Cyclical Implications of the Variable Utilization of Physical and Human Capital

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ABSTRACT

We develop a business cycle model in which consumption goods, physical capital, and human capital are produced in separate sectors. An important feature of the model is that human and machine inputs in the production process are treated symmetrically: each has both a stock and flow component. The model's representative agent is permitted to use the stocks (physical and human capital) at less than capacity by varying the utilization rate of capital and the hours of labor devoted to production. Utilizing physical capital at less than capacity slows depreciation; utilizing human capital at less than capacity frees time that may be devoted to study, resulting in more rapid human capital accumulation. We find that the model nicely characterizes cyclical properties of U.S. data on output, investment, consumption and employment, particularly at business-cycle frequencies.

Keywords: business cycles, spectral analysis

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It is well understood that characterizations of human capital accumulation are important for understanding economic growth: Lucas (1993) has illustrated this point forcefully, and it is clearly reflected in the burgeoning literature on economic growth. Here, we investigate whether human capital accumulation is useful for understanding business cycle activity. To do so, we fit a neoclassical business cycle model with endogenous human capital accumulation to U.S. data on output, investment, consumption and employment, and find the performance of the model to be impressive. The measure of fit we use is Watson's (1993): relative to standard real business cycle models, spectra calculated from the data and our model exhibit close correspondence, particularly at business cycle frequencies. Hence just as models featuring endogenously determined human capital accumulation have greatly enhanced our understanding of economic growth, we believe they can also enhance our understanding of business cycles.

The reasoning behind our effort to incorporate human capital accumulation in a business cycle framework is that it enables us to treat labor and physical capital symmetrically in the specification of the production process. This symmetric treatment is not a standard feature of business cycle models, which instead typically treat labor exclusively as a flow variable, and capital as a stock variable. In contrast, endogenous-growth models frequently incorporate a stock as well as a flow component in modeling the labor input in the production of output. The flow component is given by the quantity of hours supplied, while the stock component indexes the evolving level of labor productivity, or human capital; the combination of these components represents labor's overall contribution to the production process. We adopt such a specification for labor, and couple it with an analogous specification for physical capital: following Greenwood, Hercowitz, and Huffman (1988), we assume that the intensity with which the available stock of physical capital is utilized can be varied over time. Thus like labor, physical capital is modeled as providing a flow of inputs in the production process.[1] This completes the symmetry: the physical capital input is given by the product of the stock of capital and a variable utilization rate; the labor input is given by the product of the stock of human capital and its variable utilization rate--hours worked. We also model the evolution of the stocks of human and physical capital symmetrically: investment in these stocks, net of depreciation, determines their future levels.

We incorporate this symmetric treatment of capital and labor in an otherwise standard multi-sector business cycle model. Specifically, following Hansen and Prescott (1993), the baseline model we study features a representative agent who divides her time spent in employment between the production

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[1] Other studies which incorporate variable capital utilization specifications include Finn (1991), Greenwood, Hercowitz and Krusell (1992), Bils and Cho (1994), Burnside and Eichenbaum (1994), and DeJong, Ingram, and Whiteman (1995).

of consumption goods and physical capital (investment goods). The remainder of the agent's time is divided between leisure and study; time spent studying in period t is assumed to increase the stock of time-t human capital, which augments labor productivity in period t+1. Finally, uncertainty is introduced in the baseline model through the inclusion of a total factor productivity shock (loosely, a technology shock), as well as idiosyncratic shocks which affect the marginal productivity of resources devoted to the production of physical and human capital.

The point of the paper is to explore the internal propagation mechanism of the baseline model, as well as the model's ability to fit business-cycle characteristics of U.S. data. Following the work of Cogley and Nason (1995), Rotemberg and Woodford (1996), and Wen (1996a), it is well known that simple real business cycle models do not match many of these characteristics. Here, in exploring fit we address Watson's question "How much random error would have to be added to the data generated by the model so that the autocovariances implied by the model + error match the autocovariances of the observed data?" [p. 1012] The answer we obtain is "relatively little". In particular, Watson found that the well-known model of King, Plosser, and Rebelo (1988a,b) generates flat spectra for labor hours, and output, consumption, and investment growth at business cycle frequencies, failing to fit the data along this important dimension. In contrast, our baseline model generates hump-shaped spectra at business cycle frequencies for labor hours, output growth, and investment growth. The peaks of the spectra for these series are not as sharp as their empirical counterparts, but they occur at roughly the same frequency. (The model also does relatively well in matching standard deviation and correlation patterns observed in the data for these series, including the relative variability of hours and output.) The model's characterization of consumption growth, however, is problematic: consumption is too smooth, is too loosely correlated with output, and has a spectrum which does not peak at business-cycle frequencies.

In exploring the role that human-capital accumulation plays in enabling our baseline model to generate cyclical activity, we repeat the exercise described above by studying three alterations of the model. First, we strip the model of two of its most important features: endogenous physical-capital utilization and human capital accumulation. This leaves us with a simple model in which consumption and investment goods are produced in separate sectors using labor and physical capital; labor moves freely across sectors in response to shocks, while capital may be reallocated only with a one-period lag. Next, we restore the endogenous physical-capital utilization specification employed in the baseline model, while leaving the level of human capital fixed exogenously. Finally, we return to the stripped-down model, but restore the endogenous human capital accumulation specification, while leaving the utilization of physical capital fixed exogenously.

This exploration yields three principal findings. First, relative to single-sector models, the addition of multiple sectors is an important step in generating hump-shaped spectra: the spectrum of investment generated by the stripped-down model has a distinct hump shape, and the spectra of hours and output also have humps, albeit barely discernible ones. Second, the endogenization of both physical-capital utilization and human capital accumulation is useful for sharpening the humps in the spectra of output, hours and investment, with the endogenization of human capital accumulation being relatively most important. (The addition of endogenous human capital accumulation also increases the variability of hours relative to output, which is too low in the stripped-down model as well as in the model with variable physical-capital utilization.) Third, as we move from the relatively sparse characterization of production in the stripped-down model to the rich specification of the baseline model, the characterization of consumption deteriorates rather than improves: most notably, consumption becomes too loosely correlated with output relative to the high correlation observed in the data. It seems that an alteration of our specification of consumer preferences, which is fairly standard, is needed to address this shortcoming.

Perli and Sakellaris (1995) have undertaken a study closely related to this one: like us, they incorporate human-capital-accumulation activities in a two-sector business cycle model in order to explore whether an otherwise standard real-business-cycle model is capable of matching cyclical characteristics of U.S. data.[2] Their model consists of a goods sector and a human-capital sector. In the goods sector, physical capital and consumption goods are produced using physical capital and human-capital-augmented labor as inputs. In the human-capital sector, human capital is produced using a production technology which has an identical functional form to that employed in the goods sector, but which has a distinct parameterization. Relative to the performance of the King-Plosser-Rebelo model, they find that their model generates spectra which provide a better fit to spectra calculated for output, consumption and hours data; however, their characterization of investment is inferior to KPR's, and the spectra generated by their model for both hours and investment do not have peaks at business-cycle frequencies.

I. A Real Business Cycle Model with Human Capital and Variable Utilization

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[2] Burnside and Eichenbaum (1994) pursue a similar objective by emphasizing the importance of factor hoarding in the production process; Wen (1996b) does the same by incorporating habit formation in an otherwise standard one-sector model.

Production of consumption goods, physical capital and human capital takes place in three separate sectors. Physical capital and labor combine to produce consumption goods according to the following Cobb-Douglas technology:

$$C_t \le A_t (K_{Ct-1}\phi_{Ct})^{\alpha} (H_{t-1}N_{Ct})^{1-\alpha}, \tag{1}$$

where A_t represents an exogenous technology shock, K_{Ct-1} is physical capital available for production in the consumption goods sector in period t (constructed in period t-1), ϕ_{Ct} is the rate of utilization of this capital stock, H_{t-1} is the level of human capital, and N_{Ct} is the number of labor hours employed. The parameter α represents the share of the capital input in production.

Investment goods are produced similarly (note the equality of the capital-share parameters):

$$I_t \le A_t A_{It} (K_{It-1}\phi_{It})^{\alpha} (H_{t-1}N_{It})^{1-\alpha}, \tag{2}$$

where A_{It} represents an exogenous shock to the marginal efficiency of investment. An increase in A_{It} raises the productivity of factors of production in the investment sector relative to the consumption sector. We will refer to A_{It} as the “investment shock”.

New investment augments the physical capital stock as follows:

$$K_{It} + K_{Ct} = K_{It-1} + K_{Ct-1} - \delta(\phi_{It})K_{It-1} - \delta(\phi_{Ct})K_{Ct-1} + I_t . \tag{3}$$

There are several special features of this law of motion. First, a unit of output from the investment sector may be used to increase capital in the consumption sector *or* in the investment sector. Second, implicit in the specification (1)-(3) is the idea that capital is free to move between sectors *at the end of the period, but not during the period.*

The most important aspect of our adoption of variable utilization of physical capital is that the rate of depreciation depends on the rate of utilization: the faster the rate of utilization, the higher the rate of depreciation. Specifically, following Greenwood, Hercowitz, and Huffman (1988), we model depreciation by

$$\delta(\phi) = \frac{\delta_0}{\omega}\phi^{\omega}, \tag{4}$$

with $\omega > 1$ and $\delta_0 > 0$; note that $\delta'(\phi) > 0$ and $\delta''(\phi) > 0$, so that the marginal cost of utilization of the capital stock is increasing in the rate of utilization.

The short-run inflexibility of capital across sectors implies that production aggregation fails. Because capital cannot be moved across sectors, point-in-time marginal products may differ. As a consequence, it is not possible to write an aggregate production function in terms of aggregate capital and labor inputs; hence we work with the sectoral production functions (1) and (2). Also, the presence of the investment shock A_{It} causes the relative price of investment and consumption goods to vary over time; as a result, in computing (consumption-good denominated) output in the model, we multiply investment by the price of investment goods relative to consumption goods, and add the result to consumption.

Investment in human capital (I_{Ht}) is produced by studying, according to the technology

$$I_{Ht} = A_{Ht} S_t^{\theta}, \tag{5}$$

where S_t denotes time devoted to study, A_{Ht} is an exogenous shock which alters the marginal efficiency of study time, and $0 < \theta \leq 1$. In parallel with our specification of physical capital, human capital accumulates according to:

$$H_t = (1-\delta_h)H_{t-1} + I_{Ht}, \tag{6}$$

where δ_h is the fixed rate of human-capital depreciation.

The depreciation rate of human capital, unlike that of physical capital, is constant. But just as the marginal productivity of study time in producing human capital is variable, depending on A_{Ht} and the level of S_t, the marginal productivities of inputs to the production of physical investment goods vary according to the investment productivity shock A_{It}. In addition to this similarity in the technologies for *accumulating* the two types of capital, there is an additional sort of symmetry in their *use*. In particular, greater utilization of the stock of physical capital results in more rapid depreciation, while greater utilization of the stock of human capital through devotion of time to production reduces time available for study (other things equal) and results in less rapid accumulation.

Time may be allocated to leisure as well as production and study. The time endowment is normalized to unity, so the agent faces the constraint

$$N_{Ct} + N_{It} + S_t + \ell_t \leq 1. \tag{7}$$

Aggregate labor hours are obtained simply by adding N_{Ct} and N_{It}.

The representative agent has lifetime preferences over leisure and consumption given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t). \tag{8}$$

The parameter $\beta \in (0,1)$ represents the rate at which the agent discounts the future, and the period utility function is assumed to take the form

$$u(c_t, \ell_t) = \ln(c_t) + \ln(\ell_t). \tag{9}$$

We complete the specification of the model with the law of motion for the three exogenous shocks:

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + \gamma_A t + \varepsilon_{At}, \\ \ln A_{It} &= \rho_I \ln A_{It-1} + \gamma_I t + \varepsilon_{It}, \\ \ln A_{Ht} &= \rho_H \ln A_{Ht-1} + \gamma_H t + \varepsilon_{Ht}, \end{aligned} \tag{10}$$

The ε's are assumed to be independent and normally distributed with covariance matrix Σ, which we constrain to be diagonal.

As stated, the model consists of eight control variables (consumption, investment, hours worked and capital utilization rates in the two sectors, study time, and leisure time), and six state variables (the stocks of human capital and physical capital in the two sectors, and the three exogenous shocks). All variables in the model were normalized by their respective growth rates, and the stationary version of the model was solved using the QZ method described in Sims (1995). Let x_t denote a vector which contains logs of the ratios of the variables of the model to their steady state values, and ε_t a vector which contains the three innovations in (10). The linearized model may be written as

$$x_t = \Lambda x_{t-1} + M\varepsilon_t , \tag{11}$$

where Λ and M are functions of the parameters of the model.

II. Cyclical Properties of the Model

Several key features of the model determine its cyclical properties. One such feature is that the agent is not able to respond to a current-period shock by reallocating capital across sectors. However, the constraint is partially mitigated by the agent's ability to vary the intensity with which the fixed stocks of capital are utilized.

Another key feature of the model is the introduction of an additional margin along which the agent can respond to shocks. In standard real business cycle models, the agent smoothes consumption after the realization of a positive technology shock by accumulating physical capital; in this model the agent can accumulate human as well as physical capital. As with physical capital, the agent faces a one-period-lag constraint in adjusting along this margin: human capital accumulated in period t does not become effective in the consumption and investment sectors until period t+1.[3]

An implication of the additional flexibility provided by the introduction of human capital is that it contributes to the cyclical activity generated by the model by intensifying the model's propagation mechanism. To see this, and to further understand the implications of the additional key features of the model, it is useful to consider in detail how the representative agent responds to the three shocks incorporated in the model. This is illustrated in Figures 1 through 3, which depict the responses of nine of the variables in the model to one-standard-deviation shocks of each type, measured in own-standard-

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[3] It is possible to sharpen the lag constraint faced by the agent in adjusting along this margin by requiring the impact of study time on human capital to be realized with a longer lag. Indeed, the adoption of such a requirement would enhance the persistence of the model by spreading the impact of a given shock over time. We view the model's ability to generate hump-shaped spectra in the absence of a longer lag requirement as impressive.

deviation units. The responses are functions of the values chosen for the model’s parameters; in the figures, we use the following specification: μ = (α=0.33, β = 0.988, ω=1.5, δ_0=0.0736, δ_H=0.03, θ=1, ν_l=1 ρ_j=0.9, j=A, I, H, Σ_{ii}=0.001, 0.0015, 0.0017, i= A, I, H, γ_A=1.004, γ_I=γ_H=1,). The rationale behind this parameterization is provided in Section III.

Consider first the responses to a one-standard-deviation technology shock, illustrated in Figure 1. As in standard business cycle models in which the level of human capital is fixed, the agent smoothes consumption when confronted with this shock by investing relatively heavily in physical capital, thus sacrificing current for future consumption. This is seen by examining the responses of the capital stock, capital utilization and hours worked in the investment sector: each response is positive initially, and remains positive for approximately 7-15 periods. (Note that before returning to their steady state values, these variables “overshoot”: their responses become negative before returning to zero.) The additional hours devoted to work in the investment sector come from a reduction of hours devoted to work in the consumption sector, and to study time (although note that the response of study time is minimal in the first period, a point to which we return below); and of course, the source of the additional capital devoted to the investment sector is capital diverted from the consumption sector.

The fact that capital is substituted across sectors with a lag, and that productivity gains from study time are realized with a lag, accounts for the sharp peaks exhibited by many of the response functions in Figure 1. Consider, for example, labor hours in the investment sector. The peak response of this variable is approximately 0.25 standard deviation units, which is realized in the second period; the response in the first period is only 0.06 standard deviation units. The response is so much higher in the second period because of two effects: at this point, capital has been reallocated into this sector, thus enhancing the productivity of labor far beyond the level that existed prior to the reallocation; also, the additional human capital accumulated in the first period now comes on line, further increasing the productivity of labor. These lags also account for the delayed negative response of study hours. Hence study hours are countercyclical in this model, with a one-period lag.[4] The fact that firms can vary the utilization rate of capital helps to soften the constraint imposed by these lag restrictions, but the response functions clearly illustrate that the softening is only partial: the fact that the depreciation of physical capital is an increasing function of utilization accounts for this result.

Impulse responses generated by a one-standard-deviation investment shock are illustrated in Figure 2. With the exception of consumption, the responses of the variables to this shock mimic those

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[4] We view the countercyclicality of study time as an attractive feature of the model, as it seems to accord with the behavior of study time in the U.S. (e.g., see Dellas and Sakellaris 1996), and it is intuitive: study time should be highest when its opportunity cost is lowest.

presented in Figure 1. As for consumption, just as in the case of a positive technology shock, the agent wishes to move productive inputs to the investment sector. However, in this case there is no coincident increase in output of the consumption good, thus the response of consumption is negative over the first 7 periods. Consumption then overshoots its steady state value before gradually returning to it.

Impulse responses generated by a one-standard-deviation human-capital shock are illustrated in Figure 3. In this case, the agent finds it optimal to build up the stock of human capital at the expense of the stock of physical capital. Study hours increase, and remain above their steady state level for approximately 10 periods; this comes at the expense of hours devoted to work in both production sectors. Output in both sectors falls initially as a result, but note that the response of consumption becomes positive in the second period, following the expiration of the lag constraint imposed on the reallocation of capital between sectors, and the boost in labor productivity that results from the increase in study time. After the lag constraint expires, capital is diverted from the investment sector into the consumption sector, so the agent does indeed substitute away from the production of physical capital towards human capital: the response of investment is negative, despite the boost in labor productivity, for approximately 7 periods.

This completes our description of the behavior of the model. We now turn to a description of its fit.

IV. The Fit of the Model

We evaluate the fit of the model using U.S. postwar quarterly data ranging from 1948:I - 1994:IV. The data are described in detail in the appendix. Briefly, consumption, investment and output are measured in per capita real terms. Consumption is measured using expenditures on nondurable goods and services (we do not measure consumption of durable goods because the model is not designed to explain consumption of this type); investment is measured using nonresidential fixed investment; output is measured by computing a weighted average of consumption and investment (the weight assigned to consumption is one, and the weight assigned to investment is the investment-consumption price ratio); and hours are measured using household data on the average hours worked per quarter by the civilian noninstitutional population 16 years of age and older. All series were detrended using the Hodrick-Prescott filter.

The performance of the model is of course a function of the parameterization we select. In specifying parameters, we attempted to maintain comparability with the benchmark model of King, Plosser and Rebelo (1988), as was done, e.g., by Watson (1993). Specifically, we adopted KPR's parameterization for all parameters common to both models (with the exception of α), and made other choices with an eye toward limiting differences in the models to differences in our symmetric treatment of human and physical capital.[5] With the exception of α, the parameters common to both models are $\beta = 0.988$, $\rho_A = 0.9$, and $\gamma_A = 1.004$. In KPR's model, the technology shock provides the exclusive source of growth; we wish the same to be true in our model, hence we set $\gamma_I = \gamma_H = 1$. Also, we constrained the persistence of the shocks in our model to be identical: $\rho_I = \rho_H = 0.9$. The remaining parameters were specified as follows. We specified the physical-capital depreciation parameter ω to be 1.5 based on the estimate of this parameter obtained by DeJong, Ingram and Whiteman (1995). The human-capital depreciation rate δ_H was set at 0.03 based on Mincer's (1989) estimate. (Klenow (1992) also makes this selection, while Perli and Sakellaris (1995) choose 0.025.) The parameter δ_0 was set at 0.0736 so that the steady state rate of capital utilization would be 80 percent. (This choice has no bearing on the dynamic properties of the model). Finally, Σ was constrained to be diagonal so that the economically interesting features of the model were forced to do the bulk of the work in characterizing the comovements in the data. We chose Σ_{ii}=0.001, 0.0015, 0.0017, i=A,I,H in order to match the standard deviation of output exactly. (There are many other combinations of Σ_{ii} that would enable us to match the standard deviation of output, but that would alter the performance of the model along other margins. Extensive experimentation with alternative combinations suggests to us that the alterations would be slight.)

Figure 4 illustrates spectra of growth rates obtained from the model under this parameterization, as well as spectra estimated using the data. Following Watson (1993), it also illustrates spectra of the errors required to reconcile differences between the spectra of the model and data. Table 1 presents standard deviation and correlation patterns obtained from the model and the data, and also presents Watson's RMSAE statistics, which measure the relative mean square approximation error needed to reconcile the spectra of the model and data. The RMSAE statistics are analogous to $1\text{-}R^2$ statistics in regression analysis: smaller is better.

The performance of the model in explaining consumption is not good: consumption in the model is too smooth (the ratio of standard deviations of consumption and output is 0.635 in the data, and 0.318 in the model), is far too loosely correlated with output (0.879 in the data, 0.38 in the model), and has a spectrum which is essentially flat over business-cycle frequencies (6 through 36 quarters, or approximately 0.03 through 0.16 cycles per quarter).

The model does a far better job in characterizing the remaining series. Standard deviation ratios for investment are 3.77 in the data and 3.75 in the model; the ratios for hours are 1.09 in the data and

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[5] We made an exception in specifying α because KPR's choice (0.42) is high relative to what is thought to be reasonable in characterizing capital's share of output in the U.S. Instead, we chose 0.33 for this parameter, although the choice of 0.42 would lead to comparable results; we return to this point below.

0.945 in the model. Correlations of investment and output are 0.9 in the data and 0.97 in the model; the correlations for hours are 0.85 in the data and 0.93 in the model. So the correlations generated by the model are a bit high for these series, but much closer than that obtained for consumption.

The spectra generated by the model for investment, output and hours are hump-shaped; their peaks coincide roughly with those of the data, but are not as sharp. RMSAE's computed for these series over all frequencies are 0.375, 0.093, and 0.614; the numbers reported by Watson for KPR's model are 0.43, 0.42 and 0.73. These numbers are not directly comparable, since Watson's data differ from ours; however, the differences in data sets cannot account for the superior fit achieved by our model. RMSAE's computed over business-cycle frequencies (6 through 36 quarters) are 0.215, 0.079 and 0.298 for the three series, results we find to be impressive.

These results are fairly robust to alternative parameterizations of the model, although it is often the case that if one parameter is changed, another must also be changed in order to prevent a deterioration in the model's fit. Of particular interest to us was the robustness of our results to the adoption of KPR's specification of α (0.42). Ceteris paribus, increasing α to 0.42 causes the peaks of the spectra of output, investment and hours to shift slightly leftward; this leftshift can be offset by decreasing the persistence of the model, e.g., by decreasing ρ_A to 0.85. Of related interest is the parameter γ_A: while KPR used γ_A = 1.004, given our specification for γ_I and γ_H this implies identical growth in consumption, investment, and output, which is clearly counterfactual (and implies that simulated and actual data should be detrended by removing a common trend). Ceteris paribus, reducing γ_A to 1.0 increases the variance of output, investment and labor hours, but does not affect the location of their spectral peaks. The increased variances can of course be offset by decreasing the variances of the shocks.

V. The Relative Contributions of the Model's Key Features

In order to understand the role that human-capital accumulation plays in enabling our baseline model to generate cyclical activity, we repeated the fit exercise performed above for three alterations of the baseline model. First, we stripped the model of its two most important components: endogenous physical-capital utilization and human-capital accumulation. In terms of the equations of the baseline model, this involved eliminating study time S_t from (7), the study shock $\ln A_{ht}$ from (10), and setting H_t and the ϕ's (and hence the δ's) as constants. Second, we restored the endogenous physical-capital utilization specification employed in the baseline model, while leaving the level of human capital fixed exogenously. Finally, we restored the endogenous human capital accumulation specification , while leaving the utilization of physical capital fixed exogenously.

In parameterizing each of these alternative models, we attempted to maintain comparability with our parameterization of the baseline model to the greatest extent possible. In particular, in equating the standard deviation of output implied by each model with the data, we kept the ratio of shock variances fixed across models; i.e., when nonzero, $\Sigma_{II}/\Sigma_{AA} = 0.667$, $\Sigma_{HH}/\Sigma_{AA} = 0.588$, as in the baseline model. Also, since our choices for β and ω imply a steady state rate of depreciation for physical capital of 2.4 percent in the baseline model, we adopted this rate in the versions of the model in which physical-capital utilization was fixed exogenously. Values of the remaining parameters common to all models were held fixed across models. Tables 2 - 4 report the parameterizations we employed for each model, as well as the resulting collection of moments; Figures 5 - 7 illustrate the resulting spectra.

The results we obtained for the stripped-down model (Figure 5, Table 2) indicate that the inclusion of multiple sectors in an otherwise-standard RBC model is an important step towards generating cyclical activity: the spectrum of investment generated by this model has a distinct hump shape (the RMSAE is 0.16 over business-cycle frequencies for this series), and the spectra of hours and output have slight humps. The performance of this model is comparable to single-sector RBC models along other dimensions; in particular, the model does a good job in matching the standard deviations of consumption and investment relative to output, but generates insufficient volatility for labor hours (generating a standard-deviation ratio of only 0.395).

Adding endogenous physical-capital utilization to the stripped-down model (Figure 6, Table 3) sharpens the peaks in the spectra of output, investment and hours, although the hump observed for hours remains quite small (the RMSAE is 0.579 over business-cycle frequencies for this series). Also, the standard deviation of hours relative to output is somewhat higher in this model, but remains small (0.492) relative to its empirical counterpart. Adding endogenous human capital accumulation, keeping physical-capital accumulation fixed (Figure 7, Table 4), also sharpens the peaks in the spectra of output, investment and hours, and in this case the hump observed for hours is relatively large (the RMSAE is 0.231 in this model). Also, the standard deviation of hours relative to output is the highest among the modified models: 0.852. These results lead us to conclude that while the endogenization of both physical-capital utilization and human capital accumulation are both important in accentuating the cyclical properties of the model, the contribution of endogenous human capital accumulation is relatively more important.

Although the modified models' descriptions of consumption are less than ideal (e.g., they generate consumption-output correlations which are far too low relative to the data), each is superior to the description obtained using the baseline model. In moving from the stripped-down model to the baseline model, the volatility of consumption falls, and ends up far below that observed in the data; the correlation of consumption and output, which is low even in the stripped-down model (0.54), also falls. Hence while the endogenization of physical-capital utilization and human capital accumulation enhances the cyclical characteristics of the baseline model, and improves its description of labor hours, it does so at the expense of the description of consumption. Evidently, modifications to consumer preferences are required to improve upon the baseline model's shortcomings along this dimension.

VI. Conclusion

Our initial intention in developing this model was to explore sources of economic growth in the U.S. and other countries. To our surprise, the model turned out to produce a nice characterization of the cyclical properties of U.S. data. As we have seen, endowing agents with the ability to determine endogenously the utilization rates of the stocks of inputs to production yields a model with a rich cyclical structure. Beyond exploring the cyclical structure of various modifications of the specification of preferences in this model, our intention in future work is to determine the relative contributions of the evolving productivities of these stocks in influencing the growth patterns observed in the U.S. and elsewhere.

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Data Appendix

The data used in the paper were extracted from the CITIBASE data release of September 1995. The raw series were:

<u>Quarterly:</u>
GCNQ: personal consumption expenditures, nondurable goods
GDCN: implicit deflator for GCNQ
GCSQ: personal consumption expenditures, services
GDCS: implicit deflator for GCSQ
GINQ: gross private domestic investment, nonresidential
GDIN: implicit deflator for GINQ

<u>Monthly:</u>
LHCH: average hours of work per week (household data)
LHP16: civilian labor force participation rate, 16 yrs and over
LHUR: Civilian unemployment rate, all workers, 16 years and over
P16: population, total civilian noninstitutional

The consumption and investment series are in billions of 1987 dollars (using the fixed-weight scheme), and all series except total population are seasonally adjusted. The hours data were initially calculated as

$$\text{HOURS} = \text{LHCH}*(\text{LHP16}/100)*(1-\text{LHUR}/100).$$

These were then converted to quarterly figures by averaging.

A Laspares price index for consumption of nondurables and services was calculated as a weighted average of the two consumption deflators, using the average expenditure shares in 1987 as weights. That is,

$$PC_t = GDCN_t*S_{Nt} + GDCS_t*S_{St},$$

where PC_t is the consumption price index at time t, S_{Nt} is the 1987 nondurables expenditure share (average nominal nondurables consumption in 1987 divided by average nominal consumption of nondurables and services), and S_{St} is the 1987 services expenditure share. Then real consumption (consumption of nondurables and services) is

$$CONSUMPTION_t = [GDCN_t*GCNQ_t + GDCS_t*GCSQ_t]/ PC_t,;$$

i.e., nominal consumption of nondurables and services divided by the appropriate deflator. The investment series (nonresidential fixed) is

$$INVESTMENT_t = GINQ_t.$$

The consumption and investment series were converted to per capita terms by dividing each by quarterly population figures created from the monthly figures by averaging. The output series was created by multiplying investment by the investment-consumption price ratio (to measure it in consumption goods) and adding the result to consumption. Thus

$$C_t = CONSUMPTION_t/P16_t$$

$$I_t = INVESTMENT_t/P16_t$$

$$Y_t = C_t + (GDIN_t/PC_t)*I_t.$$

The series run from 1948:I - 1994:IV (188 quarterly observations).

Table 1

Moments Obtained From Baseline Model

Parameters: $\beta = 0.988$, $\alpha = 0.33$, $\rho_j = 0.9$ for j = A, I, h, $\gamma_A = 1.004$, $\gamma_I = \gamma_h = 1$,
$\omega = 1.5$, $\delta_h = 0.03$, $\theta = 1.0$, $\Sigma_{11} = 0.001$, $\Sigma_{22} = 0.0015$, $\Sigma_{33} = 0.0017$

		Variable			
		Y	C	I	N
Standard Deviation	Model	0.0138	0.0044	0.0518	0.0131
	Data	0.0138	0.0088	0.0520	0.0150
Relative Standard Deviation	Model	1.000	0.318	3.749	0.945
	Data	1.000	0.635	3.772	1.087
Correlation with y	Model	1.000	0.380	0.968	0.933
	Data	1.000	0.879	0.903	0.848

	RMSAE's			
	Y	C	I	N
All Frequencies	0.375	0.794	0.093	0.614
6-32 Periods	0.215	0.812	0.079	0.298

Table 2

Moments Obtained From Stripped-Down Model

Parameters: $\beta = 0.988$, $\alpha = 0.33$, $\rho_j = 0.9$ for j = A, I, $\gamma_A = 1.004$, $\gamma_I = 1$,
$\delta = 0.024$, $\Sigma_{11} = 0.00313$, $\Sigma_{22} = 0.00469$

		Variable			
		Y	C	I	N
Standard Deviation	Model	0.0138	0.0103	0.0497	0.0055
	Data	0.0138	0.0088	0.0520	0.0150
Relative Standard Deviation	Model	1.000	0.746	3.592	0.395
	Data	1.000	0.635	3.772	1.087
Correlation with y	Model	1.000	0.540	0.821	0.686
	Data	1.000	0.879	0.903	0.848

	RMSAE's			
	Y	C	I	N
All Frequencies	0.345	0.673	0.169	0.819
6-32 Periods	0.301	0.867	0.160	0.641

Table 3

Moments Obtained From Model with Variable Physical Capital Utilization, No Human Capital

Parameters: $\beta = 0.988$, $\alpha = 0.33$, $\rho_j = 0.9$ for $j = A, I$, $\gamma_A = 1.004$, $\gamma_I = 1$,
$\omega = 1.5$, $\Sigma_{11} = 0.00255$, $\Sigma_{22} = 0.00382$

		Variable			
		Y	C	I	N
Standard Deviation	Model	0.0138	0.0084	0.0552	0.0068
	Data	0.0138	0.0088	0.0520	0.0150
Relative Standard Deviation	Model	1.000	0.608	3.994	0.492
	Data	1.000	0.635	3.772	1.087
Correlation with y	Model	1.000	0.408	0.879	0.804
	Data	1.000	0.879	0.903	0.848

	RMSAE's			
	Y	C	I	N
All Frequencies	0.214	0.522	0.100	0.788
6-32 Periods	0.212	0.773	0.112	0.579

Table 4

Moments Obtained From Model with Human Capital Accumulation, Fixed Physical Capital Utilization

Parameters: $\beta = 0.988$, $\alpha = 0.33$, $\rho_j = 0.9$ for j = A, I, h, $\gamma_A = 1.004$, $\gamma_I = \gamma_h = 1$,
$\omega = 1.5$, $\delta_h = 0.03$, $\theta = 1.0$, $\Sigma_{11} = 0.00192$, $\Sigma_{22} = 0.0029$, $\Sigma_{33} = 0.0033$

		Variable			
		Y	C	I	N
Standard Deviation	Model	0.0138	0.0084	0.0479	0.0118
	Data	0.0138	0.0088	0.0520	0.0150
Relative Standard Deviation	Model	1.000	0.611	3.470	0.852
	Data	1.000	0.635	3.772	1.087
Correlation with y	Model	1.000	0.596	0.887	0.781
	Data	1.000	0.879	0.903	0.848

	RMSAE's			
	Y	C	I	N
All Frequencies	0.342	0.782	0.093	0.410
6-32 Periods	0.240	0.726	0.127	0.231