OUTPUT, EMPLOYMENT AND PRICES IN AN ECONOMY WITH ADJUSTMENT COSTS

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Abstract

This paper investigates the dynamics of output, employment and prices in an economy with costs of adjusting labor and prices. In an economy with non convex adjustments costs, firms do not adjust labor and prices continuously to accommodate every shift in demand. Rather, firms adjust employment and prices discontinuously. Employment and prices are adjusted only after demand has shifted beyond a predetermined thresholds. When adjustment is discontinuous, the dynamics of the aggregate economy is very different from the behavior of a single firm. The dynamics of the price level, aggregate employment and output depend in a crucial way on the firms' distribution along the inaction intervals. This paper develops a model with infrequent price and labor adjustments. The firms' multivariate distribution of prices and employment deviation from their optimal level is derived and is used to determine the dynamics of the price level, aggregate employment and aggregate output. It is shown that in such economy money is not neutral.

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1. Introduction

This paper addresses the fundamental macroeconomic problem how can nominal variables, such as the money supply, affect real variables such as output and employment. Fluctuations in the money supply can have real effects on the economy only if some nominal variables such as prices or wages fail to adjust immediately to their long run level. Many economists argue quite convincingly that since adjusting nominal variables does not involve any real costs, it is unclear why agents may fail to adjust nominal variables if such a policy affects their utility.

A major explanation for nominal rigidity - why agents fail to adjust -, is provided by Akerlof and Yellen (1985) and Mankiw (1985) who show that small deviations from full optimization ("near rational" in Akerlof and Yellen), or small costs of price adjustment ("menu costs" in Mankiw), can produce significant nominal rigidities in an economy where agents maximize a strictly differentiable objective function. According to Akerlof and Yellen, in such an economy the private losses to the agents from a small deviation from the optimal price are only second order while the effect on output, employment and welfare is of first order. Using Mankiw terminology, even a very small cost of adjusting prices, such as a menu cost, can result in large welfare gains (losses), i.e., the losses to a monopoly from a small deviation from its optimal price are very small, but the resulting change in its output is large. This argument seems finally to explain nominal rigidities. But the argument is flawed. The flaw in the argument, using Akerlof and Yellen terminology, is the implicit assumption that "doing nothing" means adjusting quantities but not prices. It is unclear why this definition is more reasonable than assuming that "doing nothing" means adjusting neither prices nor quantities or adjusting only prices. Similarly, in Mankiw's model it is assumed that there are small costs of changing prices but no costs of changing output or employment. It is probably more reasonable to assume that there are costs associated with hiring or firing workers, labor adjustment costs, as well as costs of adjusting prices. Furthermore, it seems likely that the costs of changing employment, which involve real terms, unlike the costs of changing price which involve only nominal terms, are larger than the costs of changing prices.

It is no longer clear that with labor adjustment costs as well as menu costs small costs of adjusting prices would result in nominal rigidities. It is easy to show that a firm would find it optimal to keep its price constant and adjust its employment (the solution in the Mankiw menu cost model) only if the costs of price adjustment are greater than the costs of labor adjustment.[1] In other words, nominal rigidity will [still] be the outcome only if the price adjustment costs are greater than the labor adjustment costs.

This paper rescues the menu cost argument by developing a dynamic stochastic model with costs of price adjustment, costs of labor adjustment and the standard U-shape short-run cost function. In the model presented, each firm is faced with costs of labor adjustment, costs of price adjustment and short run increasing marginal costs. The firms' policy is as follows: Hire (fire) workers whenever demand increases (decreases) by more than a predetermined threshold and adjust price whenever the price deviation from the optimal price is greater than a predetermined level.

In an economy where the adjustment of the microeconomic units is discontinuous, the behavior of the aggregates is very different from the behavior of the microeconomic units (see Caballero (1992)). The aggregate dynamics of output, employment and the price level is very different from the dynamics of the price, employment and output of the individual firm. The aggregate dynamics depend on the multivariate distribution of employment and prices along their inaction interval. Thus, in order to derive the aggregate response to a change in the money supply, I will derive the multivariate distribution of employment and price. I will then use the multivariate distribution to calculate the effect of a change in the money supply on employment, output, and prices.

In addition to rescuing the menu cost argument, the model explains other empirical findings and allows further insight into the dynamics of output, employment and prices. First, the model provides an explanation of the empirical observation that productivity is procyclical (see Bernanke (1986) and Hall (1986)). This observation is, of course, consistent with the short run supply curve. Firms can increase output without changing employment in several ways - e.g., employing overtime, reallocating workers between maintenance and production and contracting out some of the production during periods of high demand. As a result we observe procyclical productivity.

Second, the model gives insight about the asymmetric dynamics of recessions and recoveries: the speed of entering a recession is greater than the speed of entering a recovery (see Blanchard and Diamond (1989)). This difference can be explained by allowing asymmetric costs of hiring and firing workers[2], or by

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[1] This assumes, however, that the nominal shock is not too small in which case it will ration output.

[2] "Another line of explanation is based on asymmetric hiring and firing costs. Consider, for example, a firm that faces no cost of hiring but fixed (that is independent of the number of workers fired) costs of firing, and that faces

having an asymmetric short run cost curve.

Third, the dynamics of employment are different from the dynamics of output. During a recovery, employment keeps rising even after output begins to decline. The explanation offered by the model is as follows: following an increase in demand, the fraction of firms that are close to the threshold where they hire workers increases relative to the fraction of firms which are close to the lower threshold where they fire workers. If output stays constant at peak level, some firms experience higher demand and some firms experience lower demand. Still, employment will rise since more firms are close to the threshold where they hire new workers than to the threshold where they fire workers. As a result, the number of firms that hire workers will be greater than the number of firms that fire workers. Employment increases while output is constant.

This paper is organized as follows: Section 2 presents a simple dynamic model of monopolistic competition without any adjustments costs. In Section 3 real rigidities - labor adjustment costs - are added to the model. Section 4 discusses nominal rigidities with discontinuous price adjustments. Three cases are investigated. First, the case of constant prices, i.e., the menu costs are large relative to the labor adjustment costs so that prices are constant. This case is similar to that of Mankiw and Akerlof and Yellen with the addition of labor adjustments costs. Second, the case where prices are flexible is investigated, i.e., there are costs of labor adjustments but no costs of price adjustments. In this case money is neutral regardless of real rigidities. Third, I investigate the case where costs of adjusting prices are small and, as a result, prices are adjusted intermittently. The aggregate dynamics of output, employment and the price level when adjustment is not continuous depend on the fraction of firms that adjust their prices and employment respectively. To this end, the multivariate distribution of employment and price deviation is derived. The multivariate distribution is then used to show monetary non neutrality. Section 5 summarizes the results.

2. A simple dynamic model of monopolistic competition

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variations in demand. Such a firm will hire workers often, and more or less as it needs them (not quite myopically, as it will take into account the potential costs of firing them if the need arose), but will fire workers infrequently and in large batches. Such behavior appears to roughly fit our pattern of job creation and destruction. But we have learned to question whether such micro-asymmetries carry over to macroeconomic variables " (Blanchard and Diamond (1989) p. 115).

This section develops a dynamic model of monopolistic competition. The purpose of this section is to construct a simple economy where money is neutral in the long run or in the absence of adjustment costs. The model is then used to investigate the effects of nominal and real rigidities on the adjustments process of the microeconomic units and the dynamics of the aggregate price level, employment and output. The advantage in constructing a model where money is neutral in the long run or in the absence of adjustment costs is that it highlights the effects of adjustment costs on the adjustment process of the economy. Thus, following the development of the model I investigate the effects of real and nominal rigidities in the form of fixed costs of adjustments on the adjustment process of the microeconomic units and the aggregate economy.

2.1 The model

There are I goods in the economy, all of which are imperfect substitutes for one another. Each good i is produced by a single monopolistic competitor who chooses the good's price and the level of production given the demand for its product. The demand for each good, X_i, is the same except for a size parameter, D_i. Specifically, the demand for good i is given by[3]

$$X_i = \left(\frac{M}{\overline{P}}\right)\left(\frac{P_i}{\overline{P}}\right)^{\frac{1}{\mu-1}} D_i$$

Where X_i is the output of firm i, P_i is the price of the firm's output, $\overline{P}$ is the aggregate price level, which for simplicity is assumed to be the mean of the firms' prices, M is the nominal money supply and μ is the inverse of the markup and is assumed to be between zero and one so the firm's optimal price is well defined. The demand for each good is the same up to a specific size parameter, D_i. The size parameter of each firm, D_i, is assumed to follow a random walk with a unit step size each period:[4]

$$D_{i,t} = D_{i,t-1} + \theta_{i,t}$$

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[3] This demand function is equivalent to Mankiw's demand function if $\overline{P}$ =1.

[4] There are two related problems with this specification: first D can take negative values and second there is no steady state distribution of firms. One way to solve these problems is to restrict D to a given interval such that D is a random walk with reflecting barriers. If the interval is large enough the fraction of firms near the boundaries is negligible and we can approximate the process of each individual firm as a simple random walk. A similar solution is

where $\theta_{i,t}$ is a random variable which takes the values of 1 and -1, each with probability one half.

The production function of each firm is linear in the number of workers

$$X_i = L_i \ .$$

employed, L_i:
The number of workers is constant. The real wage, ω depends positively on the

$$\omega = g(L).$$

level of employment, L:
This specification is consistent with efficiency wage model (see, e.g., Shapiro and Stiglitz (1984) and Yellen (1984)), a fair wage model (see, e.g., Akerlof, (1990)) or a classical labor supply where workers trade off consumption and income.

Equations (1), (3) and (4) are sufficient to solve for the equilibrium in this economy. The equilibrium wage, price level, employment and output. Profit maximization implies that each firm charges a constant markup above marginal

$$P_i = \frac{\omega \overline{P}}{\mu} \ .$$

cost
Since the demand elasticity and the production function are the same for all firms, all firms charge the same price and therefor the price of each individual product is equivalent to the economy wide price level,

$$\frac{P_i}{\overline{P}} = 1 \ .$$

Thus, in equilibrium the real wage is constant and equal to μ. Consequently, the real wage determines the level of employment (from equation (4)) and real balances are determined by equations (3) and (1) respectively.

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one where firms merge when they reach the lower barrier and split when they reach the upper barrier. I do not pursue these issues since this paper focus on the short run dynamics.

To see why this is a unique equilibrium, consider the case where the real wage, ω, is greater than the equilibrium wage, μ. In such instance, each firm wishes to charge a price which is greater than the price level. Consequently, when all firms try to charge a price greater than the price level the price level increases and hence real balances and demand decrease. When demand is lower, employment and real wage decrease until the real wage reaches its equilibrium level of μ. Similarly, for example, an increase in productivity reduces marginal costs and prices. As a result real balances and demand increase until employment and the real wage reach their new level of equilibrium.

In this economy only real shocks such as changes in productivity or labor supply have any effect on real variables such as employment, output or real wage. Any change in aggregate demand such as an increase in the money supply affect nominal variables such as prices or nominal wages, but do not affect any real variables such as employment or output.

3. Labor Adjustment

The discussion above describes either the position of the economy in the long run, after all adjustments have been made, or an economy without real or nominal rigidities where adjustment to the long run equilibria is immediate. However, both economic theory and empirical findings suggest that the adjustment process of the economy when out of equilibrium is not immediate, i.e., there are real and nominal rigidities. Until recently, the standard assumption of real rigidities was of convex adjustment costs (e.g., Sargent (1978)). The advantage of assuming convex adjustment costs is that it enables us to solve the optimization path in a representative agent model. But, although models based on convex adjustment costs fit the aggregate series quite well, they don't fit very well the actual micro-level adjustment process on which they are based. Recent empirical evidence suggests that the adjustment process of the microeconomic units is not as smooth as the adjustment process of the aggregate series. Hamermesh (1989) compares models based on convex adjustment costs with models based on fixed adjustment costs by studying the dynamics of employment using plant level data. He finds that when plants are aggregated the standard convex adjustment costs model performs as well as a model based on fixed costs of adjustment. But at the plant level labor force adjustment proceeds in large jumps and hence a model based on fixed costs of adjustment outperforms a model based on convex adjustment model. Caballero, Engel and Haltiwanger (1994) study the employment flows of approximately 10,000 large U.S. manufacturing during 1972-1980. They find that employment adjustments are often either large or nil and conclude that this behavior suggests that adjustment costs are non convex.

3.1 Labor adjustment cost

Following the discussion above I introduce real rigidities to the model by assuming that there are non convex costs of adjusting labor, i.e., fixed costs of firing or hiring labor. In the presence of non convex adjustment costs, similar firms may respond very differently to the same shock. Some firms may do nothing, i.e., do not hire or fire workers, while other firms may hire or fire workers disproportionately to the shock. As a result, the dynamics of the aggregate economy is very different from the dynamics of the individual firms. The dynamics of the aggregate economy depends on the fraction of firms which hire and fire workers respectively. The decision whether to adjust or not depends on the deviation of employment from its optimal level. Hence, the dynamics of the aggregate economy is determined by the distribution of firms along the inaction interval (the interval where the deviation from the optimal level does not trigger any adjustment). In order to derive the aggregate dynamics I first specify the adjustment costs. Then, I solve for the adjustment policy of each firm given the labor adjustment cost. Finally, based on the firms' adjustment policies I derive the firms' ergodic distribution along the inaction interval. The firms' distribution combined with their adjustment policy determines the response of aggregate employment to any aggregate shock.

Assume the following costs of labor adjustment:

Assumption 1: *Each firm has to pay a fixed cost, γ, independent of the number of workers hired or fired, whenever it hires or fires workers.*

In the presence of fixed adjustment costs, firms do not adjust their employment continuously to meet any change in demand since such a policy would require paying the adjustment costs every period. Instead, firms choose to produce in ways which are less efficient but do not require continuous adjustments of employment. For example, firms may reallocate workers between maintenance and production, cleaning and maintaining during downturns and neglecting maintenance and cleaning during up turns (e.g., Fair and Medoff (1985)). Similarly firms may pay over time or contract out some production during periods of high demand and send workers on holidays, courses and so on during periods of low demand (e.g., Abraham and Taylor (1993)). Specifically, let the short run

$$C(X_i, L_i) = W X_i + \frac{b}{2} W(X_i L_i)^2$$

cost be the standard quadratic loss function:
Where W is the nominal wage and b is a parameter which represents the increase in the costs when employment is not at its optimal level. The cost function above implies a linear marginal cost function which increases (decreases) in the

$$MC(X_i, L_i) = W + bW(X_i L_i).$$

difference between output (optimal employment) and employment:

The situation is described in figure 1. In the long run, when the level of employment is optimal, marginal costs are constant and equal W. In the short run, employment is fixed at L_0. Hence, the short run marginal costs equal W when output equals L_0 , i.e., when employment is at its optimal level. Marginal costs are higher than W when production is greater than L_0 and are less than W when output is less than L_0. When employment is adjusted from L_0 to L_1, for example the firm hires new workers. The short run marginal cost curve shifts to the right where it intersects the long run marginal cost curve at the new level of employment, L_1 and W.

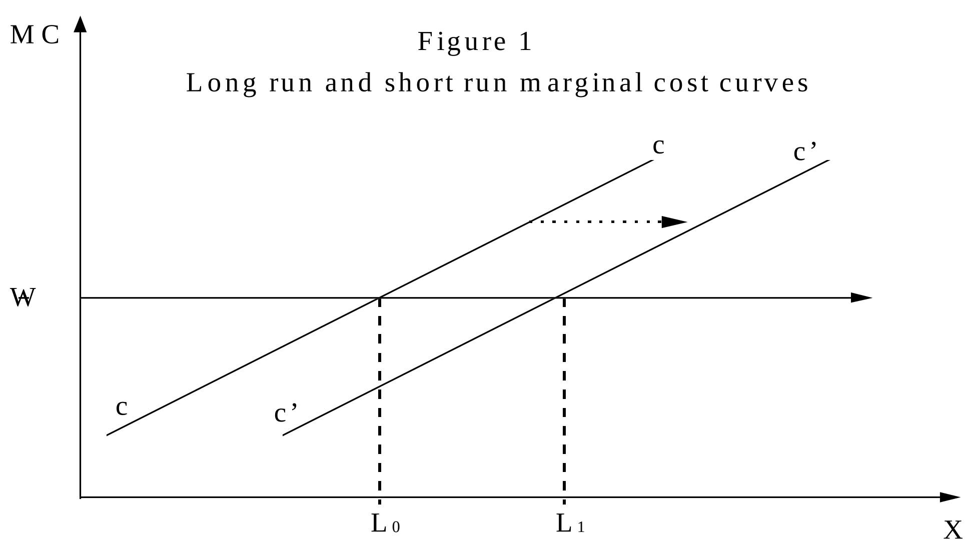

Figure 1
Long run and short run marginal cost curves

4. Price adjustment

While the debate concerning real rigidities centers around the nature of the costs, convex versus non convex, the debate concerning nominal rigidities centers around the existence of price rigidities. The nature and existence of nominal rigidities are a long and continuous source of debate among economists. One of the main reasons for the inconclusiveness of this debate is that the observed costs of nominal adjustments are only the small costs of changing prices such as printing new menus and informing agents. However, most of the costs associated with adjusting prices continuously are not the physical costs of printing new menus and informing agents, but rather as Akerlof and Yellen (1985) and Ball, Mankiw and Romer (1988) suggest, are mainly the costs of making decisions continuously rather than using a simple rule of thumb for adjusting prices.

I distinguish below between three types of price adjustment. First, I investigate the case where prices are flexible, where there are no costs associated with adjusting prices - there are real rigidities but no nominal rigidities. Second, I investigate the case where prices are constant; the costs of adjusting prices are so large that firms keep prices constant while they move along the short run marginal costs curve. This case is similar to Akerlof and Yellen (1985) and Mankiw (1985) with the addition of real rigidities to fixed prices. Finally I investigate the case where firms adjust their prices discontinuously while moving along the short run marginal cost curve, i.e., the costs of adjusting prices are such that prices are adjusted intermittently while firms move along the short run marginal costs curve.

4.1 Large menu cost

Assume that the costs of adjusting prices are large relative to the costs of adjusting labor such that firms find it optimal to adjust employment before they adjust prices. In other words, firms do not adjust their prices while moving along the short run marginal cost curve. After employment is adjusted, the price is optimal again and the situation repeats itself. The dynamics of aggregate employment depends on the adjustment policy of each firm and on the firms' distribution along the inaction interval, i.e., the fraction of firms which hire or fire workers each period.

Let u_i be the increase (decrease) in demand since the previous labor adjustment. That is, u_i is equal to K_{it} less its value the last time the firm hired (fired) workers. The firms' optimal policy can be expressed in terms of two thresholds F and H ($F<H$) in terms of u_i where workers are fired and hired

respectively. Firms neither hire nor fire workers until the increase (decrease) in their demand, u_i, reaches the adjustment thresholds. When u_i reaches H, the firms immediately hire new workers and instantaneously set u_i to zero. Likewise when u_i reaches F, the firms immediately fire workers and instantaneously set u_i to zero.

To derive the adjustment policy of each firm and consequently the firms' distribution, assume for simplicity that the discount rate is zero. The firm's decision is to construct a policy which minimizes its expected adjustment costs plus the expected opportunity costs, i.e., the expected losses from producing

$$E\left(\frac{cost}{Time}\right) = \gamma E\left(\frac{n}{Time}\right) + E\left(\frac{loss}{Time}\right) .$$

inefficiently, per unit of time
Where n is the number of labor adjustments that occur over some time interval T and γ is the fixed costs of adjustments.

Let λ be the expected duration, the expected time between adjustments. The first term in equation (8), the expected number of adjustments per unit of time approaches $1/\lambda$ as T becomes large (see Miller and Orr (1966)):

$$E\left(\frac{n}{Time}\right) = \left(\frac{1}{\lambda}\right).$$

The second term, the opportunity costs are the expectations of the second term in equation (6), the losses from being on the short run cost curve rather than on the

$$E\left(\frac{loss}{Time}\right) = E\left(\frac{b}{2} W(X_i L_i)^2\right).$$

long run cost curve:

$$X_i = \frac{M}{P} K_i$$

Since prices are constant, output is equal to:

$$L_i = \frac{M}{P}(K_i - u_i).$$

and employment is equal to the level of output at the previous adjustment: Therefor, the losses are a function of only the change in demand since the last

$$loss(u_i) = \frac{b}{2} W (\frac{M}{P} u_i)^2 .$$

adjustment, u_i:
The losses from producing inefficiently are symmetric in u_i, the change in demand since the last adjustment.

Given that the process governing u_i is Markovian, u_i can be treated as though it is governed by a repetitive process, returning to zero each time an adjustment in employment occurs. Consequently, the optimal adjustment can be described in terms of an upper threshold for u_i, (H), where workers are hired, a lower threshold for u_i, (F) where workers are fired and a returning value of 0 whenever workers are hired or fired. Moreover, since, whenever employment is adjusted, u_i returns to zero and the process repeats itself, the optimal thresholds are the same for each firm and remain optimal after future adjustment is made. (Hence, for convenience I will omit the subscript i).

The symmetry in the process governing u and the symmetry in the opportunity costs imply that the optimal solution of the lower and upper thresholds is symmetric: F=-H.[5] Thus, the duration, the expected time between adjustments is given by the duration of a random walk with a return point at zero and thresholds at $\pm$ H. Feller (1966) has shown that the duration of such a process

$$E(\frac{n}{Time}) = (\frac{1}{\lambda}) = \frac{1}{H^2} .$$

is equal to H^2 and hence:

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[5] This solution follows previous work by Barro (1972), Miller and Orr (1965), and Feller (1968).

The higher the thresholds, H, the larger the expected time between adjustments.

Next I turn to calculate the second element in equation (8), the expected

$$E(loss(u)) = \frac{b}{2} W (\frac{M}{P})^2 E(u^2).$$

opportunity costs per unit of time:

The opportunity costs depend only on $E(u^2)$. Consequently, the losses are determined by the probability distribution of u. In other words, in order to calculate the expected losses we need to derive the probability distribution of u. The probability distribution of u can be calculated from the difference equations and boundary conditions which characterize the probability distribution (using

$$f(u) = \frac{1}{2} [f(u+1) + f(u1)] if (H + 1 \leq u \leq H1 and u\_0)$$

$$f(H) = f(H) = 0$$

$$f(0) = \frac{1}{2} [f(1) + f(1) + f(H1) + f(H+1)]$$

$$\sum_{u=H}^{H} f(u) = 1$$

steady state occupancy probabilities):

The probability that u is equal to either of the thresholds, -H or H, is zero since adjustment occurs whenever u reaches any of the thresholds. For interior values, values between -H and H, the probability is equal to the probability that u is one unit less and demand takes a positive step plus the probability that u is one unit higher and demand takes a negative step. When u is equal zero, there is the additional probability that u is equal to H-1 and demand takes a positive step (which triggers adjustment to 0) plus the probability that u is equal to -H+1 and demand take a negative step (which again triggers adjustment to 0). The difference equation and the boundary conditions can be solved to yield the probability distribution function of u

$$f(u) = \frac{1}{H}(1\frac{|u|}{H})$$

$$E(u^2) = \frac{H^2}{6}.$$

and the expectations of u^2

The larger the thresholds, the greater the expected opportunity costs as the firm produces less efficiently for longer periods of time.

Substituting equation (14) and (18) in (8) yields the expected costs per unit

$$E[\frac{cost}{time}] = \frac{\gamma}{H^2} + b\omega(\frac{M}{P})^2 \frac{H^2}{6}.$$

time (in real terms):

The adjustment thresholds, H, which minimizes the expected cost per unit time

$$(H^*)^2 = \frac{1}{(\frac{M}{P})}\sqrt{\frac{\gamma}{bw}}.$$

are:

The thresholds depend positively on the labor adjustment costs, γ, and negatively on the slope of the short run supply curve, b. Higher adjustment costs decrease the frequency of adjustments and increase the size of adjustment as firms try to avoid the adjustment costs. An increase in the opportunity costs, higher b, decreases the thresholds and hence increases the frequency of adjustments as firms try to decrease the losses of producing inefficiently.

The effect of an increase in the money supply

When prices are constant as in the case above, it is obvious that an increase in the money supply would increase output. However, the adjustment process of output and employment would be different from each other, and this difference would conform to what is observed empirically. While output increases immediately to its new level, employment is sluggish in its adjustment. The dynamics of aggregate employment depends on the distribution of firms along [-H, H]. All the firms in the economy are identical in technology and demand function and all follow the same adjustment policy. Moreover, the only source of uncertainty in the economy is idiosyncratic. Hence, the probability associated with each position in [-H, H] coincide with the actual fractions of firms located in the same state (see Bertola and Caballero (1990)). In other words, the steady state distribution of firms along the inaction interval [-H, H] is given by the probability

$$f(u) = \frac{1}{H}(1\frac{|u|}{H}) \ .$$

distribution of each firm:
This distribution has its peak at zero and declines linearly to zero at -H and H. Therefore, following an increase in demand, only a small fraction of the firms - those near the upper adjustment threshold - hire new workers. All the other firms increase their output but do not adjust their labor. In other words, the increase in demand is followed by a small increase in employment but shifts the distribution of firms toward the upper threshold. The fraction of firms next to the upper threshold is now greater than the fraction of firms next to the lower threshold. As a result, employment increases over time, even after output stops increasing.

4.2 Flexible prices

Consider now the case where there are no costs associated with changing prices. Assume that each firm's employment policy is similar to the one derived above. Each firm hires workers whenever u, the change in demand since the last labor adjustment, reaches an upper threshold, H, and fire workers whenever u reaches a lower threshold, F. When u reaches H (F), the firm instantaneously hires (fires) workers and sets u to zero so employment is at its optimal level. However, in contrast to the discussion above where prices are held constant, assume now that prices are adjusted continuously while firms move along the short run marginal cost curve.

Money in this case is neutral regardless of the real rigidities. An increase in the money supply will result in a proportional increase in the price level and hence does not affect any real variables. This can be proved by contradiction. Suppose that money is not neutral. Suppose that the response in the price level (and hence the nominal wage) to an increase in the money supply is only a fraction $\alpha<1$ of the increase in the money supply (e.g., the money supply increases by 10% and the price level increases by 5%). In this case real balances and consequently demand are higher. To determined whether this is a plausible solution consider the increase in prices. First notice that since the nominal wage is proportional to the price level, the increase in the nominal wages is equal to α as well. As a result, all firms increase their prices by α. In addition, prices change further as firms move along their respective short run marginal cost curves. On the one hand, some firms reach the upper threshold, hire new workers and hence decrease their prices. On the other hand, all those firms which do not reach the threshold move along the short run marginal costs curve and therefore increase their prices. The overall increase in the price level is α only if the increase in prices by the adjusting firms exactly offset the increase in prices by all those firms which do not adjust.

Whether this is the case or not depends on the fraction of firms that adjust. In other words, it depends on the distribution of firms. Given the steady state distribution (equation (17)), the increase in prices by those firms that do not adjust is much larger than the decrease in prices by those that adjust. Therefore, the increase in the price level cannot be α. The only possible solution is one where the increase in the price level is equal to the increase in the money supply. All firms increase their prices by the increase in the money supply, no firm adjusts its employment and decreases its price or moves along the marginal cost curve and increases its price.

4.3 Small menu costs

In many cases, prices are neither adjusted continuously nor kept constant while firms move along the short run marginal costs curve. Rather, prices are adjusted intermittently while firms move along the short run marginal cost curve. The frequency of the price adjustments depends of course on the costs of price adjustments. However, most of the costs of adjusting prices are not some specific costs such as printing new menus or notifying agents, but are mainly the inconvenience of continuous revision compare to a simple rule of thumb. Thus, I assume, that firms adopt the following state-dependent policy[6].

Assumption 2: *The price is adjusted to its optimal frictionless level*

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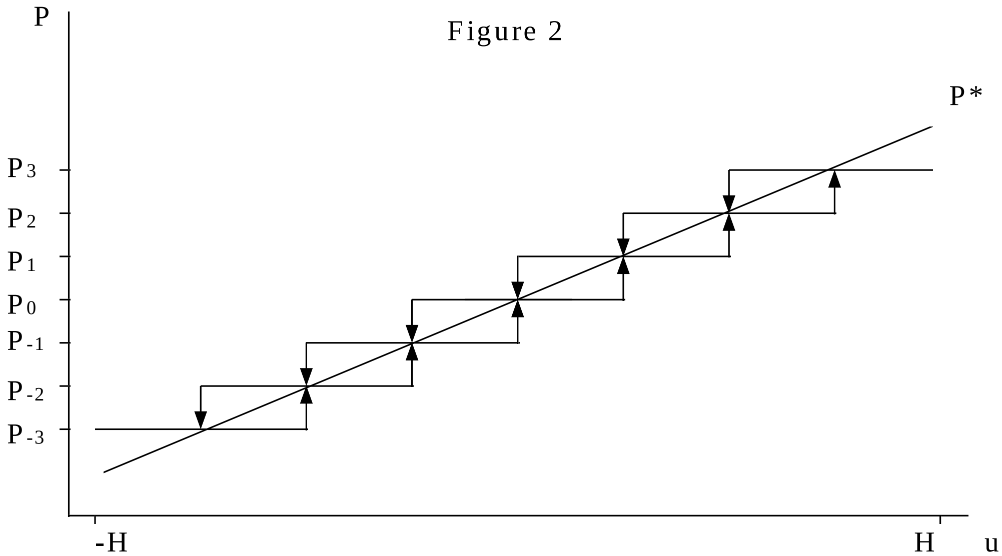
[6] As Caplin and Leahy (1991) suggest such a rule can be thought of as a state-dependent alternative to time-dependent rules such as Taylor (1982) or Fischer (1979).

whenever the deviation between the price and the optimal frictionless price is equal to g (-g) percent. Employment is adjusted to its optimal level whenever the deviation between employment and the frictionless level of employment is equal to H (-H).

This policy can be described by 2m+1 discrete prices, P_{-m}, P_{m+1} .. P_0,..P_{m-1}, P_m. The price is adjusted from p_j to p_{j+1} (p_{j-1}) only when the optimal frictionless price is equal to p_{j+1} (p_{j-1}). As long as the optimal frictionless price is between P_{j-1} and P_{j+1} the firm accommodates changes in demand by increasing its output, but keeps its employment and price unchanged (assuming of course that the price is P_j and not P_{j+1} or P_{j-1}). When the optimal price reaches P_{j+1} (P_{j-1}) the firm instantaneously increases its price to the optimal price but does not adjust its employment. Employment is adjusted only when the increase (decrease) in demand, u, reaches a predetermined thresholds, -H and H. When u reaches the upper (lower) threshold, H (-H), the firm instantaneously hires (fires) new workers such that its employment is optimal again, sets u equal to zero and adjusts its price

Figure 2



accordingly to P_0.

Figure 2 describes the policy. Suppose that employment is at its optimal level. Next consider the case where demand increases. At first, the firm moves along its short run supply curve without adjusting price or employment. When the increase in demand is such that the optimal price is P_1, the firm adjusts its price from P_0 to P_1. Further changes in demand cause the firm to move along its short run marginal cost curve but do not trigger any adjustment in price or employment. The price is adjusted downward to P_0 or upward to P_2 only when the optimal price is P_0 or P_2, respectively. When u, the increase in demand reaches the employment adjustment threshold, -H and H, employment is adjusted to its optimal level immediately, u is set to zero and the price is set back to P_0.

Let p* and p be the natural logarithms of the optimal price and the firm's actual price respectively and define Z as the deviation of the price, p, from its

$$Z = p^{*} - p$$

optimal frictionless level, p*,

The firm policy can be described in terms of Z and u. Price adjustment occurs only when the variable Z hits the trigger points -g or g., i.e., the variable Z is allowed to drift until it reaches g or -g, when Z reaches g (-g) the firm immediately increases (decreases) its price by g to its optimal level, and move Z instantaneously to zero. Employment adjustment occurs when the variable u, the increase (decrease) in demand since the last adjustment hits the trigger points H or -H., i.e., the variable u is allowed to drift until it reaches H or -H, when u reaches H (-H) the firm immediately adjusts its employment to its frictionless level and moves u instantaneously to zero.

The effect of an increase in the money supply

The discussion above describes the adjustment policy of each individual firm in terms of its employment and prices. However, the aggregate dynamics of employment, output and the price level is of course very different from the dynamics of any individual firm. The aggregate dynamics and hence the effect of an increase in the money supply on employment and output is determined by the multivariate distribution of employment and prices along their inaction interval ((-H, H) and (-g, g) respectively), i.e., the firms' distribution along the price and employment deviation. The firms' distribution along the employment inaction interval was already calculated in equation (21) above. The firms' distribution along the price deviation is derived next.

I first approximate the natural logarithm of the optimal price, p*, as a linear function of u. Then I will use this approximation to derive the stationary probability distribution of Z, the deviation of the price from its optimal frictionless level. The optimal frictionless price, P^*, is given by the price level

$$P^*(t) = argmax\ PROFITS\ (P\_\ L)\ .$$

which maximizes profit given a constant level of employment, L. That is

$$PROFITS\ (P\_\ L) = XP - XW - \frac{b}{2} W (X - L)^2$$

$$where$$

$$X = \frac{M}{\bar{P}} \left(\frac{P}{\bar{P}}\right)^{\frac{1}{1-\mu}} (D + u)$$

$$L = \frac{M}{\bar{P}} D$$

and

The optimal frictionless price is given by the first order condition of the profit

$$\mu - \frac{\omega}{P} - \frac{b\omega m}{P} \left((D + u)\left(\frac{P}{\bar{P}}\right)^{\frac{1}{\mu-1}} - D\right) = 0\ .$$

maximization:

$$\left(\frac{P}{\bar{P}}\right)^{\frac{1}{\mu-1}} \approx 1 + \frac{1}{\mu - 1}\left(\frac{P}{\bar{P}} - 1\right)$$

Using a first order approximation around the equilibrium value:

yields the optimal price as a function of u, the change in demand since the last labor adjustment:

$$P_i^* \approx \overline{P}\left(1+\frac{bmu}{1+\frac{bm(D+u)}{1-\mu}}\right)$$

Taking natural logarithms of both sides and assuming that D is large I obtain that the optimal price is a linear function of u, the increase in demand since the last

$$p^* \approx \overline{p}+\frac{(1-\mu)u}{D}=\overline{p}+Au.$$

labor adjustment:

Here A is equal to (1-μ)/D and $\overline{p}$ is the natural logarithms of the price level. A, represents the step size of the optimal price[7]. The optimal price, p*, ranges over the discrete set (p_0-AH+A . . . p_0+AH-A). That is, p* can take 2H/A-1 evenly spaced values (p^*_{-H+1}, . . . p^*_{H-1}). On the other hand, the step size of the firm's actual price is g (g>A). The actual price ranges over a subset, (p-gm ... p+gm), of the optimal price. That is, p takes 2m-1 evenly spaced value (p_{-m}, p_{-m+1} . . . p_{m-1}, p_m) from the set of the optimal price.

Next, I derive the probability distribution of the price deviation, z, in four steps: First, I express the probability of a given price deviation, z, as a sum over all possible prices, ($p_{-m} \ldots p_m$). Second, I express this probability as a conditional probability. Third, I calculate the conditional probability from the dynamic process governing it and finally I use the solution of the conditional probability and symmetry argument to calculate the probability distribution of z.

The deviation between the actual price and the optimal price is equal to z whenever the price is equal to p_j, $j\in(-m..m)$, and the optimal price is equal to p_j+z. Therefore, the probability of a price deviation z is equal to the summation over all possible prices (p_j) of the probability that the price is p_j and the optimal price is p_j+z:

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[7] For simplicity I ignore integer constraints and assume that all firms are of the same size.

$$PROB(p^* - p = z) = \sum_{j=-m}^{j=m} PROB(p^* - p_j = z;\ p = p_j).$$

Each term in the summation - the probability that the actual price is p_j and that the optimal price is p_j+z - can be expressed using a conditional probability. That is, the probability that the price is equal to p_j given that the optimal price is

$$\sum_{j=-m}^{j=m} PROB(p = p_j | p^* = p_j + z)\ PROB(p^* = p_j + z).$$

p_j+z, multiplied by the probability that the optimal price is p_j+z:

Next, I turn to calculate each of the two terms inside the summation in (31). The first term, the conditional probability, PROB(p=p_j / p*=p_j+z), is governed by the following difference equation and boundary conditions (using

$$PROB(p = p_j | p^* = p_j + z) = \begin{cases} \frac{1}{2} PROB(p = p_j | p^* = p_j + z - A) + \frac{1}{2} PROB(p = p_j | p^* = p_j + z + A) \\ 0 \ \ if\ \_z\_ \geq g \\ 1 \ \ if\ z = 0 \end{cases}$$

steady state occupancy probabilities):

In words, if the deviation between a given price, p_j, and the optimal price is greater than or equal to g the probability that the price is p_j is equal to zero since adjustment occurs whenever the deviation reaches g. If the deviation between a given price, p_j, and the optimal price is equal to 0 (that is, the optimal price is equal to p_j) then the price is p_j with probability one. To see it, notice that when the optimal price is equal to p_j, the deviation between the optimal price and any other price except p_j is at least g. Since such deviation triggers an adjustment, the only possible price is p_j. Finally, when the deviation is less than g, i.e., -g<z<g (the optimal price is between p_{j+1} and p_{j-1}) the probability is governed by a linear

difference equation. It is equal to the probability that the price is equal to p_j given that the optimal price is p_j+z+A and demand took a negative step plus the probability that the price is p_j given that the optimal price is p_j+z-A and demand took a positive step.

The linear difference equation and the boundary conditions above can be solved to yield the conditional probability of observing a given price given the

$$PROB(p = p_j | p^* = p_j + z) = 1 - \frac{|z|}{g} \;\; if \;\; -g < z < g$$
$$= 0 \qquad otherwise.$$

deviation from the optimal price:

Equation (33) is the solution of the first term of equation (31). It can be used to derive the second term in (31). First, notice that the conditional probability (33) is independent of the price, p_j. It depends only on z, the deviation from the optimal price. As a result, we can pull the conditional probability in (33) out of

$$PROB(p = p_j | p^* = p_j + z) \sum_{j=-m}^{j=m} PROB(p^* = p_j + z) = (1 - \frac{|z|}{g}) \sum_j PROB(p^* = p_j + z).$$

the summation:

Second, notice the summation of (34) over all possible values of z, is equal to one (it is a summation over all possible deviations and all possible prices). Third, the last term in equation (34), $\sum PROB(p^* = p_j + z)$, is independent of z. To see this, consider two distinct values, z_1 and z_2, $z_1>z_2>0$. The probability that the optimal price is z_1 above any of the 2m+1 possible firm's prices is equal to the sum of the probability of 2m+1 equally spaced values in the u dimension. The probability that the optimal price is z_2 above any of the 2m+1 firm's prices is equal to the sum of the probability of 2m+1 equally spaced values in the u dimension to the right of the previous values. However, the distribution of u is linear and symmetric around 0. Consequently, this shift does not affect the summation. The increase in the probability (when comparing z_2 to z_1) for values below zero (prices below p_0) is exactly offset by the decrease in the probability for values above zero (prices above p_0). Thus, we can write the summation of (34) over z as two separate

$$\left(\sum_{z=-g}^{z=g}\left(1-\frac{|z|}{g}\right)\right)\left(\sum_{j} PROB(q^* = p_j + z)\right) = 1.$$

summations:
The first term is equal to g/A since z takes the following values (-g, -g+A . . . g-

$$\sum_{j} PROB(p^* - p_j = z) = \frac{A}{g}.$$

A, g) and hence, the second term is:
Finally, substitute equation (36) in (34) to obtain the probability distribution of a

$$PROB(p^* - p = z) = \left(1 - \frac{|z|}{g}\right)\frac{A}{g}.$$

price deviation z:

To illustrate the price and employment distributions above I simulated the model with three price tiers and labor adjustment thresholds at -40 and 40. Graph 1 describes the firms' distribution along the employment deviation interval (-40, 40). This graph corresponds to the analytic solution presented in equation (17). Graph 2 shows the probability distribution of the price deviation which corresponds to the analytic solution presented in equation (36). Graph 3 presents the multivariate distribution of employment and prices.

Given the stationary probability distribution above it is clear that money is not neutral. When the economy is at the stationary distribution, the change in the price level following a change in the money supply is determined by the fraction of firms which adjust their price. When the economy is at the stationary distribution only a small fraction of the firms would adjust their prices following a change in the money supply and hence the change in the price level is smaller than the change in the money supply. As a result, output increases and a fraction of the firms hire new workers.

5. Conclusions

This paper has two main contributions. First it develops a theoretical framework to analyze an economy of heterogeneous agents who adjust discontinuously along two dimensions, prices and employment. To this end the multivariate distribution of employment and prices which determines the dynamics of the aggregate economy is derived. Second, this multivariate distribution is used to show that money is not neutral in an economy with labor adjustment costs as well as costs of price adjustment. It thus rescues the menu cost argument which is flawed in ignoring the costs associated with labor adjustment.

The theoretical analysis developed in this paper can be extended to include other variables with discontinuous adjustments at the microeconomic level (e.g., investment). The need for such a model is made clear by the accumulating empirical evidence in support of discontinuous adjustment at the microeconomic level and continuous adjustment at the aggregate level

This model is consistent with the microeconomic foundations and the aggregate macroeconomic dynamics. The model can explain the procyclical productivity, the asymmetric dynamics of recessions and recoveries and the difference dynamics of employment and output. It also provides insight into the complex dynamics of prices employment and output.

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