

# Discount Window Lending and Deposit Insurance

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## Abstract

A banking model is constructed where roles for government-provided deposit insurance and discount window lending arise when there are restrictions on branch banking. Banks arise endogenously as an efficient arrangement for sharing risk. Discount window lending permits better risk sharing by making bank assets more liquid, but is limited because of a moral hazard problem which arises from adverse selection in the loan market. Deposit insurance also creates the potential for better risk sharing, but accomplishes this through contingent transfers rather than enhancing liquidity. Banks tend to take on more risk with deposit insurance and to take less care in screening loans, but this is consistent with an increase in welfare for depositors and borrowers.

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# 1 1. Introduction

Given the attention paid in public policy discussions, particularly recently, to the role of deposit insurance in the United States, there have been surprisingly few attempts to develop formal models which are capable of evaluating deposit insurance systems. Possibly this is because, to most economists, the problems with government-provided deposit insurance in the United States are obvious, and the remedies are straightforward. In this view, most of what we need to know about deposit insurance can be learned from standard insurance models. That is, deposit insurance encourages banks to take on too much risk. If the risk associated with the bank's portfolio were costlessly observable, then this problem could be corrected with appropriately-priced deposit insurance (Kareken and Wallace 1978). However, given that there is moral hazard, i.e. risk is costly or impossible to observe, the remedies are similar to the ones which mitigate moral hazard in private insurance arrangements. Matters can be improved by building a deductible into the insurance contract (Boyd and Rolnick 1988) or constraining behavior, for example through capital requirements (Bond and Crocker 1991).

It is our contention that, while drawing an analogy to private insurance markets yields some important insights into the workings of deposit insurance, there are unique aspects of deposit insurance systems which demand an approach tailored to the idiosyncrasies of the problem. The most obvious feature that sets deposit insurance apart (from most forms of insurance, not including unemployment insurance) is that it is typically provided by the government. There have been, and continue to exist, private deposit insurance systems in the United States, but they have been notoriously unsuccessful and geographically limited (English 1993).

There have been two approaches to modeling a role for government-provided deposit insurance. The first approach begins with the argument that banking is inherently unstable, as in the banking model of Diamond and Dybvig (1983). In the Diamond and Dybvig model, given the nature of the liquidity transformation provided by banks and a sequential service constraint faced by a bank's customers, a bank run can occur as a Pareto-dominated equilibrium. Deposit insurance can prevent the bank run, as it precludes this bad equilibrium. However, Wallace (1988) shows that, if the spatial structure required to support an endogenous sequential service constraint is made explicit, then there is no role for deposit insurance in the Diamond-Dybvig model.

A second approach appeals to the ability of the government to economize on the costs of monitoring banks (for example Bond and Crocker 1991). In a world

with costly state verification (Bond and Crocker 1991, Williamson 1989), if banks are imperfectly diversified, then there will be some states of the world where banks default on their deposit liabilities and depositors must incur monitoring costs. With government deposit insurance, risk is shared efficiently and monitoring costs are lower, as the government acts as a monitoring agent for all depositors. The problem with this approach is that it does not imply a role for government-provided deposit insurance. That is, the government performs intermediation services that could be profitably provided by a private intermediary.

The fact that it has been difficult to find a welfare-enhancing role for government deposit insurance in the context of some popular models of financial intermediation may not be surprising to some. Indeed, we will take as a starting point for our analysis that there is no role for government deposit insurance in the absence of other governmental restrictions which constrain the behavior of banks. In many countries, including Canada, banking systems have experienced long periods of stability and (apparently) relative efficiency without the explicit provision of government deposit insurance. The U.S. banking system is very unusual, particularly in terms of the financial restrictions which inhibit branch banking and tend to make banks smaller and less diversified than they would otherwise be.

In the model constructed here, deposit insurance and discount window lending can improve welfare in the context of restrictions that prevent branch banking. This is a locational model, with adverse selection in the loan market and random demand for liquidity modeled as location shocks faced by deposit-holders (as in Champ, Smith and Williamson 1992). The use of a spatial approach to modeling financial frictions is in the spirit of work by Townsend (e.g. Townsend 1987). Banks arise endogenously here as an efficient arrangement for sharing risk associated with the need for liquidity, i.e. the role for banking is closely related to that in Diamond and Dybvig (1983). In the loan market there is costly screening. That is, borrower type is private information, but type can be revealed at a cost (see Wang and Williamson 1994). In equilibrium, screening costs are minimized if banks offer borrowers debt contracts. There is aggregate risk associated with private loans, and there also exists a riskless and perfectly liquid asset.

Agents who change locations require liquid assets, and so withdraw early from the bank. In a branch banking economy with full communication across locations, aggregate risk is shared perfectly between agents who withdraw early and those who do not. However, if there are branching restrictions, then there is imperfect risk-sharing. In this context, banks hold riskless assets to provide a medium of exchange for agents who withdraw early, and to give insurance against

the aggregate shock to agents who do not withdraw early.

If discount window lending and deposit insurance are introduced, each acts to promote risk-sharing, but through different means. First, discount window lending is essentially a form of government financial intermediation which makes the loan portfolios of banks more liquid. This is beneficial, as it is the illiquidity of bank portfolios that attenuates the ability of the banking system to share aggregate risk. However, discount window lending is subject to a moral hazard problem, in that the government cannot perfectly verify the quality of assets in a bank's portfolio. This implies that discount window lending cannot bring about perfect risk sharing. Second, in this model, deposit insurance is a means for effecting transfers across groups of agents in the economy, contingent on aggregate shocks. The problem here is that the government cannot observe some aggregate shocks, and therefore cannot generate perfect risk sharing with deposit insurance.

Given a special case which is amenable to analysis, we show that introducing a discount window is welfare-improving, and that introducing deposit insurance into an economy with a discount window also increases welfare. There is a tendency for banks to hold more risky assets with discount window lending and deposit insurance, but this is consistent with an improvement in welfare. As mentioned previously, it is often argued that deposit insurance encourages banks to bear more risk, and that this is a bad thing. Here, banks do not bear enough risk without deposit insurance, and deposit insurance provides a mechanism for improved sharing of aggregate risk. Discount window lending and deposit insurance also cause banks to screen borrowers less intensively; i.e. banks take less care in monitoring their portfolios. Again, this is consistent with an improvement in welfare. Banks screen less intensively because the adverse selection problem in the loan market is less severe. This saves on screening costs, which makes bank depositors better off.

In the model, there are events which can be described as banking panics, in the sense that there is a shortage of liquid assets and these assets command a premium over illiquid assets. Discount window lending acts to reduce the severity of banking panics, as does the introduction of deposit insurance.

The remainder of the paper is organized as follows. In Section 2 the model is constructed and in Section 3 an equilibrium is studied with no restrictions on banking and perfect risk sharing. Section 4 contains an examination of equilibrium when there are banking restrictions and no government intervention. In Sections 5 and 6, equilibrium is studied when there is discount window lending, and where there is discount window lending and deposit insurance, respectively. Section

7 contains an analysis of some results for a special case, including comparative statics experiments. Section 8 is a summary and conclusion.

## 2. The Model

There are two periods and  $N$  locations. In the first period, there is a continuum of agents with unit mass at each location. Agents are either lenders, type  $g$  borrowers, or type  $b$  borrowers. At each location, the fraction of lenders in the population is  $\frac{1}{2-\alpha}$ , and the fraction of borrowers who are type  $g$  is  $\alpha$ , where  $0 < \alpha \leq 1$  and  $0 < \alpha < 1$ . Each lender has one unit of an investment good in the first period and maximizes the expected value of  $u(c - e)$ . Here,  $c$  is consumption in the second period and  $e$  is effort expended in the first period in screening borrowers. Assume that  $u(\cdot)$  is strictly concave, increasing, and twice differentiable, and that  $u'(0) = \infty$ . Let borrowers be indexed by  $j \in [\frac{1}{2-\alpha}, 1]$ . A borrower maximizes the expected value of consumption in period 2, has no endowment in the first period, and has an investment project which requires one unit of the investment good to finance in the first period. For borrower  $j$ , this investment project yields  $x_j$  units of the consumption good in the second period. Here,  $x_j$  is distributed according to the probability distribution function  $F_i(x_j)$ , where  $i = g, b$  is the borrower's type. The associated probability density function is  $f_i(x_j)$ , where  $f_i(x_j)$  is strictly positive and continuous on  $[0, \bar{x}]$  for  $i = g, b$ , with  $\bar{x} > 0$ . As in Wang and Williamson (1994), assume that  $\frac{f_g(x)}{f_b(x)}$  is increasing in  $x$ .<sup>1</sup> The random variable  $\omega$  is an aggregate shock which takes on one of two values,  $\omega \in \{0, 1\}$ , where  $\Pr[\omega = 1] = \alpha$ , with  $0 < \alpha < 1$ . Letting  $\bar{r}_i$  denote the expected return on the investment project of borrower type  $i$ , conditional on  $\omega = 1$ , we assume that  $\alpha \bar{r}_i > 1$ ,  $i = g, b$ .

The aggregate shock,  $\omega$ , is common to all locations and publicly observable. We assume that  $x_j$  is publicly observable within a location, but not across locations, and that  $x_j$  is independent across borrowers,  $j$ . A borrower's type is private information, but can be revealed at a cost of  $\gamma$  units of effort in period 1.

Lenders can lend to borrowers in the first period, or they can acquire risk-free and perfectly-liquid assets, which yield a certain return in period 2. Without loss of generality, let this certain return be 1. At the end of the first period, after investment takes place, some lenders in each location move to other locations. The fraction

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<sup>1</sup>This condition implies first-order stochastic dominance of a type  $b$  investment project by a type  $g$  investment project, and is similar to the monotone likelihood ratio property often imposed in principal-agent models.

of movers in the population at each location is denoted by  $\beta$ , which is a random variable distributed according to the probability distribution function  $G(\beta)$ , with  $\beta \in [0, 1]$ . An individual mover faces the same probability ( $\frac{1}{N-1}$ ) of moving to any other location. Thus, given the law of large numbers, there is symmetry across locations. That is, in period 2 the fraction of movers at any location is  $\beta$  and the fraction of non-movers is  $1 - \beta$ . Consumption goods cannot be transported across locations, and borrowers cannot move between locations, but movers can carry risk-free assets with them to other locations. Lenders do not learn whether they are movers or nonmovers until the end of period 1.

### 3 3. Equilibrium With No Restrictions on Banking: Perfect Risk Sharing

In this section, we will assume that there is full communication across locations, or what can be interpreted as the absence of restrictions on branch banking and/or note issue by banks. As we will show, the equilibrium financial arrangement here can be interpreted as a Diamond-Dybvig (1983) - type banking system, where banks act to insure lenders against risk. Here, risk is associated with moving to another location (Champ, Smith, and Williamson 1992) rather than preference shocks, as in Diamond and Dybvig (1983).

#### 3.1 3.1 Banks

A given bank consists of  $N$  branches, one at each location. These branches can communicate, in the sense that they can coordinate deposit-taking in the first period, and each branch can identify depositors who made their deposits in other branches. Lenders make deposits of investment goods with banks in period 1, and a given bank offers depositors a pair of return schedules,  $[s_m(\beta, \cdot), s_n(\beta, \cdot)]$ . Here  $s_m(\beta, \cdot)$  is the return that a mover gets per unit deposited, in terms of second period consumption, contingent on  $\beta$  and  $\cdot$ , and  $s_n(\beta, \cdot)$  is the return to a nonmover per unit deposited. It is optimal for the bank to coordinate branches so that each branch takes the same quantity of deposits. Then, given symmetry, each branch will have the same number of depositors at its location in the first and second periods, and the fraction of depositors who moved in will be  $\beta$  in each location in the second period.

Banks maximize profits in period 2, and there is free entry into banking, so that banks earn zero profits in equilibrium. Equilibrium banking contracts will maximize the expected utility of the representative depositor subject to a zero profit constraint. Otherwise, banks could not be profit-maximizing. Let  $\ell$  denote the fraction of a bank's portfolio invested in the risk free asset,  $\ell_i$  the fraction invested in loans to type  $i$  borrowers, where  $i = g, b$ , and  $\ell_p$  the fraction invested in pooling contracts. We have  $\ell + \ell_g + \ell_b + \ell_p = 1$ . In addition, let  $r_g$ ,  $r_b$ , and  $r_p$  denote the expected returns on loan contracts with type  $g$  borrowers, type  $b$  borrowers, and on pooling contracts, respectively, conditional on  $\beta = 1$ . Here, note that the bank diversifies perfectly within a class of loan contracts, so that depositors will not bear any idiosyncratic risk. We assume that borrowers cannot observe  $\beta$ , so that loan payments, and therefore expected returns on loan contracts, are not contingent on  $\beta$ . In equilibrium, each bank chooses  $s_m(\beta, \beta)$ ,  $s_n(\beta, \beta)$ ,  $\beta = 0, 1$ , and  $\ell_j, j = \ell, g, b, p$ , to solve

$$\max \int_0^1 \left\{ \begin{array}{l} \beta [\alpha u(s_m(\beta, 1)) + (1 - \alpha)u(s_m(\beta, 0))] \\ +(1 - \beta) [\alpha u(s_n(\beta, 1)) + (1 - \alpha)u(s_n(\beta, 0))] \end{array} \right\} dG(\beta) \quad (1)$$

subject to

$$\beta s_m(\beta, \beta) + (1 - \beta) s_n(\beta, \beta) = \ell + \beta(\ell_g r_g + \ell_b r_b + \ell_p r_p), \quad (2)$$

for  $\beta = 0, 1$ . Here, (2) is the zero profit constraint for the bank.

In equilibrium, the expected returns on all loan contracts, conditional on  $\beta = 1$ , must be equalized; that is  $r_g = r_b = r_p = r$ . The solution to the above optimization problem gives  $s_m(\beta, \beta) = s_n(\beta, \beta)$  for  $\beta = 0, 1$ . That is, depositors are perfectly insured against the event that they change locations. Then, taking  $1 - \ell$  as the fraction of deposits invested in loans,  $\ell$  is determined by the following first-order condition, given  $r$  (assuming an interior solution, which will obtain in equilibrium):

$$\alpha(1 - r)u'(\ell + (1 - \ell)r) + (1 - \alpha)u'(\ell) = 0. \quad (3)$$

Also, in equilibrium we have

$$\ell = \ell, \quad (4)$$

i.e. the demand for loans equals the supply.

### 3.2 3.2 Credit Markets

As in Wang and Williamson (1994), we consider contracts between banks and borrowers which are payment-schedule/screening-probability pairs. A contract

is either a separating contract or a pooling contract. A separating contract for borrower type  $i$ ,  $[R_i(x), j_i]$ , specifies a payment schedule  $R_i(x)$  when  $\lambda = 1$  (clearly, the payment must be zero if  $\lambda = 0$ ) and a screening probability  $j_i$ . We assume that there is a technology which permits the bank to commit to a screening probability, but that outside parties cannot observe whether or not screening takes place. Thus, payments are not contingent on screening. A pooling contract specifies a payment schedule  $R(x)$  when  $\lambda = 1$ , but does not involve screening. Assume that a borrower can contact only one bank in period 1.

The equilibrium concept here is similar to the one considered by Rothschild and Stiglitz (1976). Potentially, an equilibrium could involve either separating contracts or a single pooling contract. A separating equilibrium consists of two contracts,  $[R_i(x), j_i]$ ,  $i = g, b$ , satisfying the following properties.

$$0 \leq R_i(x) \leq \bar{x}, i = g, b \quad (5)$$

$$x \geq y \implies R_i(x) \geq R_i(y); x, y \in [0, \bar{x}] \quad (6)$$

$$\int_0^{\bar{x}} R_i(x) dF_i(x) \leq (1 - j_j) \int_0^{\bar{x}} R_j(x) dF_i(x) + j_j \bar{x}; i = g, b; j \neq i \quad (7)$$

$$\int_0^{\bar{x}} R_i(x) dF_i(x) = r_i + j_i \bar{x}; i = g, b. \quad (8)$$

Here, condition (5) imposes feasibility, (6) is a monotonicity restriction,<sup>2</sup> (7) are incentive conditions, and (8) guarantees that the contract delivers an expected return of  $r_i$  to the bank, conditional on  $\lambda = 1$ . In (7) the left side of the inequality is the expected payment the borrower makes on the contract offered to her type, while the right side is the expected cost of cheating. If agent type  $i$  misreports her type as  $j$ , then with probability  $1 - j_j$  screening does not occur, and with probability  $j_j$  there is screening, cheating is discovered, and the borrower is denied a loan.<sup>3</sup> In a separating equilibrium, there must also be no other contract which yields the market expected return for that class of contracts while making the borrowers who accept it strictly better off.

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<sup>2</sup>The monotonicity restriction arises endogenously in this environment if there is a moral hazard problem, whereby the effort of the lender affects the payoff on the borrower's investment project. This fits many real-world credit relationships, and the moral hazard problem can be specified in such a way that the analysis here is unaffected (see Wang and Williamson 1994).

<sup>3</sup>Note that, if a loan is denied, the borrower cannot go to another bank, as he can contact only one bank in period 1.

A pooling equilibrium is a payment schedule  $R(x)$  satisfying

$$0 \leq R(x) \leq \bar{x} \quad (9)$$

$$x \geq y \implies R(x) \geq R(y); x, y \in [0, \bar{x}] \quad (10)$$

$$\int_0^{\bar{x}} R(x) dF_g(x) + (1 - ) \int_0^{\bar{x}} R(x) dF_b(x) = r_p. \quad (11)$$

Here, (9), (10), and (11) are analogous to (5), (6), and (8), respectively. In (11), note that the pooling contract is accepted by type  $g$  and type  $b$  agents in their population proportions. As for a separating equilibrium, a pooling equilibrium has the property that there exists no other contract which yields the market expected return for that class of contracts while yielding higher expected utility for the borrowers who accept it.

As previously noted,  $r_g = r_b = r_p = r$  in equilibrium, so that the arguments in Wang and Williamson (1994) go through virtually unchanged. If an equilibrium exists it is a separating equilibrium, and an equilibrium may not exist. In equilibrium, we have  $i_g > 0$  and  $i_b = 0$ , so that only type  $g$  borrowers are screened in equilibrium, and each borrower type receives a debt contract, i.e.  $R_i(x) = x$ , for  $x \leq R_i$ , and  $R_i(x) = R_i$  for  $x \geq R_i$ , where  $R_i$  is a constant,  $i = g, b$ .<sup>4</sup> Here,  $R_g$ ,  $R_b$ , and  $i_g$  are determined by the following three equations.

$$R_b - \int_0^{R_b} F_b(x) dx = r \quad (12)$$

$$r = (1 - i_g)[R_g - \int_0^{R_g} F_b(x) dx] + i_g \bar{b} \quad (13)$$

$$R_g - \int_0^{R_g} F_g(x) dx = r + i_g \check{\phantom{b}} \quad (14)$$

Equations (12) and (14) are versions of (8) for  $i = b, g$ , respectively, and equation (13) is the binding incentive constraint for type  $b$  borrowers [(7) with equality for  $i = b$  and  $j = g$ ]. Note that the left side of equation (12) is the expected payment made by a type  $b$  agent, simplified using integration by parts, and that similar expressions occur on the right side of equation (13) and the left side of equation

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<sup>4</sup>A debt contract is the unique equilibrium contract for type  $g$  borrowers, but there exists a continuum of equilibrium contracts for type  $b$  borrowers, one of which is a debt contract.

(14). The solution to equations (12), (13), and (14) must also have the property that  $R_g \leq R$ , where  $R$  is the solution to

$$[R - \int_0^R F_g(x)dx] + (1 - \lambda)[R - \int_0^R F_b(x)dx] = r.$$

That is,  $R$  characterizes the best alternative pooling contract which could be offered, among those that earn zero expected profits for the bank. If  $R_g > R$ , then both types prefer this pooling contract to the separating contract they are offered, and an equilibrium does not exist. Given our parameter restrictions, and following Wang and Williamson (1994), for any  $r$  an equilibrium exists if  $\lambda$  is sufficiently small. Here, we assume throughout that an equilibrium exists.

Given the solution for  $r$  and  $\lambda$  from (3) and (4) we can then solve for  $R_g, R_b$ , and  $j_g$  using (12), (13), and (14).

## 4. Equilibrium With No Communication Across Locations and No Government Intervention

Now, suppose that there is no communication across locations, so that the branch banking arrangement (or an equivalent arrangement with private note issue) studied in the previous section is not feasible. Now, movers can consume only by withdrawing risk free assets from the bank at the end of period 1 to take with them to another location. We must then have  $s_m(\beta, 1) = s_m(\beta, 0)$ , as the aggregate shock is unknown at the time agents move locations. Let  $\ell(\beta)$  denote the fraction of the quantity of liquid assets which the bank pays out to movers. Then, in equilibrium each bank meets the following two constraints.

$$s_m(\beta) = s_m(\beta, 1) = s_m(\beta, 0) = \frac{\ell(\beta)\lambda}{\beta}, \quad (15)$$

$$s_n(\beta, \lambda) = \frac{[1 - \ell(\beta)]\lambda + (\lambda_g r_g + \lambda_b r_b + \lambda_p r_p)}{1 - \beta}. \quad (16)$$

That is, movers consume some fraction of the risk free assets in the bank's portfolio, and nonmovers consume the remainder. As was the case in the previous section, we must have  $r_g = r_b = r_p = r$  in equilibrium. Banks, in equilibrium, choose  $\lambda$  and  $\ell(\beta)$  to solve (1) subject to (15) and

$$s_n(\beta, \lambda) = \frac{[1 - \ell(\beta)]\lambda + \lambda(1 - \lambda)r}{1 - \beta}.$$

The first-order conditions which characterize an equilibrium are then

$$u'(s_m(\beta)) - \alpha u'(s_n(\beta, 1)) - (1 - \alpha)u'(s_n(\beta, 0)) = 0 \quad (17)$$

and

$$\int_0^1 \{ \alpha(1 - r)u'(s_n(\beta, 1)) + (1 - \alpha)u'(s_n(\beta, 0)) \} dG(\beta) = 0. \quad (18)$$

Here, (18) has been simplified using (17). Also, in equilibrium (4) holds. Note, from (15), (16), and (17), that  $s_n(\beta, 0) < s_m(\beta) < s_n(\beta, 1)$ . That is, nonmovers consume less than movers in bad aggregate states, and more than movers in good states.

The credit market works similarly to the case in the previous section with branch banking. That is, given  $r$ , which is determined above, equations (12), (13), and (14) determine  $R_g$ ,  $R_b$ , and  $i_g$ .

With no restrictions on banking, the risk free asset is used by banks to insure agents against bad aggregate shocks. Here, where branch banking is restricted, the risk free asset plays an additional role as a transactions medium which allows movers to consume in the second period. Note that the restrictions on banking cause agents to bear some idiosyncratic risk which they did not bear when there was branch banking. That is, contingent consumption allocations are different for movers than for nonmovers, with nonmovers bearing all of the risk associated with the aggregate technology shock. In addition, consumption allocations for movers and nonmovers will vary with  $\beta$ , so that agents bear some aggregate risk which was not present with branch banking.

## 5 5. Discount Window Lending

As in Section 4, we assume that there is no communication across locations. We now suppose that there is a government agent at each location who can issue liabilities contingent on the aggregate shock, and can acquire loans as assets. That is, the government operates a discount window at the end of period 1, offering a security which will pay  $w_i$  in period 2 if  $\hat{\cdot} = 1$  and 0 if  $\hat{\cdot} = 0$ , in exchange for a loan contract of type  $i$ ,  $i = g, b, p$ . The government can observe the terms of contracts with borrowers, and abides by these terms, but it does not have access to the screening technology and it cannot observe whether screening takes place. Government liabilities can be carried by movers between locations. Thus, a mover who receives a government liability from his or her bank takes this liability to

another location where it is redeemed (contingent on the aggregate shock) by the government agent at that location.

In equilibrium, the government will not offer to discount loans which involve screening since, if it made such an offer, it would be subject to a moral hazard problem. That is, suppose the government offered to discount loans which were subject to screening. Then, banks would have no incentive to screen, and would simply exchange the loan for government securities at a profit. As in the previous section, the equilibrium must be a separating equilibrium where type  $g$  borrowers are screened and type  $b$  borrowers are not. Therefore, only loans to type  $b$  borrowers will be discounted.<sup>5</sup> Now, since type  $b$  loans will be more liquid than type  $g$  loans, we will in general have  $r_g \neq r_b$ .

In equilibrium, the bank will choose  $\ell'$ , the fraction of assets held as liquid assets,  $\ell$ , the fraction of loans made by a bank to type  $g$  borrowers,  $\ell(\beta)$ , the fraction of liquid assets paid out to movers, and  $b(\beta)$ , the fraction of loans to type  $b$  borrowers that are exchanged for government securities, to solve (1) subject to

$$s_m(\beta, 1) = \frac{\ell(\beta)\ell' + b(\beta)(1 - \ell')(1 - \beta)r_b}{\beta}$$

$$s_m(\beta, 0) = \frac{\ell(\beta)\ell'}{\beta}$$

$$s_n(\beta, 1) = \frac{[1 - \ell(\beta)]\ell' + [1 - b(\beta)](1 - \ell')(1 - \beta)r_b + (1 - \ell')r_g}{1 - \beta}$$

$$s_n(\beta, 0) = \frac{[1 - \ell(\beta)]\ell'}{1 - \beta}$$

Here, to simplify we suppose that all government securities received in exchange for loans are paid out to movers. This assumption serves only to determine  $b(\beta)$  and is otherwise inconsequential. In equilibrium, we have

$$\ell' = \ell \tag{19}$$

and

$$\ell = \ell(\beta) \tag{20}$$

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<sup>5</sup>There would be no moral hazard problem if the returns on borrowers' projects were observable in other locations, or these returns could be communicated to government agents in other locations. Then, the payoffs on government securities could be made contingent on the payoff on the loan portfolio directly backing them.

i.e. markets in loans to type  $g$  and  $b$  borrowers clear. The existence of a discount window now permits movers and nonmovers to receive the same consumption allocation for some realizations of  $\beta$ . That is, the solution to the above optimization problem gives

$$s_m(\beta, 1) = s_n(\beta, 1) = \beta' + (1 - \beta')(1 - \beta)r_b + (1 - \beta)r_g, \beta \leq \beta^*,$$

and

$$s_m(\beta, 0) = s_n(\beta, 0) = \beta', \beta \leq \beta^*,$$

where

$$\beta^* = \frac{(1 - \beta)r_b}{(1 - \beta)r_b + r_g}.$$

That is, if the fraction of movers in the population is sufficiently small, then at the end of period 1 the bank will be holding enough loans to type  $b$  borrowers that some of these loans can be discounted so as to provide movers with the same consumption as nonmovers in each aggregate state. For  $\beta \leq \beta^*$  we also have

$$\ell(\beta) = \beta$$

and

$$\ell_b(\beta) = \beta \left( \frac{(1 - \beta)r_b + r_g}{(1 - \beta)r_b} \right).$$

When  $\beta \geq \beta^*$ , we have  $\ell_b(\beta) = 1$ , that is, all loans to type  $b$  borrowers are discounted, and movers and nonmovers do not receive the same consumption allocation. That is, we have

$$s_m(\beta, 1) = \frac{\ell(\beta)' + (1 - \beta')(1 - \beta)r_b}{\beta}, \beta \geq \beta^*,$$

$$s_m(\beta, 0) = \frac{\ell(\beta)'}{\beta}, \beta \geq \beta^*,$$

$$s_n(\beta, 1) = \frac{[1 - \ell(\beta)]' + (1 - \beta)r_g}{1 - \beta}, \beta \geq \beta^*,$$

$$s_n(\beta, 0) = \frac{[1 - \ell(\beta)]'}{1 - \beta}, \beta \geq \beta^*.$$

From the above optimization problem, we have the following first-order conditions for  $\beta > \beta^*$ , and if  $r_g > r_b$  (which must hold in equilibrium).

$$\begin{aligned} & \alpha u'(s_m(\beta, 1)) + (1 - \alpha)u'(s_m(\beta, 0)) \\ & - \alpha u'(s_n(\beta, 1)) - (1 - \alpha)u'(s_n(\beta, 0)) = 0, \beta > \beta^* \end{aligned} \quad (21)$$

$$\begin{aligned} & \alpha[1 - (1 - \lambda)r_b - r_g]u'(\lambda + (1 - \lambda)(1 - \lambda)r_b + (1 - \lambda)r_g)G(\beta^*) \\ & + (1 - \alpha)G(\beta^*)u'(\lambda) \\ & + \int_{\beta^*}^1 \left[ \begin{aligned} & -\alpha(1 - \lambda)r_b u'(s_m(\beta, 1)) \\ & + \alpha(1 - r_g)u'(s_n(\beta, 1)) + (1 - \alpha)u'(s_n(\beta, 0)) \end{aligned} \right] dG(\beta) = 0 \end{aligned} \quad (22)$$

$$\begin{aligned} & (-r_b + r_g)u'(\lambda + (1 - \lambda)(1 - \lambda)r_b + (1 - \lambda)r_g)G(\beta^*) \\ & + \int_{\beta^*}^1 [-r_b u'(s_m(\beta, 1)) + r_g u'(s_n(\beta, 1))] dG(\beta) = 0 \end{aligned} \quad (23)$$

In equilibrium, we must have  $r_g > r_b$ . If  $r_g \leq r_b$ , banks would strictly prefer to make loans to type  $b$  agents rather than to type  $g$  agents, since loans to type  $b$  agents can always be discounted and are therefore perfectly liquid, while loans to type  $g$  agents are illiquid. The states of the world where  $\beta > \beta^*$  can be interpreted as banking panic states, as this is when the constraint that discount window loans be backed by “eligible securities” becomes binding. When  $\beta > \beta^*$ , all loans to type  $b$  agents made by banks are sold to the government in exchange for securities which can be used as a medium of exchange. If banks could discount more loans in these states of the world, they would, as this would provide better risk-sharing between movers and nonmovers. The only reason all loans cannot be discounted here is that the discounting of loans to type  $g$  borrowers is subject to a moral hazard problem.

## 6 6. Discount Window Lending and Deposit Insurance

The fact that discount window lending is imperfect, due to the moral hazard problem discussed in the previous section, potentially provides a role for deposit insurance in improving on the allocation of aggregate risk. When the only government intervention is discount window lending, risk sharing is imperfect in that movers consume less than nonmovers in the good aggregate state ( $\lambda = 1$ ), and movers consume more than nonmovers in bad aggregate states ( $\lambda = 0$ ) if the number of movers is sufficiently large, i.e. if the demand for liquid assets is

sufficiently high. Thus, it would seem that, if there is a mechanism which can transfer resources from movers to nonmovers in bad aggregate states, and do the reverse in good aggregate states, then this would create better risk-sharing. Such an arrangement resembles deposit insurance, in that the remaining depositors in the bank (the movers) receive compensation when the return on banks' portfolios is low, and they pay an insurance premium when this return is high.

Here, we will assume an imperfection in the deposit insurance system. That is, the government cannot observe  $\beta$ , the fraction of movers in the population. Therefore, transfers cannot be contingent on  $\beta$ . It is also assumed that the government cannot distinguish between movers and nonmovers (if it could, then clearly the government could observe  $\beta$ ). The deposit insurance system will take the form of an equal lump-sum transfer (or tax levy) for each agent, with a corresponding lump-sum tax (or transfer) on each bank, proportional to the number of depositors in the bank. These taxes and transfers are contingent on  $\ell$ . Letting  $\phi_\ell$  denote the lump-sum transfer that each agent receives in state  $\ell$ ,  $\ell = 0, 1$ , the payoffs that movers and nonmovers receive are then

$$s_m(\beta, 1) = \frac{\ell(\beta)'_\ell + b(\beta)(1 - \ell)(1 - \beta)r_b}{\beta} + \phi_1 \quad (24)$$

$$s_m(\beta, 0) = \frac{\ell(\beta)'_\ell}{\beta} + \phi_0 \quad (25)$$

$$s_n(\beta, 1) = \frac{[1 - \ell(\beta)]'_\ell + [1 - b(\beta)](1 - \ell)(1 - \beta)r_b + (1 - \ell)r_g - \phi_1}{1 - \beta} + \phi_1 \quad (26)$$

$$s_n(\beta, 0) = \frac{[1 - \ell(\beta)]'_\ell - \phi_0}{1 - \beta} + \phi_0 \quad (27)$$

In equilibrium, banks set  $\ell(\beta)$ ,  $b(\beta)$ ,  $'_\ell$ , and  $\beta$  so as to maximize (??) subject to (24)-(27), treating  $r_b$ ,  $r_g$ , and  $\phi_\ell$ ,  $\ell = 0, 1$ , as being fixed. Given  $'_\ell$  and  $\beta$ , if it is feasible for the bank to equate consumption for movers and nonmovers when  $\ell = 0$  and when  $\ell = 1$ , then this is optimal. In equilibrium we have  $'_\ell = \beta$  and  $\beta = \beta$ . For movers and nonmovers to have the same consumption in equilibrium when  $\ell = 0$ , i.e.  $s_m(\beta, 0) = s_n(\beta, 0)$ , requires, from (25) and (27), that

$$\ell(\beta) = \frac{\beta(' - \phi_0)}{'} ,$$

and since  $0 \leq \ell(\beta) \leq 1$ , we need

$$' \geq \phi_0 \quad (28)$$

and

$$\beta(\lambda - \phi_0) \leq \lambda. \quad (29)$$

Similarly, equal consumption for movers and nonmovers when  $\lambda = 1$ , i.e.  $s_m(\beta, 1) = s_n(\beta, 1)$ , requires, given (28) and (29), that

$$(1 - \lambda)[(1 - \lambda)r_b + r_g] - \phi_1 \geq 0 \quad (30)$$

and

$$\beta\{(1 - \lambda)[(1 - \lambda)r_b + r_g] - \phi_1\} \leq (1 - \lambda)(1 - \lambda)r_b. \quad (31)$$

Here, the ability of the bank to achieve perfect risk sharing is potentially limited by three factors;  $\beta$ , the number of movers in the population;  $\lambda$ , the fraction of the loan portfolio held in the form of assets which cannot be used as collateral for discount window lending; and  $\lambda'$ , the fraction of the bank's portfolio consisting of risk-free assets. Here, conditions (28) and (30) will always hold as strict inequalities in equilibrium when the government chooses  $\phi_0$  and  $\phi_1$  optimally. There is then some  $\beta^* \in (0, 1)$  such that perfect risk-sharing occurs if and only if  $\beta \leq \beta^*$ , where, from (29) and (31),

$$\beta^* = \min \left( \frac{\lambda'}{\lambda' - \phi_0}, \frac{(1 - \lambda)(1 - \lambda)r_b}{(1 - \lambda)[(1 - \lambda)r_b + r_g] - \phi_1} \right). \quad (32)$$

Here, the first-order conditions for an optimum become more complicated, particularly due to the possibility, implicit in (32), that holdings of either risk-free assets or type  $b$  loans could be the binding constraint which limits perfect risk-sharing. In the following section we will consider a special case where the analysis is more straightforward.

Given the optimizing choices of  $\lambda(\beta)$ ,  $\lambda_b(\beta)$ ,  $\lambda'$ , and  $\lambda$  by banks, the government sets  $\phi_0$  and  $\phi_1$  to maximize the expected utility of the representative depositor, treating  $r_g$  and  $r_b$  as given, which yields the following first-order conditions.

$$\int_0^1 \beta \{u'(s_m(\beta, j)) - u'(s_n(\beta, j))\} dG(\beta) = 0, j = 0, 1.$$

## 7. Results For A Special Case

To compare the properties of equilibria across regimes, we consider the following special case, where  $\beta$  can take on only two values,  $\beta_1$  and  $\beta_2$ , with  $\Pr[\beta = \beta_1] = \lambda$ ,

$0 < \beta < 1$ . Assume that  $\beta_1 < 1 - \beta < \beta_2$ . The expected utility of each agent is then

$$\beta \left\{ \begin{array}{l} \beta_1 [\alpha u(s_m(\beta_1, 1)) + (1 - \alpha)u(s_m(\beta_1, 0))] \\ + (1 - \beta_1) [\alpha u(s_n(\beta_1, 1)) + (1 - \alpha)u(s_n(\beta_1, 0))] \end{array} \right\} \\ + (1 - \beta) \left\{ \begin{array}{l} \beta_2 [\alpha u(s_m(\beta_2, 1)) + (1 - \alpha)u(s_m(\beta_2, 0))] \\ + (1 - \beta_2) [\alpha u(s_n(\beta_2, 1)) + (1 - \alpha)u(s_n(\beta_2, 0))] \end{array} \right\}, \quad (33)$$

where

$$s_m(\beta_i, 1) = \frac{i'_\ell + (1 - i'_\ell)(1 - i'_b)r_b}{\beta_i} + \phi_1, \quad (34)$$

$$s_m(\beta_i, 0) = \frac{i'_\ell}{\beta_i} + \phi_0, \quad (35)$$

$$s_n(\beta_i, 1) = \frac{(1 - i'_\ell)'_\ell + (1 - i'_\ell)[(1 - i'_b)r_b + r_g] - \phi_1}{1 - \beta_i} + \phi_1, \quad (36)$$

$$s_n(\beta_i, 0) = \frac{(1 - i'_\ell)'_\ell - \phi_0}{1 - \beta_i} + \phi_0, \quad (37)$$

for  $i = 1, 2$ . This special case has the property that the regime with discount window lending and deposit insurance gives perfect risk sharing. That is, the allocation is the same as when there is full communication across locations. The equilibrium allocation with discount window lending and deposit insurance will involve perfect risk-sharing if there exists  $i'_j$ ,  $i = 1, 2$ ,  $j = \ell, b$ , with  $0 \leq i'_j \leq 1$ ,  $i = 1, 2$ ,  $j = \ell, b$ , satisfying  $s_m(\beta_i, j) = s_n(\beta_i, j)$  for  $i = 1, 2$ , and  $j = 0, 1$ , with  $i'_\ell = i'_b$  and  $i'_\ell = i'_b$ . Using (34)-(37) to solve for  $i'_j$ ,  $i = 1, 2$ ,  $j = \ell, b$  in terms of  $i'_\ell$ ,  $i'_b$ ,  $\phi_0$ ,  $\phi_1$ ,  $r_b$ , and  $r_g$ , we get

$$i'_\ell = \beta_i \left(1 - \frac{\phi_0}{r}\right) \quad (38)$$

and

$$i'_b = \beta_i \left\{ \frac{\phi_0 - \phi_1 + (1 - i'_\ell)[r_g + (1 - i'_\ell)r_b]}{(1 - i'_\ell)(1 - i'_\ell)r_b} \right\} \quad (39)$$

for  $i = 1, 2$ . Now, if we can find values for  $\phi_0$  and  $\phi_1$  such that  $0 \leq i'_j \leq 1$ ,  $i = 1, 2$ ,  $j = \ell, b$  in (38) and (39), then this is an equilibrium. If there is perfect risk sharing, we must have  $r_g = r_b = r$ , since the only case where loans to types  $g$  and  $b$  borrowers would have different expected payoffs, conditional on aggregate shocks, is if the illiquidity of type  $g$  loans is binding in equilibrium, which it would

not be with perfect risk sharing. Manipulation of (38) and (39) gives the following conditions, which  $\phi_0$  and  $\phi_1$  must satisfy, given that  $i_\ell \in [0, 1]$ , for  $i = 0, 1$ .

$$\phi_0 \leq \phi_1 \quad (40)$$

$$\phi_0 \geq \phi_1 \left(1 - \frac{1}{\beta_i}\right), i = 1, 2 \quad (41)$$

$$\phi_1 - \phi_0 \leq (1 - \phi_1)r \quad (42)$$

$$\phi_1 - \phi_0 \geq (1 - \phi_1)r - \frac{(1 - \phi_1)(1 - \phi_1)r}{\beta_i}, i = 1, 2. \quad (43)$$

There then exists a continuum of equilibria with perfect risk sharing, as we can choose any  $\phi_0 \in [\phi_1 \left(1 - \frac{1}{\beta_1}\right), \phi_1]$ , and then choose  $\phi_1$  such that  $\phi_1 - \phi_0 \in [(1 - \phi_1)r - \frac{(1 - \phi_1)(1 - \phi_1)r}{\beta_i}, (1 - \phi_1)r]$ , then substitute in (38) and (39) to get solutions for  $i_j$ ,  $i = 1, 2$ ,  $j = \ell, b$ . The optimal deposit insurance system (i.e.  $\phi_0$  and  $\phi_1$ ) is indeterminate, as is the equilibrium liquidation and discount window policy for banks, but the equilibrium consumption allocation and  $r$  are determinate.

Note that  $\phi_0 = \phi_1 = 0$  does not satisfy (40)-(43), in particular because it is assumed that  $\beta_2 > 1 - \phi_1$  [substitute in (43)]. That is, this example has been parameterized so that, if there is no deposit insurance, then full insurance cannot be an equilibrium outcome. Note also that, in the regime with no discount window lending or deposit insurance, we have  $i_b = 0$ , which violates (39), so full insurance is not a possible equilibrium outcome without government intervention. It remains to characterize equilibria in the two cases with imperfect insurance.

## 7.1 7.1 No Intervention

In this case, we have  $\phi_0 = \phi_1 = 0$  and  $i_b = 0$ ,  $i = 1, 2$ . From (34) and (35), we then have  $s_m(\beta_i, 1) = s_m(\beta_i, 0)$  for  $i = 1, 2$ , and in equilibrium we will have  $r_g = r_b = r$ , as loans to either type of borrower yield payoffs only to nonmovers when  $\phi = 1$ . The first-order conditions characterizing an equilibrium, analogous to (17) and (18), are

$$u'(s_m(\beta_i, 1)) - \alpha u'(s_n(\beta_i, 1)) - (1 - \alpha)u'(s_n(\beta_i, 0)) = 0, i = 1, 2, \quad (44)$$

$$\begin{aligned} & \phi_1 [(1 - r)\alpha u'(s_n(\beta_1, 1)) + (1 - \alpha)u'(s_n(\beta_1, 0))] \\ & + (1 - \phi_1) [(1 - r)\alpha u'(s_n(\beta_2, 1)) + (1 - \alpha)u'(s_n(\beta_2, 0))] = 0. \end{aligned} \quad (45)$$

Given  $\phi_1$ , equations (44), (45) and (34)-(37) determine  $s_k(\beta_i, j)$ ,  $r$ , and  $i_\ell$  for  $k = m, n$ ,  $i = 1, 2$ , and  $j = 0, 1$ . We also require the equilibrium conditions (19) and (20).

## 7.2 Discount Window Lending With No Deposit Insurance

Here, we have  $\phi_0 = \phi_1 = 0$ . As shown previously, perfect insurance cannot be an equilibrium outcome in this regime. In equilibrium, banks choose  $\beta_i^i, \beta_b^i, i = 1, 2, \ell$ , and to maximize (33) subject to (34)-(37). Given that  $u'(0) = \infty$ , we must have interior solutions for  $\beta_i^i, i = 1, 2$ , and the following first-order conditions hold in equilibrium.

$$\begin{aligned} & \alpha u'(s_m(\beta_i, 1)) + (1 - \alpha)u'(s_m(\beta_i, 0)) \\ & - \alpha u'(s_n(\beta_i, 1)) - (1 - \alpha)u'(s_n(\beta_i, 0)) = 0, i = 1, 2. \end{aligned} \quad (46)$$

Also, in equilibrium we have  $\beta_\ell = \beta$  and  $\beta = \beta$ . That is, we must have interior solutions for  $\beta_\ell$  and  $\beta$ , and the following first-order conditions must hold, using (46) to simplify.

$$\begin{aligned} & \beta \left\{ \begin{array}{l} -(1 - \beta_b)r_b \alpha u'(s_m(\beta_1, 1)) \\ + [1 - (1 - \beta)(1 - \frac{1}{\beta})r_b - r_g] \alpha u'(s_n(\beta_1, 1)) \\ + (1 - \alpha)u'(s_n(\beta_1, 0)) \end{array} \right\} \\ & + (1 - \beta) \left\{ \begin{array}{l} -(1 - \beta_b)^2 r_b \alpha u'(s_m(\beta_2, 1)) \\ + [1 - (1 - \beta)(1 - \frac{2}{\beta})r_b - r_g] \alpha u'(s_n(\beta_2, 1)) \\ + (1 - \alpha)u'(s_n(\beta_2, 0)) \end{array} \right\} = 0, \end{aligned} \quad (47)$$

$$\begin{aligned} & \beta \left\{ -\frac{1}{\beta} r_b u'(s_m(\beta_1, 1)) + [r_g - (1 - \frac{1}{\beta})r_b] u'(s_n(\beta_1, 1)) \right\} \\ & + (1 - \beta) \left\{ -\frac{2}{\beta} r_b u'(s_m(\beta_2, 1)) + [r_g - (1 - \frac{2}{\beta})r_b] u'(s_n(\beta_2, 1)) \right\} = 0. \end{aligned} \quad (48)$$

If banks chose interior solutions for  $\beta_b^i$  in equilibrium,  $i = 1, 2$ , then we would have, using (33) and (34)-(37),  $u'(s_m(\beta_i, 1)) = u'(s_n(\beta_i, 1))$  for  $i = 1, 2$ , that is  $s_m(\beta_i, 1) = s_n(\beta_i, 1)$  for  $i = 1, 2$ . Then, (46) implies a perfect insurance result where consumption is equated for movers and nonmovers in each state. Since perfect insurance cannot be an equilibrium outcome, it therefore must be the case that  $\beta_b^i = 1$  or  $\beta_b^i = 0$  for some  $i$ . Suppose that  $\beta_b^i = 0$ , which implies that  $s_n(\beta_i, 1) \leq s_m(\beta_i, 1)$ , and, from (34) and (35), that  $s_m(\beta_i, 1) = s_m(\beta_i, 0)$ . Also, from (36) and (37), we have  $s_n(\beta_i, 1) > s_n(\beta_i, 0)$ . But then (46) does not hold, a contradiction. Therefore  $0 < \beta_b^i \leq 1$  for all  $i$ . We then have  $s_m(\beta_i, 1) \leq s_n(\beta_i, 1)$  for all  $i$ . In (48), then, we either have  $\beta_b^i = 1$  and  $s_m(\beta_i, 1) \leq s_n(\beta_i, 1)$  or  $0 < \beta_b^i < 1$  and  $s_m(\beta_i, 1) = s_n(\beta_i, 1)$ . But this implies that, if  $r_g \leq r_b$ , then the left side of (48) is negative, a contradiction. Therefore,  $r_g > r_b$ .

Now, suppose that  $0 < \frac{i}{b} < 1$ , which implies that  $s_m(\beta_i, 1) = s_n(\beta_i, 1)$  and  $s_m(\beta_i, 0) = s_n(\beta_i, 0)$ . From (34)-(37), we then have

$$\frac{i}{b} = \frac{\beta_i[(1 - \alpha)r_b + r_g]}{(1 - \alpha)r_b}. \quad (49)$$

But, since  $\beta_2 > 1 - \alpha$ , (49) then implies that  $\frac{i}{b} > 1$ , a contradiction. Therefore,  $\frac{i}{b} = 1$ . That is, when  $\beta = \beta_2$  the fraction of movers in the population is large enough relative to the fraction of bad types in the population of borrowers, that there is an insufficient quantity of liquid assets for banks to provide perfect insurance. When  $\beta = \beta_1$ , it is not clear whether  $\frac{i}{b} = 1$  or  $0 < \frac{i}{b} < 1$ . However, we know that, if there are only lenders in the population, i.e.  $\alpha = 1$ , then banks hold only risk-free assets in equilibrium, there is perfect insurance, and  $r_b = r_g = r$ . Then, by continuity, and given that  $\beta_1 < 1 - \alpha$ , (49) implies that  $0 < \frac{i}{b} < 1$  if  $\alpha$  is sufficiently close to 1.

### 7.3 Comparative Statics

Here, we will use a comparative statics approach to compare risk premia on loans, welfare, and the severity of banking panics in the three different regimes, for the special case discussed above. Here, the risk premium on a loan made to a type  $i$  borrower is  $\alpha r_i - 1$ , and this premium captures the willingness of banks to hold risky loans. The conditional expected returns,  $r_g$  and  $r_b$ , are also important in determining  $j_g$ , the screening probability for type  $g$  borrowers, which is a measure of the care taken by banks in screening loans. Intuitively, it seems that risk premia should fall, welfare should increase, and the severity of banking panics should decrease with government interventions which promote risk-sharing. These are the results we will obtain, but we need to do some work to show this.

As a benchmark, consider the case where all agents in the population are lenders, i.e.  $\alpha = 1$ . Here, all investment is made in the risk free asset, and each agent consumes one unit in the second period. Banks are purely transparent in this case, but in an equilibrium with banks we will have  $\frac{i}{b} = \beta_i$ , i.e. the fraction of assets paid out to movers at the end of the first period is just a proportional share in the bank's total assets. Risky loans are priced according to  $r_g = r_b = \frac{1}{\alpha}$ , that is, since no agent bears any aggregate risk, the expected return on all assets is unity, and risk premia are zero. Consumption allocations and prices are identical across the three regimes.

The comparative statics experiment we will conduct amounts to introducing a small quantity of risky assets into the above risk-free economy, and determining

the effects on asset prices and welfare across all three regimes. That is, we totally differentiate the relevant set of equations determining equilibrium in each regime, and then evaluate derivatives in the equilibrium where  $\delta = 1$ . The details of the comparative statics here are quite tedious, and available from the author on request.

### 7.3.1 7.3.1 Effects on Risk Premia

We will use superscripts on variables to denote regimes, for example  $r_g^j$  is the conditional expected return to the bank from a loan to a type  $g$  borrower in regime  $j$ , where  $j = A$  denotes the regime with no intervention,  $j = B$  denotes the regime with discount window lending only, and  $j = C$  denotes the regime with discount window lending and deposit insurance. With no intervention, we obtain

$$\frac{dr_g^A}{d\delta} \Big|_{\delta=1} = \frac{dr_b^A}{d\delta} \Big|_{\delta=1} = \frac{(1-\alpha)u''(1)}{\alpha^2 u'(1)} \left( \frac{\beta}{1-\beta_1} + \frac{1-\beta}{1-\beta_2} \right). \quad (50)$$

In the regime with discount window lending only, we have

$$\frac{dr_g^B}{d\delta} \Big|_{\delta=1} = \frac{(1-\alpha)u''(1)}{\alpha^2 u'(1)} \left( \beta + \frac{(1-\beta)}{1-\beta_2} \right), \quad (51)$$

$$\frac{dr_b^B}{d\delta} \Big|_{\delta=1} = \frac{(1-\alpha)u''(1)}{\alpha^2 u'(1)} \left( \beta + \frac{(1-\beta)(1-\beta)}{\beta_2} \right). \quad (52)$$

Finally, with discount window lending and deposit insurance,

$$\frac{dr_g^C}{d\delta} \Big|_{\delta=1} = \frac{dr_b^C}{d\delta} \Big|_{\delta=1} = \frac{(1-\alpha)u''(1)}{\alpha^2 u'(1)}. \quad (53)$$

From (50)-(53), we then have that  $\frac{dr_g^A}{d\delta} \Big|_{\delta=1} < \frac{dr_g^B}{d\delta} \Big|_{\delta=1} < \frac{dr_g^C}{d\delta} \Big|_{\delta=1} < 0$  and  $\frac{dr_b^A}{d\delta} \Big|_{\delta=1} < \frac{dr_b^B}{d\delta} \Big|_{\delta=1} < 0$ . Introducing a small amount of aggregate risk into the riskless economy, i.e. decreasing  $\delta$  a small amount from  $\delta = 1$ , results in an increase in the risk premia on loans. For type  $g$  loans, the increase in the risk premium is greatest when there is no government intervention, smaller when there is discount window lending only, and smallest when there is discount window lending and deposit insurance. That is, discount window lending and deposit insurance contribute to better sharing of aggregate risk in this environment, and this lowers the risk premium on type  $g$  loans. The story is somewhat different for type  $b$  loans because

of the special role that these loans play, in the regime with discount window lending only, as the only bank asset which is discounted. The risk premium on type  $b$  loans is lower in regime  $B$  than in regime  $C$ , because type  $b$  loans provide liquidity in the discount window regime while type  $g$  loans do not.

The above results are broadly consistent with the notion that deposit insurance encourages banks to take on more risk. Here, deposit insurance tends to reduce risk premia, i.e. banks are more willing to hold risky loans. The exception to this is in moving from regime  $B$  to regime  $C$ , in that the risk premium on type  $b$  loans rises in this case, i.e. the demand for type  $b$  loans falls in this case because these loans become less important for the sharing of aggregate risk.

### 7.3.2 7.3.2 Effects on the Welfare of Depositors

We will let  $W_j$  denote the expected utility of lenders in regime  $j$ , where  $j = A, B, C$ . When  $\theta = 1$ , we have  $W_A = W_B = W_C = u(1)$ . Comparative statics gives

$$\frac{dW_A}{d\theta} \Big|_{\theta=1} = \frac{dW_B}{d\theta} \Big|_{\theta=1} = \frac{dW_C}{d\theta} \Big|_{\theta=1} = 0,$$

but evaluating second derivatives for  $\theta = 1$ , we obtain

$$\frac{d^2 W_A}{d\theta^2} \Big|_{\theta=1} = -\alpha u'(1) \frac{dr^A}{d\theta} \Big|_{\theta=1}, \quad (54)$$

$$\frac{d^2 W_B}{d\theta^2} \Big|_{\theta=1} = -\alpha u'(1) \left[ (1 - \alpha) \frac{dr_b^B}{d\theta} \Big|_{\theta=1} + \frac{dr_g^B}{d\theta} \Big|_{\theta=1} \right], \quad (55)$$

$$\frac{d^2 W_C}{d\theta^2} \Big|_{\theta=1} = -\alpha u'(1) \frac{dr^C}{d\theta} \Big|_{\theta=1}. \quad (56)$$

We then have, substituting using (50)-(53) in (54)-(56),

$$\begin{aligned} \frac{d^2 W_A}{d\theta^2} \Big|_{\theta=1} &= -\frac{(1 - \alpha)u''(1)}{\alpha} \left( \frac{\theta}{1 - \beta_1} + \frac{1 - \theta}{1 - \beta_2} \right), \\ \frac{d^2 W_B}{d\theta^2} \Big|_{\theta=1} &= -\frac{(1 - \alpha)u''(1)}{\alpha} \left( \theta + (1 - \theta) \left( 1 + \frac{(1 - \alpha - \beta_2)^2}{\beta_2(1 - \beta_2)} \right) \right), \\ \frac{d^2 W_C}{d\theta^2} &= -\frac{(1 - \alpha)u''(1)}{\alpha} \end{aligned}$$

Given that  $\beta_2 > 1 - \beta_1$ , we then have  $\frac{d^2 W_A}{d^2 \beta_1} |_{\beta_1=1} > \frac{d^2 W_B}{d^2 \beta_1} |_{\beta_1=1} > \frac{d^2 W_C}{d^2 \beta_1} |_{\beta_1=1} > 0$ . Therefore, for  $\beta_1$  close to 1, i.e. given a small quantity of risky assets in this economy, welfare is highest in regime *C*, lower in regime *B*, and lowest in regime *A*. Thus, adding each layer of government intervention increases welfare for lenders. These results cannot be obtained trivially, as the deposit insurance scheme is set up so as to maximize the welfare of depositors, treating market loan rates as given, but in general equilibrium loan rates are altered by the existence of deposit insurance. The discount window acts to make a subset of assets more liquid, but without deposit insurance markets are in some sense incomplete. Thus, it is not immediately obvious that discount window lending would increase welfare for depositors.

### 7.3.3 7.3.3. Loan Market Effects

Effects on loan market interest rates and screening intensities are determined by  $r_g$  and  $r_b$ , the terms on which banks will hold the supplies of risky assets. The following equations, similar to (12)-(14), determine  $R_g$ ,  $R_b$ , and  $i_g$ , given  $r_g$  and  $r_b$ .

$$R_b - \int_0^{R_b} F_b(x) dx = r_b \quad (57)$$

$$r_b = (1 - i_g)[R_g - \int_0^{R_g} F_b(x) dx] + i_g \bar{b} \quad (58)$$

$$R_g - \int_0^{R_g} F_g(x) dx = r_g + i_g \check{v} \quad (59)$$

From (57)-(59), we obtain the following comparative statics results.

$$\frac{dR_g}{dr_b} = \frac{-\check{v}}{\nabla} > 0 \quad (60)$$

$$\frac{dR_g}{dr_g} = \frac{-[\bar{b} - R_g + \int_0^{R_g} x dF_b(x)]}{\nabla} > 0 \quad (61)$$

$$\frac{dR_b}{dr_b} = \frac{1}{1 - F_b(R_b)} > 0 \quad (62)$$

$$\frac{dR_b}{dr_g} = 0 \quad (63)$$

$$\frac{di_g}{dr_b} = \frac{-[1 - F_g(R_g)]}{\nabla} > 0 \quad (64)$$

$$\frac{dj_g}{dr_g} = \frac{(1 - j_g) [1 - F_b(R_g)]}{\nabla} < 0 \quad (65)$$

In (60)-(65),

$$\nabla = -(1 - j_g)[1 - F_b(R_g)] - [1 - F_g(R_g)] \left[ -r_b - R_g + \int_0^{R_g} x dF_b(x) \right] < 0$$

Therefore, from (50)-(53), since  $r_g$  and  $r_b$  fall in moving from regime *A* to regime *B*, the interest payments and principal paid by type *g* and type *b* borrowers,  $R_g$  and  $R_b$  respectively, must fall. All borrowers are therefore better off in regime *B* than in regime *A*, and since depositors are better off as well, regime *B* is a Pareto improvement on regime *A*. Similarly, interest rates faced by borrowers are lower in regime *C* than in regime *A*, and *C* Pareto dominates *A*. However, regimes *B* and *C* are Pareto non-comparable. That is, since  $r_b^B < r_b^C$  and  $r_g^B > r_g^C$ , type *b* borrowers are better off in regime *B* than in regime *C*, and type *g* borrowers are better off in regime *C*, while depositors are better off in regime *C*.

To determine the effects on screening intensity,  $j_g$ , we use (50)-(53), (64) and (65). In moving from regime *A* to regime *B*, both  $r_b$  and  $r_g$  fall, but  $r_b$  falls more than does  $r_g$ . Therefore,  $j_g$  falls (given first-order stochastic dominance, that is  $F_g(x) < F_b(x)$ ), i.e. banks screen less intensively because the adverse selection problem is less severe. Both borrower types receive a more favorable loan contract, but the improvement is greater for type *b* borrowers, so they are less likely to misreport their type. Similarly,  $r_b$  and  $r_g$  fall in moving from regime *A* to regime *C*, and both fall by the same amount. Therefore, the screening intensity,  $j_g$ , decreases. A change from regime *B* to regime *C* implies an increase in  $r_b$  and a decrease in  $r_g$ , which acts to increase  $j_g$ , since this makes the adverse selection problem in the loan market more severe.

### 7.3.4 Effects on the Severity of Banking Panics

In this model, as in Champ, Smith, and Williamson (1992), a banking panic occurs when there is a shortage of liquid assets, relative to what is needed to provide full insurance. Here, one approach to obtaining a measure of the scarcity of liquid assets is to permit banks to trade assets in each location on a market which opens at the end of the first period. In equilibrium, there will be no trade on this market, but the price of a risky asset in terms of the riskless asset will capture the scarcity of the liquid asset.

Let  $q_i(\beta)$  denote the price of a loan to a type  $i$  agent in terms of riskless assets, for  $i = g, b$ .<sup>6</sup> If  $\beta = 1$ , then given that movers and nonmovers consume one unit in all states of the world, and  $r_g = r_b = 1$  no matter what regime is in place, we have  $q_i(\beta) = 1$  for all  $i$ . As for the comparative statics experiments in the previous sections, we will take this as the benchmark case. A banking panic will be defined as a state where  $q_i(\beta) < 1$  for any  $i$ .

In the regime with no intervention, loans to type  $g$  and  $b$  borrowers are equivalent from the point of view of the bank, so we have  $q_g^A(\beta) = q_b^A(\beta) = q^A(\beta)$  for  $\beta = \beta_1$  and  $\beta = \beta_2$ . Given optimization by banks over loan sales at the end of period 1 and the equilibrium condition that loan sales be equal to zero, we obtain the following asset pricing relationship:

$$q^A(\beta_i)u'(s_m(\beta_i, 1))\beta_i + \alpha u'(s_n(\beta_i, 1)) [q^A(\beta_i)(1 - \beta_i) - r] + (1 - \alpha)u'(s_n(\beta_i, 1)) q^A(\beta_i)(1 - \beta_i) = 0,$$

for  $i = 1, 2$ , or, using (44) to simplify the above expression,

$$q^A(\beta_i)u'(s_m(\beta_i, 1)) - \alpha u'(s_n(\beta_i, 1))r = 0, \quad (66)$$

for  $i = 1, 2$ . From (66), comparative statics then gives

$$\frac{dq^A(\beta_1)}{d\beta} \Big|_{\beta=1} = \frac{u''(1)(1 - \alpha)(1 - \beta_2)(\beta_2 - \beta_1)}{\alpha u'(1)(1 - \beta_1)(1 - \beta_2)} < 0,$$

$$\frac{dq^A(\beta_2)}{d\beta} \Big|_{\beta=1} = \frac{u''(1)(1 - \alpha)(\beta_1 - \beta_2)}{\alpha u'(1)(1 - \beta_1)(1 - \beta_2)} > 0.$$

Therefore, for  $\beta$  close to 1,  $q^A(\beta_1) > 1$  and  $q^A(\beta_2) < 1$ , i.e. a banking panic occurs in the state where the fraction of movers in the population is relatively large, but not in the other state. When  $\beta = \beta_2$ , banks are willing to pay a premium to obtain liquid assets.

With discount window lending only, loans to type  $b$  borrowers are more liquid than are loans to type  $g$  borrowers, so that  $q_g^B \neq q_b^B$ . In a manner similar to what was done above for regime A, we obtain the following asset pricing relationships for the case where  $\beta = \beta_1$ .

$$q_j^B(\beta_1) [\alpha u'(s_m(\beta_1, 1)) + (1 - \alpha)u'(s_m(\beta_1, 0))] - \alpha r_j u'(s_m(\beta_1, 1)) = 0,$$

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<sup>6</sup>Note that there would be a moral hazard problem involved in trading loans to type  $g$  agents, for the same reason that there was moral hazard in discount window lending. The determination of “asset prices” at the end of period 1, is simply the determination of shadow prices for the purposes of measuring the severity of banking panics.

for  $j = g, b$ . For  $\beta = \beta_2$ , we obtain

$$\begin{aligned} -r_b \alpha u'(s_m(\beta_2, 1)) + q_b^B(\beta_2) [\alpha u'(s_n(\beta_2, 1)) + u'(s_n(\beta_1, 0))] &= 0, \\ -r_g \alpha u'(s_n(\beta_2, 1)) + q_g^B(\beta_2) [\alpha u'(s_n(\beta_2, 1)) + u'(s_n(\beta_1, 0))] &= 0. \end{aligned}$$

Comparative statics then gives

$$\begin{aligned} \frac{dq_b^B(\beta_1)}{d'} \Big|_{'=1} &= \frac{u''(1)(1-\alpha)(1-!)}{\alpha u'(1)} \left( \frac{1}{\beta_2} - 1 \right) < 0, \\ \frac{dq_g^B(\beta_1)}{d'} \Big|_{'=1} &= \frac{u''(1)(1-\alpha)(1-!)}{\alpha u'(1)} \left( \frac{1}{1-\beta_2} - 1 \right) < 0, \\ \frac{dq_b^B(\beta_2)}{d'} \Big|_{'=1} &= \frac{u''(1)(1-\alpha)}{\alpha u'(1)} \left( \frac{-(1-\beta_2)}{\beta_2} \right) < 0, \\ \frac{dq_g^B(\beta_2)}{d'} \Big|_{'=1} &= \frac{u''(1)(1-\alpha)!}{\alpha u'(1)} \left( \frac{1-\beta_2-}{1-\beta_2} \right) > 0. \end{aligned}$$

Here, for  $'$  close to 1 we have  $q_j^B(\beta_1) > 1$  for  $j = g, b$ ,  $q_b^B(\beta_2) > 1$ , and  $q_g^B(\beta_2) < 1$ . That is, risky assets command a premium over the riskless asset when  $\beta = \beta_1$ , but when  $\beta = \beta_2$  loans to type  $g$  borrowers sell at a discount in terms of the riskless asset and loans to type  $b$  borrowers sell at a premium. In this regime, the discount window mechanism affects what is classed as a liquid asset. Riskless assets and loans to type  $b$  borrowers are liquid, and in the state where the fraction of movers in the population is high ( $\beta = \beta_2$ ), there is a banking panic ( $q_g^B(\beta_2) < 1$ ).

In regime  $C$ , where there is perfect risk sharing, we have  $q_b^C(\beta) = q_g^C(\beta) = q^C(\beta)$ , and we obtain the following asset pricing relationship

$$q^C(\beta) [\alpha u'(' + (1 - ')r) + (1 - \alpha)u'(')] - r \alpha u'(' + (1 - ')r) = 0. \quad (67)$$

But, the following first-order condition holds, from the bank's optimization problem

$$\alpha(1 - r)u'(' + (1 - ')r) + (1 - \alpha)u'(') = 0. \quad (68)$$

Therefore, substituting in (67) using (68) gives

$$q^C(\beta) = 1,$$

so that banking panics do not occur in regime  $C$ , i.e. a premium on liquid assets can arise only if risk sharing is imperfect.

Now, comparing results across regimes  $A$  and  $B$ , we obtain

$$\frac{dq^A(\beta_2)}{d'} \Big|_{\beta=1} - \frac{dq_b^B(\beta_2)}{d'} \Big|_{\beta=1} > 0$$

and

$$\frac{dq^A(\beta_2)}{d'} \Big|_{\beta=1} - \frac{dq_g^B(\beta_2)}{d'} \Big|_{\beta=1} = \frac{u''(1)(1-\alpha)[\beta_1(1-\beta_2) - (1-\alpha)(1-\beta_1)]}{\alpha u'(1)(1-\beta_1)(1-\beta_2)} > 0.$$

Therefore, banking panics are more severe in regime  $A$ , where there is no intervention, than in regime  $B$ , where there is discount window lending. This is due to the fact that discount window lending permits better risk sharing. As a result, the riskless asset commands a lower premium in the state of the world where there is a high demand for liquidity.

## 8. Summary and Conclusion

In the model constructed here, banks arise as an efficient mechanism for sharing the risk associated with random individual needs for liquidity, much as in Diamond and Dybvig (1983). An important difference here is that random liquidity needs are modeled as relocation shocks. In this context, banking panics arise due to government restrictions on banks, interpreted here as branch banking restrictions or restrictions on private banknote issues. Discount window lending and deposit insurance act to promote aggregate risk sharing, but through different means. The discount window is a form of financial intermediation which makes the discountable assets in a bank's portfolio more liquid, while deposit insurance is a mechanism for effecting contingent transfers between groups of economic agents.

In the model, there is adverse selection and costly screening in the loan market. With discount window lending, there is a moral hazard problem, in that the government cannot screen borrowers and therefore cannot verify the quality of some types of bank assets. Thus, the government will discount only some assets, and the ability of a discount window institution to promote risk sharing is limited. Similarly, the deposit insurer has limited information on aggregate variables so that, in general, the first best allocation cannot be achieved, even with government deposit insurance and discount window lending in place. It would be preferable in this model to eliminate all restrictions on banking, in which case

deposit insurance and discount window lending would be redundant. However, in the context of other regulations, deposit insurance and discount window lending are both welfare-improving.

Here, with deposit insurance banks have a greater preference for holding risky assets, and they screen borrowers less intensively. However, this is consistent with an increase in welfare. In fact, in the absence of discount window lending or deposit insurance, banks hold a greater quantity of liquid assets than they would in the first-best optimum, as liquid assets are required to satisfy the random liquidity needs of depositors, and to insure depositors against bad aggregate shocks.

Our model appears consistent with the observation made by Boyd and Gertler (1993), that during the 1980s large banks held riskier portfolios and experienced worse performance than small banks. Boyd and Gertler interpreted this as an inefficiency associated with the implicit government guarantees which back the liabilities of large banks because these banks are “too big to fail.” However, our model would predict that large banks should hold riskier portfolios than small banks. The lower returns realized by big banks in the 1980s could simply reflect the fact that large banks are more sensitive to bad aggregate shocks (i.e. aggregate shocks that are bad for banks, in this case).

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