Mathematical Model of Interdependency between Production and Price Fluctuations

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Abstract

Presented here a mathematical model with one commodity that describes the mutual relationship between two sets of differential equations generating respectively the commodity's production and price fluctuations.

JEL classification: E 32

Keywords: Business fluctuations

1. Introduction

In present article author advances the model of commodity's production fluctuations (see Krouglov, 1997) by including in it the commodity's price which serves as an indicator to producers whether to increase or to decrease the commodity's production. Besides obtained production fluctuations it also gives us the commodity's price fluctuations. Will be shown that both sets of differential equations are interrelated.

2. Model's Description

Consider model with one commodity, and denote the volumes of production V_p and consumption V_c respectively. Designate r_p and r_c to be production and consumption rates. Assume also that introduced values at initial time were V_p^0, V_c^0, r_p^0, and r_c^0 respectively.

As in Krouglov (1997) presume that acceleration of commodity's production is a linear function of the commodity's deficit (surplus) on the market,

$$\frac{d^2V_p}{dt^2} = -\lambda \cdot \left(V_p - V_c\right) \tag{1}$$

where λ is a constant and $\lambda > 0$.

Here we are introducing the price P of commodity which will play the intermediary role for the producers' behavior. It means that Eq. (1) is rewritten into the following set,

$$\frac{d^2V_p}{dt^2} = \lambda_1 \cdot \frac{dP}{dt} \tag{2}$$

$$\frac{dP}{dt} = \lambda_2 \cdot \left(V_c - V_p\right) \tag{3}$$

where λ_1, λ_2 are constants, and $\lambda = \lambda_1 \cdot \lambda_2$.

Let us integrate Eq. (2) and differentiate Eq. (3). We will get,

$$\frac{dV_p}{dt} = \lambda_1 \cdot P + C_1 \tag{2'}$$

$$\frac{d^2P}{dt^2} = \lambda_2 \cdot \left(\frac{dV_c}{dt} - \frac{dV_p}{dt}\right) \tag{3'}$$

where C_1 is an integration's constant.

It will produce the following differential equation for the commodity's price fluctuations,

$$\frac{d^2P}{dt^2} = \lambda_2 \cdot \left(\frac{dV_c}{dt} - \lambda_1 \cdot P - C_1 \right) = -\lambda_1 \cdot \lambda_2 \cdot P + \lambda_2 \cdot \frac{dV_c}{dt} - \lambda_2 \cdot C_1$$
$$= -\lambda \cdot P + \lambda_2 \cdot \frac{dV_c}{dt} + C_2 \tag{4}$$

where $C_2 = -\lambda_2 \cdot C_1$ is the constant.

Constant C_2 serves for reconciliation between commodity's production rate r_p and its price P at initial time $t = 0$.

3. Model's Analysis

Let us examine the price fluctuations for the case of commodity's consumption development with a fixed rate r_c (as in Krouglov, 1997).

That is,

$$V_c = r_c \cdot t + V_c^0, \quad t \in [0, +\infty) \tag{5}$$

where V_c^0 is the volume of consumption at the initial time $t = 0$.

Then Eq. (4) transforms into

$$\frac{d^2P}{dt^2} = -\lambda \cdot P + \lambda_2 \cdot r_c + C_2 = -\lambda \cdot P + C_3 \tag{6}$$

where C_3 is the constant:

$$C_3 = \lambda_2 \cdot r_c + C_2 = \lambda_2 \cdot (r_c - r_p^0) + \lambda \cdot P^0$$

here r_p^0 and P^0 are respectively commodity's production rate and its price at time $t = 0$.

We use the change of variables $y = P - \frac{C_3}{\lambda}$, therefore Eq. (6) becomes

$$\frac{d^2 y}{dt^2} = -\lambda \cdot y \qquad (7)$$

and its solution is *the equation of harmonic oscillations* (e.g. see Piskunov, 1965),

$$y = A \cdot \sin(\beta \cdot t + \vartheta_0)$$

or for the previous variable P ,

$$P = A \cdot \sin(\beta \cdot t + \vartheta_0) + \frac{C_3}{\lambda} \qquad (8)$$

where $\beta = \sqrt{\lambda}$,

$$A = \frac{1}{\lambda_1} \cdot \sqrt{\lambda \cdot (V_p^0 - V_c^0)^2 + (r_p^0 - r_c)^2} \ ,$$

$$\vartheta_0 = \arctan\left[(r_p^0 - r_c) / \left(\beta \cdot (V_p^0 - V_c^0)\right)\right] .$$

Let us conclude with the finding of initial price P^0 which guarantees that commodity’s prices will always reside in the positive domain.

This is fulfilled when

$$P_{min} = -A + \frac{C_3}{\lambda} > 0 \ ,$$

therefore

$$P_{min}^0 = \frac{1}{\lambda_1} \cdot \left(\sqrt{\lambda \cdot (V_p^0 - V_c^0)^2 + (r_p^0 - r_c)^2} + (r_p^0 - r_c) \right) .$$

And for all $P^0 > P_{min}^0$ commodity’s price will remain positive.

4. Conclusion

We have shown here the interconnection between differential equations describing commodity’s production and price fluctuations. The actual equation of price fluctuations was obtained for the case of commodity’s consumption development with a fixed rate, and the value of commodity’s initial price securing that all prices will always remain positive was found for the same case.

5. References

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