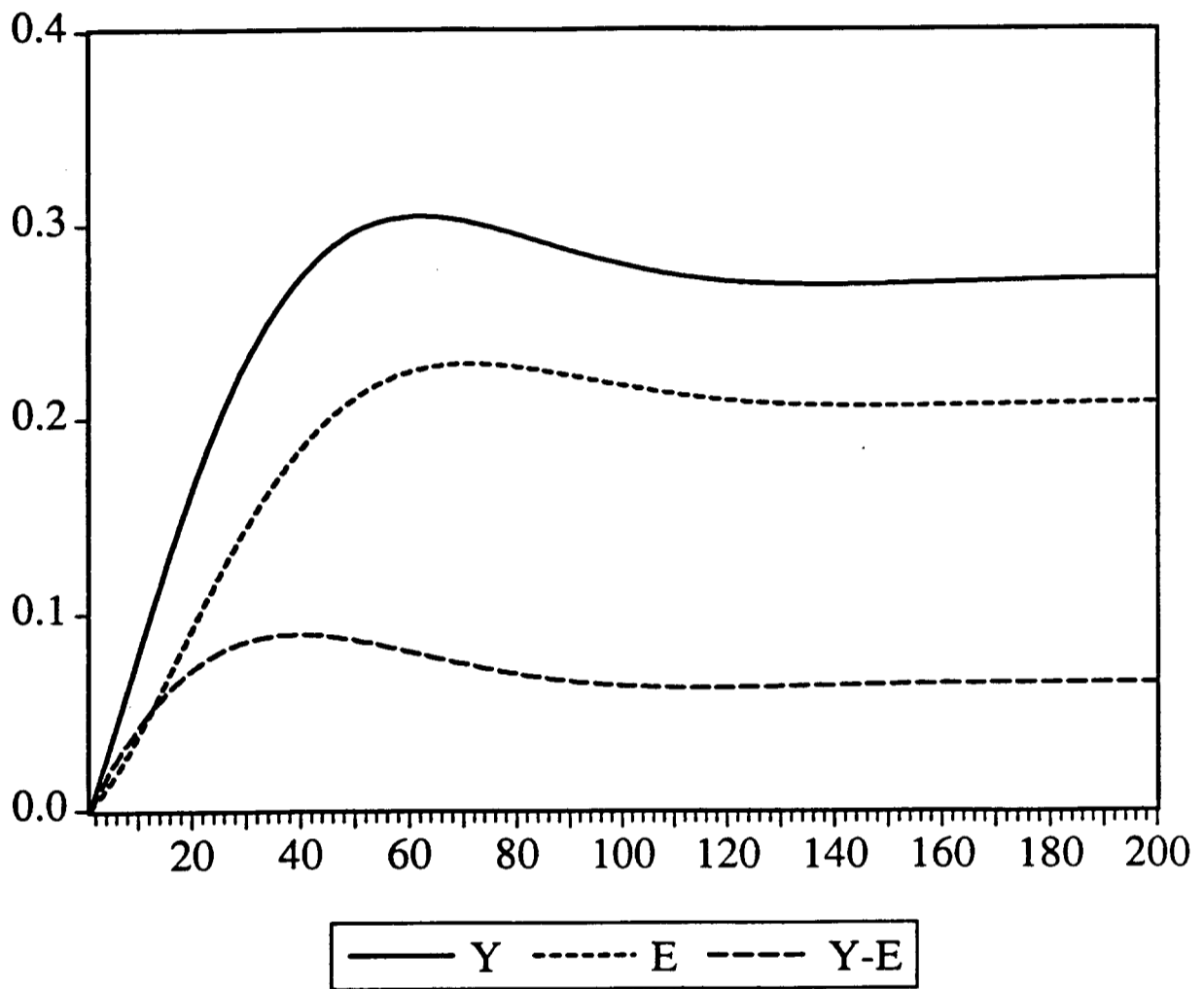


Figure 9

the long run equilibrium; however, a rough estimate of the persistence rates can be obtained by application of the formula

$$r_x = \frac{dX - dDX}{dX}$$

to the percentage changes in the levels of output and employment (assumed to be essentially achieved after 200 years), dX , and the initial changes in the growth rates of output and employment, dDX . This procedure basically “smooths out” the oscillations in output and employment and, as such, can be taken as a rough approximation of the underlying dynamics of the model economy. For output--using a 27.2 percent increase in the long run level of output along with a 0.8 percent per year initial increase in output growth, we obtain a persistence rate of 97.1 percent. Similarly, for employment--employing a 20.8 percent increase in the long run level of employment with a 0.3 percent per year increase in employment growth--we find a persistence rate of 98.6 percent. Finally, for productivity--factoring in a 6.4 percent increase in the long run level of productivity against a 0.5 percent per year increase in initial productivity growth--we arrive at a persistence rate of 92.2 percent.

These results may be compared to the results of empirical work, such as Barro and Sala-i-Martin (1991, 1995) dealing with the *convergence* of state economies. In this work, a single equation in output (or income) per capita such as

$$Dy = \beta \cdot y + \gamma \cdot z$$

is estimated, where β = convergence rate and z denotes one or more variables which are thought to

affect state economic performance (such as government consumption spending, weather, agricultural intensity, etc.). In this context, it can be shown that the relationship between the persistence rate (as defined here) and the convergence rate is given by $r_y = 1 - \beta$. Furthermore, in the literature the convergence rate is typically estimated at 2 or 3 percent, yielding a persistence rate of 97 or 98 percent--in the same range as those estimated here in a two equation system in output and employment. Thus, the results obtained in the present paper appear to be in line with the previous results in the literature.

Second, the results pertain to a *permanent* increase in the public capital ratio which, in turn, requires (at least for a time) an *increasing* level of public investment to match the induced rise in the private capital stock. A more realistic policy, however, might involve a permanent or persistent increase in public investment which, in the context of a growing private capital stock, translates into a temporary increase in the public capital ratio. Figure 10 shows the assumed nature of the temporary rise in the public capital ratio; an initial rise of 5 percentage points (as in the case of the permanent increase) gradually diminishes at a rate of 0.1 percentage points per year over a 50 year period.

Figure 11 illustrates the impact of the temporary rise in the public capital ratio on the growth rates of output, employment, and productivity. The initial effects on the growth rates of output (at 0.8 percent per year), employment (at 0.3 percent per year), and productivity (at 0.5 percent per year) are (nearly) the same as in the case of a permanent increase in the public capital ratio. However, the growth rate of output begins to fall after 2 years (compared to 9 years in the case of a permanent rise

PERMANENT & TEMPORARY INCREASES IN PUBLIC CAPITAL
(baseline: $KG/K = .45$)

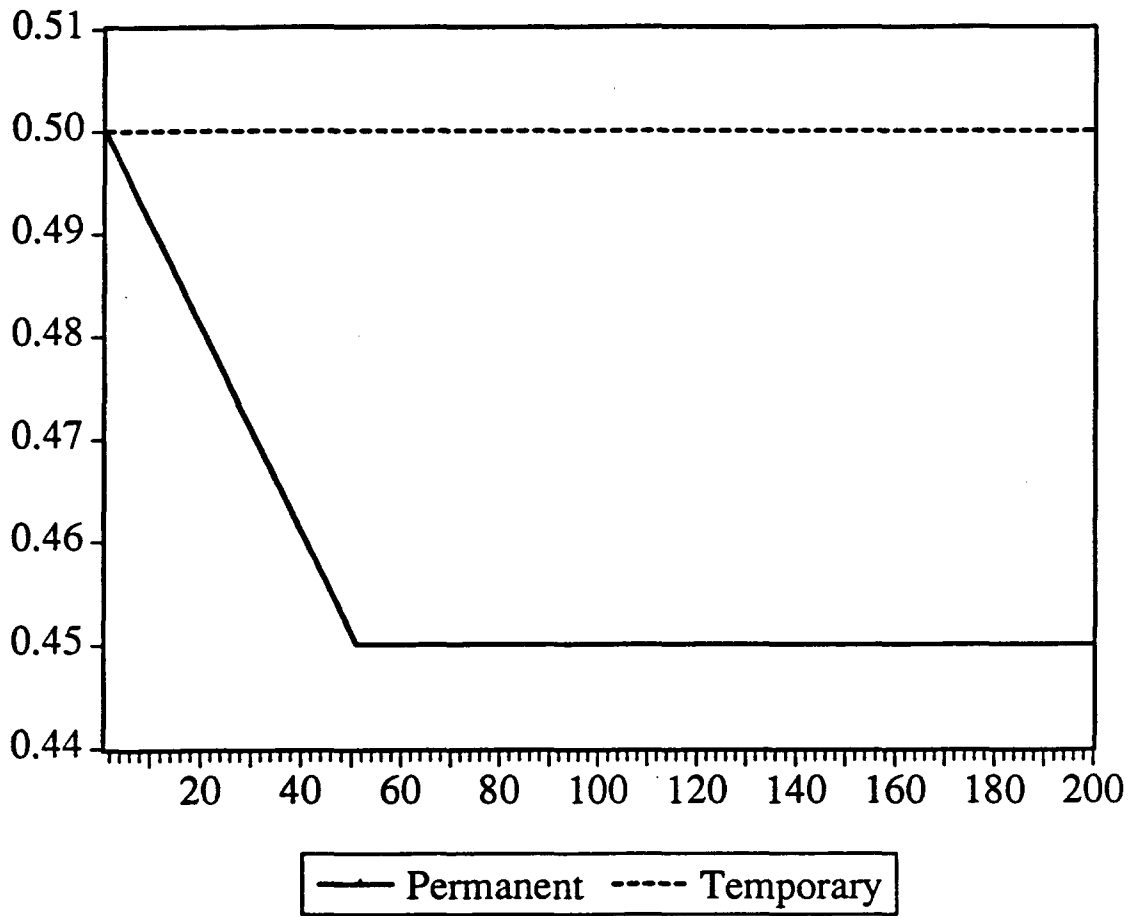


Figure 10

GROWTH EFFECTS OF PUBLIC CAPITAL
(temporary increase in public capital)

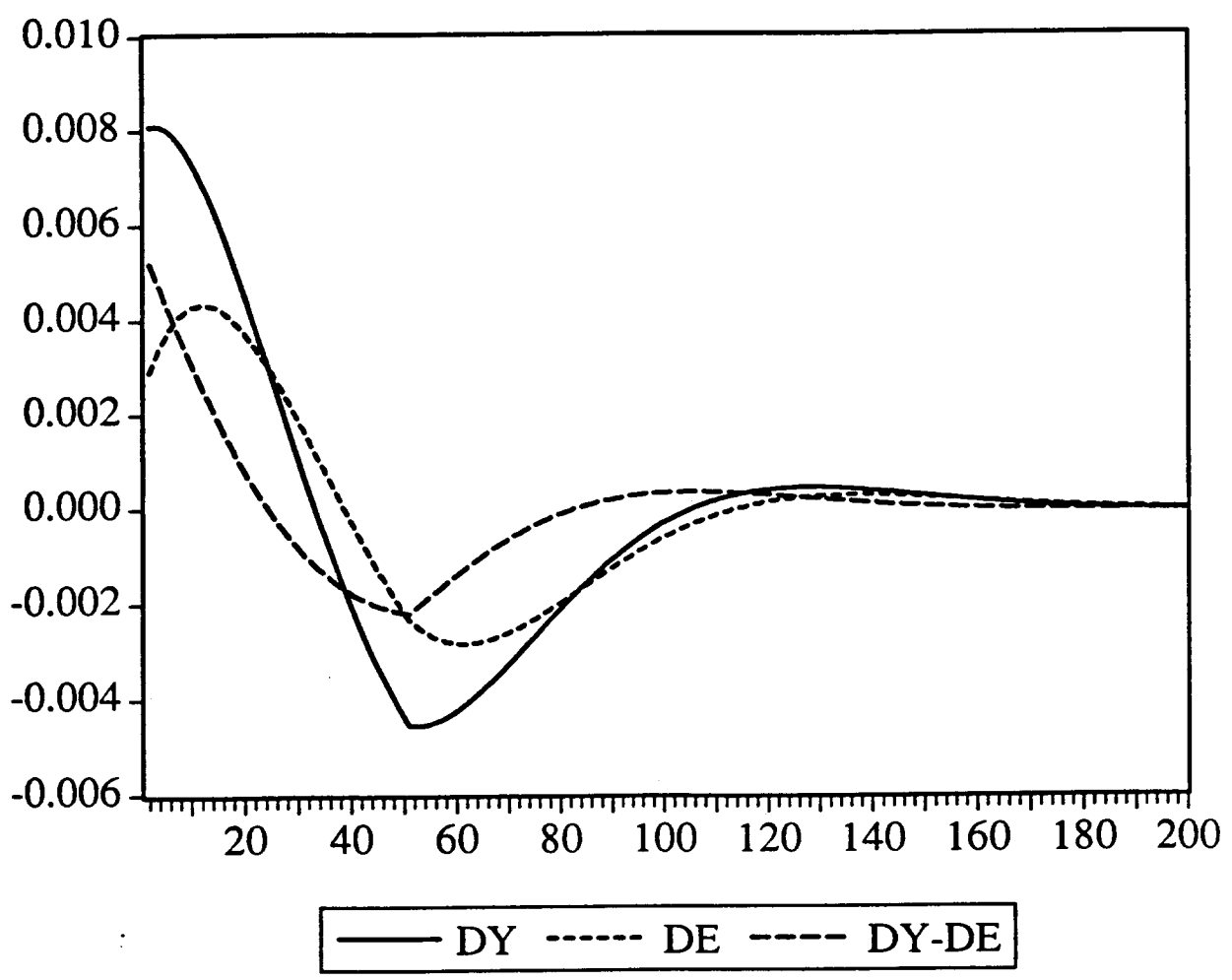


Figure 11

in the public capital ratio) while the growth rate of employment builds to only 0.4 percent per year over 12 years (compared to 18 years). Output growth and employment growth turn negative after 33 and 38 years, respectively, more than a decade before the public capital ratio returns to its original level. Output growth and productivity growth begin to rise almost immediately after the public capital ratio has stabilized at its original level, while employment growth begins to rise after 61 years.

Figure 12 shows the corresponding impacts on the levels of output, employment, and productivity. The maximum cumulative gains equal 15.7 percent for output (after 33 years), 11.0 percent for employment (after 38 years), and 5.6 percent for productivity (after 24 years). Thereafter, output, employment, and productivity each decline and then, in damped fashion, oscillate around their original levels. As a result of the stable nature of the model, there are no long run effects of temporary changes in the public capital ratio on either growth rates or levels of the endogenous variables of the model.

Third, much of the increase in output in the case of the permanent increase in the public capital ratio can be attributed to an increase in employment which, in turn, can be traced to an increase in the labor force. Such an increase in the labor force can be considered reasonable if the rise in the public capital stock is isolated in a particular state which may then draw from the labor force of all other states. However, if the rise in the public capital stock is regional or even national in scope--the result of a broad scale public policy attempt to boost regional or national growth--then a much more modest increase in the labor force can be deemed reasonable.

CUMULATIVE GROWTH EFFECTS OF PUBLIC CAPITAL
(temporary increase in public capital)

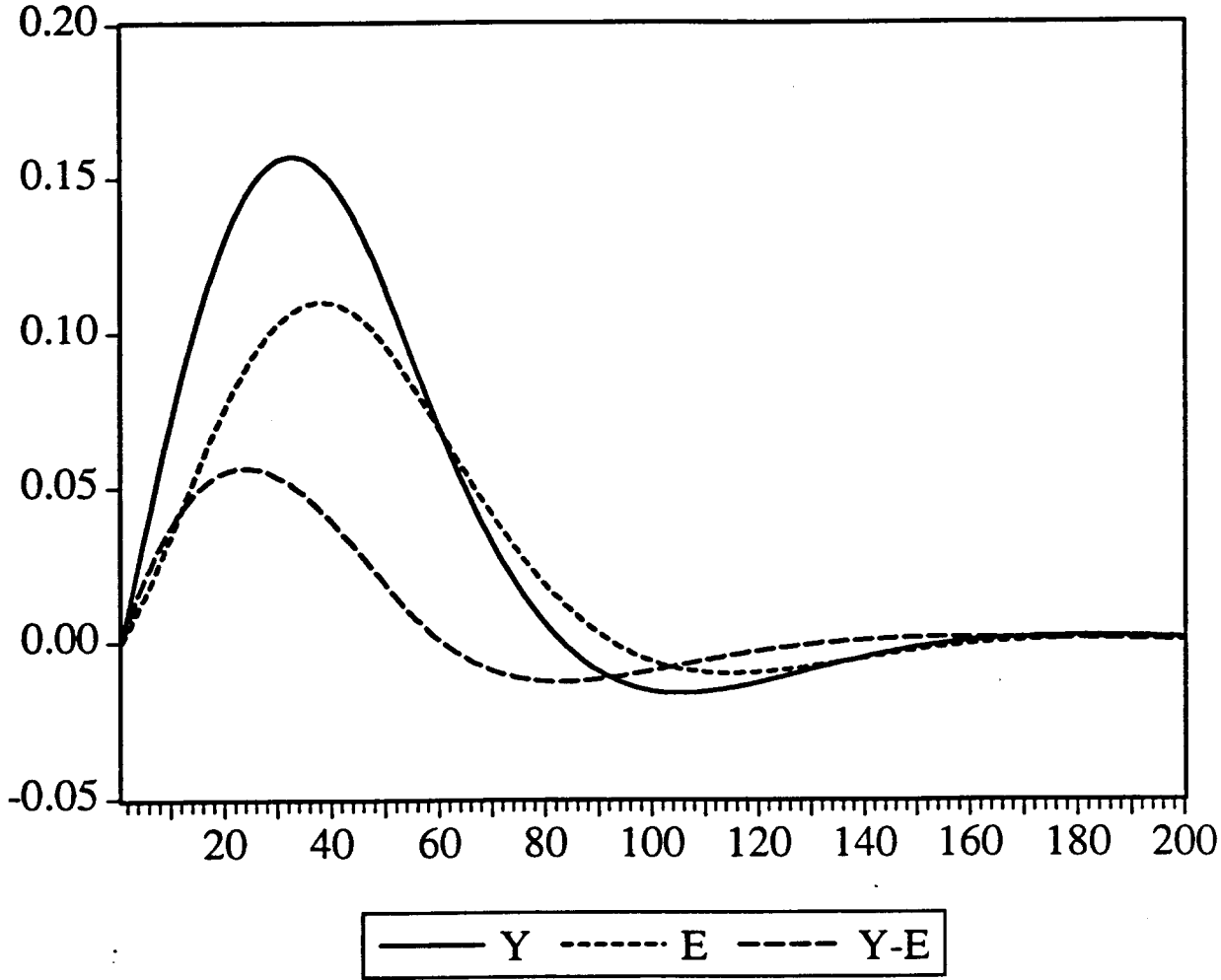


Figure 12

Figures 13, 14, and 15 compare the effects of a permanent rise in public capital on the growth rates of output, employment, and productivity for an *endogenous labor force* and *exogenous labor force* model, respectively. The endogenous labor force model is that represented previously; its key characteristic is that it holds fixed the unemployment rate; since, by definition

$$\ln(1 + U) = L - E \quad \rightarrow \quad U = L - E$$

increases in employment must be associated with increases in the labor force. The exogenous labor force, on the other hand, holds fixed the labor force so that increases in employment must be associated with decreases in the unemployment rate.

As is evident in the figures, the impact of a rise in public capital on the growth rates of output, employment, and productivity is much more modest in the exogenous labor force model than in the endogenous labor force model. While the initial increases in each variable are the same in each model, in the exogenous labor force model neither output nor employment rise over time--as they do in the endogenous labor force model--and both variables decline at a much faster rate. For instance, in the exogenous labor force model, output growth declines from 0.8 percent per year in the first year, to 0.4 percent per year after 10 years, to 0.2 percent per year after 20 years, and to 0.1 percent per year after 30 years.

Figures 16, 17, and 18 compare the cumulative effect of the increase in public capital on the output, employment, and productivity in each of the two models. As can be seen from the figures, the

OUTPUT GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

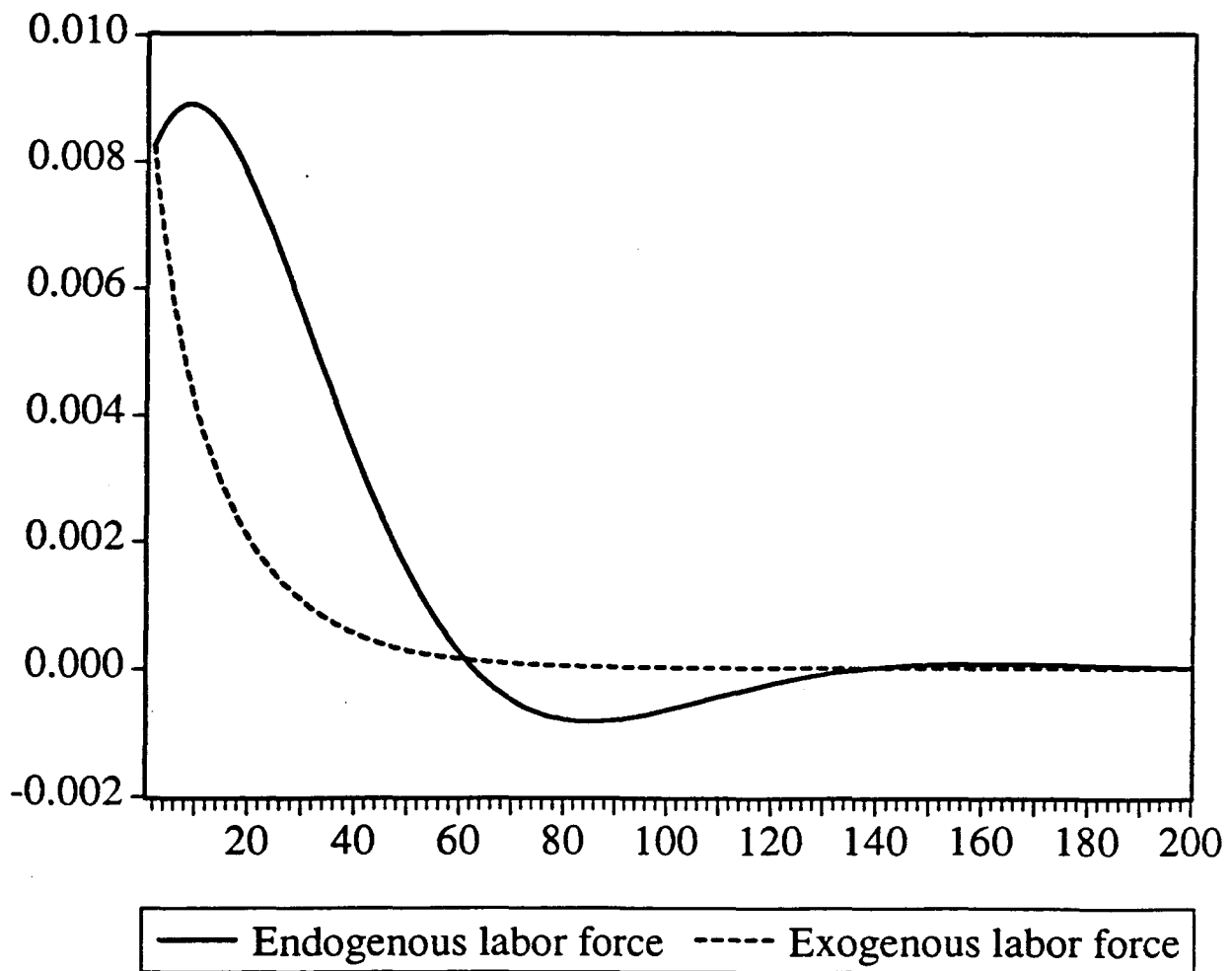


Figure 13

EMPLOYMENT GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

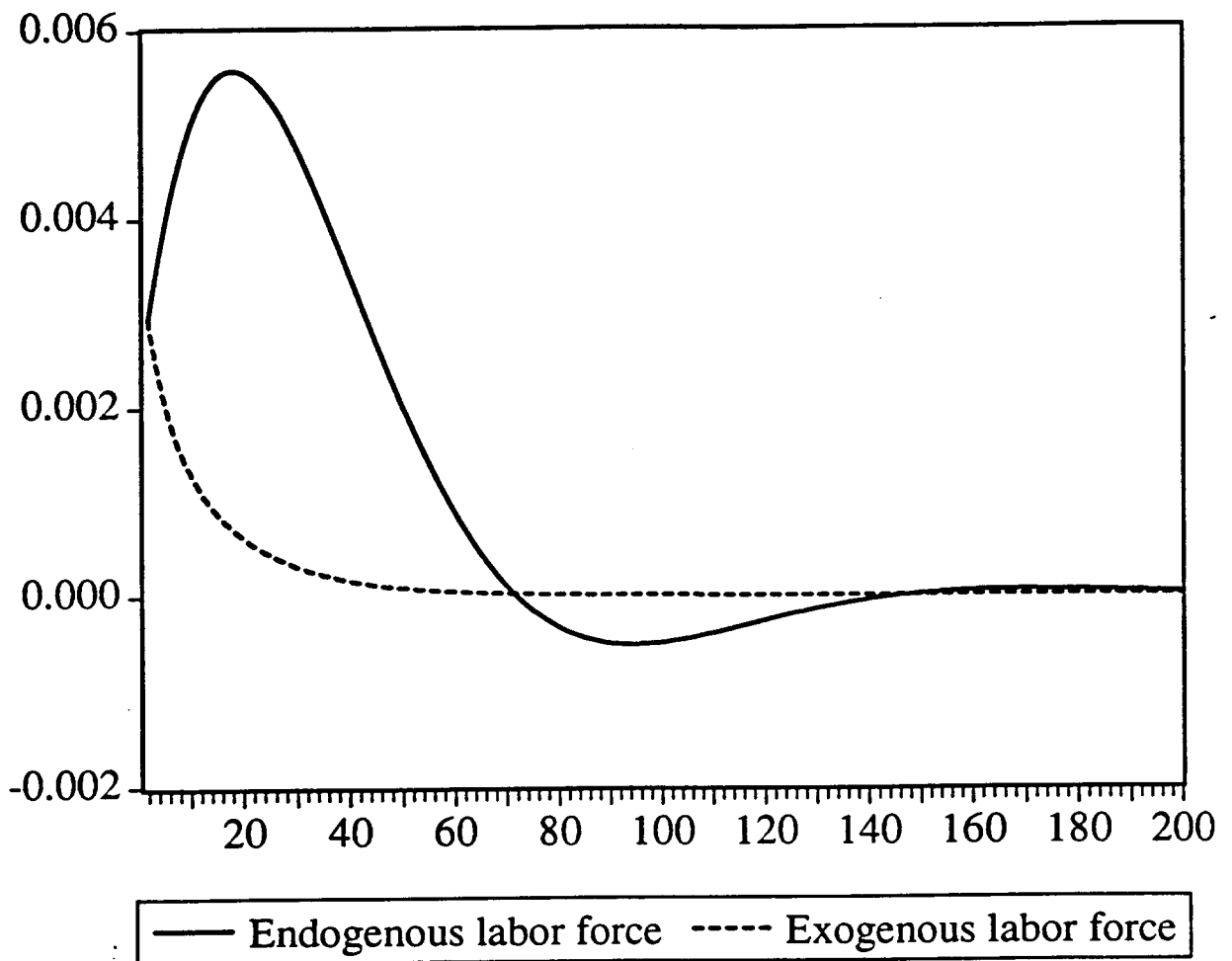


Figure 14

PRODUCTIVITY GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

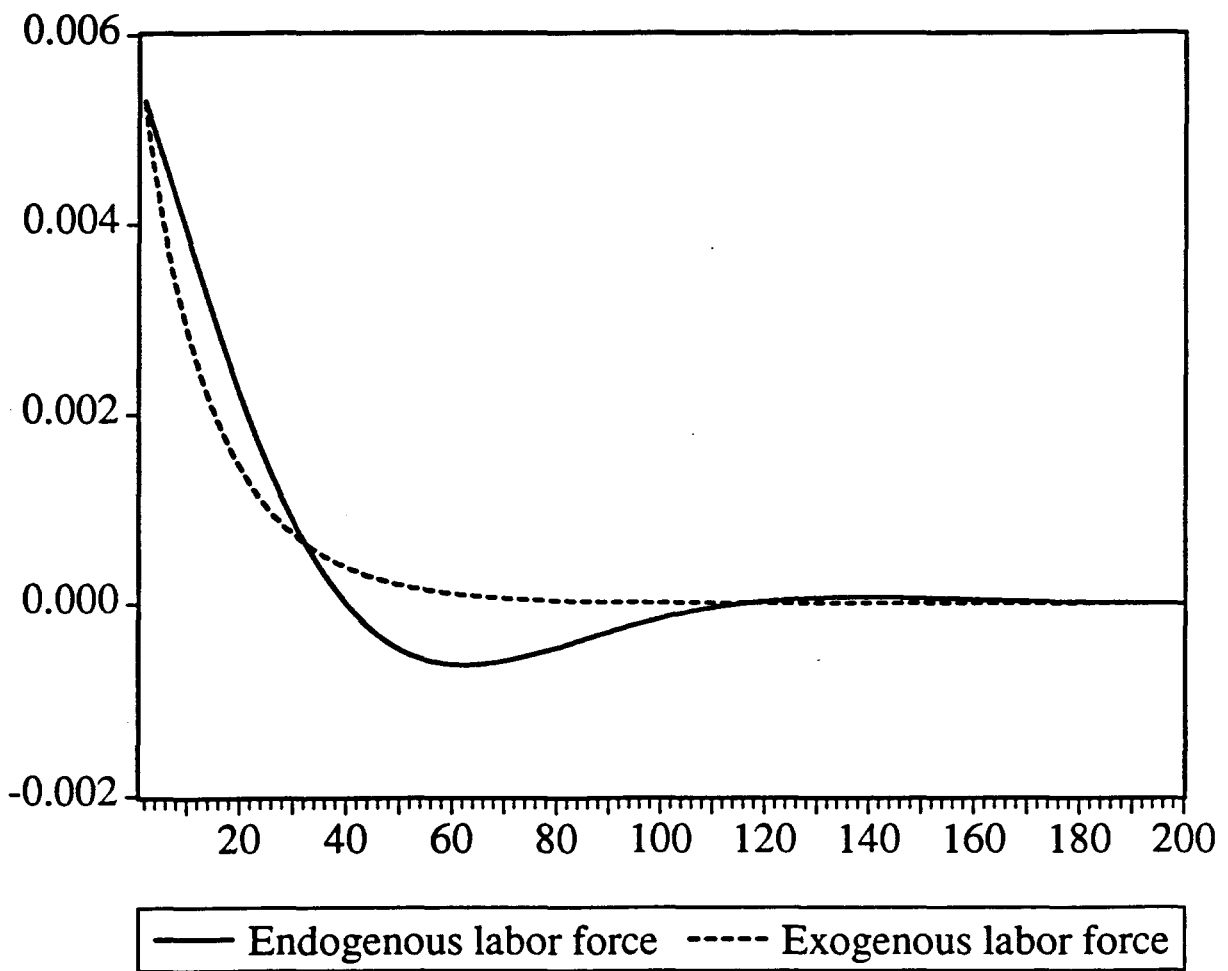


Figure 15

assumptions regarding the source of employment increases--expansions of the labor force versus reductions in unemployment--is of critical importance to the magnitude of the cumulative impacts on the endogenous variables of the model. Specifically, in the exogenous labor force model the cumulative gains in output and employment are 11.2 percent and 3.5 percent, respectively, some 16.0 and 16.2 percentage points below the corresponding gains in the endogenous growth model.⁸ Interestingly, however, in the exogenous labor force model the 7.6 percent gain to productivity actually exceeds the 6.4 percent gain in the endogenous labor force model.

Persistence rates for output, employment, and productivity in the exogenous labor force model may be computed on the basis of the cumulative gains achieved by these variables after 200 years. These rates are 92.9 percent for output, 85.7 percent for employment, and 93.4 percent for productivity, to be compared to the previously computed rates in the endogenous labor force model of 97.1 percent, 98.6 percent, and 92.2 percent, respectively. These persistence rates in the exogenous labor force model imply that the *half lives* of the increase in growth rates--the period of time which must pass before the particular growth rate returns half way to zero--are 9.4, 4.5, and 10.2 years, respectively, for output, employment, and productivity. When compared to the corresponding half lives for the endogenous labor force model of 23.5, 49.2, and 8.5 years, these represent much quicker convergence for employment (by a factor of 10), somewhat speedier convergence for output (by a factor of 2), and marginally slower convergence for productivity.

CUMULATIVE OUTPUT GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

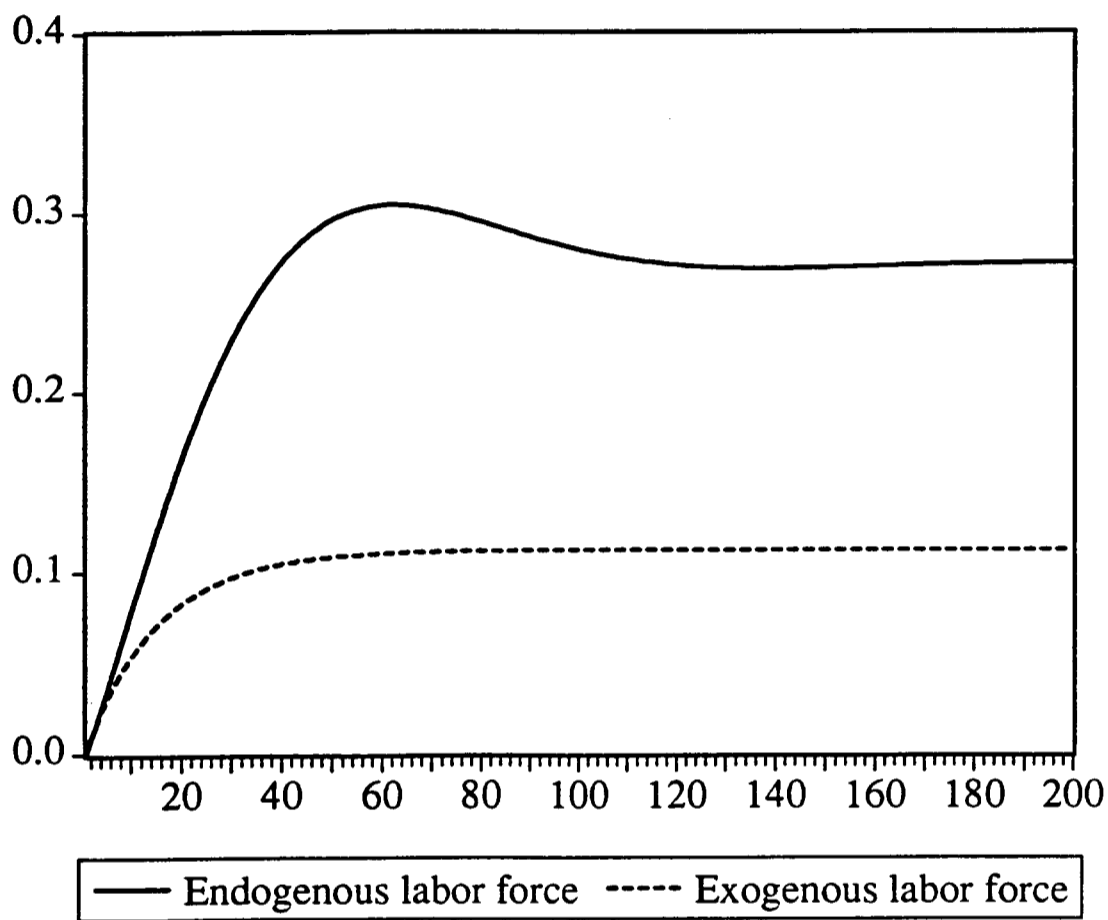


Figure 16

CUMULATIVE EMPLOYMENT GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

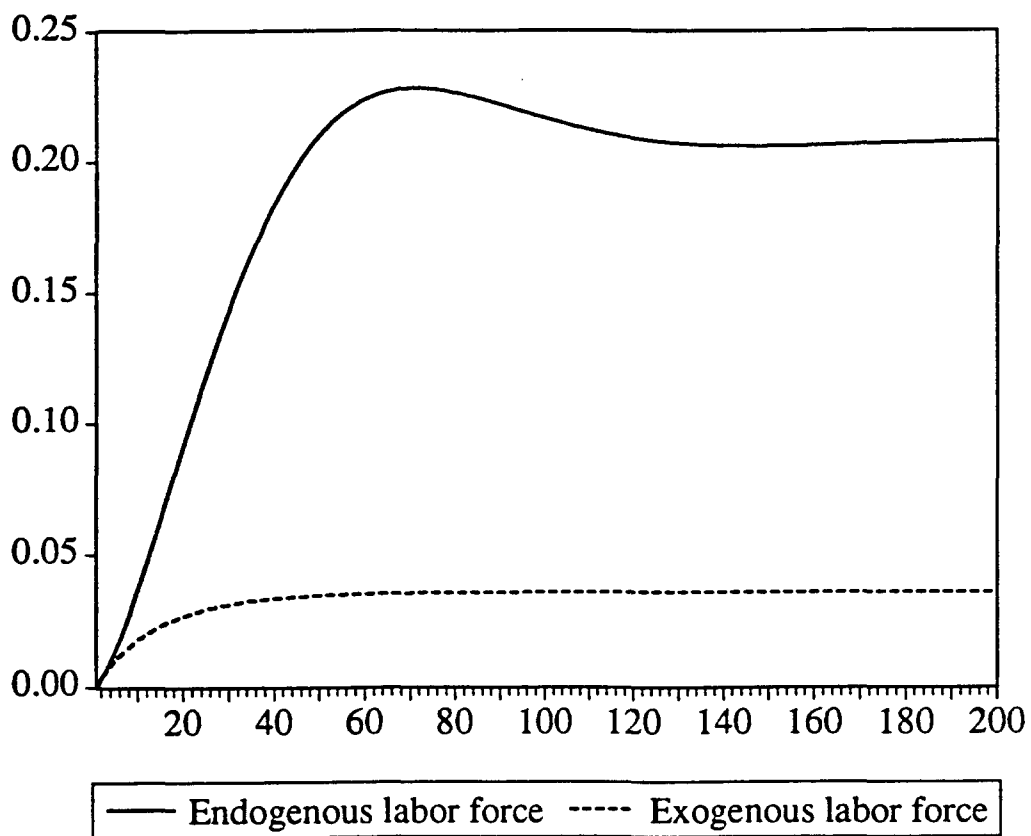


Figure 17

CUMULATIVE PRODUCTIVITY GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital)

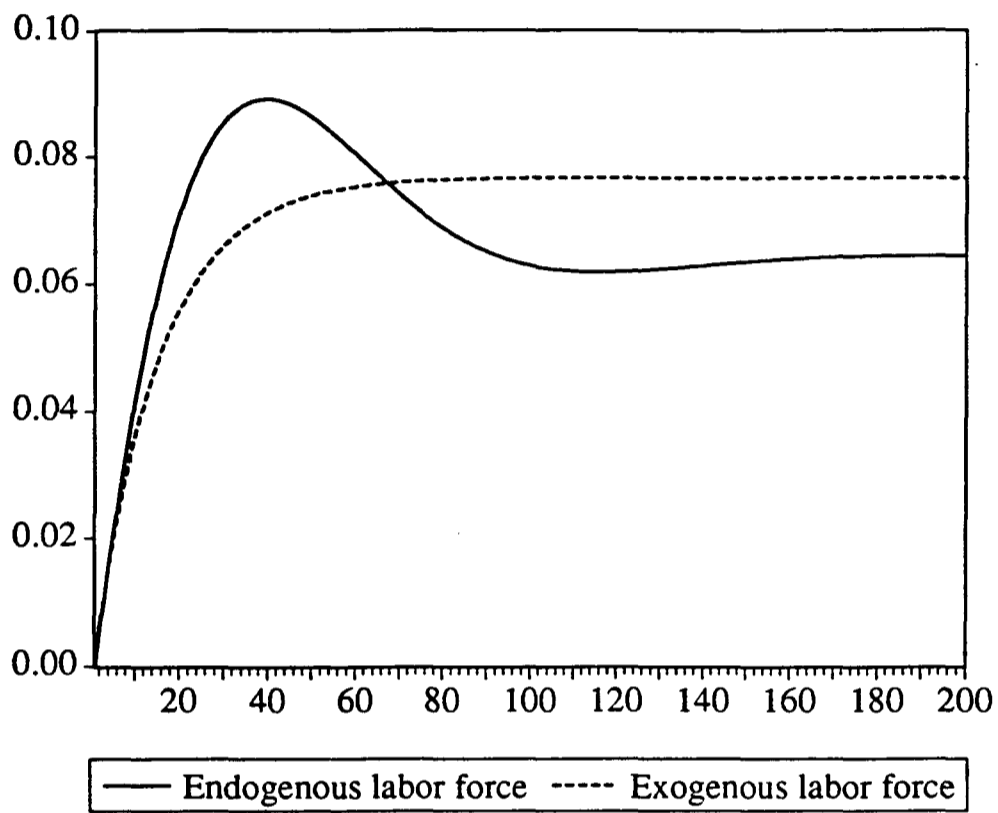


Figure 18

V. Dynamic Output and Employment Effects of Public Capital, Debt, and Taxes

Public capital must be financed in some manner: by debt, taxes, grants, or user fees. Most, in not all, of these financing techniques can be expected to have an adverse impact on economic growth. Consequently, the *net* effect of public capital on economic performance may turn out to be somewhat smaller than the *gross* effects estimated and analysed in the previous sections.

Here it is assumed that the financing of public capital involves: the issuance of debt to cover the original investment, so

$$dB = dKG$$

where B represents debt; and the levying of taxes to cover the necessary maintenance of public capital against physical depreciation as well as effective depreciation arising due to economic growth, so

$$dT = (dep + DY) \cdot dKG$$

where T represents taxes and dep represents a physical depreciation rate.

Table V.1 presents estimates of the growth impacts of public capital, debt, and taxes. Here, public debt is measured as total state debt, taxes is measured as total own source revenues, and the physical depreciation rate is assumed to equal 2.5 percent per year.⁹ The output maximizing public capital ratio, m , is estimated at a somewhat higher level than in Table III.1, ranging between 0.614 and 0.720 in the output growth expression and 0.614 and 0.699 in the employment growth expression. As

Table V.1
Output and Employment Growth Effects of Public Capital, Taxes and Debt

$$DX = l_x \left(\frac{KG}{K} \right) \left(1 - \frac{1}{2 \cdot m_x} \left(\frac{KG}{K} \right) \right) + a_x z + b_x \left(\frac{BG}{K} \right) + t_x \left(\frac{T}{K} \right) + e_x$$

$$X = Y, E$$

	OLS		WLS [√y]		WLS [√Y]		WLS [√E]		SUR	
	DY	DE	DY	DE	DY	DE	DY	DE	DY	DE
l_x	.873 (.164)	.387 (.093)	.901 (.156)	.415 (.099)	.664 (.164)	.285 (.075)	.643 (.169)	.278 (.073)	.873 (.107)	.387 (.057)
m_x	.672 (.053)	.614 (.035)	.667 (.051)	.616 (.035)	.710 (.054)	.699 (.052)	.720 (.059)	.693 (.053)	.672 (.026)	.614 (.026)
b_x	-.156 (.047)	-.045 (.025)	-.157 (.049)	-.045 (.027)	-.103 (.050)	-.050 (.025)	-.102 (.047)	-.049 (.025)	-.156 (.036)	-.045 (.019)
t_x	-.580 (.320)	-.275 (.089)	-.593 (.326)	-.215 (.151)	-.309 (.251)	-.161 (.118)	-.316 (.245)	-.160 (.115)	-.580 (.184)	-.275 (.068)
Y	.066 (.027)	.089 (.018)	.069 (.026)	.096 (.018)	.084 (.022)	.096 (.012)	.075 (.025)	.090 (.014)	.066 (.023)	.089 (.012)
E	-.075 (.037)	-.113 (.024)	-.077 (.037)	-.121 (.024)	-.117 (.037)	-.118 (.019)	-.109 (.040)	-.112 (.031)	-.075 (.027)	-.113 (.014)
U	.376 (.182)	.275 (.089)	.422 (.190)	.291 (.093)	.150 (.188)	.233 (.075)	.119 (.189)	.229 (.070)	.376 (.128)	.275 (.068)
R ²	.680	.870	.710	.893	.857	.938	.850	.933	.680	.870
SER x10(-3)	9.529	5.101	9.512	5.278	7.914	4.012	7.901	3.962	9.529	5.101
SSR x10(-3)	3.632	1.041	3.619	1.114	2.505	.644	2.497	.628	3.632	1.041

Notes: standard errors are in parentheses; all equations include individual state and decade fixed effects.

before, the estimate of m pertaining to output growth consistently exceeds that pertaining to employment growth. The estimates of l are positive and significantly different from zero, although no clear pattern of difference with the previous results is evident. Together, these estimates imply substantive growth effects of public capital on states' economies.

The results in Table V.1 also indicate that state economic performance is adversely affected by both means of financing public capital. A one percentage point increase in debt is associated with reductions in output growth of between 0.102 and 0.157 percent per year and employment growth of between 0.045 and 0.50 percent per year. An increase in taxes is associated with even larger impacts on growth; a one percentage point increase in tax revenues is related to diminished rates of growth of output and employment, respectively, of between 0.309 and 0.593 percent per year and 0.160 and 0.275 percent per year. Together, these estimates suggest substantially larger *gross* effects than *net* effects of public capital on both output and employment growth.

As before, output and employment growth are positively related to initial output and negatively related to initial employment. Compared to the results of Tables III.1 and III.2, however, the dynamics of the model are much more complex. In particular, with only one exception, the estimated coefficients on initial output and employment suggest that the endogenous labor force model is unstable. As the first panel of Table V.2 indicates, all equations possess a negative trace (consistent with stability) but, except for the WLS [\sqrt{Y}] model, also a negative determinant (inconsistent with stability). Furthermore, in the unstable cases, the nature of the dynamics is such that output and

Table V.2
Stability: Public Capital, Debt, Taxes and Economic Growth

	OLS	WLS [\sqrt{y}]	WLS [\sqrt{Y}]	WLS [\sqrt{E}]	SUR
Endogenous Labor Force Model					
<i>tr</i>	-0.047	-0.052	-0.021	-0.013	-0.047
CI	-0.008	-0.009	0.002	-0.001	-0.008
$tr^2 - 4 \cdot CI $	-0.030	-0.033	0.007	-0.004	-0.030
Exogenous Labor Force Model					
<i>tr</i>	-0.322	-0.343	-0.267	-0.266	-0.322
CI	0.014	0.020	-0.003	-0.005	0.014
$tr^2 - 4 \cdot CI $	0.048	0.038	0.059	0.050	0.048

employment follow undamped, oscillatory paths.

Still, it should be noted that these stability results are *knife-edged* in the sense that a null hypothesis of a zero determinant cannot be rejected at conventional significance levels. This represents an interesting extension to the *convergence* controversy in studies of economic growth. Many authors studying economic growth have found that state, regional, and national economies converge; specifically, in an equation such as

$$Dy = \beta \cdot y + \gamma \cdot z$$

they find that β is negative and significantly different from zero. However, other authors--using different data sets and/or different empirical methodologies--have found evidence that of considerably weaker or even non-existent convergence effects. The picture which arises from the study of the evolution of output and employment--of tenuous convergence (or stability)--seems to be consistent with these conflicting results in the literature.

Of course, if we are hoping to obtain some indication of the effects of a regional or national policy of increased public investment, the exogenous labor force model may be more useful. In contrast with the endogenous labor force model, the estimated coefficients for the exogenous labor force model are, for the most part, consistent with non-oscillatory, stable paths for output and employment.¹⁰ Specifically, all of the estimated exogenous labor force models, with the sole exception of the WLS [\sqrt{Y}] model, have both a negative trace and a positive determinant.

Figure 19 makes use of the OLS/SUR coefficient estimates for the exogenous labor force model to illustrate the impact of a permanent, 5 percentage point increase in public capital on the growth rates of output, employment, and productivity. The initial increases in output, employment, and productivity growth are equal to 0.4, 0.2, and 0.2 percent per year, respectively. After a few years of rather volatile movements, the growth rates follow smooth, non-oscillatory paths to the long run equilibrium.

The corresponding paths for the cumulative effects of the public capital increase on the levels of output, employment, and productivity--shown in Figure 20--indicate that these variables rise by some 6.0, 1.9, and 4.1 percent, respectively. It is evident that the *net* effects of public capital--depicted here--are much diminished relative to the *gross* effects of public capital--as illustrated in Figures 16, 17, and 18 above. The rise in the long run levels of output employment, and productivity is about one-half (*i.e.*, 53 percent) as much as previously. Of this reduction, about one-third can be attributed to diminished *initial* growth rates while about two-thirds can be attributed to quicker convergence (or lower persistence) rates.

VI. Conclusions and Directions for Further Research

This paper has investigated, in some depth, the importance of public capital to the long run performance of the economy. A main theme is that to be complete, an analysis of the long run impact of public capital must be decomposed into (a) the impact of public capital on *initial* growth rates and (b) the subsequent increases in output and employment due to the underlying dynamics of the model

GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital/tax & debt finance)

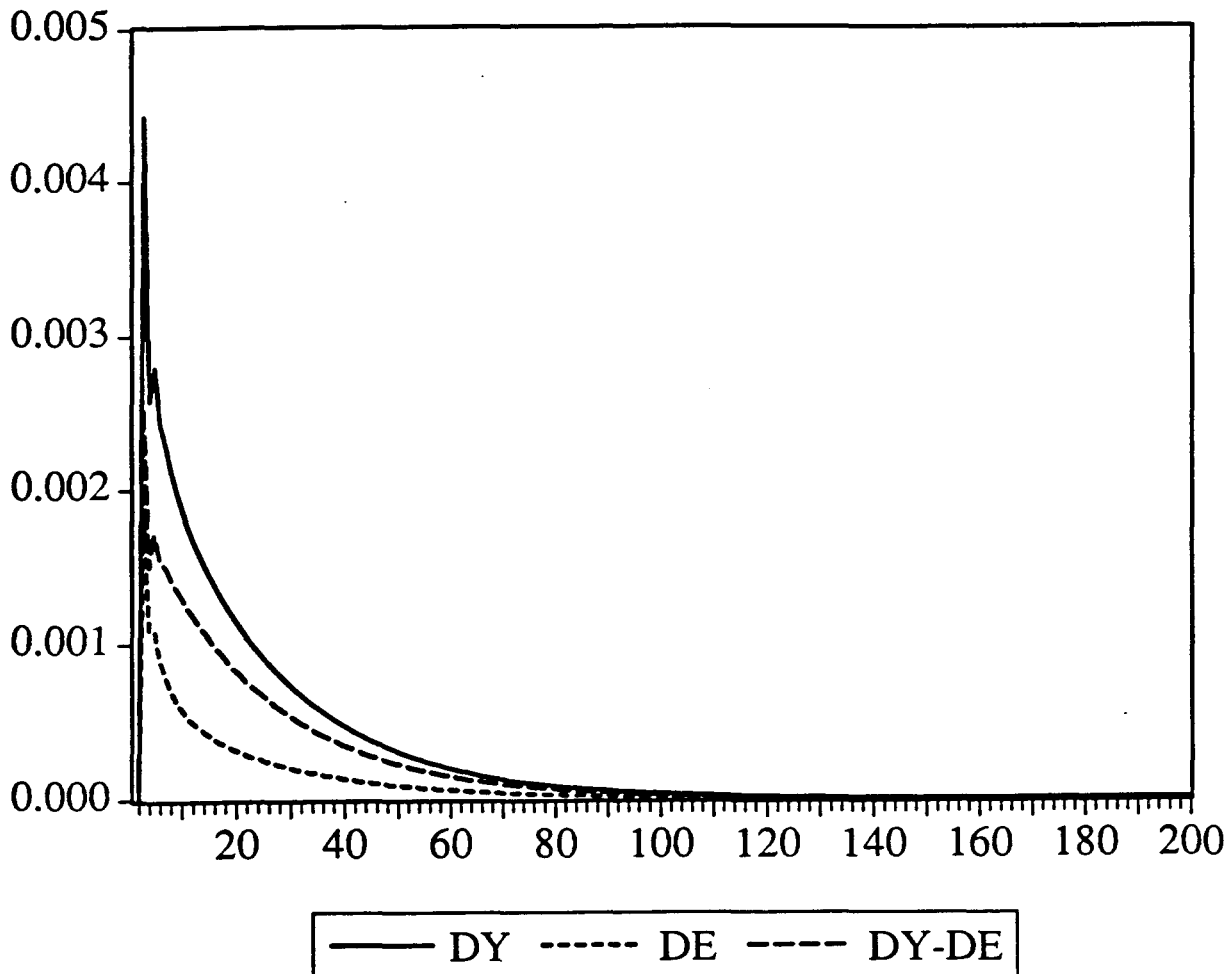


Figure 19

CUMULATIVE GROWTH EFFECTS OF PUBLIC CAPITAL
(permanent increase in public capital/tax & debt finance)

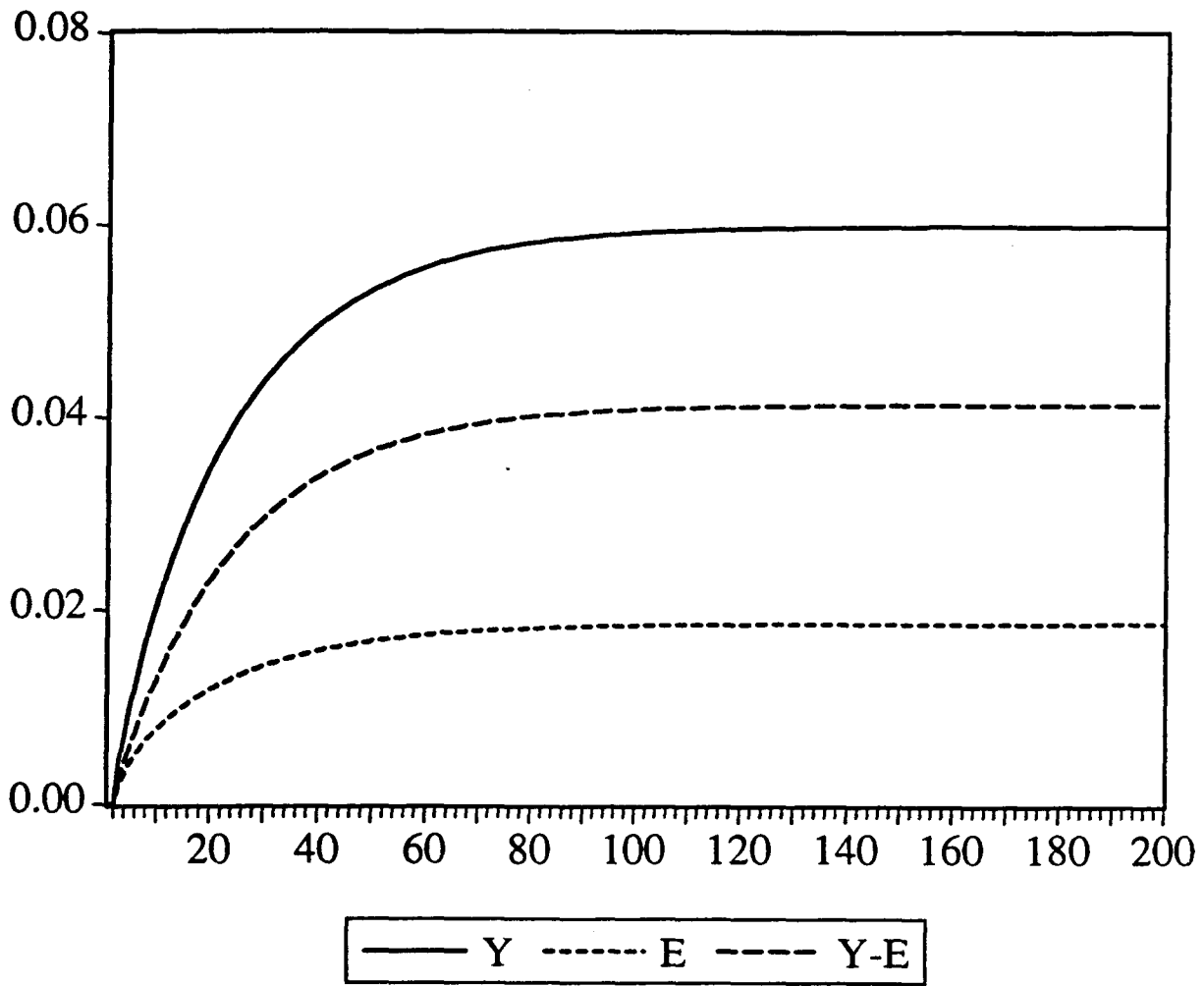


Figure 20

economy. In particular, depending on the potency of feedback effects of output and/or employment on subsequent output and employment growth, a small (respectively, large) initial effect of public capital can cumulate to a large (respectively, small) long run effect. We have found that the cumulative effects of public capital are much larger for an endogenous labor force model than for an exogenous labor force model. We have also found that the cumulative effects are significantly reduced if we take into account the fact that public capital, for the most part, must be financed by debt and/or taxes, and that the latter variables have adverse effects on state level economic performance.

Given these results, it would be of interest to pay more attention to the distinction between the endogenous and exogenous labor force specifications of the model. The endogenous labor force model carries the implication that increases in public capital translate into significant increases in the labor force at the state level. Consequently, it no doubt would be informative to investigate the implications of the provision of public capital for the inter-regional and inter-state migration of labor. It would also be of interest to pay more attention to the means of financing public capital. The model also carries the implication that increases in particular types of public capital which are financed by federal grants (the majority of the funds for which are collected outside of a specific state) translate into significant increases in output at the state level. Future research may usefully be pointed in one or both of these directions.

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Endnotes

1. The mean value of the public capital to private capital ratio is 0.446; one standard deviation is 0.136.

2. These estimates imply a rise in productivity growth of 1.1 percent per year, somewhat lower than the 1.4 percent per year in Aschauer (1997a). The difference--which is not statistically significant--can be ascribed to slightly different estimating equations in the two papers.

3. For this sort of path, the roots of the characteristic equation

$$x^2 - tr \cdot x + |C| = 0$$

x_1 and x_2 must be real and negative. This, in turn, requires that

$$tr^2 > 4 \cdot |C|.$$

4. For this sort of path, the roots of the characteristic equation, x_1 and x_2 , must be complex with negative real parts. This, in turn, requires that

$$tr^2 < 4 \cdot |C|.$$

5. The characteristic roots are real and negative and (approximately) equal to (-.056, -.235).

6. The characteristic roots are complex with a negative real part equal to -.05.

7. Thus, we assume that $c_{YE} = c_{EY} = 0$ and $c_{YY} < 0$, $c_{EE} < 0$.

8. The implied drop in unemployment of 3.5 percentage points appears reasonable. However, a linear extrapolation would suggest that a significantly larger increase in the public capital ratio might require a much larger drop in unemployment--which would reduce the unemployment rate to "unreasonable" levels (such as, perhaps, 1 or 2 percent). From this perspective, it might be preferable to include the unemployment rate in the model in a non-linear fashion to allow a diminishing impact of growth on unemployment. However, including the unemployment rate in a linear way--as is done in the present paper--eases computations based on the equation

$$U = L - E$$

where, it will be recalled, L represents the natural logarithm of the labor force and E represents

the natural logarithm of employment.

9. The low rate of depreciation reflects the fact that public capital, as opposed to private capital, consists mainly of structures with effective lives of 40 years or more.

10. These coefficients involve subtracting the coefficient on unemployment from the coefficient on the (natural logarithm of) employment in accordance with the definition that

$$U = L - E$$

which, when substituted for U , allows us to transform the estimating equations in such a way as to treat the labor force as an exogenous variable.