

Will The New \$100 Bill Decrease Counterfeiting? *

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Abstract

A current U.S. policy is to introduce a new style of currency that is harder to counterfeit, but not immediately to withdrawal from circulation all of the old-style currency. This policy is analyzed in a random-matching model of money, and its potential to decrease counterfeiting in the long run is shown. For various parameters of the model, three types of equilibria are found to occur. In only one does counterfeiting continue at its initial high level. In the other two, both genuine and counterfeit old-style money go out of circulation—immediately in one and gradually in the other. There are objectives and expectations that can reasonably be imputed to policymakers, under which the policy that they have chosen can make sense.

Earlier this year, the Federal Reserve System and the U.S. Treasury introduced \$100 bills that are printed in a new style. These new-style bills are much more difficult and expensive to counterfeit convincingly than the old-style bills, and the main reason for introducing them is the desire to decrease counterfeiting. The U.S. government is emphasizing that old-style bills will still be honored, though. They are being removed from circulation by a process that could take years to complete.¹ Superficially, then, it seems that this policy—introducing new-style bills but not aggressively withdrawing old-style bills from circulation—might not achieve its aim of decreasing counterfeiting until the last genuine old-style bill is gone. Although we do not show that the current policy will necessarily be effective in the near term, we do show that a long-term failure cannot be taken for granted. Thus the U.S. policy is not self defeating as it seems on first sight. There are objectives and expectations that can reasonably be imputed to policymakers, under which the policy that they have chosen can make sense.

* Any views expressed in this paper are solely those of the authors, and do not necessarily represent the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

¹Old-style bills are being replaced by new-style bills as they come into the Federal Reserve Banks for processing, but no deadline for turning in old-style bills is being imposed. Since between fifty and seventy percent of the U.S. currency stock is held abroad, partly as a long-term store of value rather than as a medium of exchange, some old-style bills are likely to be outstanding for a long time. (The estimate of 50–70% is due to Porter and Judson (1996).)

Specifically, we analyze the effects of the introduction of new-style money on the counterfeiting of old-style money in a *random matching model*, where genuine old-style money is acceptable as legal tender forever. We find that three types of equilibrium exist in this model economy. In one, counterfeiting persists; in others, counterfeiting ceases, either immediately or after some period of time.² We find conditions under which the U.S. policy has the best chance to be effective, although we cannot say unconditionally that the policy will lead to the elimination of counterfeiting. Moreover, we show that even a successful policy may not have an immediate effect. Counterfeit money may continue to be produced for some time after the policy is introduced, and counterfeit money may be acceptable in trade forever, even though it will asymptotically stop circulating.

Since counterfeiting persists in some equilibria but not in others, what can we learn from the equilibrium analysis? Actually, we learn three things of interest. First, as we have already pointed out, we learn that an equilibrium does exist in which counterfeiting stops at some date. Thus the failure of the U.S. policy is not inevitable. Moreover, the realization that counterfeiting can stop eventually, although it does not stop immediately, may prevent people from wrongly making a premature judgment that the introduction of new-style money has failed to achieve its purpose.

Second, we learn that a necessary condition for the existence of an equilibrium with persistent counterfeiting is that the probability of confiscation cannot be too high.³ In other words, an aggressive effort to confiscate counterfeit bills can stop counterfeiting, and such government effort may be necessary. Thus, the model shows that, at least for some parameter values, continued confiscation of old-style counterfeit is an essential complement to the introduction of new-style money. However, our analysis also shows that the level of confiscation effort needed to stop counterfeiting may be lower when new-style money is introduced concurrently than it would have to be otherwise.

Third, we learn from the model that the introduction of the new-style money does not necessarily mean that old-style money will immediately go out of circulation in the sense of being refused in transactions. In fact, we show a case in which the old-style money always remains in circulation in this sense. Thus, the analysis shows that the old-style money can be withdrawn from circulation on a smooth time path, so that the quantity of money acceptable in trade does not decrease abruptly. This is perhaps not directly relevant to the U.S. domestic economy. However, the large holdings of U.S. currency in some foreign countries adds a further dimension to the problem. For instance, a U.S. foreign policy objective is to foster economic stability in Russia, where more than a quarter of the real value of the total currency stock consisted of old-style U.S. \$100 bills at the start of 1996.⁴ Pulling these bills from circulation abruptly would be an even more extreme monetary contraction than the severe one that occurred in the U.S. at the beginning of the 1930s depression. Many policymakers would worry deeply about the macroeconomic con-

²For the equilibrium in which counterfeiting stops immediately, see Proposition 3 (specifically, the discussion of the case when the value of parameter λ_{01} is zero) in the Technical Appendix.

³The other necessary conditions are not as interesting, because they concern things that we assume to be outside the control of the government at the initial date.

⁴Numerous news reports, such as *Los Angeles Times* (1995), suggest that the proportion is at least this high.

sequences of such a contraction, especially in a country where there are already public-finance difficulties would already complicate the use of fiscal policy to mitigate the shock. In view of such concern by policymakers, our model can help explain how the new U.S. currency policy can have been chosen rationally.

The Environment

To study the new policy, we formulate and analyze a *random matching model* of money, in which agents are randomly matched into pairs and use money to make trades that would otherwise not be made.⁵

There are two types of agents. One type is *private agents*, or *traders*, each of whom is able to (costlessly) produce and store one commodity but wants to consume only another commodity.⁶ We assume that there are T types of traders, who are indexed by the commodity they want to consume. Specifically, a trader of type j wants to consume only commodity j and can costlessly produce a unit of type $j + 1$, which can then be traded for money in the future. (We adopt the convention that $T + 1 = 1$.)

The other type is *government agents*, who do not consume anything and do not maximize their own utility (or even have a utility function). Rather, these agents follow a prescribed rule for replacing genuine old-style money with new-style money and confiscating counterfeit money in a way more fully described below. The fraction of agents in the economy who are government agents is S .

The use of money is essential for trade to occur in this model. Barter is ruled out, because the seller of a commodity will never want to consume the specific commodity the buyer could provide in return. Our assumption about storage also makes it infeasible for a trader to carry inventories of all the various commodities that are traded, so that only intrinsically worthless (but easily storable and transferable) fiat objects can become universally acceptable in trade. A seller accepts such objects, which are the monies in our model, if they can be given in turn to another seller who offers what the current seller desires to consume. This trade takes place when the current seller is subsequently paired with an appropriate trading partner and takes the role of buyer. We assume that both commodities and money objects are indivisible.

In our model, three types of money objects might serve as fiat money: genuine old-style money (denoted G); counterfeit, or bad, old-style money (B); and new-style money (N). We assume that government agents can identify all three types with perfect accuracy. Traders can

⁵Our analysis of a *random matching model* follows Kiyotaki and Wright's (1989) in its main respects. Kultti (1995) uses such a model independently to address counterfeiting questions. Our model includes *government agents*, which are first introduced by Aiyagari, Wallace, and Wright (1995).

⁶This is a highly stylized assumption. One might try to motivate it by the idea that a basic commodity such as food both is enjoyed in its own right and is necessary for a person to be productive. However, this and several other highly stylized assumptions are clearly hard to view as photographic representations of an actual economy. Rather, one should think of this sort of model as a kind of science fiction world that shares some salient features with the actual economy and that is simple enough so that the logic of its equilibrium can be understood explicitly.

identify new-style money, but we assume that they are completely unable to distinguish between genuine and counterfeit old-style money when either is presented in trade. If traders do accept counterfeit money, though, then they are able to recognize it after making a close inspection. Based on some news reports, we believe that this assumption accurately reflects the predicament of the public in places like Russia and the Middle East today.⁷

All agents are infinitely-lived, and they are randomly matched into pairs at each date. Because there are infinitely many agents, no pair ever meets twice. Whenever two private agents are matched with each other, they must decide whether or not to trade the objects (commodities or money) they are holding. Trade occurs only if both traders agree to it. When a trader succeeds in buying a unit of the desired commodity, that trader enjoys an amount u of utility from its consumption. Traders each maximize the expected discounted utility of the random consumption streams they get participation in the trading process. The discount factor is $1/(1 + \rho)$, corresponding to a real interest rate of ρ .

In our model, a trader's life is basically a repetitive sequence of producing a unit of a commodity the trader does not want to consume, exchanging it for a money object with someone who does want to consume it, exchanging the money object for a unit of a commodity that the trader does want to consume, and then producing another unit of the first commodity as a consequence.

To this description of a trader's life, we add a description of what happens in meetings with government agents. Whenever a trader is matched with a government agent and the trader is holding either genuine or counterfeit old-style money, the government agent confiscates it. The government agent then gives a unit of new-style money to the trader if the trader was holding genuine old-style money, but gives nothing to a trader who was holding counterfeit. If a trader's counterfeit is confiscated by a government agent, the agent can either replace the counterfeit or not. Replacement requires the trader to pay a utility cost c , which is borne by the trader at the time the old counterfeit is confiscated. We assume that a trader who chooses not to produce a new unit of counterfeit can never trade again, because that trader has neither money nor a commodity. What determines whether or not a trader chooses to produce replacement counterfeit after confiscation is the essence of what we study here.

States, strategies, and equilibria

As a trader participates in the process of matching and trading we have just described, that trader goes through a sequence of states that are defined by what object is being held. At any time, the trader might be holding his produced commodity (state 0), genuine old-style money (state

⁷News reports to this effect were prominent during the months preceding the introduction of the new U.S. \$100 bill. Representative accounts are Ghattas 1995 and Specter 1995. A report issued this year by the U.S. General Accounting Office (U.S. Congress 1996, pp. 10, 14) confirms that "Recently, very sophisticated counterfeiters have been producing very high-quality notes. . . [that] are difficult for the public to discern. . . . [M]any foreign law enforcement and financial organization officials had inconsistent and incomplete information on how to detect the Superdollar [a particularly high-quality counterfeit produced abroad]. Thus, financial institutions abroad may be recirculating the Superdollars."

G), counterfeit money (state B), or new-style money (state N). The trader might also be holding nothing, if previously held counterfeit has been confiscated and has not been replaced.

A trader's *exchange strategy* at a given time is a policy that specifies, for each type of object possibly being held, what other types of objects the trader is willing to exchange for it. Most importantly, the exchange strategy specifies which types of money object the trader is willing to exchange for the produced commodity. (Money objects are simply old-style and new-style money, since he cannot distinguish between genuine and counterfeit old-style money.) Let $\lambda_{ij} = 1$ denote that the trader is willing to move from state i to state j ; $\lambda_{ij} = 0$, otherwise. For example, $\lambda_{01} = 1$ indicates that a trader is willing to trade a commodity for old-style money and $\lambda_{G0} = 0$ indicates that a trader would not be willing to trade a unit of genuine money for a commodity.

Besides having an exchange strategy, at each time, a trader must have a *counterfeiting strategy* to determine whether or not to make a new unit of counterfeit if the trader is in the situation of holding neither money nor a commodity. (Presumably, this situation would be caused by having had counterfeit confiscated by a government agent.) Let $\gamma = 1$ be decision by a trader to produce a new counterfeit after having existing counterfeit confiscated in a meeting with a government agent; $\gamma = 0$, otherwise. A trader's *comprehensive strategy* is an exchange strategy and a counterfeiting strategy to be followed by each trader.

A *Nash equilibrium* is a comprehensive strategy that each individual trader would adopt if that trader were sure that every other trader had also adopted it. A *steady-state equilibrium* is one in which traders' strategies do not change over time. Whenever we refer below to an equilibrium of our model, we mean specifically a steady-state Nash equilibrium. The way in which we solve for an equilibrium is shown in the Technical Appendix.

A Model Without New-Style Money

As a starting point for our analysis of counterfeiting, consider an economy with only one type of genuine money, which traders cannot distinguish from counterfeit. Assume that government agents confiscate counterfeit, but that they do nothing when they meet a trader holding genuine money. Except for these simplifications, this economy works just like the more general one that we mainly intend to study. In particular, traders cannot distinguish genuine money from counterfeit when they make purchases, and traders whose counterfeit is confiscated have to decide whether or not to replace it.

Since we want to use this simplified model as a starting point for the analysis of the effects of introducing new-style money, we will consider only an economy for which these two conditions are satisfied: (1) There is a unique equilibrium with strictly positive stocks of both genuine and counterfeit money in which sellers accept money in exchange for commodities. (2) In this equilibrium, a trader holding counterfeit always chooses to replace it after confiscation. We require this condition in order to have a positive stock of counterfeit money in existence in the steady state.

In this economy, the value to a trader of having a unit of counterfeit, V_B , given that money is

acceptable in trade, is

$$V_B = \frac{(\rho + g + b)(\rho + k)ku - [(\rho + b)(\rho + k) + \rho g]Sc}{\rho(\rho + g + b + k)(\rho + k)} > 0 \quad (1)$$

where g , b , and k are the fractions of traders of a given type holding genuine money, counterfeit money, and commodities, respectively. The following proposition, which is proved in the Technical Appendix, shows that parameter values exist for which traders will replace confiscated counterfeit in such an economy:

Proposition 1 *If*

$$V_B > c, \quad (2)$$

then a steady-state Nash equilibrium exists with money offered and accepted in trade ($\lambda_{01} = \lambda_{G0} = \lambda_{B0} = 1$) and with confiscated counterfeit money replaced ($\gamma = 1$).

Given our assumptions about the environment that rule out barter and that force traders to engage in trade in order to enjoy any utility, accepting money for one's produced commodity is the only option for participation in exchange. Thus, by itself, the acceptability of money implies no restriction on the parameter values for the economy.

In contrast, traders' willingness to replace confiscated counterfeit is restrictive. It requires condition (2) to be satisfied in Proposition 1. In deciding whether or not to make a replacement, traders weigh the expected utility from the consumption they can get with a unit of counterfeit, (V_B), against the immediate utility cost, c , of making the replacement. The higher c is, the more likely it is that this cost will be higher than the expected utility and that traders will choose not to replace the counterfeit. Further, traders' expected utility depends negatively on the fraction of agents in the economy who are government agents, because the larger S is, the more likely it is that traders will have their counterfeit confiscated before being able to trade it for commodities. Thus, the higher S is, the less likely it is that traders will be willing to replace confiscated counterfeit.

... And With New-Style Money

We now turn to our main model, in which government agents exchange new-style money for genuine old-style money in their randomly paired meetings with traders. Eventually, genuine money will be perfectly distinguishable from counterfeit under this scheme, because in the limit, the stock of genuine money becomes new-style money. Here we start from the steady state described in the last section, in which confiscated counterfeit is being replaced, so that (2) is satisfied. We show two possible outcomes, both of which depend on the parameters of the economy: either the introduction of new money will have no effect on counterfeiting or it will lead to the eventual elimination of counterfeiting.

The following proposition, which is proved in the Technical Appendix, shows the conditions under which the introduction of the new-style money might not eliminate counterfeiting of old-style money. Let n be the fraction of agents holding new-style money who are of a given type. Since in the steady state, all old-style genuine money will be replaced by new-style money after its introduction, $n = g$.

Proposition 2 *If (2) is satisfied and*

$$\frac{(\rho + k)ku}{(\rho + n + k)S} > c, \quad (3)$$

then a steady-state Nash equilibrium exists with both old- and new-style money offered and accepted in trade ($\lambda_{O1} = \lambda_{G0} = \lambda_{B0} = \lambda_{N0} = 1$) and with counterfeit money produced ($\gamma = 1$), although it may not be unique.

This proposition shows that two conditions must be satisfied in order for counterfeiting to continue after the new-style money is introduced. The first Condition (2) is that traders find it in their interest to replace counterfeit after it has been confiscated. This condition is satisfied after the introduction of new-style money, because we have assumed that the economy started from a steady state in which it was.

In order for the introduction of new-style money to have no effect on counterfeiting, sellers must have an incentive to accept counterfeit money, even though in the steady state they know that they are getting counterfeit. Condition (3), guarantees that this will be true. Why would sellers knowingly accept counterfeit? Recall that in this economy traders only obtain utility if they are able to trade their commodities for money and then trade money for the commodities they want to consume. Recall also that waiting for consumption is costly. If there is not much genuine money in the economy, then a seller would expect to wait a long time before meeting a trader with a unit of it. In such a case, a seller might knowingly accept a unit of counterfeit and accept the possibility of it being confiscated, rather than bear the cost of waiting to encounter a buyer with new-style money. Therefore, the smaller n is, the more likely it is that condition (3) will be satisfied.

We have demonstrated that under certain conditions, the introduction of the new money may have no effect on counterfeiting. We now examine cases in which the introduction of the new-style money could lead to the elimination of counterfeiting. One case is that in which traders would not knowingly accept counterfeit; that is, the parameters of the economy do not satisfy condition (3). In this case, the introduction of new-style money must lead to the elimination of counterfeiting in the steady state. Why? Suppose that confiscated counterfeit continues to be replaced as new-style money replaces genuine old-style money. Eventually, traders will know that any old-style money being offered for trade must be counterfeit. Thus, in the steady state, no old-style money will be accepted in trade, which would make it worthless. Obviously, utility-maximizing traders would not pay the cost c to replace something worthless, so confiscated counterfeit would not be replaced, which contradicts the supposition. Inspection of condition 3) shows

that the larger is the fraction of genuine old-style money when the new money is introduced, the more likely this outcome is to occur. (Recall that $n = g$.) Also more likely is the possibility that a trader will encounter a government agent and have counterfeit confiscated.⁸

However, the introduction of new-style money could also lead to the elimination of counterfeiting even if traders would knowingly accept counterfeit. This is shown in the following proposition:

Proposition 3 *If*

$$V_B < c \tag{4}$$

and

$$\rho(\rho + k) > nS \tag{5}$$

then a steady-state Nash equilibrium exists with both old- and new-style monies offered and accepted in trade ($\lambda_{01} = \lambda_{G0} = \lambda_{B0} = \lambda_{N0} = 1$) but without replacement of confiscated counterfeit ($\gamma = 0$).

Condition (4) is that replacing confiscated counterfeit does not pay. Since we started from an economy in which (2) is satisfied, it may seem as if (4) cannot be. That is not so. If counterfeit is replaced, then V_B in the steady state is given by (1) with b equal to whatever the quantity of counterfeit happens to be. However, if counterfeit is not replaced, then V_B in the steady state is given by (1) with b equal to zero, since there will be no counterfeit in the steady state in such cases. Thus, as long as $ku > (\rho + S)c$, both conditions can be satisfied. This is why we said that the equilibrium in Proposition 2 was not necessarily unique.

Condition (5) is that a seller will accept old-style money even knowing it is counterfeit. This condition is more likely to be satisfied the smaller the stock of new-style money and the smaller the probability of a seller meeting a government agent (the slower the rate at which the old-style money is being replaced).

From Propositions 1 and 3, we see that before the introduction of the new-style money, the economy could be in a steady state in which money is used in trade, and even though the government is confiscating counterfeit at rate S , it is being replaced as rapidly as it is confiscated. From Proposition 2, we see that if condition (3) is satisfied, the economy could remain in this steady state after new-style money is introduced. From Proposition 3, however, we see that in the same circumstances, the economy can move to a steady state in which old-style money continues to be acceptable in trade, but in which counterfeiting no longer takes place.

If the economy moves to the no-counterfeiting steady state, will the transition be immediate, or will it take some time? We are not able to answer this question analytically, but we have computed equilibrium paths of the economy for various parameter values. The details of the

⁸In random matching models of money, an equilibrium always exists in which one or more monies are not acceptable in trade. Here we are asserting something stronger than that. Not only is there some equilibrium where old-style money is not acceptable in trade, but when condition (3) is not satisfied, all equilibria are characterized by the nonacceptability of old-style money in trade.

simulation are given in the Technical Appendix. Here we discuss some features of a typical simulated equilibrium path. This path is charted in Green and Weber 1996. Here we summarize the salient features.

- The probability that traders are willing to exchange their produced commodities for old-style money is one at all times, since we choose the parameter values such that condition (5) is always satisfied.
- The probability that a trader will replace confiscated counterfeit is one until the critical date 426, after which it is zero.
- Over time, the stock of counterfeit remains constant at the initial level until the critical date, since counterfeit is being replaced until then, but thereafter the stock falls sharply, because counterfeit is being confiscated without replacement.
- The values of holding counterfeit, V_B , and of holding other (new and genuine old-style) money all decline markedly after the critical date. The values also decline from the initial date to the critical date, although the rate of decline is barely perceptible.

There is, of course, a relationship between the behavior of V_B and the time path of γ . As long as V_B is greater than c , traders will replace confiscated counterfeit and $\gamma = 1$. Once V_B falls below c , however, traders no longer replace confiscated counterfeit, and $\gamma = 0$. In our example, this switch occurs at the critical date 426.

This simulation shows that in order for the eventual elimination of counterfeiting to occur, V_B must decline over time. We can explain, intuitively, why this decline would occur. Until the critical date, the total money stock remains constant, because genuine old-style money is being replaced one-for-one with new-style money, and counterfeit is being replaced whenever it is confiscated. However, the critical date is approaching, so the expected discounted present value V_B weights the utility of participation in the economy after the critical date more and more heavily. If the utility of participation declines after the critical date, then the weighting causes it to decline before the critical date as well. The utility of participation (and hence V_B) does decline after the critical date, because the total money stock is falling after the critical date due to the nonreplacement of confiscated counterfeit. Because of this decline, the number of traders holding money is decreasing, while the number of traders holding commodities is not increasing correspondingly, because the traders who suffer confiscation live in autarky thereafter. (Note that the decline in the number of money holders due to the falling nominal stock of counterfeit reflects indivisibility.) Therefore, finding trading partners takes progressively longer. This deterioration of the trading environment causes the value of every phase of participation in the economy, including the holding of counterfeit, to decline.

Conclusion

This study has been motivated by a desire to understand the new U.S. policy: the introduction of a new-style bill that is more difficult to counterfeit and lack of any deadline for private holders to exchange old-style money for new-style. Superficially this policy combination seems to do nothing to decrease the continued counterfeiting of old-style bills. We find that, despite this appearance, it can potentially help to decrease counterfeiting in a way consistent with foreign-policy goals. Three equilibria might occur for various parameters of the simple model economy we formulate to analyze the effectiveness of this policy, but in only the first equilibrium does counterfeiting continue at its initial, high level.

In a second equilibrium, both genuine and counterfeit old-style money go out of circulation immediately when new-style money is introduced. This is an equilibrium outcome essentially because of self-fulfilling expectations. That is, fiat money is only accepted if it will subsequently be accepted by someone else.⁹

But again, the abrupt transition that would occur in this second equilibrium might well be a problem for some foreign economies where U.S. currency is widely used. From this perspective, the existence of a third equilibrium—one in which both genuine and counterfeit money disappear gradually from circulation—is especially significant. In this equilibrium, counterfeiting eventually stops because it is unprofitable, despite the willingness of traders to accept counterfeit.

A noteworthy feature of the third equilibrium is that counterfeiting may not stop immediately after the introduction of new-style money, even though it does stop at some later time. In view of this possibility, current U.S. policy should not be judged a failure too quickly if its initial results are not dramatic.

The third equilibrium involves an enforcement effort against counterfeiting in an essential way. In the face of sufficiently aggressive enforcement, counterfeiting would stop even if new-style money were not to be introduced. The relevance of introducing new-style money is that it reduces the level of enforcement required for success.

⁹Self-fulfilling expectations also make it an equilibrium in this model for new-style money not to be accepted in trade. We ignore this equilibrium because it is so counterintuitive. Li and Wright (1996) show how the model could be modified in agreeable ways that would get rid of it.

Technical Appendix

The model environment

Our model is a version of the Kiyotaki-Wright model modified by assuming that agents can only store the output of their own production. In our model there are two types of agents. *Private agents*, or *traders*, resemble the Kiyotaki and Wright's agents. *Government agents* do not consume anything and do not maximize their own utility (or even have a utility function). Rather they follow a prescribed rule for replacing genuine old-style money with new-style money and confiscating counterfeit in a way more fully described below.

Suppose that there are T commodities, and that each trader is able to produce and store one commodity but wants only to consume another commodity. In other words, there are traders of T types, and a trader of type j only wants to consume good j and can costlessly produce a unit of type $j + 1$, which can then be traded for money in the future. (We adopt the convention that $T + 1 = 1$.) Each trader has an infinite lifetime, and would like to consume at each date $0, 1, 2, \dots$. Each time that a type- j trader consumes a unit of commodity j , the trader receives an amount u of utility. This utility is discounted by discount factor $1/(1 + \rho)$, corresponding to a real interest rate of ρ .¹⁰

In addition to commodities, there are money objects. A money object is an object that does not intrinsically provide utility to anyone, but that people might potentially be willing to accept in trade for a commodity that does provide utility in consumption. Money objects are of three types: new, genuine old-style, and counterfeit old-style. Government agents can distinguish among all three types of money with perfect accuracy. Private agents can identify new-style money, but we assume that they are completely unable to distinguish between genuine old-style money and counterfeit when either is presented in trade. We assume that both commodities and money objects are indivisible.

We assume an equal "number" of traders being of each type. Rather than discussing various groups of agents in terms of their absolute size, we will consider their proportion to the total population. Let the fraction of public agents be S . Then the fraction of traders of each type will be $(1 - S)/T$.

Agents are randomly matched into pairs at each date. Because there are infinitely many agents, no pair ever meets twice. Whenever two private agents are matched with one another, they must decide whether or not to trade the objects (commodities or money) that they are holding. Trade occurs only if both paired traders agree to it. Whenever a private agent is matched with a government agent, and the private agent is holding old-style money either genuine or counterfeit, the government agent confiscates it. The government agent then gives the private agent a unit of new-style money if the private agent was holding genuine old-style money, but gives nothing to a private agent who was holding counterfeit. If a trader holds counterfeit and it is confiscated by a government agent, then the trader can make a new unit of counterfeit which requires a utility

¹⁰This correspondence reflects Fisher's equation of equilibrium in the loan market.

“cost” c .

If a trader’s counterfeit is confiscated by a government agent and subsequently the trader elects not to produce a new unit of counterfeit, then he has neither money nor a commodity, so he can never trade again. What determines whether or not a trader in this position exercises the option to produce new counterfeit is the essence of what we will study here.

Now we present some notation to describe the distribution of agents holding commodities and money objects at each time t .

$G(t)$ is the fraction of traders holding genuine old-style money at time t .

$B(t)$ is the fraction of traders holding counterfeit (bad) old-style money at time t .

$N(t)$ is the fraction of traders holding new-style money at time t .

$K(t)$ is the fraction of traders holding commodities at time t .

Above we assumed that traders who do not produce replacement counterfeit after existing counterfeit is confiscated still meet other agents but cannot trade. That is, these traders without possessions remain in the population, and thus the size of the entire population remains constant through time. The fraction of agents holding commodities (we will refer to these agents as sellers) will also be constant over time, so that $K(t) = K$. Further, the distribution of agents holding either genuine old-style money or new-money must remain constant over time since government agents replace one with the other. Since fraction of meetings in which a trader with genuine old-style money meets a government agent is $SG(t)$, we must have that the fractions of agents with these two types of money move over time (letting $t' = t + 1$) according to

$$G(t') = G(t) - SG(t) \tag{6}$$

and

$$N(t') = N(t) + SG(t). \tag{7}$$

The way in which the fraction of agents holding counterfeit moves over time will depend upon whether or agents decide to produce new counterfeit when old counterfeit is confiscated. If they do decide to produce new counterfeit, then the stock of counterfeit will remain constant; otherwise it will decline at the same rate as genuine old-style money:

$$B(t') = B(t) - [1 - \gamma(t)]SB(t). \tag{8}$$

States, strategies, and equilibrium

As a trader participates in the process of matching and trading that we have just described, he goes through a sequence of states that are defined by what object he is holding. At any time, he

might be holding his production good (state 0), genuine old-style money (state G), counterfeit (bad) old-style money (state B), or new-style money (state N). Besides these states, we will also refer to a composite state 1, in which the trader holds a unit of old-style money that might be either genuine or counterfeit.

If a trader is in state 0 at date t and is paired with another trader who wants to trade money for his production good, then the trader has an opportunity to move to state 1 if the trading partner is holding an old-style money object, or to state N if the trading partner is holding a new-style money object. In the former case, if the trader does move to state 1, he will actually be in state G with probability $G(t)/(G(t) + B(t))$ and in state B with probability $B(t)/(G(t) + B(t))$.

Given that the trading partner does want to make an exchange, the trader can accept or reject the exchange. These options correspond to moving to the new state with probability 1 or 0, respectively.¹¹ Let $\lambda_{ij}(t) = 1$ denote that the trader at time t decides to trade to move from state i to state j ; $\lambda_{ij}(t) = 0$, otherwise. A trader's *exchange strategy* at t is a quintuple $\lambda(t) = (\lambda_{01}(t), \lambda_{0N}(t), \lambda_{G0}(t), \lambda_{B0}(t), \lambda_{N0}(t))$ of numbers that denote the trader's decisions on whether to make the following state transitions, if the opportunity would arise.

- $\lambda_{01}(t)$ sell production good for old-style money;
- $\lambda_{0N}(t)$ sell production good for new-style money;
- $\lambda_{G0}(t)$ purchase consumption with genuine old-style money;
- $\lambda_{B0}(t)$ purchase consumption with counterfeit old-style money;
- $\lambda_{N0}(t)$ purchase consumption with new-style money.

Besides having an exchange strategy, at each time the trader must have a *counterfeiting strategy* to determine whether or not he will make a new unit of counterfeit if he is in the situation (presumably caused by having had counterfeit confiscated from him by a government agent) of holding neither money nor a commodity. Let $\gamma(t) = 1$ be decision by a trader to produce a new counterfeit after having existing counterfeit confiscated in a meeting with a government agent at time t ; $\gamma(t) = 0$, otherwise.

A trader's *comprehensive strategy* is an infinite sequence $((\lambda(0), \gamma(0)), (\lambda(1), \gamma(1)), \dots)$ that specifies both an exchange strategy and a counterfeiting strategy to be followed at every time.

A *Nash equilibrium* is a comprehensive strategy that every trader would adopt if he were sure that every other trader had also adopted it. Whenever we refer below to an equilibrium of our model, we mean specifically a Nash equilibrium.

¹¹More generally, we could permit a trader to base his response to the partner's offer on the outcome of a randomizing device such as a coin toss – for example, accept if heads but reject if tails. (The coin need not be a fair one, but rather the trader can select a coin with any bias from 0 to 1.) However, adding such generality would not contribute enough to the analysis to warrant the additional level of complexity it would add.

An old-style money only economy

As a means of setting up a starting point for our analysis of whether counterfeiting will continue and to provide a vehicle for better understanding how the introduction of new money will affect the economy we will briefly analyze an economy which is identical to that just described with the exception that only old-style money exists. As described above, meetings between traders with counterfeit and government agents result in confiscation of the counterfeit; meetings between traders with genuine money and government agents result in nothing happening. Thus, the fraction of traders with genuine money does not change over time, so that (6) becomes

$$G(t) = G. \quad (1')$$

The problem that a trader faces is to choose comprehensive strategies to maximize the discounted flow of expected utility over his infinite lifetime. Although this is a very complicated problem, it can be simplified by using a representation of discounted expected utility known as the Bellman equations. These equations express the trader's expected discounted utility from being in each of the possible states at time t as a function of the expected discounted utility of being in the various possible states in the future. Thus, the Bellman equations reflect the thought processes that traders go through in order to determine their optimal actions.

We first state the Bellman equations for a seller, a buyer with genuine money, and a buyer with counterfeit money and then provide a brief discussion of the first two. In order to state these equations compactly, let $g = G/T$, $b(t) = B(t)/T$, and $k = K/T$. Then

$$V_0(t) = \frac{1}{1 + \rho} \max_{\lambda_{01}(t')} \{g\lambda_{01}(t')\lambda_{G0}(t')V_G(t') + b(t')\lambda_{01}(t')\lambda_{B0}(t')V_B(t') + [1 - g\lambda_{01}(t')\lambda_{G0}(t') - b(t')\lambda_{01}(t')\lambda_{B0}(t')]\lambda_{01}(t')V_0(t')\} \quad (9)$$

$$V_G(t) = \frac{1}{1 + \rho} \max_{\lambda_{G0}(t')} \{k\lambda_{G0}(t')\lambda_{01}(t')[V_0(t') + u] + [1 - k\lambda_{G0}(t')\lambda_{01}(t')]\lambda_{G0}(t')V_G(t')\} \quad (10)$$

$$V_B(t) = \frac{1}{1 + \rho} \max_{\gamma(t'), \lambda_{B0}(t')} \{k\lambda_{B0}(t')\lambda_{01}(t')[V_0(t') + u] + S\gamma(t')[V_B(t') - c] + [1 - k\lambda_{B0}(t')\lambda_{01}(t') - S]\lambda_{B0}(t')V_B(t')\} \quad (11)$$

Since we want to use this model with only old-style money as a place for starting the analysis of the effects of the introduction of new-style money, we will consider only parameter values for the economy for which the following conditions are satisfied: (i) There is a unique steady-state equilibrium with strictly positive stocks of both genuine and counterfeit money in which sellers

accept money in exchange for goods. (ii) In this equilibrium, a trader holding counterfeit money always chooses to replace it after confiscation. Since steady state equilibria are ones in which nothing changes over time, we require the second condition in order to have a positive stock of counterfeit money in existence in the steady state.

These considerations lead us to the following definition:

Definition 1 *Steady state with money offered and accepted in trade and counterfeit money produced.*

Given K , G , S , and $B(t) = B$, $V = (V_0, V_G, V_B)$ and $(\lambda_{01}, \lambda_{G0}, \lambda_{B0}, \gamma) = (1, 1, 1, 1)$ which satisfy the Bellman equations (9) - (11) and the incentive compatibility conditions

$$[g(V_G - V_0) + b(V_B - V_0)] > 0 \quad (\lambda_{01})$$

$$(V_0 + u - V_G) > 0 \quad (\lambda_{G0})$$

$$(V_0 + u - V_B) > 0 \quad (\lambda_{B0})$$

$$(V_B - c) > 0 \quad (\gamma)$$

is a steady state with money offered and accepted in trade and counterfeit money produced.

The meaning of the incentive compatibility conditions can be seen by examining that for λ_{01} . From the Bellman equation (9) it can be seen that a seller will only choose to accept money in trade for commodities if the incentive compatibility condition (λ_{01}) is satisfied. If $g(V_G - V_0) + b(V_B - V_0)$ were less than zero, then a seller would maximize the term in braces by choosing $\lambda_{01} = 0$, which is not consistent with the definition.

The following proposition shows that a steady state with money offered and accepted in trade exists if the parameters of the model satisfy a certain condition.

Proposition 4 *If*

$$(\rho + g + b)(\rho + k)ku - [\rho(\rho + g + b + k)(\rho + k + S) + bkS]c > 0, \quad (12)$$

then a steady state equilibrium with money offered and accepted in trade and counterfeit money produced exists.

Proof: The solution to the Bellman equations in this steady state is

$$V_0 = \frac{(b+g)ku - bcS}{\rho(\rho + b + g + k)}$$

$$V_G = \frac{k[(\rho + b + g)(\rho + k)u - bcS]}{\rho(\rho + k)(\rho + b + g + k)} > V_0$$

$$V_B = \frac{[(\rho + b)(\rho + k) + \rho g](ku - cS) + k^2gu}{\rho(\rho + k)(\rho + b + g + k)}$$

Condition (12) is sufficient for $V_B > c$, so the incentive compatibility condition (γ) is satisfied. It is also sufficient for $V_0, V_G > 0$. Since $g(V_G - V_0) + b(V_B - V_0) = \rho V_0$ and $V_0 + u - V_G = \rho V_G/k$, this condition is also sufficient for the incentive compatibility conditions (λ_{01}) and (λ_{G0}) to be satisfied. Since

$$V_0 + u - V_B = \frac{(\rho + b + g)(\rho + k)u + (\rho + g + k)cS}{\rho(\rho + k)(\rho + b + g + k)} > 0,$$

the incentive compatibility condition (λ_{B0}) is also satisfied. ■

An economy with distinguishable genuine and counterfeit monies

One way to think about how the introduction of a new-style money will affect the economy in the steady state is to think of it as making genuine money distinguishable from counterfeit. In other words, the introduction of new-style money will have the same effect on the economy as would a technological innovation that would allow sellers to distinguish genuine old-style money from counterfeit old-style money. Thus, we now consider the economy with only one style of genuine money considered above, but for the case in which sellers can distinguish genuine money from counterfeit.

This change in distinguishability of genuine and counterfeit money changes the decision problem for a seller. Before, a seller only had a single strategy variable $\lambda_{01}(t)$ in each time period. Now, a seller has two strategy variables – $\lambda_{0G}(t)$ and $\lambda_{0B}(t)$ – because he can now decide separately whether or not to accept genuine money and whether or not to accept counterfeit money. Separate decisions are possible because the monies are distinguishable. As a result, the Bellman equation for a seller becomes:

$$V_0(t) = \frac{1}{1 + \rho} \max_{\lambda_{0G}(t'), \lambda_{0B}(t')} \{g\lambda_{0G}(t')\lambda_{G0}(t')V_G(t') + b(t')\lambda_{0B}(t')\lambda_{B0}(t')V_B(t') + [1 - g\lambda_{0G}(t')\lambda_{G0}(t') - b(t')\lambda_{0B}(t')\lambda_{B0}(t')]V_0(t')\} \quad (13)$$

Since we are interested in establishing that the introduction of new-style money may have no effect on counterfeiting, we consider the following type of steady-state equilibrium:

Definition 2 *Steady state with genuine and counterfeit monies distinguishable, both types offered and accepted in trade, and counterfeit produced.*

Given K , G , S , and $B(t) = B$, $V = (V_0, V_G, V_B)$ and $(\lambda_{0G}, \lambda_{B0}, \lambda_{G0}, \lambda_{B0}, \gamma) = (1, 1, 1, 1, 1)$ which satisfy the Bellman equations (13), (10), and (11) and the incentive compatibility conditions

$$(V_G - V_0) > 0 \quad (\lambda_{0G})$$

$$(V_B - V_0) > 0 \quad (\lambda_{0B})$$

(λ_{G0}) , (λ_{B0}) , and (γ) is a steady state with genuine and counterfeit monies distinguishable, both types offered and accepted in trade, and counterfeit produced.

The change in the incentive compatibility conditions from before is that there are now separate conditions for λ_{0G} and λ_{0B} . From the Bellman equation (13) it can be seen that a seller will only choose to accept genuine money in trade for commodities if the incentive compatibility condition (λ_{0G}) is satisfied; if $(V_G - V_0)$ were less than zero, then a seller would maximize the term in braces by choosing $\lambda_{0G} = 0$, which is not consistent with the definition. The same reasoning holds for (λ_{0B}) .

The following proposition shows that such a steady state exists if the parameters of the model satisfies certain conditions:

Proposition 5 *If (12) holds and*

$$(\rho + k)ku - (\rho + g + k)Sc > 0, \quad (14)$$

then a steady state equilibrium with genuine and counterfeit monies distinguishable, both types offered and accepted in trade, and counterfeit produced.

Proof: The solution to the Bellman equations in this steady state is the same as those given above. From this and the proof of Proposition (1) we know that the incentive compatibility conditions (λ_{G0}) , (λ_{G0}) , (λ_{B0}) , and (γ) are satisfied. Condition (14) is sufficient for the incentive compatibility condition (λ_{B0}) to be satisfied. ■

In an economy that satisfies the parameters of Proposition (2), counterfeit money would be offered and accepted for trade even if traders were able to distinguish it from the genuine money. However, since condition (12) can be written as

$$(\rho + g + b)[(\rho + k)ku - (\rho + g + k)]Sc + (\rho + g + b + k)[\rho(\rho + k) - gS]c > 0,$$

the parameters can satisfy (12) and not satisfy (14). Thus, there are regions of the parameter space in which counterfeit money would be produced and accepted in trade if it were indistinguishable from genuine money, but where it would not be acceptable in trade if it were distinguishable. The following lemma implies that if counterfeit is not acceptable for trade, then it would also not be produced.

Lemma 1 *If $\lambda_{0B} = 0$, then $V_B = 0$.*

Proof: Follows immediately from (11) with λ_{01} replaced by $\lambda_{0B} = 0$. ■

To summarize what we have learned from the economy with only old-style money we engage in the thought of experient of seeing what happens to a steady state in which genuine and counterfeit monies are indistinguishable and introduce a technology that makes them instantly distinguishable. From Propostion 2 we infer that if the parameters of the economy are such that conditions (12) and (14) are satisfied, then nothing would happen as a result of this innovation. However, if the parameters of the economy are such that (12) is satisfied but condition (14) is not, then the result of the innovation would be that counterfeiting would cease immediately because only genuine money would be offered and accepted in trade.

Analysis with both old-style and new-style monies

As mentioned above, the introduction of new-style money acts like a technology that permits genuine and counterfeit monies to be distinguishable, but it does it in a way that gradually permits a larger and larger fraction of money to be distinguishable over time. This is in contrast to the technology discussed above which permits the entire money stock to be distinguishable instantly. We now show that regions of the parameter space exist in which the introduction of the new money will necessarily cause counterfeiting to cease although the effect may not be immediate.

Specifically, we start from the economy discussed above in which there is only old-style money, and we consider regions of the parameter space in which (12) is satisfied but (14) is not. That is, we consider regions in which counterfeit old-style money is produced and accepted for trade if it is indistinguishable from genuine but in which it is not produced or accepted if it is distinguishable. We then introduce new-style money into this economy and proceed as follows. We first show that in such regions of the parameter space, counterfeiting cannot continue forever. Then we show that a steady-state exists for this economy in which both old-style and new-style monies are accepted in trade but counterfeit is not produced. Since we started from the case in which there was counterfeiting and moved to a steady-state without out it, these results show that the introduction of the new money will cause counterfeiting to be discontinued, even though the effect may not be immediate. (Of course, if we had started from a region of the parameter space

in which both (12) and (14) were satisfied, then the introduction of the new-style money would have not effect on counterfeiting.)

The Bellman equations when both old- and new-style exist are the following:

$$\begin{aligned}
V_0(t) = & \frac{1}{1 + \rho} \max_{\lambda_{01}(t'), \lambda_{0N}(t')} \{ \lambda_{01}(t') [\lambda_{G0}(t') g(t') V_G(t') + \lambda_{B0}(t') b(t') V_B(t')] \\
& + \lambda_{0N}(t') \lambda_{N0}(t') n(t') V_N(t') \\
& + [1 - \lambda_{01}(t') \lambda_{G0}(t') g(t') - \lambda_{01}(t') \lambda_{B0}(t') b(t') - \lambda_{0N}(t') \lambda_{N0}(t') n(t')] V_0(t') \}
\end{aligned} \tag{15}$$

$$\begin{aligned}
V_G(t) = & \frac{1}{1 + \rho} \max_{\lambda_{G0}(t')} \{ \lambda_{G0}(t') \lambda_{01}(t') k [V_0(t') + u] + S V_N(t') \\
& + [1 - \lambda_{G0}(t') \lambda_{01}(t') k - S] V_G(t') \}
\end{aligned} \tag{16}$$

$$\begin{aligned}
V_B(t) = & \frac{1}{1 + \rho} \max_{\lambda_{B0}(t'), \gamma(t')} \{ \lambda_{B0}(t') \lambda_{01}(t') k [V_0(t') + u] + \gamma(t') S [V_B(t') - c] \\
& + [1 - \lambda_{B0}(t') \lambda_{01}(t') k - S] V_B(t') \}
\end{aligned} \tag{17}$$

$$V_N(t) = \frac{1}{1 + \rho} \max_{\lambda_{N0}(t')} \{ \lambda_{N0}(t') \lambda_{0N}(t') k [V_0(t') + u] + [1 - \lambda_{N0}(t') \lambda_{0N}(t') k] V_N(t') \}, \tag{18}$$

where $g(t') = G(t')/T$, $n(t') = N(t')/T$, and $b(t')$ and k are defined as before.

Once again we want to consider only steady state equilibria. However, after the introduction of the new-style money, the stock of genuine old-style money would always be decreasing and the stock of new-style money would always be increasing. Further, if no counterfeiting were going on, the stock of counterfeit old-style money would also be decreasing over time. For these reasons, we have to modify the way in which we think about steady state equilibria. We now have to define them as the equilibria that the economy would approach as time goes to infinity.

The first steady state equilibrium we consider is one in which counterfeiting continues forever.

Definition 3 *Steady state equilibrium with counterfeiting and with both old- and new-style monies offered and accepted for trade.*

For a given K , S , $\lim_{t \rightarrow \infty} G(t) \rightarrow 0$, $\lim_{t \rightarrow \infty} B(t) \rightarrow B$, $\lim_{t \rightarrow \infty} N(t) \rightarrow N = G > 0$, $\lim_{t \rightarrow \infty} \frac{B(t)}{G(t)+B(t)} \rightarrow 0$, $V = (V_0, V_G, V_B, V_N)$ and $(\lambda_{01}, \lambda_{G0}, \lambda_{B0}, \lambda_{N0}, \gamma) = (1, 1, 1, 1, 1)$ which satisfy the Bellman equations (15) – (18) and the incentive compatibility conditions (λ_{G0}) , (λ_{B0}) , (γ)

$$(V_B - V_0) > 0 \quad (\lambda'_{01})$$

$$(V_0 + u - V_N) > 0 \quad (\lambda_{N0})$$

is a steady state with counterfeiting and with both old- and new-style monies offered and accepted for trade.

We now demonstrate that if (12) is satisfied but (14) is not, then the steady state equilibrium described above cannot exist.

Proposition 6 *If (14) is not satisfied, then a steady state equilibrium with counterfeiting and old-style and new-style monies offered and accepted for trade cannot exist.*

Proof: The solution to the Bellman equations for this steady state are the same as for the steady state with counterfeiting in the economy with only old style money except that g is replaced by n . The proof follows immediately from Proposition 2. ■

We have now shown that if before the introduction of the new-style money the economy starts from a steady state in which counterfeit money would not have been acceptable if it had been distinguishable, then the economy cannot go to a steady state in which counterfeit will still be produced. It remains to show that the economy can go to a steady state in which counterfeit will not be produced. This is done in the next proposition.

Definition 4 *Steady state equilibrium with no counterfeiting and with both old- and new-style monies offered and accepted for trade.*

For a given K, S , $\lim_{t \rightarrow \infty} G(t) \rightarrow 0$, $\lim_{t \rightarrow \infty} B(t) \rightarrow 0$, $\lim_{t \rightarrow \infty} N(t) \rightarrow N = G > 0$, $\lim_{t \rightarrow \infty} \frac{B(t)}{G(t)+B(t)} \rightarrow \theta > 0$, $V = (V_0, V_G, V_B, V_N)$ and $(\lambda_{01}, \lambda_{G0}, \lambda_{B0}, \lambda_{N0}, \gamma) = (1, 1, 1, 1, 0)$ which satisfy the Bellman equations (15) – (18) and the incentive compatibility conditions (λ_{G0}) , (λ_{B0}) , (γ) , (λ_{N0}) , and

$$(1 - \theta)(V_G - V_0) + \theta(V_B - V_0) > 0 \quad (\lambda''_{01})$$

is a steady state with no counterfeiting and with both old- and new-style monies offered and accepted for trade.

Note that in this definition we have the amount of old-style money of both kinds going to zero over time. The genuine money is going to zero because it is being replaced as traders holding it are matched with government agents. The counterfeit is going to zero because it is being confiscated as traders holding it are matched with government agents and the traders decided to

not replace it ($\gamma = 0$). In the limit, the quantity of new-style money approaches the quantity of genuine old-style money that was in existence before the new-style money was introduced.

The incentive compatibility condition (λ''_{01}) requires some explanation. Although as time passes, the quantity of old-style money is disappearing, it never totally disappears. Thus, sellers always face some positive probability of being matched with a buyer with old style money. The parameter θ is the probability the a unit of old-style money will be counterfeit as the quantity of old style money becomes arbitrarily small. (All of the limits in the definition should be thought of as being taken along the economy's transition path.) Thus, the incentive compatibility condition states that in the limit the weighted gains from accepting a unit of old-style money, $(1 - \theta)V_G + \theta V_B$, must be greater than what he will be giving up, V_0 , in order for a seller to be willing to accept a unit of old-style money.

Proposition 7 *If*

$$\theta(\rho + n) > Sn - \rho(\rho + k) \quad (19)$$

and

$$(\rho + k)(\rho + n)ku < \rho(\rho + n + k)(\rho + k + S)c, \quad (20)$$

then there is a steady state with no counterfeiting and with both old- and new-style monies offered and accepted for trade.

Proof: The solution to the Bellman equations in this steady state is

$$V_0 = \frac{nku}{\rho(\rho + k + n)} \quad (21)$$

$$V_G = V_N = \frac{(\rho + n)ku}{\rho(\rho + k + n)} \quad (22)$$

$$V_B = \frac{\rho + k}{\rho + k + s} V_N < V_N \quad (23)$$

Condition (19) is necessary and sufficient for the incentive compatibility condition for λ_{01} to be satisfied, and condition (20) is necessary and sufficient for $V_B - c < 0$. The other incentive compatibility conditions are satisfied since:

$$V_N - V_0 = \frac{ku}{\rho + k + n} > 0$$

$$V_0 + u - V_N = \frac{(\rho + n)u}{\rho + k + n} > 0$$

and $V_B < V_G = V_N$. ■

The conditions for this proposition to hold are consistent with (12) holding and (14) being violated. Thus, it is possible for an economy with indistinguishable old-style money being counterfeited and offered and accepted in trade to move to a steady state in which no counterfeiting will occur after the introduction of new style money.

Simulation

The numerical simulations to generate Figures 1 - 4 were performed by solving (15) - (18) and (6) - (7) backward in time with a terminal date of $T = 600$. In these solutions we set $V_0(T)$, $V_G(T)$, $V_N(T)$, and $V_B(T)$ equal to their steady state values as given by (21) - (23), respectively. Since we could not set $b(T)$ and $g(T)$ either to their steady state values of 0, however, since this would have yielded solutions in which the values of these values would always have been equal to 0. Therefore, we set them as $b(T) = 0.001$ and $g(T) = 0.0005$. The values chosen for the other parameters are:

$$\begin{aligned}n(T) &= 0.01 \\u &= 1.5 \\c &= 9 \\s &= 0.01 \\k &= 0.15 \\\rho &= 0.01\end{aligned}$$

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