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Budget Deficit Persistence and the Twin Deficits Hypothesis*

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Abstract:

This paper gauges the causal relationship between the external and budget deficits by using Blanchard's overlapping generations model. This model nests the twin deficits hypothesis (*i.e.* there is a positive relationship between the deficits) and the Ricardian equivalence hypothesis (*i.e.* there is no link between the deficits). This model also implies that consumers forecast future budget deficits by using (at least) the history of the two deficits. This implication is used to derive time series restrictions, which are testable for given consumers' planning horizons. For the relevant consumers' horizons, the response of the external deficit to an increase in the budget deficit is computed. Empirically, the relevant Canadian consumers' horizons (which can be long) and the persistence of the Canadian budget deficit produce responses that are statistically positive. In contrast, the relevant U.S. consumers' horizons (which can be long) and the dynamic behavior of the U.S. budget deficit yield responses that are statistically insignificant.

Résumé:

Cette étude évalue la relation de causalité entre les déficits extérieur et budgétaire à partir du modèle de générations imbriquées proposé par Blanchard. Ce modèle emboîte l'hypothèse des déficits jumeaux (*i.e.* il y a une relation positive entre les déficits) et l'hypothèse d'équivalence ricardienne (*i.e.* il n'y a aucun lien entre les déficits). Aussi, ce modèle implique que les consommateurs prévoient les déficits budgétaires futurs en considérant l'évolution des deux déficits. Cette implication permet de dériver des restrictions pouvant être testées pour certains horizons de planification des consommateurs. Pour les horizons pertinents, la réponse du déficit extérieur à une augmentation du déficit budgétaire est calculée. Pour le cas canadien, les horizons pertinents (pouvant être longs) et la persistance du déficit budgétaire impliquent des réponses qui sont statistiquement positives. Pour le cas américain, les horizons pertinents (pouvant être longs) et le comportement dynamique du déficit budgétaire impliquent des réponses qui ne sont pas statistiquement positives.

1. INTRODUCTION

One of the most striking features of the behavior of Canadian and U.S. deficits is the positive correlation between the external and budget deficits (Figure 1). In particular, these deficits have increased significantly during the 1980s. This has led several analysts to attribute a significant portion of the deterioration in the external deficits to the emergence of record budget deficits. This causal relationship is termed the twin deficits hypothesis. One implied policy prescription is a tax increase. Applying such a policy would directly decrease the budget deficit and would indirectly reduce the external deficit, due to the reduction of imports induced by the decline of private after-tax incomes.

This traditional view is however challenged by adherents of the Ricardian equivalence hypothesis. This hypothesis implies that a tax increase would contract the budget deficit but would not alter the external deficit. This reflects the notion that simply altering the means that goverments use to finance their expenditures should not affect private spending.

This controversy has motivated many empirical investigations for the Canadian and U.S. cases.[1] Roubini (1988) has estimated that the external deficit is significantly influenced by the budget deficit. Andersen (1990) has found that a decrease of the budget deficit induces a reduction of the external deficit which is roughly half as large. Johnson (1986) has shown that intertemporal models are valid only when the government debt is treated as private financial wealth, so that tax policies can be important determinants of the external deficit. Otto (1992) and Sheffrin and

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[1] The Ricardian equivalence hypothesis has also been tested for other economies (Ahmed 1986, 1987; Hercowitz 1986; Leiderman and Razin 1988).

Woo (1990) have refuted the validity of the current account behavior induced by the Ricardian equivalence hypothesis, while Ghosh (1994) has obtained mixed results. In contrast, Enders and Lee (1990), Haug (1990), and Katsaitis (1987) have shown that Ricardian equivalence models cannot be rejected. Poterba and Summers (1987) have found (among other things) that the distinction between overlapping generations and infinite horizon models is of little practical importance for evaluating the impact of budget deficits. Finally, Chen and Haug (1993), Evans (1988, 1990, 1993), and Evans and Hasan (1994) have estimated from Blanchard's (1985) overlapping generations model that the consumers' planning horizon is long, so that Ricardian equivalence represents an adequate description of the Canadian and U.S. economies. However, none of these analyses has formally taken into account the stochastic properties of the budget deficit, which can affect the relationship between the two deficits.

This paper re-examines the link between the external and budget deficits by gauging not only the consumers' horizon, but also the persistence of the budget deficit. To this end, a tractable version of Blanchard's model is solved. The solution states that (for a given stream of government spending) the contemporaneous domestic external deficit is determined by a weighted present value of current and expected future domestic budget deficits. The weight reflects the fraction of the contemporaneous budget deficit that domestic consumers expect not to reimburse during their lifetime. Thus, consumers having a one-period horizon do not finance the budget deficit at all (the weight is equal to unity), so that there is a one-to-one relationship between the two domestic deficits. In contrast, consumers characterized by an infinite horizon reimburse the budget deficit entirely (the weight is equal to zero), and consequently, the two domestic deficits are not related at all. Even though consumers with long (but finite) horizons finance a substantial part of the domestic budget deficit, the impact of this variable on the domestic external deficit can be large. This is because consumers form their decisions by taking into account the dynamic behavior of the budget deficit. For example, a decrease of the current budget deficit can signal future declines of this variable (*i.e.* future tax increases, given that government spending is not altered), so that these "rainy days" can lead to a significant increase of private savings, and consequently, to a substantial reduction of the external deficit. Actually, numerical examples reveal that a decrease of one dollar in the domestic budget deficit can yield a reduction of the domestic external deficit which is larger than one dollar.

The solution also establishes that the contemporaneous domestic external deficit reflects consumers' early notice of domestic budget deficits. This means that consumers forecast future domestic budget deficits by using (at least) the history of the two domestic deficits. This implication is used to derive smoothness-orthogonality restrictions, which are testable for given consumers' horizons. For the horizons for which these restrictions are satisfied, the response of the domestic external deficit to a change in the domestic budget deficit is computed.

For the Canadian economy, the relevant horizons can be as long as 83 years. It is tempting to argue that these horizons are long enough to insure that Ricardian equivalence is an accurate abstraction, and thus, that the relationship between the two deficits is insignificant. However, given the persistence of the budget deficit, these horizons imply that a decrease of one dollar in the budget deficit leads to a reduction of the external deficit of between \$0.12 and \$0.48. These responses are always statistically smaller than unity, and consequently, the one-to-one relationship between the two deficits is refuted. Moreover, these responses are systematically significant, so that Ricardian equivalence is rejected. Thus, taking into account the stochastic properties of the budget deficit allows one to explain the apparently contradictory previous findings obtained for the Canadian case, *i.e.* there is a significant relationship between the two deficits altough the consumers' horizon can be long.

For U.S. data, the relevant horizons can also be long. Moreover, these horizons and the dynamic behavior of the budget deficit yield responses that are almost always insignificant. This suggests that the persistence of the budget deficit does not substantially affect the relationship between the two deficits. Consequently, Ricardian equivalence seems to be a useful approximation of the U.S. economy.

This paper is organized as follows. Section 2 presents the model. Section 3 outlines the empirical method. Section 4 discusses the results. Section 5 concludes.

2. THE MODEL

In order to investigate the twin deficits hypothesis, this section presents a tractable open economy version of Blanchard's (1985) overlapping generations model. First, the behavior of the individual consumers is described. Second, the aggregate behavior for the economy is analyzed. Finally, the solution establishing the potential relationships between the external and budget deficits is derived.

2.1 Individual Behavior

It is assumed that, in period t, each domestic consumer born at time s solves the following problem:

$$\max_{\{C_{s,t+j}\}} -\frac{1}{2}E_t\sum_{j=0}^{\infty}(C_{s,t+j}-\beta_s)^2(1+r)^{-j}(1-p)^j, \tag{1}$$

$$\text{s.t.} \quad (B_{s,t+1}+F_{s,t+1}) = (1+\eta)(B_{s,t}+F_{s,t}+Y_{s,t}-T_{s,t}-C_{s,t}), \tag{2}$$

where E_t represents the conditional expectation operator, $C_{s,t}$ is the (real) consumption, $Y_{s,t}$ corresponds to the (real) noninsurable stochastic labor income, $T_{s,t}$ represents (real) lump-sum taxes, $B_{s,t}$ is the (real) purchases of one period bonds issued by the domestic government, $F_{s,t}$ is the (real) purchases of foreign one period bonds, and β_s is a bliss point.

Also, $(1-p)$ is the probability of being alive next period and $(1/p)$ corresponds to the consumer's planning horizon. The term $(1+\eta)$ represents the gross (real) return on individual nonhuman wealth $(B_{s,t}+F_{s,t})$. Under the existence of a zero-profit condition of insurance firms which make (receive) every period an annuity payment to (from) each domestic consumer holding positive (negative) nonhuman wealth and inherit this wealth at consumer's death, then $(1+\eta)=(1+r)/(1-p)$— where $(1-p)^{-1}$ is the gross annuity rate and $(1+r)$ is the gross (real) return on one period domestic bonds (Blanchard 1985, Yaari 1965).

Fixing the discount factor to $(1+r)^{-1}$ and specifying a quadratic period utility function in (1) allow one to simplify the exposition (Gali 1990). In this environment, the expected consumption path is $E_tC_{s,t+j}=C_{s,t}$ (*i.e.* consumption is a martingale) and the consumption function is:

$$C_{s,t} = \frac{\eta}{1+\eta}\Big[(B_{s,t}+F_{s,t}) + E_t\sum_{j=0}^{\infty}(Y_{s,t+j}-T_{s,t+j})(1+\eta)^{-j}\Big]. \tag{3}$$

2.2 Aggregate Behavior

The aggregate domestic variable X_t corresponds to $\sum_{s=-\infty}^{t} P_{s,t} X_{s,t}$, where $P_{s,t}$ is the size in period t of the domestic cohort born at time s. As is standard practice, it is postulated that p domestic individuals are born each period ($P_{s,s} = p$) so that $P_{s,t} = p(1-p)^{(t-s)}$ and that the total domestic population is normalized to unity in every period. Note that when p approaches zero, the domestic economy is described by an infinitely-lived representative consumer model. When $p = 1$, the domestic environment is represented by a sequence of static economies, *i.e.* each domestic cohort lives for one period and is fully replaced in the subsequent period by a different cohort.

The demographic structure just described and the assumption that the domestic consumers born at different times supply similar labor services in period t (*i.e.* there is no life-cycle behavior) imply that $Y_t = Y_{s,t}$, for $t \geq s$ (Gali 1990). Similarly, the idea that domestic agents born at different dates face equivalent lump-sum taxes in period t yields $T_t = T_{s,t}$, for $t \geq s$. In this environment, the aggregate private consumption for the domestic economy is:

$$C_t = \frac{\eta}{1+\eta}\Big[(B_t + F_t) + E_t \sum_{j=0}^{\infty} (Y_{t+j} - T_{t+j})(1+\eta)^{-j}\Big]. \tag{4}$$

Also, it can be shown that $(B_{t+1} + F_{t+1}) = \sum_{s=-\infty}^{t} P_{s,t}(1-p)(B_{s,t+1} + F_{s,t+1})$, given that each domestic consumer has a zero nonhuman wealth at birth. Moreover, substituting $(B_{s,t+1} + F_{s,t+1})$ by the intertemporal budget constraint (2) yields:

$$(B_{t+1} + F_{t+1}) = (1+r)(B_t + F_t + Y_t - T_t - C_t). \tag{5}$$

Equation (5) captures the notion that, in the aggregate, the gross return on non-human wealth is $(1+r)$ instead of $(1+\eta)$, since annuity payments represent pure transfers among consumers.

Finally, the public sector of the domestic country faces the following intertemporal budget constraint:

$$(B_{t+1} + B^*_{t+1}) = (1+r)(B_t + B^*_t + G_t - T_t), \tag{6}$$

where B^*_t is the (real) value of foreign purchases of one period domestic bonds and G_t represents the domestic government's (real) expenditure on goods and services.

2.3 Twin Deficits Hypothesis

The (real) current account for the domestic economy just described corresponds to $Z_t = [(F_{t+1} - F_t) - (B^*_{t+1} - B^*_t)]/(1+r)$. Substituting the terms involved in this definition by using the aggregate intertemporal budget constraints (5) and (6) yields:

$$\begin{aligned} Z_t &= S_t - D_t, \\ &= \Big[I_t + Y_t - T_t - C_t\Big] - \Big[\frac{r}{1+r}(B_t + B^*_t) + G_t - T_t\Big], \end{aligned} \tag{7}$$

where D_t represents the domestic (real) budget deficit (including the service of the debt) and S_t is the domestic (real) saving. This last term corresponds to the domestic total income—that is, the sum of domestic interest income $I_t = \frac{r}{1+r}(B_t + F_t)$ and labor income—less the sum of domestic lump-sum taxes and consumption.

To highlight the twin deficits hypothesis, it is instructive to rewrite equation (6) as $(B_{t+j}+B^*_{t+j}) = (B_t+B^*_t)+(1+r)\sum_{k=0}^{j-1} D_{t+k}$. Using this expression as well as the definition of D_t to substitute T_{t+j} by $\left[\frac{r}{1+r}(B_t+B^*_t+\sum_{k=0}^{j-1} D_{t+k})+G_{t+j}-D_{t+j}\right]$ in the aggregate consumption function (4) and replacing this modified equation in (7) leads to:

$$Z_t = \Big(Q_t - \frac{p}{r}I_t\Big) - \frac{\eta}{1+\eta}E_t\Big[\sum_{j=0}^{\infty}\Big(Q_{t+j} + \Big(\frac{\eta - r}{\eta}\Big)D_{t+j}\Big)(1+\eta)^{-j}\Big], \qquad (8)$$

where $Q_t = (Y_t - G_t)$ is denoted the domestic net output and $\big(Q_t + (\frac{\eta-r}{\eta})D_t\big)$ represents a measure of the domestic (real) after-tax labor income. More precisely, it can be shown that the expression between brackets in (8) corresponds to the present value of domestic labor incomes net of the taxes required to finance all domestic government expenditures, except the service of the current debt $\left[\frac{r}{1+r}(B_t + B^*_t)\right]$. Also, $-\frac{p}{r}I_t = (\frac{r}{1+r} - \frac{\eta}{1+\eta})(B_t + F_t)$ captures the absorption of the domestic nonhuman wealth — that is, the difference between interest income and the fraction of nonhuman wealth devoted to private consumption. More generally, equation (8) reflects the intertemporal approach to the current account analysis because it describes the absorption of domestic ressources by taking into account that consumption and government decisions result from forward-looking solutions (Obstfeld and Rogoff 1994).

Equation (8) is attractive because it nests in a simple way the following polar cases. At one extreme, $p = 1$ ($\eta \to \infty$) implies that $(\frac{\eta-r}{\eta}) \to 1$, where this weight reflects the fraction of the contemporaneous domestic budget deficit that consumers expect not to reimburse during their lifetime. In this environment, the domestic after-tax labor income corresponds to the sum of domestic net output and budget deficit $(Q_t + D_t)$. Also, equation (8) reduces to $Z_t = -D_t$,[2] so that an increase of one dollar in the domestic budget deficit decreases the domestic current account by one dollar, and consequently, raises the domestic external deficit by one dollar. This one-to-one relationship is consistent with the twin deficits hypothesis, which states that the budget deficit positively affects the external deficit. Moreover, this one-to-one relationship is explained by the idea that, for a sequence of static economies, there is no saving. At the other extreme, $p = 0$ $(\eta = r)$ implies that $(\frac{\eta - r}{\eta}) = 0$. Consequently, the domestic after-tax labor income is strictly equal to the domestic net output (Q_t). Moreover, the domestic external deficit is not related at all to the domestic budget deficit, so that the twin deficits hypothesis is invalid. This prediction relies on the notion that, for an infinitely-lived representative consumer economy, an increase of one dollar in the domestic budget deficit implies that domestic saving is augmented by one dollar in order to prepare for future increases in taxes. This is a consequence of the Ricardian equivalence hypothesis.

When $0 < p < 1$ $(r < \eta < \infty)$, the evaluation of the relationship between the two domestic deficits must take into account the impact of the contemporaneous budget deficit on expected future net outputs and budget deficits. This can be done by constructing expectations from linear time-series processes for changes in net output and in budget deficit, where these changes are stationary (as will be shown in section 4.3). Given these processes, it is convenient to reformulate equation (8) as:

$$\left[Z_t + \frac{p}{r} I_t + \left(\frac{\eta - r}{\eta}\right) D_t\right] = -E_t \sum_{j=1}^{\infty} \left[\Delta Q_{t+j} + \left(\frac{\eta - r}{\eta}\right) \Delta D_{t+j}\right] (1 + \eta)^{-j}, \qquad (9)$$

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[2] Note that $p{=}1$ implies that $I_t{=}I_{t,t}{=}\frac{r}{1+r}(B_{t,t}{+}F_{t,t}){=}0$. This is because domestic cohorts, which live for one period only, do not invest in domestic and foreign bonds.

where Δ is the first difference operator.

Combining equation (9) with the univariate processes $\Delta Q_t = \delta \Delta Q_{t-1} + u_{qt}$ and $\Delta D_t = \gamma \Delta D_{t-1} + u_{dt}$, for example, produces a response of the domestic external deficit to an increase in the domestic budget deficit that is equivalent to $R = -(\partial Z_t / \partial D_t) = (\frac{\eta - r}{\eta})(1+\theta)$—where $\theta = \gamma(1+\eta)^{-1}\big[1 - \gamma(1+\eta)^{-1}\big]^{-1}$. As expected, $p = 1$ implies that $R = 1$ and $p = 0$ yields $R = 0$—regardless of the value of γ (Figure 2). Moreover, $0 < p < 1$ induces a positive response which is smaller than unity when $\gamma \leq 0$ and produces a positive response which can be larger than one when $\gamma > 0$. This last case can occur when the domestic budget deficit is highly persistent (*i.e.* θ is large). In this context, an increase of the current domestic budget deficit signals future rises in this variable, so that domestic consumers expect significant increases of their future after-tax labor incomes $\big[Q_{t+j} + \big(\frac{\eta - r}{\eta}\big) D_{t+j}\big]$, and consequently, the domestic current account decreases substantially, or synonymously, the domestic external deficit rises significantly. Interestingly, this behavior can arise although domestic consumers have relatively long (but finite) horizons, and thus, finance a substantial portion of the domestic budget deficit. This suggests that, not only the domestic consumers' horizon, but also the stochastic properties of the domestic budget deficit can be crucial for understanding the relationship between the two domestic deficits.

3. EMPIRICAL METHOD

This section explains the empirical method used to test the twin deficits hypothesis. To be fully consistent with the model, expectations of future domestic net outputs and budget deficits are first constructed from a multivariate specification, rather than univariate processes. Second, this specification is used to derive a set of restrictions capturing smoothness and orthogonality conditions. Third, these restrictions are tested for various domestic consumers' planning horizons, so that the relationship between the domestic external and budget deficits can be evaluated for the relevant horizons.

3.1 Specifying Expectations of Future Variables

As pointed out previously, when $0 < p < 1$ the evaluation of the response R requires the specification of the formation of expected future variables. Although the calculations presented in Figure 2 are useful to illustrate the properties of R, they are subject to an important objection: they assume that ΔQ_t and ΔD_t are appropriately modelled as univariate stochastic processes.

Unfortunately, using univariate processes can lead to a mismeasurement of the impact of the domestic budget deficit on the domestic external deficit. Put differently, it is possible to obtain a response R that is statistically zero (positive), not because the twin deficits hypothesis is invalid (valid), but simply because expected future variables are computed from univariate processes rather than a multivariate process involving ΔQ_t and ΔD_t. Moreover, it seems likely that domestic consumers form their expectations using a richer information set than just the history of domestic net output and budget deficit.

However, even when the history of many variables is taken into account, it is not certain that all the domestic agents' relevant information is captured. To circumvent this problem, domestic consumers' own behavior is used to reveal their expectations (Campbell 1987; Campbell and Deaton 1989). To do so, first denote

$A_t = \left[Z_t + \frac{p}{r}I_t + \left(\frac{\eta - r}{\eta}\right)D_t\right]$ the domestic adjusted saving because it is closely related to saving behavior. For example, equation (9) states that $p = 1$ leads to $A_t = 0$, so that there is no saving for a sequence of static economies. In contrast, $0 \leq p < 1$ implies that an increase (decrease) in the current domestic adjusted saving signals declines (rises) in expected future domestic after-tax labor incomes. This behavior reflects the notion of saving for a "rainy day" (Campbell 1987). Moreover, this behavior implies that the contemporaneous domestic adjusted saving summarizes the trajectory of expected future changes in domestic after-tax labor income, and consequently, helps in forecasting this variable.

Thus, to be fully consistent with the structure of the model, the following τ-order vector autoregression (VAR) process (stacked into a first-order system) is specified:

$$\begin{pmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-\tau+1} \end{pmatrix} = \begin{pmatrix} \Gamma_1 & \Gamma_2 & \ldots & \Gamma_{\tau-1} & \Gamma_\tau \\ I & 0 & \ldots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \ldots & I & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-\tau} \end{pmatrix} + \begin{pmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad (10)$$

where $x_t = \left[A_t, \Delta Q_t, \Delta D_t\right]'$ is a vector of current domestic (demeaned) variables, u_t is a vector of innovations, and the typical element of the submatrix Γ_k is denoted $\gamma_{ij,k}$.[3] When $p = 0$, $\gamma_{13,k} = \gamma_{23,k} = \gamma_{31,k} = 0$ $(\forall k)$ is imposed to ensure that the contemporaneous domestic adjusted saving summarizes entirely and exclusively the expected future domestic after-tax labor incomes — where the after-tax labor income corresponds to the net output (section 2.3). In such a case, the system (10) allows the possibility of observing a negative feedback from A_{t-k} $(k > 0)$ to ΔQ_t. This Granger-causality is consistent with the behavior of saving for a "rainy day"

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[3] The case where x_t contains more than three variables is analyzed in section 4.2.

induced by $p = 0$. When $0 < p < 1$, no restriction on $\gamma_{ij,k}$ is imposed. Again, this guaranties that the current domestic adjusted saving encapsulates all the relevant information to forecast future domestic after-tax labor incomes — where the after-tax labor income is determined by the net output and the budget deficit. Thus, negative feedback effects from A_{t-k} to ΔQ_t and from A_{t-k} to ΔD_t can be obtained, as predicted by equation (9) with $0 < p < 1$.

The system (10) is rewritten more compactly as:

$$X_t = \Gamma X_{t-1} + U_t, \tag{11}$$

where $X_t = (x_t, x_{t-1}, \ldots, x_{t-\tau+1})'$, $U_t = (u_t, 0, \ldots, 0)'$, and $E[U_t U_t'] = \Sigma$. Provided that expectations can be approximated by linear projections, the VAR forecasts imply that $E_t \Delta Q_{t+j} = e_2' \Gamma^j X_t$ and $E_t \Delta D_{t+j} = e_3' \Gamma^j X_t$ — where $e_1' = (1, 0, 0, 0, \ldots, 0)$, $e_2' = (0, 1, 0, 0, \ldots, 0)$, and $e_3' = (0, 0, 1, 0, \ldots, 0)$.

3.2 Deriving Restrictions

The VAR forecasts of ΔQ_{t+j} and ΔD_{t+j} ($j > 0$) can be weighted together to form the right-hand side of (9), which can then be compared with the left-hand side. This yields the following restrictions:

$$\begin{aligned} e_1' &= -\sum_{j=1}^{\infty} \Big[e_2' + \Big(\frac{\eta - r}{\eta}\Big) e_3' \Big] \Gamma^j (1+\eta)^{-j}, \\ &= -\Big[e_2' + \Big(\frac{\eta - r}{\eta}\Big) e_3' \Big] \Gamma (1+\eta)^{-1} \Big[I - \Gamma (1+\eta)^{-1} \Big]^{-1}, \end{aligned} \tag{12.1}$$

where I is the identity matrix. The second equality follows by evaluating the infinite sum, noting that it converges as long as A_t, ΔQ_t, and ΔD_t are of mean exponential order less than $(1+\eta)^{1/2}$ (Hansen and Sargent 1980). The 3τ nonlinear restrictions involved in (12.1) imply that the smoothness of the domestic adjusted saving (measured by the variance in its innovation) induced by the model, *i.e.* $\Omega\Sigma\Omega'$ where Ω is the right-hand side of (12.1), matches the actual smoothness of the domestic adjusted saving, *i.e.* $e_1'\Sigma e_1$.

Equivalently, the restrictions (12.1) can be rewritten as:

$$e_1'\left[I-\Gamma(1+\eta)^{-1}\right]=-\left[e_2'+\left(\frac{\eta-r}{\eta}\right)e_3'\right]\Gamma(1+\eta)^{-1}. \tag{12.2}$$

From the structure of the matrix Γ, the constraints imposed by (12.2) on individual coefficients are $(1+\eta)=\left[\gamma_{11,1}-\gamma_{21,1}-(\frac{\eta-r}{\eta})\gamma_{31,1}\right]$ and $\left[\gamma_{1j,k}-\gamma_{2j,k}-(\frac{\eta-r}{\eta})\gamma_{3j,k}\right]=0$, where $j=2,3$ and $k=1$ or $j=1,2,3$ and $k=2,\ldots,\tau$. Applying the transformation $\left[A_t-\Delta Q_t-(\frac{\eta-r}{\eta})\Delta D_t\right]$ yields an alternative interpretation of these 3τ linear restrictions, namely that the expression $\epsilon_t=\left[A_t-(1+\eta)A_{t-1}-\Delta Q_t-(\frac{\eta-r}{\eta})\Delta D_t\right]=\left[e_1'-e_2'-(\frac{\eta-r}{\eta})e_3'\right]U_t$ is orthogonal to information dated before period t. Thus, the rejection of the restrictions (12) means the failure of the smoothness and the orthogonality conditions.

3.3 Testing Restrictions

The restrictions just derived are tested by applying the following procedure.

Step 1. The term $(1+\eta)=\left[(1+r)/(1-p)\right]$ and the variables $A_t=\left[Z_t+\frac{p}{r}I_t+(\frac{\eta-r}{\eta})D_t\right]$ and $\epsilon_t=\left[A_t-(1+\eta)A_{t-1}-\Delta Q_t-(\frac{\eta-r}{\eta})\Delta D_t\right]$ are constructed for $r=1.5\%$ per quarter and for a given value of p.

Step 2. The VAR process (11) is estimated by OLS for the appropriate lag structure.

The lag length is determined by applying the likelihood ratio (LR) test [corrected for small sample (Sims 1980)] and the criterion proposed by Akaike (A). This criterion involves choosing the lag structure which maximizes the likelihood after imposing the penalty $(2\tau n^2)$ — where τ is the number of lags per variable, $n = 3$ is the number of variables in x_t, and T is the sample size.

Step 3. The 3τ nonlinear restrictions involved in equation (12.1) are jointly tested. To do so, the $\chi^2(3\tau)$ distributed Wald test statistic W_1 is computed by using numerical derivatives.

Step 4. The 3τ linear restrictions implied in (12.2) are jointly tested. To this end, the $\chi^2(3\tau)$ distributed Wald test statistic W_2 is calculated. The statistic W_2 is easy to obtain because it is numerically identical to the Wald statistic testing that all the coefficients associated with the regression of ϵ_t on A_{t-k}, ΔQ_{t-k}, and ΔD_{t-k} $(k = 1, \ldots, \tau)$ are jointly insignificant. However, the statistic W_2 is not necessarily identical to W_1, even though equations (12.1) and (12.2) are algebraically equivalent restrictions. This is because nonlinear transformations can change the numerical values of Wald statistics and can alter their power (Gregory and Veall 1985).

Step 5. The Wald test statistic W_2 is decomposed by evaluating the statistics $W_{2,A}$, $W_{2,q}$, and $W_{2,d}$. $W_{2,A}$ tests the τ linear restrictions that the coefficients associated with A_{t-k} in the regression of ϵ_t on A_{t-k}, ΔQ_{t-k}, and ΔD_{t-k} $(k = 1, \ldots, \tau)$ are jointly insignificant. Similarly, $W_{2,q}$ tests the τ linear restrictions that the coefficients affecting $q_{t-k} = \Delta Q_{t-k}$ in the regression just described are jointly statistically null. Finally, $W_{2,d}$ tests the τ linear restrictions that the coefficients related to $d_{t-k} = \Delta D_{t-k}$ are jointly insignificant. As will become clear later on, $W_{2,A}$, $W_{2,q}$, and $W_{2,d}$ will prove to be useful to understand the intuition behind the empirical results.

Steps 1 to 5 are performed for several values of p, where $0 \leq p \leq 1$. This exercise allows one to find the values of p for which the smoothness-orthogonality conditions hold. For these relevant values of p, the relationship between the domestic external and budget deficits is evaluated. To do this, first note that equations (9) and (11) imply that $A_t = -\big[e_2' + \big(\frac{\eta-r}{\eta}\big)e_3'\big]\Theta X_t$—where $\Theta = \Gamma(1+\eta)^{-1}\big[I - \Gamma(1+\eta)^{-1}\big]^{-1}$. Then, using the definition of A_t and rearranging lead to $R = -(\partial Z_t/\partial D_t) = \big(\frac{\eta-r}{\eta}\big) + \big[\theta_{23} + \big(\frac{\eta-r}{\eta}\big)\theta_{33}\big]/\big[1 + \theta_{21} + \big(\frac{\eta-r}{\eta}\big)\theta_{31}\big]$, where θ_{ij} represents the (i,j) element of the matrix Θ.

When $\tau = 1$, there is no interaction between ΔQ_t and ΔD_t, and A_t does not Granger-cause ΔQ_t and ΔD_t ($\gamma_{21,1} = \gamma_{23,1} = \gamma_{31,1} = \gamma_{32,1} = 0$), then this response reduces to that obtained from the univariate processes $\Delta Q_t = \delta \Delta Q_{t-1} + u_{qt}$ and $\Delta D_t = \gamma \Delta D_{t-1} + u_{dt}$ (section 2.3). When $\tau \geq 1$ and $p = 1$, the response is numerically equal to one, so that the one-to-one relationship between the two domestic deficits holds. When $\tau \geq 1$ and $p = 0$, the response is numerically equal to zero, and thus, the Ricardian equivalence hyppothesis is valid. In general, the twin deficits hypothesis cannot be rejected when R is statistically positive, while Ricardian equivalence cannot be refuted when R is insignificant. This statistical analysis requires the computation of the variance ($\Phi'\Psi\Phi$) of R—where Φ is the vector of numerical derivatives of R with respect to the VAR parameters ($\gamma_{ij,k}$) and Ψ is the variance-covariance matrix of the VAR parameters.

4. RESULTS

This section applies the empirical method to evaluate the validity of the twin deficits hypothesis for Canadian and U.S. data. Testing this hypothesis for the Canadian case is appealing because this is a small open economy, so that the assumption that the return on one period bonds is exogenous is likely to be adequate (section 2). Also, testing the twin deficits hypothesis for the U.S. economy is instructive because most empirical analyses have focused on this case. This section first gauges the relationship between the external and budget deficits for the consumers' planning horizons for which the smoothness-orthogonality conditions are satisfied. Then, the robustness of the results is verified by using several values of the return on one period bonds, various subsamples, and different specifications to construct expected future budget deficits. Finally, the validity of the consumers' horizons used to compute the responses R is cross-checked by investigating the stochastic properties of the time series.

4.1 Basic Results

The basic results are obtained from quarterly seasonally adjusted Canadian and U.S. data for the flexible exchange rate regime, 1970-II to 1993-II (see the appendix). These data are used to test the restrictions (12) associated with the appropriate lag structure of the VAR process (11). For Canadian and U.S. data, the criteria A and LR indicate that the relevant lag length is $\tau = 2$ — regardless of the value of p.

For this lag structure, the linear restrictions involved in the Wald test statistics $W_{2,A}$, $W_{2,q}$, and $W_{2,d}$ are sequentially tested for several values of p. Figure 3 displays the levels of significance at which each set of restrictions is rejected. For the Canadian economy, the level of significance associated with the restrictions captured in $W_{2,d}$ is larger than 10% — whatever the value of p. This means that ϵ_t

and ΔD_{t-k} $(k = 1, 2)$ are always orthogonal. Moreover, $W_{2,A}$ indicates that ϵ_t and A_{t-k} are correlated (at all conventional levels of significance) for $0.23 \leq p \leq 1.0$. In contrast, $W_{2,q}$ reveals that ϵ_t and ΔQ_{t-k} are nonorthogonal (at all conventional levels of significance) for $0 \leq p \leq 0.002$. Thus, $W_{2,A}$ and $W_{2,q}$ suggest that the smoothness-orthogonality conditions (12) may be invalid when p is in the neighborhood of zero and when p is large. Put differently, it is plausible that some values of p are large enough to guaranty that ϵ_t is orthogonal to ΔQ_{t-k} but are small enough to insure that ϵ_t is uncorrelated with A_{t-k}, so that the smoothness-orthogonality conditions cannot be rejected for Canadian data. Figure 3 suggests that this intuition also holds for the U.S. economy.

This intuition is checked formally by testing the nonlinear restrictions (12.1) contained in the Wald test statistic W_1 and the linear restrictions (12.2) embodied in the Wald test statistic W_2. Figure 4 shows the levels of significance at which these restrictions are rejected. For the Canadian case, W_1 indicates that the smoothness-orthogonality conditions hold at the 1%, 5%, and 10% levels of significance for $0.006 \leq p \leq 0.019$, $0.008 \leq p \leq 0.016$, and $0.010 \leq p \leq 0.014$, respectively. Also, W_2 suggests that these restrictions cannot be refuted at the 1%, 5%, and 10% levels of significance for $0.003 \leq p \leq 0.030$, $0.006 \leq p \leq 0.026$, and $0.008 \leq p \leq 0.024$. Overall, these findings reveal that the relevant values of p for the Canadian economy are between 0.003 and 0.030. For the U.S. case, W_1 suggests that the smoothness-orthogonality conditions hold at the 1% and 5% levels of significance for $0.001 \leq p \leq 0.010$ and $0.002 \leq p \leq 0.005$. Moreover, these restrictions cannot be rejected at the 1% level of significance for $0.001 \leq p \leq 0.018$. Thus, the relevant values of p for the U.S. economy are between 0.001 and 0.018.

To summarize, the Canadian consumers' horizon can be as short as 33 quarters (obtained from $p = 0.030$) and as long as 333 quarters (when $p = 0.003$). From this wide range of horizons, it is difficult to discriminate between the competing hypotheses. For example, it can be argued that a horizon of 33 quarters is probably short enough to guaranty that the twin deficits hypothesis is valid. In contrast, it is tempting to conclude that a horizon of 333 quarters is long enough to insure that Ricardian equivalence is an accurate abstraction, and thus, that the relationship between the external and budget deficits is insignificant. However, taking into account the persistence of the budget deficit yields an estimate (standard deviation) of the response R which is equal to 0.1174 (0.0557) when $p = 0.003$ and to 0.4813 (0.1144) when $p = 0.030$. These responses are systematically statistically smaller than unity, so that the one-to-one relationship between the two deficits is rejected. Moreover, these responses are always statistically larger than zero (at the 5% level of significance), and consequently, Ricardian equivalence is rejected. Overall, these results suggest that the persistence of the Canadian budget deficit is a crucial determinant of the relationship between the two Canadian deficits.

For the U.S. economy, recall that the relevant values of p are between 0.001 and 0.018. The estimate (standard deviation) of R is -0.0715 (0.0718) when $p = 0.001$ and is 0.2741 (0.1671) when $p = 0.018$. Thus, the one-to-one relationship between the two deficits is always refuted. However, Ricardian equivalence can never be rejected. This means that the persistence of the U.S. budget deficit does not substantially affect the relationship between the two U.S. deficits. This contrasts greatly with the Canadian case.

4.2 Robustness

So far, the results have been derived by fixing the return on one period bonds

(r) to 1.5% per quarter. Glick and Rogoff (1993) have argued that the results can be potentially sensitive to the choice of r . Empirically, using $r = 0.5\%$ or $r = 2.5\%$ (per quarter) does not however alter the conclusions already obtained for the Canadian and U.S. economies (Table 1, Panels A and B).

The robustness of the results also is checked by estimating the relationship between the two deficits for the period 1981-I to 1993-II. Recall that during this period the Canadian and U.S. deficits have increased significantly (Figure 1). Thus, this exercise allows one to verify whether the deterioration in the external deficits is mainly due to the emergence of record budget deficits. For the Canadian case, the results (derived with $r = 1.5\%$) are similar to that obtained from the full sample (Table 1, Panel C). For the U.S. economy, $p = 0$ implies that there is no relationship between the two deficits, while $p = 0.058$ produces a response that is statistically positive. Therefore, it is difficult to discriminate between the Ricardian equivalence and the twin deficits hypotheses. This contrasts with the findings obtained from the full sample.

Finally, the robustness of the results is investigated by taking into account that Canada is an open economy which interact massively with the U.S. economy. This suggests that some U.S. variables can help in forecasting future Canadian budget deficits, and consequently, these U.S. variables should be incorporated in the Canadian consumers' information set. It is assumed that this information set contains the history of the vector $x_t = \left[A_t, \Delta Q_t, \Delta D_t, \Delta Z_t^*, \Delta I_t^*, \Delta Q_t^*, \Delta D_t^*, \Delta e_t\right]'$ — where the asterisk denotes U.S. variables and e_t is the (real) exchange rate. This specification is attractive because it includes all the variables involved in the U.S. adjusted saving (A_t^*), although it does not require the evaluation of the U.S. consumers' horizon. A similar exercise is performed for the U.S. case. To do so, x_t

includes the usual U.S. variables as well as changes in German and Japanese current accounts, gross domestic products, budget deficits, and exchange rates. Interestingly, the responses (computed for $r = 1.5\%$ and the full sample) for the Canadian and U.S. economies are close to that obtained from the basic specification of x_t (Table 1, Panel D).

4.3 Stochastic Properties of the Time Series

An alternative way to check the robustness of the results is to verify whether the horizons used to compute the responses R are consistent with the stochastic properties of Canadian and U.S. time series. For example, if the net ouput and budget deficit are first-order integrated time series, then the adjusted saving must be stationary in levels; this is because equation (9) states that A_t is just a linear combination of ΔQ_t and ΔD_t. Moreover, an infinite horizon ($p = 0$) implies that $A_t = Z_t$, so that the current account must be stationary in levels. In contrast, a finite horizon ($0 < p < 1$) yiels $A_t = \left[Z_t + \frac{p}{r} I_t + \left(\frac{\eta - r}{\eta}\right) D_t\right]$, so that Z_t, I_t, and D_t must be cointegrated (given that D_t is assumed to be integrated). For Canadian data, applying Dickey-Fuller and Phillips-Perron tests on the full sample reveals that the unit root hypothesis cannot be rejected (at all conventional levels of significance) for the budget deficit, the net output, the current account, and the interest income series (Table 2). Furthermore, Phillips-Ouliaris tests indicate that the no cointegration hypothesis between the Canadian current account, budget deficit, and interest income series is refuted (at the 5% or 10% level of significance)—as long as Z_t or D_t is the regressand in the cointegrating regression (Table 3). These results suggest that the Canadian consumers' horizon is potentially finite. For the U.S. case, all the time series are integrated, but the cointegration relationship is never observed. Thus, unit root tests and cointegration tests do not permit one to determine whether the U.S consumers' horizon is finite or infinite.

Next, equation (9) predicts that domestic agents with early notice of after-tax labor income declines (raises) react in advance by increasing (decreasing) their adjusted saving — where the after-tax labor income corresponds to the net output when $p = 0$ and is determined by the net output and budget deficit when $0 < p \leq 1$ (section 3.1). For the Canadian case, Granger-causality tests (performed on the full sample and with $r = 1.5\%$) reveal that $p = 0$ implies that there is a positive feedback from A_{t-k} $(k > 0)$ to ΔQ_t — unlike the prediction of the model (Table 4). Interestingly, $0 < p < 1$ produces negative feedbacks from A_{t-k} to ΔD_t and from A_{t-k} to ΔQ_t, which is consistent with the behavior of saving for a "rainy day". Again, this indicates that the Canadian consumers' horizon is finite. For the U.S. economy, the negative feedback effects are exclusively observed from A_{t-k} to ΔQ_t, which is only consistent with an infinite horizon.

To summarize, unit root tests, cointegration tests, and Granger-causality tests reveal that the Canadian consumers' horizon is likely to be finite. Moreover, $p = 0$ implies that the predicted adjusted saving is the mirror image of the historical series (Figure 5).[4] Interestingly, $p = 0.01$ yields a predicted adjusted saving which captures quite well the actual series. In contrast, $p = 1$ implies that there is no adjusted saving (because there is a sequence of static economies), while an upward trend is observed in the data. Thus, this exercise confirms that the relevant values of p is around 0.01 — as already concluded by testing the smoothness-orthogonality

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[4] The predicted adjusted saving corresponds to $-E_t \sum_{j=1}^{\infty} \left[\Delta Q_t + \left(\frac{\eta - r}{\eta}\right) \Delta D_t\right] (1+\eta)^{-j} = -\left[e_2' + \left(\frac{\eta - r}{\eta}\right) e_3'\right] \Gamma (1+\eta)^{-1} \left[I - \Gamma (1+\eta)^{-1}\right] X_t$ [see equations (9) and (11)] and is obtained from $x_t = [A_t, \Delta Q_t, \Delta D_t]'$, $\tau = 2$, and $r = 1.5\%$.

conditions (section 4.1). For the U.S. economy, unit root tests indicate that the horizon is finite, while cointegration tests and Granger-causality tests reveal that the horizon is infinite. However, an infinite horizon yields the most accurate predictions. In fact, $p = 0$ leads to a projected adjusted saving which tracks remarkably well the historical series. This suggests that Ricardian equivalence is a useful approximation of the U.S. economy. This is consistent with the findings obtained by computing the responses R.

5. CONCLUSION

This paper gauges the causal relationship between the external and budget deficits by taking into account the consumers' planning horizon and the stochastic properties of the budget deficit. To do so, a tractable open economy version of Blanchard's overlapping generations model is solved. The solution nests the twin deficits and the Ricardian equivalence hypotheses. This solution also states that consumers' expectations of future budget deficits are constructed from multivariate processes. These processes are used to derive smoothness-orthogonality restrictions, which are testable for given consumers' horizons. For the relevant horizons, the response of the external deficit to an additional dollar of budget deficit is computed.

Empirically, Canadian consumers' are characterized by horizons that can be long, but finite. The validity of these horizons is generally confirmed by performing unit root tests, cointegration tests, and Granger-causality tests. These horizons and the dynamic behavior of the budget deficit produce responses that are statistically positive but smaller than unity, so that Ricardian equivalence and the one-to-one relationship between the two deficits are rejected. Moreover, these responses are robust to several values of the return on one period bonds, to various subsamples (which isolate different magnitudes of deficits), and to different specifications (including those involving foreign variables) used to compute expected future budget deficits. Overall, these results reveal that taking into account the persistence of the budget deficit permits one to reconcile the apparently contradictory previous findings obtained for the Canadian case, *i.e.* there is a significant relationship between the external and budget deficits altough the consumers' horizon can be long.

For U.S. data, the relevant horizons can also be long. Whether these horizons are finite or infinite cannot be clearly determined by investigating the stochastic properties of the time series. However, these horizons and the dynamic behavior of the budget deficit yield responses that are almost always statistically insignificant. This suggests that the persistence of the budget deficit is not a major determinant of the relationship between the two deficits, so that Ricardian equivalence seems to be a useful approximation of the U.S. economy.

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Appendix: The Data

All the quarterly series (except exchange rates) are seasonally adjusted at annual rates. The following Canadian nominal series are used: (i) the current account (Cansim D72002 $*$ 4); (ii) the budget deficit (Cansim D20168 $* -1$) and (iii) the purchases of goods and services (Cansim D20162) of all levels of government as well as hospitals, Canada Pension Plan, and Québec Pension Plan; (iv) the personal interest, dividend, and other investment incomes (Cansim D20096); and (v) the labor income, that is, the sum of wages, salaries and supplementary labor incomes (Cansim D20088), net incomes of farm operators (Cansim D20093), and incomes (net of rents) of nonfarm unincorporated business (Cansim D20094 - D20095). These nominal series are deflated by P_t to construct the real variables (i) Z_t, (ii) D_t, (iii) G_t, (iv) I_t, and (v) Y_t. The implicit price deflator P_t is obtained by dividing the nominal expenditures on nondurable goods (Cansim D20121) and services (Cansim D20126) by the real expenditures (in 1986 Canadian dollars) on nondurable goods (Cansim D20498) and services (Cansim D20504).

For U.S. data, the following nominal series are used: (i) the current account (Citibase BPCR $*$ 4 / 1000); (ii) the budget deficit (Citibase (GGFNET + GGSNET) $* -1$) and (iii) the purchases of goods and services (Citibase GGFE + GGSE) of all levels of government; (iv) the personal interest incomes (Citibase GPINT) and dividend incomes (Citibase GPDIV); and (v) the labor income, that is, the sum of wage and salary disbursements (Citibase GW), other labor incomes (Citibase GPOL) and proprietor's incomes (Citibase GPROJ). These nominal series are deflated by P_t^* to construct the real variables. P_t^* is computed by dividing the nominal expenditures on nondurable goods (Citibase GCN) and services (Citibase GCS) by the real expenditures (in 1987 U.S. dollars) on nondurable goods (Citibase

GCNQ) and services (Citibase GCSQ).

The quarterly nominal exchange rate (U.S. dollars per Canadian dollar) corresponds to the monthly nominal exchange rate (Cansim B3400) observed for the third month of each quarter. This nominal exchange rate times P_t divided by P_t^* yields the real exchange rate e_t. Finally, the nominal series for Germany and Japan are obtained from the OECD and the Bank of Japan data sets. These nominal series are deflated by the CPI to construct the real variables.

Table 1. Robustness of the Response R

Canada		U.S.	
	A. r = 0.5%		
p	R	p	R
.001	.1165 (.0516)	.000	.0000 (.0000)
.017	.5543 (.1240)	.010	.2100 (.2218)
	B. r = 2.5%		
p	R	p	R
.005	.1182 (.0519)	.003	-.0812 (.0829)
.035	.4254 (.1179)	.023	.2103 (.1794)
	C. 1981-I to 1993-II		
p	R	p	R
.004	.0831 (.0389)	.000	.0000 (.0000)
.074	.4444 (.0569)	.058	.4726 (.1572)
	D. Augmented x_t		
p	R	p	R
.002	.1145 (.0563)	.001	-.0632 (.0874)
.037	.4687 (.1225)	.024	.2914 (.1701)

The estimates (standard deviations) of the response R are computed for $\tau = 2$ and for the lower and upper bounds of p for which the smoothness-orthogonality conditions hold. When $p = 0$, $\gamma_{13,k} = \gamma_{23,k} = \gamma_{31,k} = 0$ $(\forall k)$ is imposed so that $\theta_{23} = 0$, and consequently, the estimate and the standard deviation of the response R are exactly equal to zero (see section 3).

Table 2. Unit Root Tests

Canada

		D_t			Q_t	
κ	PP_1	PP_2	DF	PP_1	PP_2	DF
1	-14.61	-2.88	-2.61	-5.64	-1.64	-2.33
2	-14.23	-2.92	-2.37	-6.35	-1.51	-2.48
4	-15.15	-2.82	-2.42	-7.58	-1.35	-3.07
8	-15.29	-2.81	-2.11	-8.45	-1.26	-2.75
12	-16.78	-2.67	-3.17	-8.41	-1.26	-2.42

		Z_t			I_t	
κ	PP_1	PP_2	DF	PP_1	PP_2	DF
1	-11.96	-2.69	-2.33	-2.27	-0.22	-1.14
2	-12.11	-2.67	-2.33	-3.65	-0.21	-1.82
4	-11.50	-2.75	-1.78	-4.87	-0.20	-1.55
8	-11.12	-2.79	-1.19	-4.98	-0.20	-1.78
12	-13.21	-2.55	-1.75	-4.26	-0.20	-1.06

U.S.

		D_t			Q_t	
κ	PP_1	PP_2	DF	PP_1	PP_2	DF
1	-11.99	-2.42	-2.27	-8.14	-1.77	-2.74
2	-13.29	-2.48	-2.77	-9.38	-1.81	-2.77
4	-14.83	-2.56	-3.09	-11.64	-1.88	-3.29
8	-14.45	-2.54	-2.77	-11.68	-1.88	-2.82
12	-13.00	-2.47	-2.79	-9.80	-1.82	-2.96

		Z_t			I_t	
κ	PP_1	PP_2	DF	PP_1	PP_2	DF
1	-6.55	-1.72	-1.67	1.74	0.48	-0.77
2	-5.69	-1.64	-1.16	0.88	0.15	-0.42
4	-6.62	-1.72	-1.79	-0.10	-0.15	-0.09
8	-7.82	-1.83	-2.94	-0.70	-0.32	-0.26
12	-8.80	-1.91	-3.18	-0.81	-0.35	-0.23

PP_1 and PP_2 are the Phillips-Perron standardized bias-statistic and t-statistic of $\alpha = 0$ in $\Delta V_t = \text{constant} + \alpha V_{t-1} + \beta t + \text{res}_t$ ($V_t = D_t, Q_t, Z_t$, or I_t), where a triangular Bartlett window with a truncation parameter κ is used. DF represents the Dickey-Fuller t-statistic of $\alpha = 0$ in $\Delta V_t = \text{constant} + \alpha V_{t-1} + \beta t + \sum_{j=1}^{\kappa} \omega_j \Delta V_{t-j} + \text{res}_t$. MacKinnon's asymptotic critical values for PP_1 associated with the 10%, 5%, and 1% levels are -18.2, -21.7, and -29.4. MacKinnon's asymptotic critical values for PP_2 and DF associated with the 10%, 5%, and 1% levels are -3.13, -3.41, and -3.96.

Table 3. Cointegration Tests

Canada

Regressand:		Z_t			D_t	
κ	PO_1	PO_2	EG	PO_1	PO_2	EG
1	-29.21	-4.49	-3.57	-30.50	-4.58	-3.67
2	-28.81	-4.52	-3.07	-29.38	-4.69	-2.94
4	-30.92	-4.34	-2.90	-32.49	-4.41	-3.02
8	-32.08	-4.25	-1.97	-33.86	-4.30	-2.41
12	-32.65	-4.20	-1.80	-32.72	-4.39	-2.70

Regressand:		I_t	
κ	PO_1	PO_2	EG
1	-9.25	-1.88	-1.74
2	-10.04	-1.83	-2.08
4	-11.84	-1.72	-2.82
8	-12.14	-1.70	-2.17
12	-10.26	-2.20	-2.10

U.S.

Regressand:		Z_t			D_t	
κ	PO_1	PO_2	EG	PO_1	PO_2	EG
1	-11.58	-2.43	-2.48	-12.22	-2.49	-2.42
2	-10.91	-2.37	-2.07	-13.41	-2.54	-2.89
4	-12.66	-2.53	-2.74	-14.73	-2.60	-3.13
8	-13.89	-2.63	-3.85	-14.46	-2.59	-2.82
12	-13.66	-2.61	-3.46	-13.07	-2.53	-2.77

Regressand:		I_t	
κ	PO_1	PO_2	EG
1	-4.18	-1.08	-1.70
2	-4.97	-1.26	-1.64
4	-6.68	-1.62	-1.87
8	-7.58	-1.79	-2.09
12	-6.82	-1.65	-1.84

PO_1 and PO_2 are the Phillips-Ouliaris standardized bias-statistic and t-statistic of $\alpha = 0$ in $V_{1t} = \text{constant} + \delta_1 t + \delta_2 V_{2t} + \delta_3 V_{3t} + v_t$ and $\Delta v_t = \alpha v_{t-1} + \text{res}_t$ — where the regressand V_{1t} is Z_t, D_t or I_t, the regressors V_{2t} and V_{3t} are the other two variables, and a triangular Bartlett window with a truncation parameter κ is used. EG represents the Engle-Granger t-statistic of $\alpha = 0$ in $\Delta v_t = \alpha v_{t-1} + \sum_{j=1}^{\kappa} \omega_j \Delta v_{t-j} + \text{res}_t$. MacKinnon's asymptotic critical values for PO_1 associated with the 10%, 5%, and 1% levels are -28.5, -32.8, and -42.0. MacKinnon's asymptotic critical values for PO_2 and EG associated with the 10%, 5%, and 1% levels are -3.84, -4.12, and -4.66.

Table 4. Granger-Causality Tests

	Canada		U.S.	
p	ΔD_t	ΔQ_t	ΔD_t	ΔQ_t
0.000	—	.0398	—	-.0353
	—	(.2239)	—	(.4177)
0.005	-.0714	-.0043	.0786	-.0818
	(.2075)	(.8969)	(.2371)	(.1743)
0.010	-.0419	-.0211	.0338	-.0285
	(.2460)	(.3153)	(.3059)	(.3425)
0.050	-.0053	-.0087	.0047	-.0039
	(.5239)	(.0650)	(.3974)	(.4332)
0.100	-.0022	-.0047	.0023	-.0020
	(.6117)	(.0514)	(.4010)	(.4114)
0.300	-.0006	-.0017	.0008	-.0007
	(.6990)	(.0419)	(.4011)	(.3925)
0.500	-.0003	-.0010	.0005	-.0004
	(.7206)	(.0399)	(.4008)	(.3883)
0.700	-.0002	-.0007	.0003	-.0003
	(.7304)	(.0389)	(.4005)	(.3865)
0.999	-.0001	-.0005	.0002	-.0002
	(.7379)	(.0383)	(.4004)	(.3851)

The entries under the column ΔD_t are the estimates (levels of significance) of the sum of the coefficients affecting lagged adjusted savings in the budget deficit changes equation of the VAR process (11) with $\tau = 2$. The entries under the column ΔQ_t are the estimates (levels of significance) of the sum of the coefficients affecting lagged adjusted savings in the net output changes equation of the VAR. For $p = 0$, there is no lagged adjusted saving included in the budget deficit changes equation in order to be consistent with equation (9) — see section 3.1.