

**New Result in Theory of Consumption:
Changes in Savings and Income Growth**

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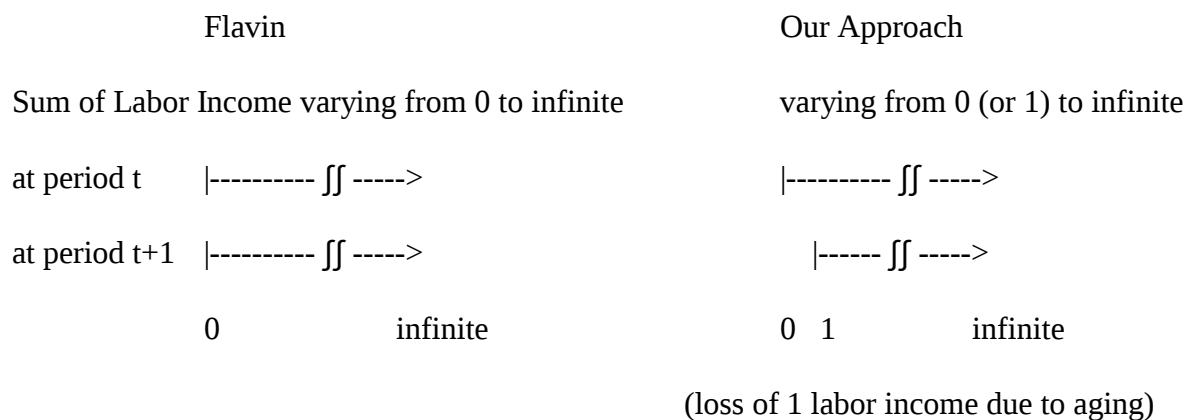
Introduction

In extending the Life Cycle-Permanent Income hypothesis, where an agent maximizes its utility based upon its assets, present and expected future labor income, Hall (1978) theorized that future consumption could be written only as a function of its current consumption; in effect, weakening the link between changes in income and consumption. One consequence of his result is that consumption theory has been slow to systematically incorporate research on factors affecting income, such as productivity and foreign trade. Our article examines this discontinuity. To help explain changes in savings, we show the Modigliani and Brumberg's proposition (1952, published 1979) that changes in savings are also a function of income growth.

Following the works of Hall (1978) and Sargent (1978), Flavin (1981) made a "convincing" derivation showing that, under the permanent income hypothesis (PIH) and rational expectations, changes in permanent income, or consumption should depend only upon revisions in expectations of future income (when these terms are assumed to be zero, Hall's martingale holds). However, from the start, this theoretical result was questioned by Flavin (1981) - and Hayashi (1982) - who found evidence that consumption is "excess-sensitive" to current income. Similarly, Deaton (1987) found evidence that changes in consumption are "excess-smooth" compared with changes in labor income. The causes of this discrepancy have been the subject of an extensive literature. Some authors, such as Deaton (1987) on aggregate consumption behavior and indefinitely lived agent, Zeldes (1989) on liquidity constraints, Campbell and Mankiw (1987) and Koenig (1990) on nonseparabilities of the

utility function of consumers and Deaton (1991) on rate of time preference and interest rates, have examined the microfoundations of the PIH's and rational expectations; while others, Hall and Mishkin (1982) on current consumption response to an innovation in income, Quah (1991) on agent's information of its permanent and transitory components of labor income and Deaton (1991) on income growth as the income process, have emphasized the nature of the income process.

Since many of the above mentioned authors have employed Flavin's derivation as the starting point, we also think appropriate to examine this increasing divergence between theory and evidence based upon her original approach. In section I, we show her three basic equations, and in section II, we replaced one of her equations. Under the same initial conditions, we show that the permanent income for the period $t+1$ should have the sum of incomes varying from period 1 to infinite instead of 0 to infinite, i.e. the loss of one labor income should be partially compensated for by an increase in assets (in the Appendix, we derive a generic permanent income equation and also discuss Flavin's possible misspecification). Graphically,



With this adjustment for the period $t+1$, we show that changes in consumption should also depend upon labor income and change in assets. In section III, we apply this result to savings and show that its change may depend upon interest rate and growth of labor income.

I The Three Basic Equations

Following Flavin (1981), an agent's permanent income can be expressed as

$$(1) \quad y_t^p = r \left[A_t + \sum_{j=0}^{\infty} \delta^{j+1} E_t y_{t+j} \right]$$

A_t is the agent's real non-human wealth at the beginning of the period t ,

r is the real rate of return, assumed constant,

δ is $(1 + r)^{-1}$

y_t is labor income at the end of period t ,

E_t is the agent's expectation at period t .

Assets varying according to:

$$(2) \quad A_{t+1} = (1 + r) A_t + y_t - c_t$$

And, Flavin (1981) considered agents consuming exactly their permanent income,

$$(3) \quad c_t = y_t^p$$

where consumption, c_t , is also paid at the end of period t .

(From the seminal works of Friedman (1957), Modigliani and Brumberg (1954), Muth (1961) and the already mentioned authors, these equations basically assume a quadratic utility with constant risk, infinitely lived and forward-looking agents, little or no liquidity constraints, and rate of time preference equal to interest rate. It also assumes rate of interest and income exogenous to consumption.)

II What, However, Is Permanent Income for Period t+1?

While Flavin (1981) wrote permanent income at t+1 as,

$$(4) \quad y_{t+1}^p = r \left[A_{t+1} + \sum_{j=0}^{\infty} \delta^{j+1} E_{t+1} y_{t+j+1} \right],$$

in the Appendix A, we show that, assuming interest rates were to remain constant, an agent's permanent incomes over time should be:

$$(5) \quad \begin{aligned} y_t^p &= r \left[A_t + \delta E_t y_t + \delta^2 E_t y_{t+1} + \delta^3 E_t y_{t+2} + \dots + \delta^{n+1} E_t y_{t+n} + \dots \right] \\ y_{t+1}^p &= r \left[A_{t+1} + \delta^2 E_{t+1} y_{t+1} + \delta^3 E_{t+1} y_{t+2} + \dots + \delta^{n+1} E_{t+1} y_{t+n} + \dots \right] \\ y_{t+2}^p &= r \left[A_{t+2} + \delta^3 E_{t+2} y_{t+2} + \dots + \delta^{n+1} E_{t+2} y_{t+n} + \dots \right] \\ &\dots \end{aligned}$$

Specifically, under an "Euler equation approach," if there is a change in the reference system, from t to t+1, permanent income at t+1 should be written as

$$(6) \quad y_{t+1}^p = r \left[A_{t+1} + \sum_{j=1}^{\infty} \delta^{j+1} E_{t+1} y_{t+j} \right]$$

If, as in eq. (3), the change in consumption with stochastic, or transitory components of consumption were,

$$(7) \quad c_{t+1} - c_t = y_{t+1}^p - y_t^p + \epsilon_{t, t+1}, \quad (\text{in Flavin, } \epsilon_{t, t+1} = - (1 + r) \mu_t + \mu_{t+1})$$

Substituting eq. (6) and (1) into (7), we find that,

$$(8) \quad c_{t+1} - c_t = r \left[A_{t+1} - A_t + \sum_{j=1}^{\infty} \delta^{j+1} E_{t+1} y_{t+j} - \sum_{j=0}^{\infty} \delta^{j+1} E_t y_{t+j} \right] + \epsilon_{t, t+1}$$

and since,

$$\sum_{j=0}^{\infty} \delta^{j+1} E_t y_{t+j} = \delta E_t y_t + \sum_{j=1}^{\infty} \delta^{j+1} E_t y_{t+j}$$

eq. (8) can be written as

$$(9) \quad c_{t+1} - c_t = r \left[A_{t+1} - A_t - \delta E_t y_t + \sum_{j=1}^{\infty} \delta^{j+1} E_{t+1} y_{t+j} - \sum_{j=1}^{\infty} \delta^{j+1} E_t y_{t+j} \right] + \epsilon_{t, t+1}$$

Eq. (9) shows that changes in consumption depend upon more than just changes caused by revisions in expectations on income. Note that, by working out the variances on eq. (9), it easy to see that the addition of the changes in assets less current income may help explain why changes in consumption are "excessively smooth" compared with the changes in income, i.e. the Deaton's paradox (see Flavin 1992).

Furthermore, if we simplify eq. (9) assuming that,

$$(10) \quad \sum_{j=1}^{\infty} \delta^{j+1} E_{t+1} y_{t+j} - \sum_{j=1}^{\infty} \delta^{j+1} E_t y_{t+j} \approx 0$$

then changes in consumption are given by

$$(11) \quad c_{t+1} - c_t = r \left[A_{t+1} - A_t - \delta E_t y_t \right] + \epsilon_{t, t+1}$$

III Was the Decline of Savings Caused by a Slower Growth of Income?

From the definition of total income or "measured" income,

$$(12) \quad y_{mt} = r A_t + y_t \quad (\text{with } A_t \text{ defined at the beginning of the period}),$$

eq. (11) can be written as

$$c_{t+1} - c_t = y_{mt+1} - y_{mt} - r \delta y_t - y_{t+1} + y_t + \epsilon_{t, t+1}$$

thus,

$$(13) \quad \text{savings}_{t+1} - \text{savings}_t = \left(y_{t+1} - \frac{y_t}{1 + r} \right) - \epsilon_{t, t+1}$$

From eq. (13), we could conclude that, for constant rates, the recent decline of savings may, in theory, be partially attributed to a slower growth in labor income. This result was anticipated by Modigliani and Brumberg (1952, published 1979). The striking innovation of their model over earlier ones - and advantage over more recent ones - was to rationalize zero-negative saving in terms of the stationary-dynamic state of the economy.

For the past two decades, this decline in growth seems to be supported by a growing number of research. Among them, Levi (1989) found evidence of slower growth of family income and inequalities in income distribution; Bosworth, Burtless and Sabelhaus (1991), after an extensive discussion on demographics, income distribution and capital gains, also raised the possibility of savings decline being related to a slower growth of income. In practice, to derive its optimal consumption model, Deaton (1991) has also accepted the growth of income as the income process.

Appendix A

A.1 A consumer maximizes

$$(i) \quad \sum_{t=0}^{\infty} b^t \left[\mu_0 + \mu_1 c_1 - \frac{\mu_2}{2} c_t^2 \right], \text{ subject to}$$

$$(ii) \quad A_{t+1} = R \left[A_t + y_t - c_t \right], \text{ where } y_t, \text{ under a stochastic process, is } E_t y_t.$$

Thus, under the "Euler equation approach," which is found in many textbooks, including Sargent (1987) - chapters IX and XII, optimal consumption for period t is given by

$$(iii) \quad c_t = (1 - R^{-2}b^{-1}) A_t - \frac{\mu_1}{\mu_2} \frac{(R^{-1}b^{-1}L - 1)}{1 - R} + \frac{(1 - R^{-2}b^{-1})}{1 - L^{-1}R^{-1}} E_t y_t$$

($R^{-1} = \delta$, L is the lag operator and y is at the beginning of period.)

Obviously, repeating the same optimization technique applied for eq. (iii), one should find that consumption, $c_{t+1}, c_{t+2}, \dots, c_{t+n}$, should be given by,

$$(iv) \quad c_{t+1} = (1 - R^{-2}b^{-1}) A_{t+1} - \frac{\mu_1}{\mu_2} \frac{(R^{-1}b^{-1}L - 1)}{1 - R} + \frac{(1 - R^{-2}b^{-1})}{1 - L^{-1}R^{-1}} E_{t+1} y_{t+1}$$

...

$$(vi) \quad c_{t+n} = (1 - R^{-2}b^{-1}) A_{t+n} - \frac{\mu_1}{\mu_2} \frac{(R^{-1}b^{-1}L - 1)}{1 - R} + \frac{(1 - R^{-2}b^{-1})}{1 - L^{-1}R^{-1}} E_{t+n} y_{t+n} \quad A$$

SS

uming $Rb = 1$, the difference between (vi) and (iii) is,

$$(vii) \quad c_{t+n} - c_t = (1 - R^{-1}) \left[A_{t+n} - A_t + \frac{E_{t+n}y_{t+n} - E_t y_t}{1 - L^{-1}R^{-1}} \right]$$

and since,

$$(viii) \quad \frac{E_{t+n}y_{t+n}}{1 - L^{-1}R^{-1}} = \sum_{j=n}^{\infty} R^{-j} E_{t+n}y_{t+j} \quad , \text{ where } n \text{ may be interpreted as age}$$

writing equation (vii) in terms of their power series results in,

$$(ix) \quad c_{t+n} - c_t = (1 - R^{-1}) \left[A_{t+n} - A_t + \sum_{j=n}^{\infty} R^{-j} E_{t+n}y_{t+j} - \sum_{j=0}^{\infty} R^{-j} E_t y_{t+j} \right]$$

A.2 On the other hand, though Flavin provided no explanation on how she arrived at equation (4), it would be a misspecification to reach her conclusion based upon "shifting forward one period and writing out the polynomial in L explicitly." That is, dividing eq. (iii) by L results in

$$(x) \quad L^{-1} c_t = (1 - R^{-1}) L^{-1} A_t + \frac{(1 - R^{-1})}{1 - L^{-1}R^{-1}} L^{-1} E_t y_t$$

or,

$$(xi) \quad c_{t+1} = (1 - R^{-1}) \left[A_{t+1} + \sum_{j=1}^{\infty} R^{-j+1} E_{t+1} y_{t+j} \right] =$$

$$= (1 - R^{-1}) \left[A_{t+1} + (E_{t+1} y_{t+1} + R^{-1} E_{t+1} y_{t+2} + \dots + R^{-n} E_{t+1} y_{t+n+1} + \dots) \right]$$

which may be confused with Flavin's equation (4):

$$\begin{aligned}
(xii) \quad c_{t+1} &= (1 - R^{-1}) \left[A_{t+1} + \sum_{j=0}^{\infty} R^{-j} E_{t+1} y_{t+j+1} \right] = \\
&= (1 - R^{-1}) \left[A_{t+1} + (E_{t+1} y_{t+1} + R^{-1} E_{t+1} y_{t+2} + \dots + R^{-n} E_{t+1} y_{t+n+1} + \dots) \right]
\end{aligned}$$

Eq. (xii) is clearly not a particular case of eq. (vi). The flaw is that, **multiplying eq. (iii), which is a *particular* optimal consumption, either by a constant or variable, or worse, utilizing the lag operator definition to establish beforehand that there is a relationship between c_t and c_{t+1} , will not necessarily result in another optimal solution.**

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