

**The U.S. Federal Intertemporal Budget Constraint:  
Restoring Equilibrium Through Increased Revenues or Decreased Spending?**

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Abstract

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The literature concerning the U.S. federal deficit and its implications for the federal government's intertemporal budget constraint seems to have reached a tentative conclusion that starting with the Reagan administration the U.S. federal government has violated its intertemporal budget constraint. Recently the popular press has been debating the issue of whether this violation is due to a fall in tax revenues, that must eventually be made up in the future with (presumably) higher tax rates, or whether expenditures have risen to unsustainable levels and must eventually fall. I review the evidence on the stability of the U.S. federal intertemporal budget constraint and find that there was a break from the equilibrium that occurred about the fourth quarter of 1981. Analyzing the deviations of real revenues and real expenditures from their common stochastic trend reveals that expenditures account for a greater percentage of the deviation from equilibrium than do revenues. Further evidence demonstrates that while real revenues per unit of real GNP have grown at a relatively constant rate over the post-war period, real expenditures per real GNP have undergone an increase in trend growth starting in approximately 1976.

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JEL: H6, E6, C3

\*- A previous version circulated under the title "Revenue Growth, Expenditure Growth and the U.S. Federal Intertemporal Budget Constraint."

The U.S. federal budget has been in almost perpetual deficit for the last forty-five years but not until the unusually large deficits of the 1980s has the economics profession addressed the important issues involved. In a seminal article, Hamilton and Flavin (1986) demonstrate that the federal government cannot run a permanent deficit exclusive of interest payments on the debt, but may have a constant deficit when interest payments are included. The key result is that the real rate of interest at which the government borrows must be greater than the growth rate of real debt so that the discounted present value of future government debt goes to zero. Furthermore, they claim that if the government budget is to be balanced in present value terms, the surplus (deficit) inclusive of interest payments should be stationary. Hamilton and Flavin present evidence that this condition is satisfied using annual data from 1960 to 1984. But Kremers (1988) demonstrates that this result is not robust to lag specification in the test.

Trehan and Walsh (1988) generalize the Hamilton and Flavin result to show that government expenditures, inclusive of interest payments, and revenues should be cointegrated with a cointegration vector equal to  $[1 \ -1]'$ . They present evidence that supports this restriction. Hakkio and Rush (1991a) point out that the restriction that the cointegrating vector between government expenditures, inclusive of interest payments, and revenues equal  $[1 \ -b]'$ , such that  $b = 1$ , is a sufficient condition for intertemporal budget balance but not, strictly speaking, a necessary condition. Quintos (1995) expands on Hakkio and Rush (1991a) by stating the necessary and sufficient conditions for deficit sustainability as  $0 < b \leq 1$ . She notes, however, "that  $0 < b < 1$  is sufficient for the deficit to be sustainable, it is inconsistent with the governments ability to market its debt in the long run." (p. 410) Thus as Hakkio and Rush (1991a) state,  $b = 1$  "is probably necessary" (p. 433) for the government

to remain solvent.

Hakkio and Rush (1991a) conduct an analysis to see if the results are sensitive to sample period. Using a sample that spans from the first quarter of 1952 to the fourth quarter 1988 they find that the intertemporal budget is balanced in sub-samples up until 1976, but that for the sub-sample 1976:1 to 1988:4 the constraint is violated. They suggest that "(t)his indicates that the behavior of spending and revenue may well have changed recently, within, say, the last one hundred quarters." (Hakkio and Rush (1991a) pg.437).

Wilcox (1989) also finds evidence of a change in the stance of U.S. fiscal policy in that the market value of government debt appears to be greater than the sum of expected future surpluses. Wilcox cites 1974 as the year in which the disequilibrium is first observed which is somewhat earlier than the period suggested by Hakkio and Rush (1991a). The implications as stated by Wilcox are that "these results suggest that the structure of fiscal policy (as summarized by the joint behavior of debt and deficits) must eventually change." (Wilcox (1989) pg. 292). This has been a resounding theme in the political discourse over the last ten years in the U.S. Controlling the expanding U.S. government budget deficits has recently become the single most important issue on the national legislative agenda, with the proposed 1995 balanced budget Constitutional amendment failing in the U.S. Senate by only one vote. The issue has become very highly politicized as different sides debate the "best" way to achieve a balanced budget. Raising tax rates is very unpopular and it is questionable whether such a strategy in fact raises revenues. In a recent *Wall Street Journal* editorial, Hauser provides some evidence that tax revenues as a percentage of GNP have remained fairly constant over the last thirty years even though tax rates have been unstable over this

same period.<sup>1</sup>(*WSJ* December 7, 1994). Spending reductions are also unpopular especially among the constituencies that benefit most directly from these expenditures.

Haug (1995) presents results that are in contrast to those of Hakkio and Rush (1991a). Using a recursive test of the stability of the cointegrating vector,  $[1 \ -b]'$ , he finds very little evidence of a structural change in the conduct of fiscal policy over the last four decades. However, Quintos (1995) presents evidence that there was a shift in the deficit policy in the U.S., first around 1975 and then again around 1980. Her statistical procedure was very similar to that used by Haug (1995) with only the null hypothesis being different.

This study reexamines whether the U.S. federal government has practiced a sustainable fiscal policy over the postwar period. Using a new recursive test conducted within the framework of the maximum likelihood estimator, the stability of the U.S. intertemporal budget equilibrium is examined. The results are in line with those of Hakkio and Rush (1991a), Wilcox (1989) and Quintos (1995) in that I find the federal budget constraint was obeyed until about 1980 at which point the intertemporal budget constraint has been violated.

Given that the U.S. budget is currently not on a sustainable path, I address the issue of how to achieve intertemporal federal government budget balance, i.e., through revenue increases or spending reductions.<sup>2</sup> Using the equilibrium implied by the present value

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<sup>1</sup> In fact, Crowder (1996) presents evidence that the average marginal tax rate in the U.S. from 1947 to 1991 has evolved as an integrated process.

<sup>2</sup> Bohn (1991) used an error correction model (ECM) to describe how intertemporal budget imbalances were corrected in the past. Although this study uses similar methodology, the question addressed is somewhat different. Specifically, this study asks, "given the implied intertemporal equilibrium between expenditures and revenues and the fact that this equilibrium is being violated currently, do the data indicate which of the two components is most responsible for the disequilibrium and thus which of the two should make the greater

constraint on government borrowing, the common trend between real revenues and real expenditures (inclusive of interest) is estimated. This common trend can be viewed as the equilibrium path of the two budget components. Analyzing the deviations of each component from this common trend reveals that while both series contribute to the recently high deficits, i.e., real revenues are below trend and real expenditures are above trend, real expenditures make a relatively greater contribution to the disequilibrium. This result is reinforced by analyzing the ratios of real revenues and real expenditures to real GDP. The deterministic trend components of these two ratios, which must also be equal for intertemporal balance, display evidence of separating temporarily in 1976 and then permanently in 1981. Further analysis reveals that this break in the deterministic equilibrium between revenues/GDP and expenditures/GDP is due to an increase in the trend in the expenditures/GDP component.

The preceding discussion suggests several hypotheses of interest that can be tested. Two preliminary issues that should be addressed are the appropriate measure of the real rate of interest and whether this measure is consistent with the restriction that the growth rate of debt is smaller than the real interest rate. The next issue concerns the existence of a long-run equilibrium relationship between real expenditures inclusive of interest payments and real revenues. Of specific interest is whether the cointegration vector is  $[1 \ -1]'$  as is necessary for the government to be able to market its debt. If this restriction does not hold for the entire sample, are there sub-samples for which it is true? If a specific break point can be found, which of the two budget components, if either, can be identified as the proximate cause of the break in equilibrium?

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adjustment to restore the equilibrium."

The rest of paper is organized as follows: section 1 describes the theoretical restrictions implied by the intertemporal federal government budget constraints. Section 2 describes the econometric results using quarterly data from 1950:1 to 1994:2. Section 3 concludes with a discussion. Econometric methodologies are described in the appendix.

## I. The Government Intertemporal Budget Constraint

In every period the government must meet a budget constraint. Assuming that all government debt has one-period maturity, the government budget constraint can be written as,

$$\mathbf{G}_t + (1+i_t)\mathbf{B}_{t-1} = \mathbf{R}_t + \mathbf{B}_t \quad (1)$$

where  $G_t$  is the government's expenditures net of interest payments in period  $t$ ,  $B_t$  are the funds raised by the government through bond sales in period  $t$ ,  $i_t$  is the interest rate on the bonds sold in period  $t-1$ , and  $R_t$  is the government's revenue in period  $t$ .<sup>3</sup> The variables in equation (1) can be nominal, real or deflated by population or GDP.<sup>4</sup> Solving (1) forward yields the governments intertemporal budget constraint given in (2),

$$\mathbf{B}_0 = \sum_{t=1}^{\infty} \gamma^{t+1} (\mathbf{R}_t - \mathbf{G}_t) + \lim_{j \rightarrow \infty} \gamma^{j+1} \mathbf{B}_{t+j} \quad (2)$$

where  $\gamma \equiv (1+i_t)^{-1}$ .<sup>5</sup> If the limit term in (2) equals zero then the current stock of outstanding

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<sup>3</sup>  $R_t$  includes seignorage generated by central bank through money creation.

<sup>4</sup> The interpretation of the interest rate in equation (1) depends upon how the other variables are measured. See Hakkio and Rush (1991a) fn. 2.

<sup>5</sup> The discussion treats the nonstochastic economy. The stochastic version of the model differs from the nonstochastic case only in the discount factor. The choice of discount factor does not alter the time series implications of the model (see Bohn (1993)).

debt,  $B_0$ , is just equal to the discounted value of future government surpluses. This condition is often called the "No Ponzi Game" condition and implies that the government cannot retire its debt simply by issuing new debt perpetually. Equation (2) also highlights the important concept that the relevant surplus measure is net of interest payments.

If it is assumed that the interest rate is stationary with mean of  $i$ , then (1) can be rewritten as,

$$G_t + (i_t - i)B_{t-1} + (1+i_t)B_{t-1} = R_t + B_t \quad (3)$$

which must hold every period. Define  $E_t = G_t + i_t B_{t-1}$ , i.e., government spending inclusive of interest payments. Substituting and solving (3) forward yields equation (4).

$$B_{t-1} = \sum_{j=0}^{\infty} \gamma^{j+1} (R_{t+j} - E_{t+j}) + \lim_{j \rightarrow \infty} \gamma^{j+1} B_{t+j} \quad (4)$$

Using equation (1) and taking first differences of (3), equation (4) can then be rewritten as,

$$E_t - R_t = \sum_{j=0}^{\infty} \gamma^{j-1} (\Delta R_{t+j} - \Delta E_{t+j}) + \lim_{j \rightarrow \infty} \gamma^{j+1} \Delta B_{t+j} \quad (5)$$

where  $E_t$ , and  $R_t$  are integrated of order one so that their first differences on the right hand side of (5) are stationary. Imposing the further restriction that the growth rate of debt,  $\Delta B$ , is

stationary implies that the limit term in (5) goes to zero.<sup>6</sup> Thus the restriction implied by the government's intertemporal budget constraint is that  $E_t - R_t$  must be stationary or that these two series are cointegrated with cointegrating vector of  $[1 \ -1]'$ .<sup>7</sup> Furthermore, the existence of a cointegrating relationship itself implies that  $E_t$  and  $R_t$  share a common stochastic trend (Stock and Watson, 1988). It is possible to estimate this common stochastic trend using a multivariate Beveridge-Nelson decomposition by restricting the trend component to be a pure random walk with possible drift. One can then interpret the deviations from this common trend as deviations from the government's intertemporal budget constraint. Analyzing these deviations of both the spending and revenue series from the common trend may reveal which series is more "responsible" for the current deficit.

Several observations should be made at this point. First, since the empirical evidence regarding the stationarity of the nominal interest rate in the U.S. over the post-war period is not supportive, equation (5) should not be analyzed in nominal terms<sup>8</sup>. Second, if the source of nonstationarity is due to deterministic trends, say in the relative-to-output revenues and

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<sup>6</sup> This is actually a stronger restriction than that which is necessary for the limit term to vanish as demonstrated by Quintos (1995). But relaxing this restriction implies eventual government default on its outstanding stock of debt. Given this outcome, if the growth rate of debt is not stationary, the federal government might be unable to market its debt. Another way to see this point is to consider equation (5) with variables defined relative to real output. Since the growth rate of real output is stationary, if  $\Delta B_t$  is nonstationary the debt-to-output ratio is growing. Since taxes are, presumably, bounded as a percentage of real output, a growing debt-to-output ratio implies eventual default by the government.

<sup>7</sup> Hakkio and Rush (1991a) and Quintos (1995) demonstrate that the cointegration vector  $[1 \ -b]'$ , where  $0 > b > 1$ , is consistent with deficit sustainability since the limit term in (5) would still go to zero, but this would imply eventual government default on its debt, assuming no change in fiscal policy.

<sup>8</sup> Hakkio and Rush (1991a) make this point as well.

expenditures measures, the deterministic trend in the revenue-to-GDP ratio must be the same as that in the expenditure-to-GDP ratio (inclusive of interest payments) or the deficit will grow unboundedly as a proportion of GDP. Equation (5) then implies a common deterministic trend.

## **II. Empirical Results**

### **Data**

The data used in this study are quarterly from 1950:1 to 1994:2. The government expenditures net of interest and revenues are defined as in Hakkio and Rush (1991a). The market value of U.S. government debt was provided by Michael Cox of the Federal Reserve Bank of Dallas. The real interest rate used to calculate real interest payments on the debt is the ex-post real rate on the three-month U.S. treasury bill. The method used to calculate this real rate is described in Crowder and Hoffman (1996).<sup>9</sup> The use of the three-month rate differs from previous studies, i.e. Hakkio and Rush (1991a), that use the five-year treasury note rate. The problem with the five-year rate is that it is not consistently defined over the entire sample (see Hakkio and Rush (1991a)) while the three-month rate is. As it turns out, there is no qualitative difference between the calculated three-month real interest rate and the five-year real rate. The means are not statistically different from each other and the correlation coefficient between the two real rates is 0.94.

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<sup>9</sup> Using the long-run equilibrium relationship between nominal interest rates and inflation implied by the after-tax Fisher relation allows the real rate to be estimated by cointegration, implying a stationary real rate. The Fisher effect estimate over this sample is 1.43 which implies a marginal tax rate on interest income of approximately 30%. See Crowder and Hoffman (1996) for details of the estimation.

Quintos states that "the necessary and sufficient condition (*for deficit sustainability*) being that the debt process grow slower than the growth rate of mean interest rates" (1995, p. 410). Figure 1 presents graphs of the growth rate of debt and the ex-post real interest rate. A simple test of means yields a z-test statistic of 2.47 which is significant at the 1% level for a one-sided null.<sup>10</sup> This suggests that the U.S. may be violating its intertemporal budget. Figure 2 displays the same series but they are now adjusted for real GDP growth. The z-test of the null that the ex-post real rate, adjusted for the growth rate of real GDP, is greater than the debt-to-GDP growth rate is 0.06 which is insignificant at conventional levels of significance. This test implies that the U.S. federal government fiscal policy may be on a sustainable path.<sup>11</sup> An analysis of spending and revenues may shed more light on this issue.

### **Tests of the Intertemporal Restriction**

Real federal expenditures (inclusive of interest payments) and real federal revenues are plotted in figure 3. A visual inspection of the data demonstrates that the federal budget was close to balance from the end of the Korean War until the recession of 1974-75. But it isn't until about 1981 that the "permanent" divergence between real expenditures and real revenues occurs.

A more rigorous analysis reveals that each of the series is consistent with a unit root,

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<sup>10</sup> The z-statistic is calculated as,

$$Z_{\bar{X}_1 - \bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$$

which has student-t distribution since the variances must be estimated.

<sup>11</sup> Note that the mean adjusted real rate is less than the debt-to-GDP growth rate, -0.0039 versus -0.0037, respectively, but not statistically significantly so.

as evidenced by conventional unit root tests.<sup>12</sup> Using the procedure introduced by Johansen (1988), I tested for the existence of cointegration between the two series over the entire sample. The calculated trace test statistic associated with the null of zero cointegrating vectors is 21.83 which can be compared to critical values in table 1 of Osterwald-Lenum (1992).<sup>13</sup> This null hypothesis has a marginal significance level greater than 1%. The hypothesis that there is at most one cointegrating vector yields a test statistic of 0.59 which has a marginal significance level of approximately 55%. The estimated cointegration vector normalized on the revenues variable is  $[1 \ -0.872]'$ . A likelihood ratio (LR) test of the restriction that the cointegration vector is  $[1 \ -1]'$  yields a statistic of 11.09 which is distributed as a  $\chi^2(1)$  variate. Thus, the restriction implied by equation (5) above is violated using data for the entire sample. These results are consistent with those obtained by Hakkio and Rush (1991a) for their entire sample.

Hakkio and Rush (1991a) break their sample into sub-samples and conduct the cointegration analysis on these sub-samples. Although problematic (discussed below), I also

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<sup>12</sup> The ADF test with and without a time trend are 0.31 and -2.95 for the real revenues series and -0.78 and -2.43 for the real expenditures series, implying non-rejection of the unit root null at conventional significance levels. Both tests include one lag of the change in the dependent variable. Higher lag orders in the model do not overturn the results.

<sup>13</sup> The specification of the VAR in the Johansen estimation procedure included eight lags, the minimum necessary to remove residual serial correlation. The specification is consistent with a model that allows deterministic trend components in the data that are eliminated by the cointegration vector. I tested this specification against the more general model where the deterministic trends are not eliminated by the cointegration vector. These correspond to hypotheses  $H_1(r)$  vs.  $H_2^*(r)$  in Johansen (1994), with appropriate critical values given in tables 1 and 2\* in Osterwald-Lenum. The restriction that the cointegration vector also eliminates the deterministic trends could not be rejected, yielding a test statistic of 0.001 which has a  $\chi^2(1)$  distribution.

analyzed two sub-samples using the Johansen procedure. The first sub-sample is from 1950:1 to 1973:4. The second sub-sample is from 1974:1 to 1994:2. The choice for the period of demarcation is based upon the results from Wilcox (1989).<sup>14</sup> Over the period 1950:1 to 1973:4 the trace test for zero cointegration vectors is 25.99 which is significant at higher than the 1% level. The estimated normalized cointegration vector is  $[1 \ -0.966]'$ . The hypothesis that the cointegrating vector is  $[1 \ -1]'$  yields a LR statistic of 0.648 which is not significant at any conventional level.<sup>15</sup> The results from the same analysis over the sub-sample 1974:1 to 1994:2 yielded a trace statistic for the null hypothesis of zero cointegrating vectors of 10.11 which has an approximate marginal significance level of 25%. Thus, over the second sub-sample there is virtually no evidence that the two series are cointegrated!

The problem with this strategy is that the accuracy of the estimates of the long-run parameters are quite sensitive to the sample span. It is important to use as long a sample as possible to estimate and test the long run cointegrating parameters. Hakkio and Rush (1991b) use a Monte Carlo study to demonstrate this fact. To overcome the potential pitfalls of using sub-sampled estimates, I use a procedure introduced by Hansen and Johansen (1993) to test the stability of the cointegration vector over the entire sample. The strategy is to fix the short run dynamics at their full sample values by concentrating them out of the log likelihood and recursively estimate and test the long-run parameters. This yields a series of LR tests that

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<sup>14</sup> Using Hakkio and Rush's (1991a) sub-samples of 1950:1 to 1976:3 and 1976:4 to 1994:2 yielded qualitatively identical results.

<sup>15</sup> The lag length in the (levels) VAR of five was again chosen such that it was the minimum lag necessary to eliminate any significant serial correlation in the errors. The trace statistic for the null hypothesis of no more than one cointegrating vector is 0.0002 which is insignificant.

have an asymptotic  $\chi^2(1)$  distribution as demonstrated by Hansen and Johansen (1993). The restriction imposed is that the cointegration vector is  $[1 \ -1]'$  at each iteration.<sup>16</sup> The first ten years of data are used as starting values. Figure 4 plots the recursive test statistics normalized by the appropriate 5% critical value so that values greater than one imply significance at the 5% level. The restriction implied by equation (5) is not rejected until the third quarter of 1982. If a 10% level of significance is used, then this break occurs in 1982:1. That is, the federal government appears to have been obeying its intertemporal budget constraint until 1982. This date conforms reasonably well to the a priori belief that the deficit "exploded" in the early part of the Reagan administration.

### **Identifying the Source of Imbalance**

In an effort to identify which of the two budget components is most responsible for the change in the U.S. federal budget policy, the implied common stochastic trend between real revenues and real expenditures is calculated.<sup>17</sup> Figure 5 plots the two series along with the estimated common trend (the solid line in each panel). Note that this common stochastic trend is estimated under the restriction that the cointegration space,  $\beta$ , spans the space of the vector  $[1 \ -1]'$  which is not true for the sample after 1981:4 (see figure 4). The drift in the common random walk component is the drift estimated over the sub-sample 1950:1 to 1981:4. Thus, this estimated common trend can be thought of as the equilibrium path of the two

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<sup>16</sup> This vector,  $[1 \ -1]'$ , is the theoretical equilibrium vector and as such it was chosen as the basis for the stability tests. Bohn (1991) also uses the implied theoretical values, not the estimated vector, in his analysis.

<sup>17</sup> The technical appendix discusses the identification and estimation of this common trend.

budget components had the restriction that  $\beta = [1 \ -1]'$  been true over the entire sample. The question arises, "In what sense is this not accurate?" A short digression is necessary in order to address this issue.

The problem manifests itself in the ECM estimation of the vector of error correction coefficients  $\alpha$ . If  $\beta$  is misspecified, then the estimates of  $\alpha$  will be biased. This becomes important in the calculation of the C(1) matrix when finding the common stochastic trend.<sup>18</sup> The common trend is given by the null space of  $\alpha$ , i.e.,  $\alpha_{\perp}$ . A poor or biased estimate of  $\alpha$  implies a poor or biased estimate of  $\alpha_{\perp}$ . Consider an extreme case where the true error correction coefficient,  $\alpha$ , in the real expenditures (revenues) equation of the ECM is zero. This implies that the 'source' of the common trend is the real expenditures (revenues) series since  $\alpha_{\perp} = [1 \ 0]'$ , assuming the variables are ordered [rev exp]' ([exp rev]') in the VAR. Thus all deviations from the equilibrium path must be due to the real revenues (expenditures) series since the real expenditures (revenues) series is by definition the equilibrium path or common trend. Thus, using a value for  $\beta$  that is not true over the entire sample may lead to estimates of  $\alpha$  that do not impose the (true) restriction that the error correction coefficient in the expenditures (revenues) equation is zero. This allows the estimator of the common stochastic trend to "pull" the common trend towards the real revenues (expenditures) path.

I can think of two ways to gauge how much of a problem this presents in the present analysis. The first is to estimate the ECM over the full-sample using the full-sample estimate

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<sup>18</sup> See the technical appendix for further elaboration on the relation between  $\alpha$ ,  $\beta$  and C(1).

of  $\beta$  which is  $[1 \ -0.872]'$ , and see what the ECM estimates are.<sup>19</sup> The second is to estimate the ECM over the sub-sample in which the restriction that  $\beta = [1 \ -1]'$  cannot be rejected and find the estimates of  $\alpha$  over this period and assume that if the deficit process followed this equilibrium path that these estimates of  $\alpha$  would prevail over the entire sample.

Implementing the first strategy yields an error correction coefficient estimate in the real revenues equation of the ECM of -0.074 and in the real expenditures equation of 0.277. Using a likelihood ratio test to test the restriction that  $\alpha_{\text{rev}} = 0$  from the maximum likelihood estimate of the ECM using Johansen's method yields a test statistic of 3.021 which is distributed as a  $\chi^2(1)$  variate. A test of the analogous restriction on  $\alpha_{\text{exp}}$  yields a statistic of 16.962. Clearly, the full-sample results favor the weak exogeneity of the real revenues over the real expenditures thus implying that accumulated revenue innovations are the common stochastic trend.<sup>20</sup> The second strategy to assess the importance of misspecifying  $\beta$  in estimating the common trend yields similar results. Testing the joint restriction from Johansen's MLE that  $\beta = [1 \ -1]$  and  $\alpha_{\text{rev}} = 0$  over the sub-sample 1950:1 to 1981:4 yields a test statistic of 3.906 which is distributed as a  $\chi^2(2)$  variate. The analogous restriction on  $\alpha_{\text{exp}}$  yields a test statistic of 19.968. It is quite evident that the revenues series is more likely to be the common trend component than the expenditures series, indeed over the sub-sample in which intertemporal balance is maintained, the revenues series is the common trend.

What the above analysis has demonstrated is that the results based upon the full

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<sup>19</sup> This presents problems of its own since this estimate of  $\beta$  is not consistent with intertemporal budget balance. The value of  $\beta$  should be  $[1 \ -1]'$  as discussed in section I.

<sup>20</sup> A sufficient condition for real revenues to be weakly exogenous with respect to the long-run parameter matrices  $\alpha$  and  $\beta$  is that  $\alpha_{\text{rev}} = 0$ . See Dolado (1992).

sample, to be given subsequently, will be biased toward finding a larger role for the real revenues series in explaining the violations of the U.S. federal government's intertemporal budget constraint than it has actually played. As it turns out, this makes the subsequent conclusions stronger.

Returning to the analysis at hand, figure 6 plots the deviations from the implied equilibrium path of the common stochastic trend in figure 5. Note that from this graph it appears that both components contribute to the rise in the real federal deficit beginning in the early 1980s. Beginning in 1982:1, the mean of the deviation of real revenues from its equilibrium path is -76.47 billion dollars with a standard deviation of \$2.65 billion. The mean of the deviation of real expenditures over this same period is 93.82 billion dollars with a standard deviation of \$4.43 billion. Both are significantly different from zero at very high levels of significance. Over the prior sample of 1950:1 to 1981:4, both means are insignificantly different from zero with the real revenues mean of \$3,800 (s.d. of \$1.62 billion) and the real expenditures mean of \$810 (s.d. of \$2.23 billion). Using a simple difference in means test, the z-statistic calculated over the sub-sample 1950:1 to 1981:4 implies that the means of the two deviations are not statistically different from one another.<sup>21</sup> But over the sub-sample 1982:1 to 1994:2 the positive deviations of real expenditures from the equilibrium path are statistically larger than the negative deviations of real revenues from the common stochastic trend. The calculated z-statistic associated with the null hypothesis that the means are equal in absolute value is 23.77 which is significant at a very high level. It can be concluded that the evidence presented thus far implies that the both components of

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<sup>21</sup> The calculated z-statistic is -0.000012.

the federal budget have contributed to the violation of the U.S. government intertemporal balance. It should be remembered, however, from the discussion above that this is a conservative estimate that gives more weight to the revenues component than the ECM estimates over 1950:1 - 1981:4 would suggest is appropriate.

Figure 7 presents the ratios of real expenditures to real GDP and real revenues to real GDP for the full sample. From this figure one can see the change in the expenditures/GDP ratio that occurred first in the mid-1970s and then again in the early 1980s. Augmented Dickey-Fuller tests reveal that the two series are only stationary after removing a deterministic time trend.<sup>22</sup> For intertemporal balance to be achieved, the trends in each series must be the same, i.e, each of the ratios must grow at the same rate.

Figure 8 plots the two ratios but eliminates the four observations in 1950 since the price controls of the Korean War seem to create an anomalous measure in the expenditures/GDP series. A casual analysis of figure 8 reveals that the two ratios appear to share the same trend growth until the mid-1970s or early 1980s. The results of a formal analysis of the restriction that the two ratios have equal deterministic trends is presented in figure 9. The series plotted in figure 9 is generated by estimating univariate time series models of the revenues/GDP and expenditures/GDP series. Each model includes six lags of the dependent variable, a constant, and a time trend and the two models are estimated jointly by SUR. The estimation is done recursively, with 1951:1 to 1955:4 used as initial values,

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<sup>22</sup> For the revenues/GDP series the ADF  $\tau_{\mu}$  test statistic, with four lags in the regression, is -2.58 and the  $\tau_{\tau}$  test is -4.39. For the expenditures/GDP series the analogous test statistics are -2.72 and -4.28. The four lagged changes in the regression are the minimum necessary to whiten the residuals.

where a likelihood ratio of the restriction that the two trend coefficients are equal across equations is calculated at each iteration. This statistic is then normalized by its 95% critical value and plotted in figure 9. Therefore, values of the normalized statistic greater than one imply significance at the 5% level. The test reveals that the trends are basically equal, except at the beginning of the sample which may be due to the few observations available, until the early 1980s, at which point the trend coefficients begin to diverge.

It is clear from figure 8 that the trend in expenditures/GDP is greater than that for revenues/GDP from about 1980 until the end of the sample. The question is, which of the two ratios is responsible for the divergence in trend growth rates. To answer this question, I calculate a recursive test for the stability of the trend coefficient in each univariate model of the two ratios. This is essentially a Chow test where only part of the parameter space is restricted. The results from this recursive analysis are plotted in figure 10. The series in figure 10 are the marginal significance levels of this recursive test of trend stability. The dotted line represents the 5% level so that values of the test that fall below this line are significant at the 5% level. Note that the real revenue-to-real GDP trend coefficient has been stable over the entire sample. But the real expenditure-to-real GDP trend coefficient underwent significant changes from 1970 to 1982. This implies that the divergence of the two ratios' deterministic trends is due to the changing expenditures/GDP trend. Once again, this suggests that it is the expenditures component of the federal budget that has been the main culprit in the large budget deficits of the last fifteen years implying that expenditures should probably fall by a larger percentage than revenues rise if the federal government is to restore intertemporal balance to its budget.

### **III. Conclusions**

This study has used some recent advances in the testing of the stability in long-run equilibrium relationships to analyze which of the two U.S. federal budget components is more responsible for the recent violations of intertemporal balance and thereby identify the component which must make the greater adjustment in the future in order to restore intertemporal budget balance. The results presented concerning the relationship between real expenditures (inclusive of interest payments) and real revenues conform closely to the results from earlier studies. I extend the results by extending the often used cointegration methodology to estimate the implied common stochastic trend between the two budget components. This common trend represents the equilibrium path for the intertemporally balanced federal budget. Therefore, the deviations of the two budget components from this trend can be interpreted as departures from the long-run equilibrium path.

The results strongly imply that it is the real expenditures component of the U.S. federal budget that has violated the long-run equilibrium implied by the intertemporal budget constraint. These results are reinforced by an analysis of the revenues/GDP and expenditures/GDP ratios. In this case, the change in the deterministic trend in the expenditures/GDP series explains all of the divergence of trends in the two ratios.

The implications of the analysis seem quite clear. In order for the government to restore its intertemporal budget balance, real federal expenditures must begin to grow at a lower rate than that at which real revenues must rise.

## Technical Appendix

The estimation of the  $r$  cointegrating relationships between real federal revenues and expenditures is carried out using the maximum likelihood procedure introduced by Johansen (1988). The estimation is based upon the  $n$ -dimensional error correction model in (A1),

$$\Delta \mathbf{X}_t = \boldsymbol{\mu} + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{X}_{t-1} + \sum_{j=1}^{k-1} \boldsymbol{\Gamma}_j \Delta \mathbf{X}_{t-j} + \boldsymbol{\varepsilon}_t \quad (\text{A1})$$

where  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  are  $n \times r$  matrices of the error correction space and the cointegration space, respectively. The  $\boldsymbol{\Gamma}_j$  are the  $n \times n$  parameter matrices of the short run dynamics and  $\boldsymbol{\mu}$  contains the deterministic part of the vector process. Following Johansen, let  $\mathbf{Z}_{0t} = \Delta \mathbf{X}_t$ ,  $\mathbf{Z}_{1t} = \mathbf{X}_{t-1}$ , and  $\mathbf{Z}_{2t} = (\Delta \mathbf{X}_{t-1}', \dots, \Delta \mathbf{X}_{t-k+1}', 1)'$ . Then stack the parameters  $(\boldsymbol{\Gamma}_1, \dots, \boldsymbol{\Gamma}_{k-1}, \boldsymbol{\mu})$  in  $\boldsymbol{\Gamma}$ . The model in (A1) can then be written as in (A2).

$$\mathbf{Z}_{0t} = \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{Z}_{1t} + \boldsymbol{\Gamma} \mathbf{Z}_{2t} + \boldsymbol{\varepsilon}_t \quad (\text{A2})$$

Maximum likelihood estimation of (A2) consists of regressing  $\mathbf{Z}_{0t}$  on  $\mathbf{Z}_{1t}$  corrected for  $\mathbf{Z}_{2t}$ .

The residuals from this regression are given as,

$$\begin{aligned} \mathbf{R}_{0t} &= \mathbf{Z}_{0t} - \mathbf{M}_{02} \mathbf{M}_{22}^{-1} \mathbf{Z}_{2t} \\ \mathbf{R}_{1t} &= \mathbf{Z}_{1t} - \mathbf{M}_{12} \mathbf{M}_{22}^{-1} \mathbf{Z}_{2t} \end{aligned} \quad (\text{A3})$$

where

$$\mathbf{M}_{ij} = \mathbf{T}^{-1} \sum_{t=1}^T \mathbf{Z}_{it} \mathbf{Z}_{jt}' \quad (\text{A4})$$

The remaining analysis is then based upon the model with the short run dynamics concentrated out of the log likelihood is in (A5).

$$\mathbf{R}_{0t} = \alpha \beta' \mathbf{R}_{1t} + \tilde{\varepsilon}_t \quad (\text{A5})$$

Equation (A5) forms the basis for the recursive test of the stability or constancy of the parameter space in  $\beta$ . In this model, all of the short run dynamics have been fixed at their full-sample values. Recursively estimating (A5) and testing the restriction that  $\beta = [1 \ -1]'$  at each iteration yields a set of likelihood ratio tests which are asymptotically distributed as  $\chi^2(1)$  variates.

The common trends representation of  $X_t$  is given by inverting (A1) and decomposing the MA lag polynomial,  $C(L)$ , into the permanent and transitory components by noting that  $C(L) = C(1) + (1-L) C^*(L)$ .

$$\mathbf{X}_t = \mathbf{C}(1)(\boldsymbol{\mu} + \boldsymbol{\varepsilon}_t)(1-L)^{-1} + \mathbf{C}^*(L) \boldsymbol{\varepsilon}_t \quad (\text{A6})$$

Johansen (1991) demonstrates that  $C(1) = \beta_{\perp}(\alpha_{\perp}'\Pi^*(1)\beta_{\perp})^{-1}\alpha_{\perp}'$ , where  $\alpha_{\perp}'\alpha = \beta_{\perp}'\beta = 0$  and  $\Pi^*(1) = \Gamma_1 + \dots + \Gamma_{k-1}$ . Structural identification of the MA representation can be achieved by suitable restrictions on the parameter spaces of  $\alpha_{\perp}$  and/or  $\beta_{\perp}$  as demonstrated by Crowder, et. al. (1996).

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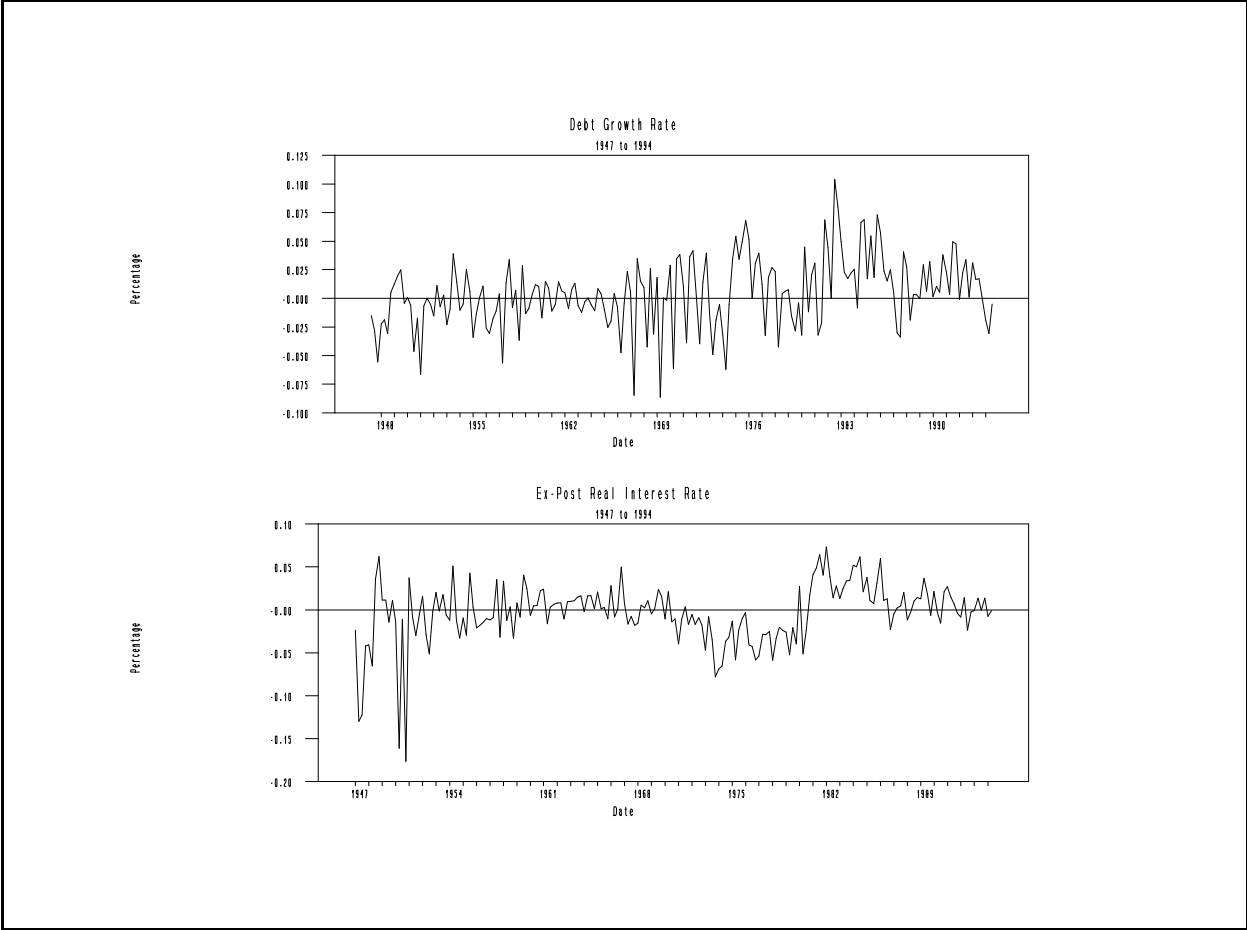


Figure 1

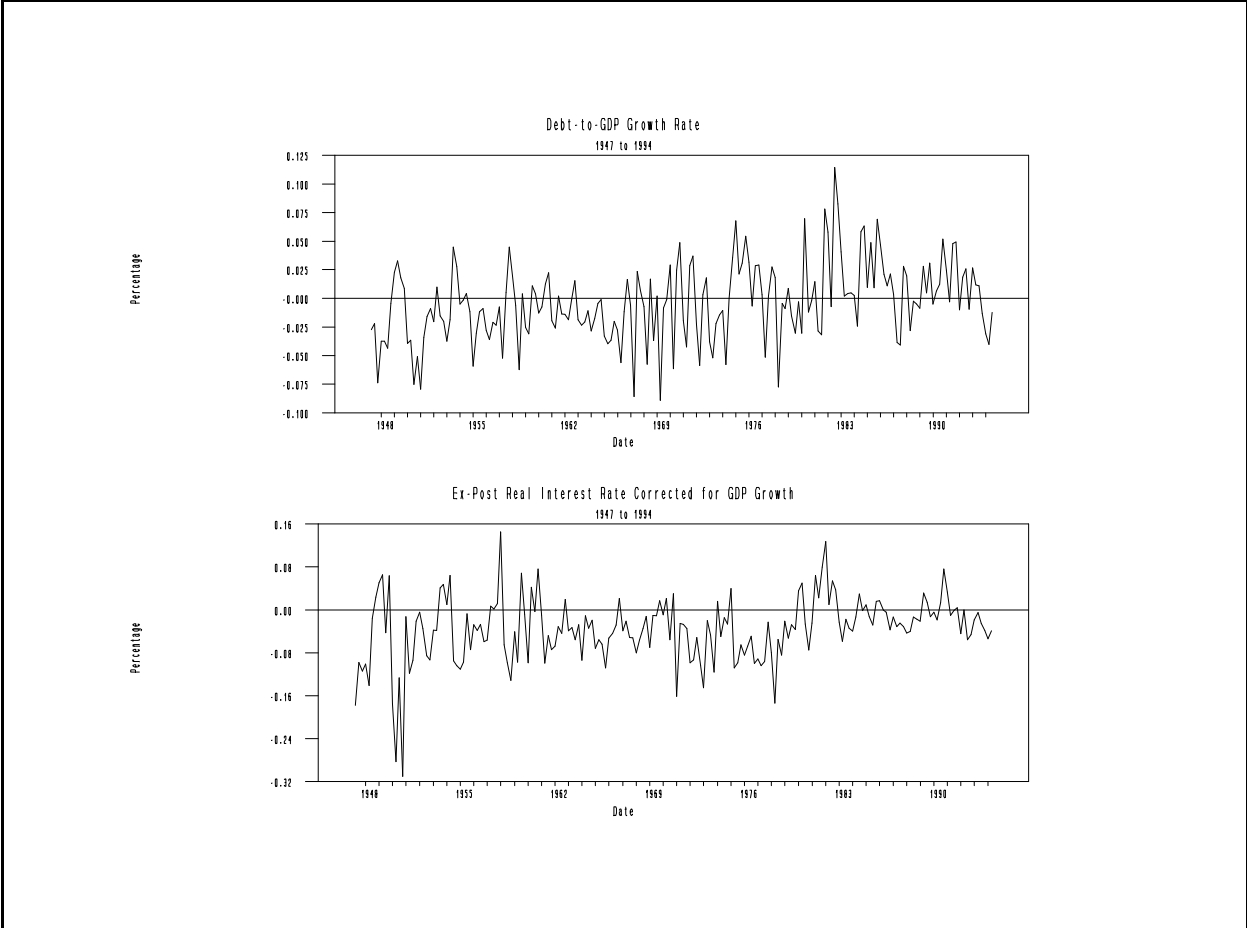


Figure 2

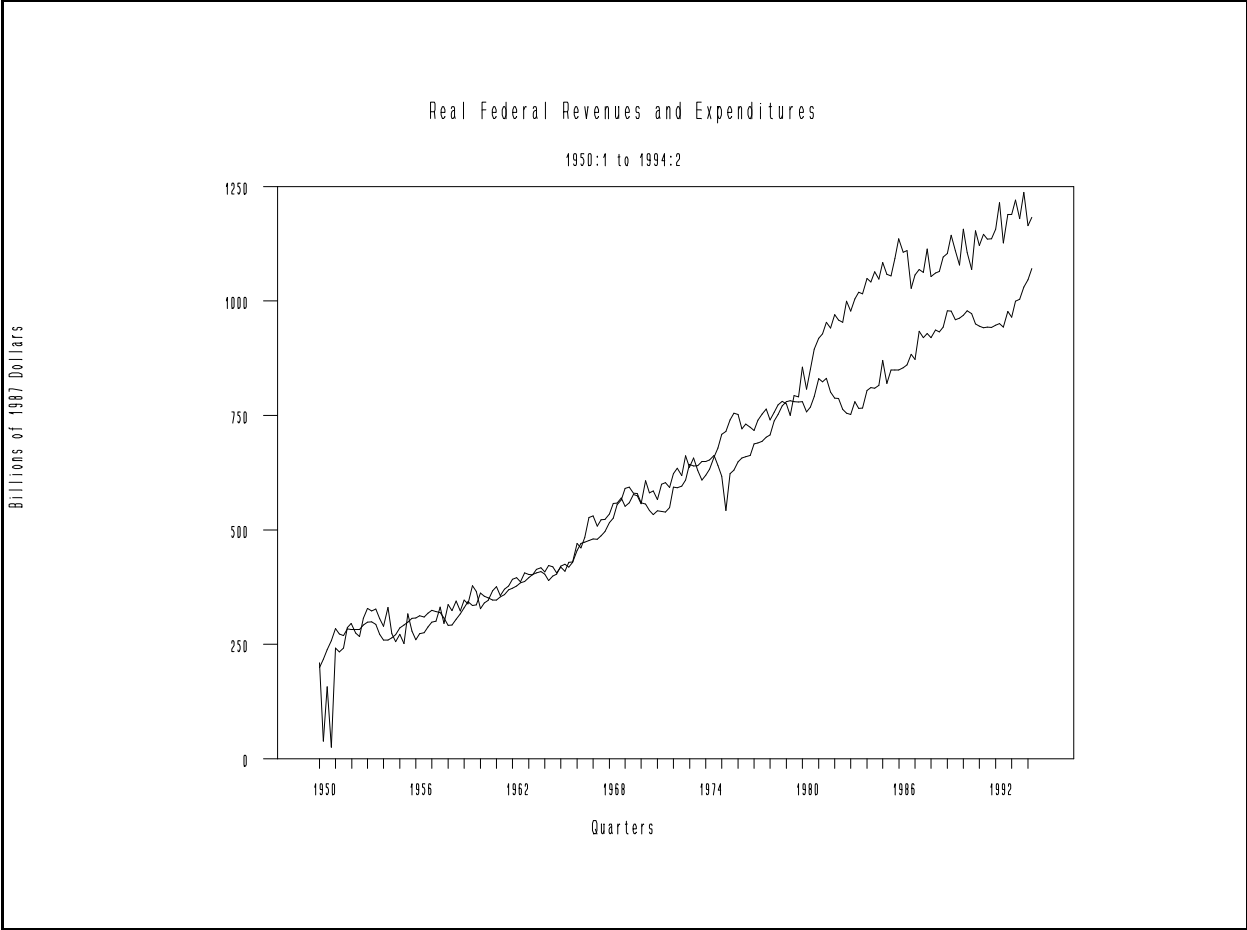
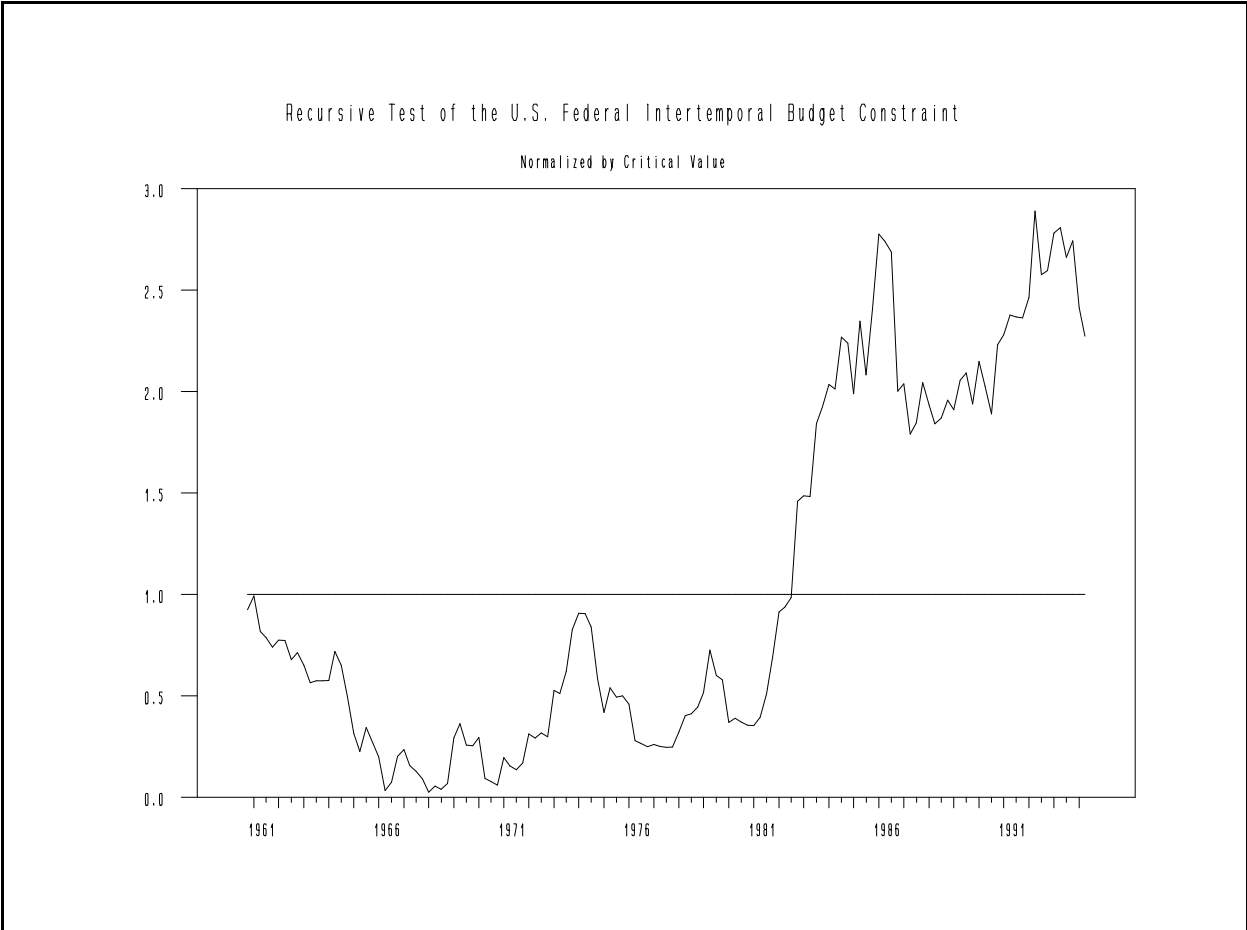


Figure 3



**Figure 4**

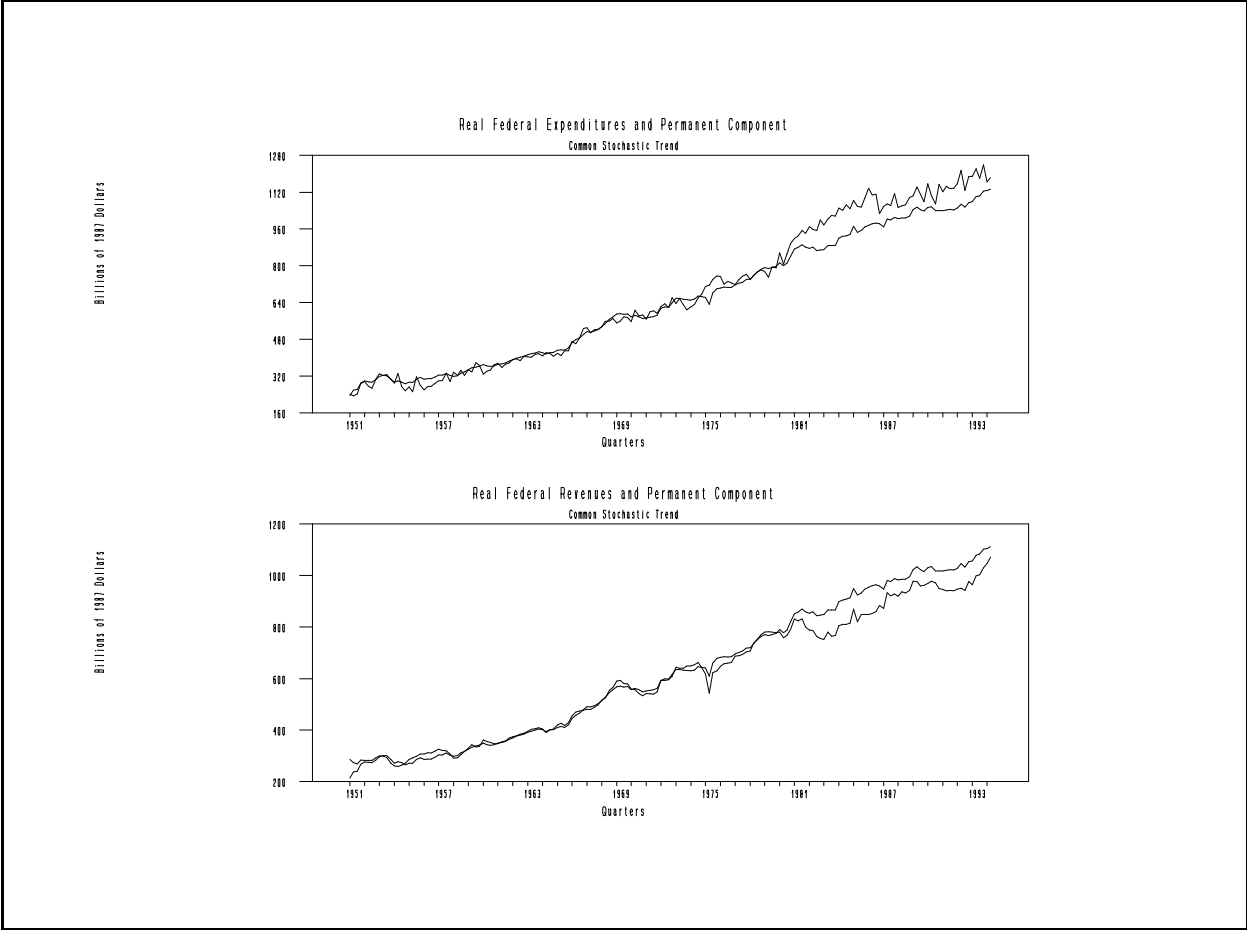


Figure 5

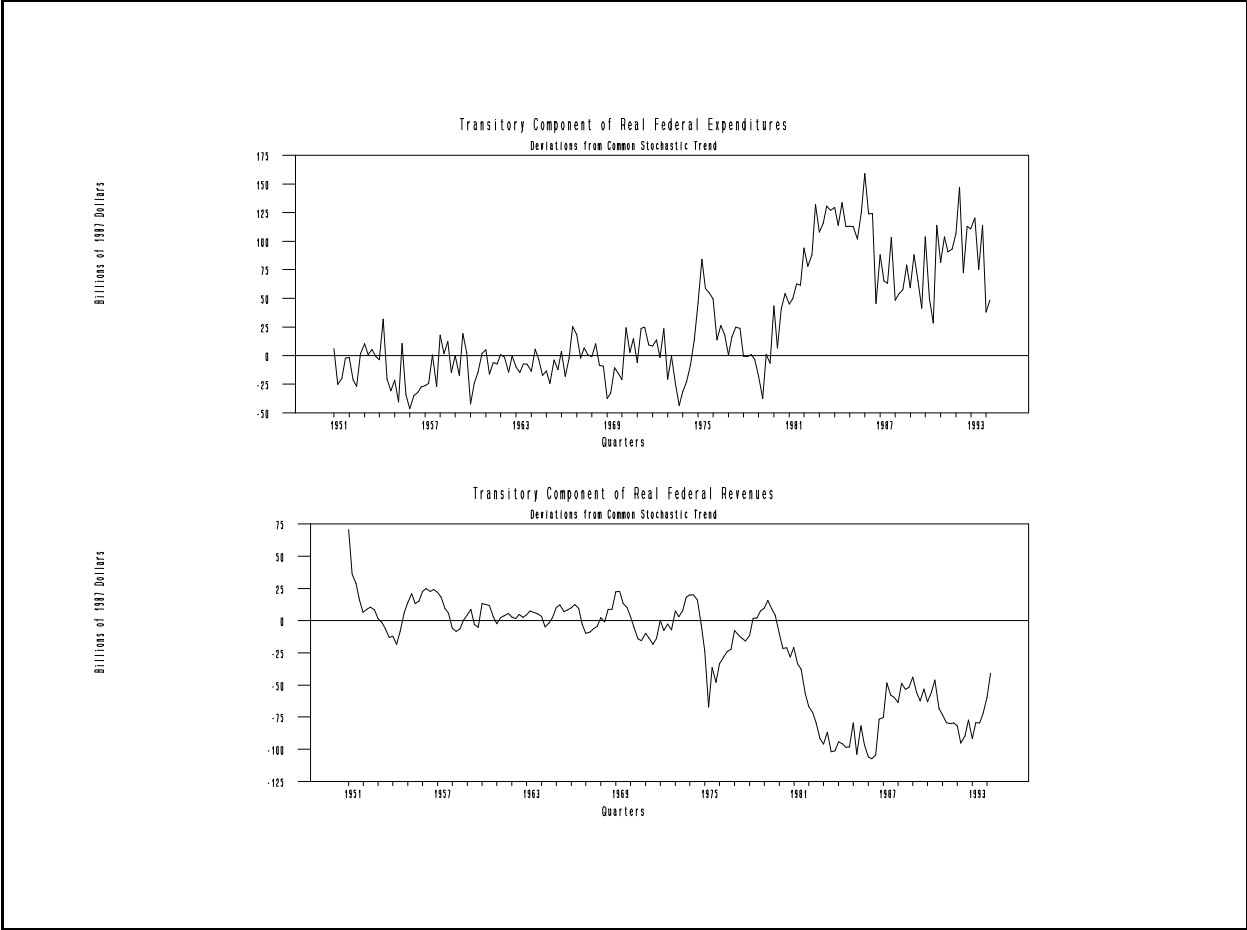


Figure 6

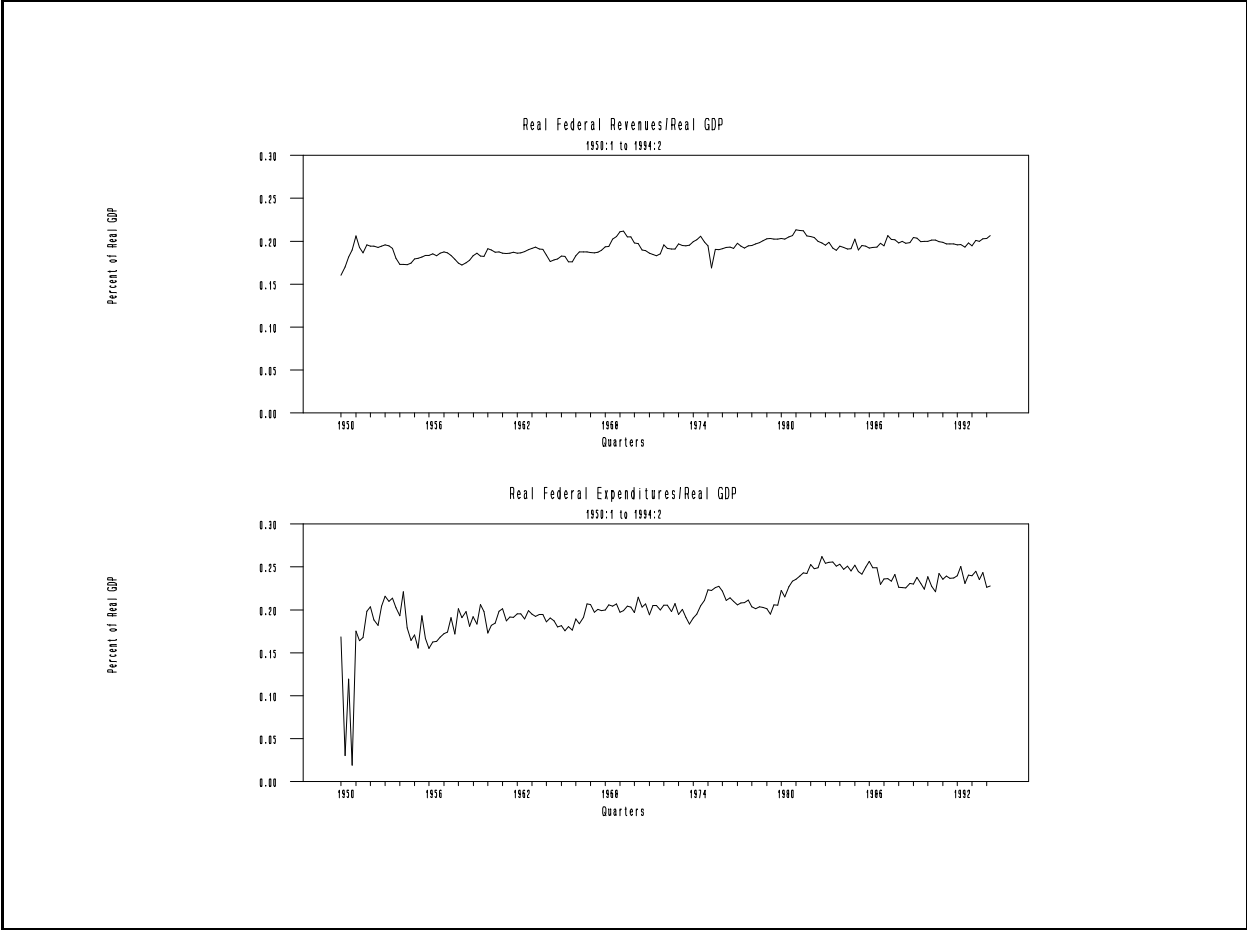


Figure 7

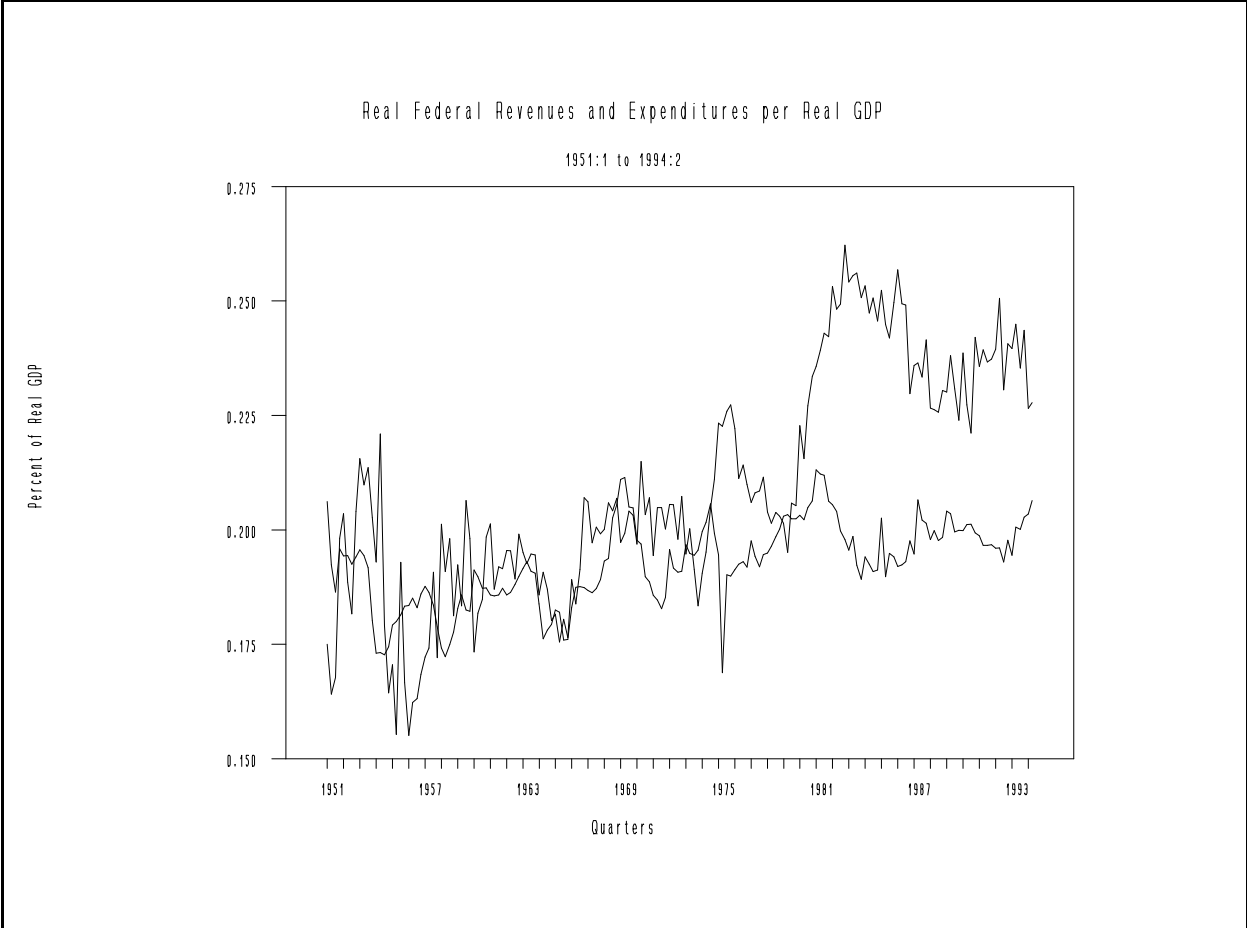
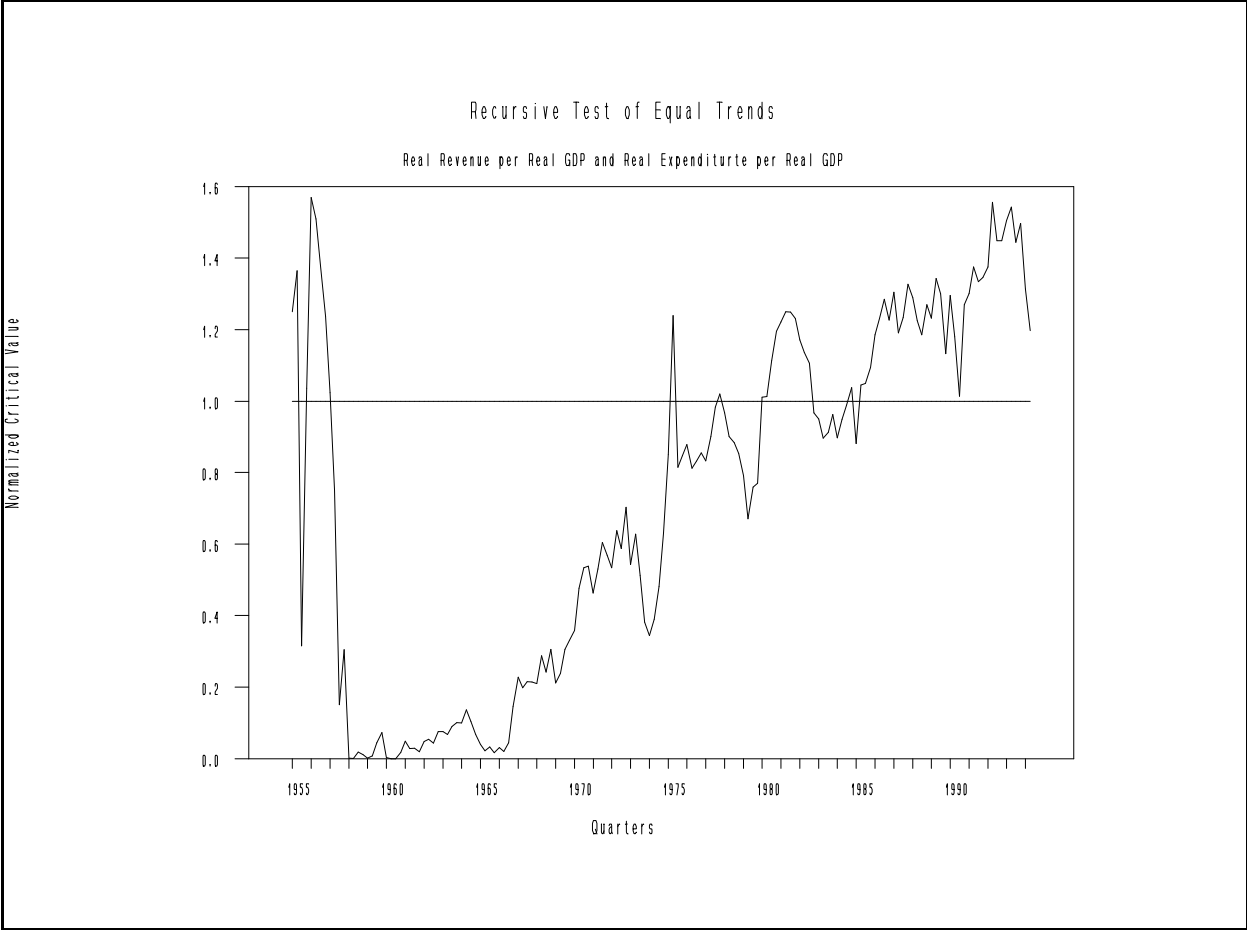


Figure 8



**Figure 9**

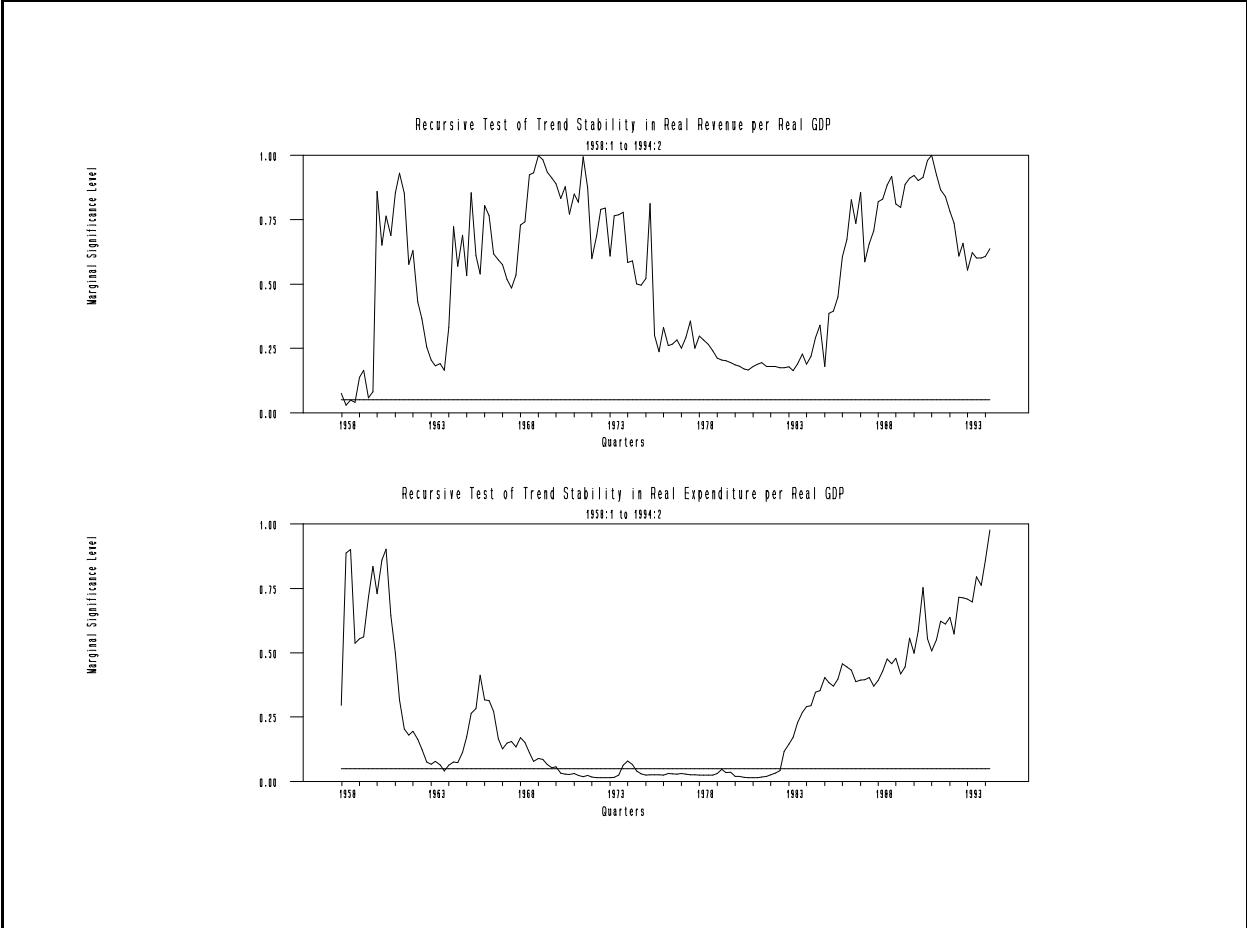


Figure 10