

Revised November 1993

# ACTIVE INTERMEDIATION IN A MONETARY OVERLAPPING GENERATIONS ECONOMY\*

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## ABSTRACT

It is now widely believed that government intervention is essential to ensure Pareto efficiency in the standard overlapping generations economy with nonaltruistic agents. This paper argues that the normal profit-seeking activities of *private intermediaries*—missing from the standard overlapping generations economy—would tend to eliminate the need for such government intervention. A private earnings-driven corporate intermediary is introduced into a standard monetary overlapping generations economy which, in the absence of the intermediary, generates Pareto inefficient equilibria. The intermediary issues unsecured corporate debt and maximizes its market value in direct accordance with the interests of its successive shareholders. The significant impact of private intermediation is demonstrated by establishing that all equilibria for the resulting “Corporate Economy” are Pareto efficient. Dynamic properties of equilibrium paths are also determined. It is shown, for example, that endogenous cyclic equilibria are possible for the Corporate Economy in the absence of gross substitutability.

**Keywords:** Financial Intermediation, Overlapping Generations, Pareto Efficiency, Endogenous Cycles

**J.E.L. Number:** E44

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\*Previous versions of this paper have been presented at the University of Iowa and at the American Mathematical Society Meeting. The authors are grateful to seminar participants for helpful comments. Please address correspondence to L. Tesfatsion, Department of Economics, Iowa State University, Ames, IA 50011-1070.

## 1. INTRODUCTION

The conventional definition of a competitive equilibrium, originally developed for economies with finitely many consumers and goods, does not ensure a Pareto efficient outcome for the overlapping generations economy [Samuelson (1958), Gale (1973)]. This well-known finding has widely been interpreted to mean that some form of government intervention is essential to ensure efficiency in dynamic open-ended economies with nonaltruistic finite-lived agents. Recently this “folk theorem” has even begun to make its way into graduate and undergraduate textbooks; see, for example, Azariadis (1993, pp. 270-271) and Champ and Freeman (1994, pp. 206-207).

It is therefore important to ask whether the conventional definition of a competitive equilibrium, as applied to an overlapping generations economy, is truly satisfactory in the sense that private agents under this definition are permitted to exploit all available profit opportunities. In particular, even if government intervention can eliminate inefficiency by an appropriate manipulation of monetary or fiscal policy instruments, is it possible that the normal profit-seeking activities of *private intermediaries*—missing from the standard overlapping generations economy—would tend to eliminate the need for any such government intervention?

Preliminary support for this conjecture is provided in Pingle and Tesfatsion (1991a,b). The basic argument is that the profit opportunities associated with intermediation per se are not exploited by private agents under the conventional definition of a competitive equilibrium because private agents are assumed to be narrowly focused on consumption and production opportunities, taking prices (terms of trade) as given. Using Samuelson’s (1958) overlapping generations economy as an illustration, it is shown that the introduction of an earnings-driven private intermediary willing and able to arbitrage profit opportunities on terms of trade has a dramatic impact on the efficiency properties of the economy. For example, regardless of the precise form assumed for the earnings objective of the intermediary, Pareto inefficient outcomes are ruled out as equilibria since the intermediary necessarily perceives unbounded earnings opportunities.

Extending this prior work, the present paper introduces an earnings-driven corporate intermediary into the basic monetary overlapping generations economy studied by Grandmont and Laroque (1973), Balasko and Shell (1981), and Grandmont (1985), among others. The corporate intermediary issues unsecured corporate debt (i.e., debentures) and strives to maximize its market value in direct accordance with the interests of its successive shareholders. The conventional definition of a monetary equilibrium is generalized to include this corporate objective.

We first derive a necessary and sufficient condition for a consumption allocation for this modified “Corporate Economy” to be Pareto efficient. We then establish that all equilibria for the Corporate Economy are Pareto efficient, yielding the golden rule consumption allocation, if consumer preferences satisfy gross substitutability; and the set of such equilibria is not empty. In particular, the corporate intermediary has a viable way to enter the economy and earn a windfall return by issuing unsecured debt whenever the value of the initial real money balances held by consumers is “too low.” We further show that a similar first welfare theorem and existence theorem are obtained in the absence of gross substitutability if the offer curve of each consumer satisfies certain curvature restrictions of the type studied by Grandmont (1985). Without these curvature restrictions, it is shown that the corporate intermediary faces an interesting time inconsistency problem.

We also investigate the dynamic properties of Corporate Economy equilibria. Given gross substitutability, the golden rule consumption allocation is immediately obtained. In the absence of gross substitutability, only three types of dynamic behavior are possible in equilibrium: either (a) the economy enters immediately into a cycle with a period 2 orbit; or (b) the economy converges to a limit cycle with a period 2 orbit; or (c) the economy converges cyclically to the golden rule consumption allocation. In contrast, Grandmont (1985) establishes for the basic monetary overlapping generations economy that, in the absence of gross substitutability,  $k$  periodic ( $k > 2$ ) or even aperiodic equilibria can exist. The introduction of an earnings-driven corporate intermediary thus eliminates these more complex (and Pareto inefficient) equilibria, although an endogenous

“business cycle” is still possible.

The work closest in spirit to our own is the seminal paper by E. Thompson (1967). Thompson asserts (p. 1205) that if interest rates were forever too low for Pareto efficiency, then private corporate firms would proceed to bid up interest rates by issuing new debts to finance current new expenditures. Despite Thompson’s important insight, most researchers using the overlapping generations framework continue to exclude private corporate intermediaries. The current paper supports Thompson’s claim that, to be a fully articulated competitive equilibrium model, the standard overlapping generations economy must be extended to permit the entry of private corporate intermediaries willing and able to take advantage of unexploited profit opportunities through private debt issue.

The basic structure of the Corporate Economy is described and motivated in sections 2 through 4. Section 5 sets out a definition of equilibrium for the Corporate Economy, and section 6 investigates the efficiency and dynamic properties of these equilibria. Concluding comments are given in section 7. Proofs of propositions are provided in an appendix.

## **2. CONSUMER OPTIMIZATION IN THE CORPORATE ECONOMY**

Consider a pure exchange overlapping generations (OG) economy that begins in period 1 and extends into the infinite future. The economy’s population growth rate is equal to zero, and each generation consists of one two-period lived consumer. The economy contains a single perishable consumable resource that provides consumers with utility. The resource available during period  $t$  will be referred to as good  $t$ .

Consumers born in periods  $t \geq 1$  are identical aside from time of birth. The “generation  $t$ ” consumer is born at the beginning of period  $t$  and lives through the end of period  $t + 1$ . Each generation  $t$  consumer is endowed with  $w^y > 0$  units of good  $t$  and  $w^o > 0$  units of good  $t + 1$ . Letting  $c_t^y$  and  $c_{t+1}^o$  denote the young and old age consumption levels of the generation  $t$  consumer, it is assumed that his lifetime consumption preferences are measured by a utility function  $U(c_t^y, c_{t+1}^o)$

that is twice continuously differentiable, strictly increasing, strictly quasi-concave, and satisfies  $U(0, c_{t+1}^o) = U(c_t^y, 0) = U(0, 0)$ . Letting  $U_1$  and  $U_2$  denote the partial derivatives of the utility function with respect to  $c_t^y$  and  $c_{t+1}^o$ , respectively, it is also assumed that

$$MRS(w^y, w^o) \equiv \frac{U_1(w^y, w^o)}{U_2(w^y, w^o)} < 1. \quad (1)$$

The implications of these utility function regularity conditions will be clarified below.

Consumers in the Corporate Economy can hold both government-issued fiat money and stock shares issued by a private corporation. Let  $M_t$  denote the quantity of money held by the generation  $t$  consumer from period  $t$  to period  $t + 1$ , and let  $P_t$  denote the price of good  $t$  in terms of fiat money. The generation  $t$  consumer obtains the  $M_t$  units of money by trading away  $M_t/P_t$  units of the good  $t$  endowment. In period  $t + 1$ , the  $M_t$  units of money can be used to purchase  $M_t/P_{t+1}$  units of good  $t + 1$ . Thus, as long as the prices  $P_t$  and  $P_{t+1}$  are not infinite, the generation  $t$  consumer is able to transfer wealth from period  $t$  to period  $t + 1$  by choosing to obtain and hold fiat money.

Let  $\theta_t$  denote the number of stock shares purchased or sold short in period  $t$  by the generation  $t$  consumer, and let  $v_t \geq 0$  denote the price of a share in period  $t$ , measured in terms of good  $t$ . The generation  $t$  consumer purchases or sells short the  $\theta_t$  shares in return for  $v_t \theta_t$  units of good  $t$ . In period  $t + 1$ , the consumer then receives or pays out  $[v_{t+1} + d_{t+1}^e] \theta_t$  units of good  $t + 1$ , where  $d_{t+1}^e$  denotes the expected per share dividend. Thus, in addition to saving through money holding, the generation  $t$  consumer is able to save or borrow from period  $t$  to period  $t + 1$  through share transactions.

Given these specifications, the lifetime utility maximization problem facing the generation  $t$  consumer can be represented as

$$\max U(c_t^y, c_{t+1}^o) \quad (2)$$

with respect to  $c_t^y$ ,  $c_{t+1}^o$ ,  $M_t$ , and  $\theta_t$  subject to the budget and nonnegativity constraints

$$c_t^y = w^y - [M_t/P_t] - v_t \theta_t ;$$

$$\begin{aligned}
c_{t+1}^o &= w^o + [M_t/P_{t+1}] + [v_{t+1} + d_{t+1}^e]\theta_t ; \\
0 &\leq c_t^y, c_{t+1}^o, M_t .
\end{aligned}$$

The generation  $t$  consumer takes as given the (possibly infinite) positive goods prices  $P_t$  and  $P_{t+1}$ , the finite nonnegative share prices  $v_t$  and  $v_{t+1}$ , and the finite nonnegative expected per share dividend  $d_{t+1}^e$ .

The regularity conditions on the utility function  $U(\cdot)$  guarantee that each consumer in generation  $t \geq 1$  will choose  $c_t^y > 0$  and  $c_{t+1}^o > 0$ . Since  $\theta_t$  is unrestricted in sign, the first-order conditions for problem (2) require that

$$MRS(c_t^y, c_{t+1}^o) = q_t , \quad (3)$$

where

$$q_t \equiv \frac{v_{t+1} + d_{t+1}^e}{v_t} \quad (4)$$

denotes the expected rate of return on holding shares from period  $t$  to  $t + 1$ .

Let  $s_t \equiv w^y - c_t^y$  denote the savings of the generation  $t$  consumer. For later purposes, it will now be shown that the budget constraints for problem (2) can be simply expressed in terms of  $s_t$  and the share rate of return  $q_t$  over the range  $0 < q_t < +\infty$ . The proof of the following proposition (and all subsequent propositions) can be found in an appendix to this paper.

**Proposition 2.1.** *Suppose  $0 < q_t < +\infty$ . Then a finite solution exists for problem (2) if and only if either  $0 \leq P_t/P_{t+1} \leq q_t$  or  $P_t$  and  $P_{t+1}$  are both infinite. In either case the budget constraints for problem (2) can be expressed without loss of generality as*

$$c_t^y = w^y - s_t ; \quad (5)$$

$$c_{t+1}^o = w^o + q_t s_t ; \quad (6)$$

$$0 \leq c_t^y, c_{t+1}^o ; \quad (7)$$

and the optimal consumption and savings levels of the generation  $t$  consumer are uniquely determined as functions  $(c^y(q_t), c^o(q_t), s(q_t))$  of the period  $t$  share rate of return  $q_t$ , where  $s(q_t) \geq 0$  if and only if  $q_t \geq MRS(w^y, w^o)$ .

In the initial period 1 the Corporate Economy consists of one generation 1 young consumer and one generation 0 old consumer. The generation 0 old consumer is endowed with  $w^o$  units of good 1 and a positive amount  $M_0$  of fiat money issued (once and for all time) by government. The generation 0 old consumer is the entrepreneur who starts the corporation, hence he is also endowed with an initial quantity of stock shares,  $\theta_0 > 0$ . To retain symmetry with other consumers, it is assumed that the generation 0 old consumer plans to sell these shares at the unit price  $v_1$  and expects a per share dividend payment  $d_1^e$ .

The utility of the generation 0 consumer in period 1 is assumed to increase with increases in his consumption level  $c_1^o$ . Thus, the generation 0 old consumer chooses  $c_1^o$  to satisfy

$$c_1^o = w^o + [M_0/P_1] + [v_1 + d_1^e]\theta_0 . \quad (8)$$

Note from (8) that the introduction of fiat money and corporate stock shares gives the generation 0 old consumer a potential wealth windfall.

### 3. THE CORPORATION

A distinguishing feature of the corporate form of business is that a corporation can outlive any particular shareholder and generally has no foreseeable date of termination. As an approximation to this reality, we suppose that the corporation has an infinite planning horizon spanning all successive generations of its shareholders. Moreover, since the focus of the present study is on the ability of corporations to incur and roll over debt, we simplify the analysis by abstracting from the production process. That is, we assume that the corporation has no capital assets and employs no labor, and hence produces no physical output by which to generate earnings. Nonetheless, the corporation can borrow. As will be clarified, below, this permits in principle the continual payout of positive dividends financed by successive debt accumulation and roll over, hence the shares of the corporation need not be valueless.

It is also assumed that the corporation acts in the interests of its shareholders. Examining the budget constraints (5) and (6), it is seen that the optimized lifetime utility of the generation  $t$  consumer is an increasing function of  $q_t$ , the expected rate of return on holding shares from period  $t$  to period  $t + 1$ , over the range  $q_t \geq MRS(w^y, w^o)$  where the consumer's optimal savings level  $s(q_t)$  is nonnegative. Examining the budget constraint (8), it is seen that the utility of the generation 0 consumer is an increasing function of  $[v_1 + d_1^e]\theta_0$ , his expected windfall return from stock share ownership. To what extent can the corporation control these quantities?

By definition (4), the expected share rate of return  $q_t$  depends upon the share prices  $v_t$  and  $v_{t+1}$  and the expected dividend  $d_{t+1}^e$ . It follows that the corporation's control over  $q_t$  depends upon its control over share prices and expected dividends. In reality, corporations can and do influence their stock share prices by buying and selling their own shares. Here it is assumed that the corporation sets its own share prices by agreeing to buy or sell any quantity of shares at the share prices it desires to support.

In particular, at the beginning of period 1 the corporate intermediary announces a sequence  $\mathbf{v} = (v_1, v_2, \dots)$  of finite nonnegative share prices  $v_t$  together with a sequence  $\mathbf{d}^e = (d_1^e, d_2^e, \dots)$  of finite nonnegative expected dividend payments  $d_t^e$ . In announcing this pair of sequences  $\mathbf{I} = (\mathbf{v}, \mathbf{d}^e)$ , henceforth referred to as a *corporate plan*, the corporation takes goods prices as given. It is assumed that the announced corporate plan is known to all current and prospective shareholders. Also, while actual dividends can differ from expected, it is assumed that the dividend expectations of current and prospective shareholders coincide with the dividend expectations of the corporation as reflected in its announced corporate plan.

The corporation must of course be concerned about the feasibility of supporting any particular corporate plan. Since the corporation has no physical assets and no earnings capacity from physical production, the shares that it issues represent unsecured debt. The quantity  $v_1\theta_0$  measures the value of corporate debt which matures in period 1. The only way that the corporation can repay this debt is by rolling it over. In order for the corporation to remain solvent, the value of debt that it issues

in period 1,  $v_1\theta_1$ , must be at least as great as the debt maturing in period 1,  $v_1\theta_0$ . More generally, the incremental change in the value of corporate shares outstanding in any period  $t \geq 1$ , measured in terms of good  $t$ , is given by

$$\pi_t \equiv v_t[\theta_t - \theta_{t-1}]; \quad (9)$$

and it follows by a simple induction argument that the corporation is viable in period  $t$  only if  $\pi_t \geq 0$ .

If  $\pi_t > 0$ , i.e., if  $\theta_t > \theta_{t-1}$ , the corporation is issuing new shares in period  $t$  to cover an increase in the demand for its shares. In this case the corporation has positive net receipts equal to  $\pi_t$  units of good  $t$ . Because good  $t$  is perishable, any net receipts held by the corporation become worthless at the end of the period. It is therefore assumed that the corporation pays out all net receipts in the form of dividends to its shareholders. Letting  $d_t$  denote the good  $t$  dividend per share paid to the (old aged) shareholder in period  $t$ , who holds  $\theta_{t-1}$  shares, it follows that

$$\pi_t = d_t\theta_{t-1}. \quad (10)$$

Note that all of the corporation's dividends are financed by incurring new debt. From an empirical standpoint, this is an extreme case. However, many actual corporations do occasionally borrow to support a dividend distribution when earnings are low.<sup>1</sup> Our model is an abstraction that allows us to focus on the efficiency and stability implications of debt roll over.

By assumption, the corporation announces share prices and expected dividend payments in the form of a corporate plan and then agrees to buy or sell shares in accordance with consumer preferences. As seen in (9), actual period  $t$  net receipts  $\pi_t$ , and hence actual dividend payments, depend on the actual consumer share demands  $\theta_t$  and  $\theta_{t-1}$ . These share demands in turn depend on the share prices and expected dividend payments announced in the corporate plan. It is assumed

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<sup>1</sup>Interestingly, Kane (1989, p. 36) argues that "zombie firms now constitute roughly 25 percent of the FSLIC-insured thrift industry," where a zombie thrift is characterized as a thrift with zero enterprise-contributed capital that must rely on FSLIC guarantees to keep attracting new deposits to pay off previously accumulated debts. The possible need for guarantees to ensure the viability of the corporation currently under consideration is taken up in section 4.

that the corporate plan truthfully reveals to potential shareholders the dividend payments that the corporation actually expects to make. This “dividend consistency” condition will now be formalized.

Suppose the corporation in period 1 is able to form estimates for the share demands  $\theta_t$ ,  $t \geq 1$ , conditional on a goods price sequence  $\mathbf{P}$  and a corporate plan  $\mathbf{I} = (\mathbf{v}, \mathbf{d}^e)$ . Letting  $\theta_t^e = \theta_t(\mathbf{I}, \mathbf{P})$  denote the quantity of shares that the corporation expects the generation  $t$  consumer to purchase, conditional on  $\mathbf{P}$  and  $\mathbf{I}$ , the corporation’s period  $t$  expected net receipts are given by

$$\pi_t^e = v_t[\theta_t^e - \theta_{t-1}^e], \quad (11)$$

where  $\theta_0^e \equiv \theta_0$ . The period  $t$  expected dividend per share implied by (11) is then given by

$$\mathbf{d}_t^{e*} = \frac{\pi_t^e}{\theta_{t-1}^e}. \quad (12)$$

If the expected dividend payments (12) were to differ from the expected dividend payments announced in the corporate plan  $\mathbf{I}$ , the corporation could be accused of deliberately deceiving its potential shareholders. It is assumed that the corporation does not engage in this behavior. Specifically, it is assumed that the corporate plan  $\mathbf{I}$  announced by the corporation exhibits *dividend consistency* in the sense that the expected dividend sequence  $\mathbf{d}^e$  appearing in  $\mathbf{I}$  coincides with the expected dividend sequence  $\mathbf{d}^{e*}$  implied by relation (12).

Finally, the corporation is assumed to be aware that the total economy resources  $w^y + w^o$  available in each period  $t$  constitute a bounded sequence, implying that the sequence of real share demands,  $v_t \theta_t$ , must also be bounded over time. Consequently, the corporation is assumed to limit its choice of a corporate plan to those plans entailing a *bounded* sequence of expected real share demands  $(v_t \theta_t^e)$ .

The behavior of the corporation can now be summarized. The corporation in period 1 takes the sequence  $\mathbf{P} = (P_1, P_2, \dots)$  of current and future goods prices as given, as do its consumer-shareholders. Conditional on this information, the corporation chooses a corporate plan  $\mathbf{I} = (\mathbf{v}, \mathbf{d}^e)$

for periods  $t \geq 1$  consisting of a finite-valued nonnegative share price sequence  $\mathbf{v}$  and a finite-valued nonnegative expected dividend sequence  $\mathbf{d}^e$ .

The corporation desires to exist indefinitely, implying that it only considers corporate plans that are perceived to be viable. In particular, then, the corporation only considers corporate plans that generate nonnegative expected net receipts in each period, that exhibit dividend consistency, and that imply a bounded sequence of expected real share demands. The corporation is only interested in the subset of viable corporate plans that are in accordance with the interests of its successive shareholders. In particular, the corporation first identifies the subset of viable corporate plans yielding the highest expected wealth windfall for the generation 0 old consumer. This subset is then further winnowed down by requiring the successive maximization of the expected share rates of return  $q_t$ ,  $t \geq 1$ . The (possibly empty) set of corporate plans that remains will be referred to as the corporation's *optimal choice set*, denoted by  $\mathcal{I}(\mathbf{P})$ . It is assumed that the corporation is indifferent among all corporate plans in  $\mathcal{I}(\mathbf{P})$ .

The success of a modern corporation is often judged by the market value of its outstanding shares. The behavior of our corporation is consistent with this viewpoint. To see this, note that it follows from the definition (11) for expected net receipts, the definition (12) for the expected per share dividend, and dividend consistency that  $d_1^e \theta_0 = v_1[\theta_1^e - \theta_0]$ , hence  $[v_1 + d_1^e]\theta_0 = v_1\theta_1^e$ . This last relation shows that, by maximizing the expected windfall return  $[v_1 + d_1^e]\theta_0$  to the generation 0 old consumer, the corporation also maximizes  $v_1\theta_1^e$ , the expected market value of its outstanding shares at the end of period 1. Furthermore, using the expected dividend definition (12) to eliminate  $d_{t+1}^e$  from the expression (4) for  $q_t$ , it follows that

$$q_t \equiv \frac{v_{t+1}\theta_{t+1}^e}{v_t\theta_t^e}. \quad (13)$$

Consequently, by maximizing the expected share rate of return  $q_t$  for the generation  $t$  consumer, the corporation also maximizes the expected incremental increase in the market value of its shares from period  $t$  to period  $t + 1$ .

#### 4. VIABILITY OF THE CORPORATION

The only way the corporation described in section 3 can viably enhance the welfare of its shareholders is by incurring debt and rolling it over forever. Although  $v_t\theta_t$  measures the market value of the corporation's stock shares during period  $t$ , it also measures the market value of the corporation's debt during period  $t$ . Thus, in attempting to increase the rate of return  $q_t$  that the generation  $t$  consumer receives for holding its shares, the corporation also increases the rate at which it assumes unsecured debt. This has led some to conclude that a privately owned firm such as the one described here is not viable. There are three basic arguments.

One argument, discussed by Lerner (1959; p. 523), is that such a firm operates as an illegal "chain letter" or "Ponzi scheme." However, as Lerner also recognized, if the economy does extend into the infinite future, then in fact no one need be hurt by the chain letter aspect of the operation because correspondents need not run out. More precisely, our corporation can viably operate in an economy with an infinite time horizon as long as it attracts enough shareholders in each period  $t$  to fund the obligations it incurs in period  $t - 1$ .

The second argument is related to the corporation's net worth position. Note that the net worth of the corporation is negative at the end of each period  $t$  because the corporation has no assets (all receipts are paid out in the form of either share redemptions or dividends) but still has a liability equal to  $v_t\theta_t$ . Recognizing this situation, Cass and Yaari (1966; p. 360) conclude that "it is certainly the case that a privately owned financial intermediary will not rescue the economy from inefficiency." They arrive at this conclusion by noting that, while it has a negative net worth whenever it operates, the firm can guarantee itself a zero net worth by not operating. They argue that a privately-owned firm would therefore choose not to operate.

This argument assumes that a corporation will use net worth per se to gauge its proper course of action. However, the owners of the corporation—its shareholders—only care about net worth to the extent that it affects the total net returns (capital gains or losses plus dividend distributions) that

are associated with holding shares in the corporation. Hence, a negative net worth per se will not necessarily convince a corporation to cease its operations. Indeed, in our context, a negative net worth for the corporation is *necessary* in order for the corporation to distribute positive dividends.

The third argument against the existence of the type of corporation we are considering is that the corporation's survival might be threatened by the entry of other organizations which also seek to incur and roll over debt. In our context, the generation 0 old consumer who starts the corporation receives a wealth windfall. However, this windfall can only be obtained if the generation 1 consumer can be persuaded to buy shares in the corporation. Suppose that the generation 1 consumer "gets smart" and refuses to purchase shares in the corporation started by the generation 0 consumer, opting instead to start a similar corporation at the beginning of period 2. If successful, the generation 1 consumer would receive the wealth windfall rather than the generation 0 consumer. Of course, the generation 1 consumer would face the same problem that the generation 0 consumer faced. Thus, there is a real possibility that the corporation could never get started. Moreover, even if the corporation manages to stay in existence for some number of periods  $t$ , there is always the possibility that it will be bankrupted in period  $t + 1$  if the generation  $t + 1$  consumer refuses to buy the corporation's shares, opting instead to start his own corporation.

Samuelson (1958; pp. 480-482) discusses at some length the potential usefulness of social compacts for enhancing social welfare in situations characterized by a "prisoner's dilemma." Roughly described, a prisoner's dilemma occurs when all agents are better off if all agents cooperate rather than mutually defect, yet it is always privately rational for individual agents to defect. This is the type of dilemma that undermines the viability of the private corporation introduced in the current context. Even if all consumers are potentially better off with the corporation in existence, each individual consumer always perceives a private advantage to be gained by "defecting" from the existing corporation and starting a new one.

Note, however, that this same dilemma undermines the viability of fiat money. An agent might anticipate that a wealth windfall could be obtained by refusing to accept the fiat money currently

in use, and by instead issuing a new form of fiat money. It was this fragility of fiat money that motivated the discussion in Samuelson (1958) regarding the significance of social compacts. By agreeing to accept fiat money issued by government as “legal tender” in exchange for goods, private agents make it possible for this fiat money to act as a store of value and hence also as a medium of exchange. A violation of this social compact could lead to the demise of fiat money in any form [Bryant (1981)].

This discussion makes a simple but important point: An organization will only be able to incur and roll over debt from one period to the next if it obtains and maintains the confidence of savers. In the model presented here, the confidence that savers have in fiat money allows a government to incur and roll over a debt, while the confidence that savers have in share values allows a corporation to incur and roll over a debt. If confidence is lost in either financial asset for whatever reason, then the ability to roll over debt is lost.

In reality, various regulations protect government’s monopoly over the issue of fiat money, just as chartering and other types of regulations restrict entry into the private intermediation sector. In this paper we abstract from the issue of entry. We are concerned with the efficiency implications of government intermediation through fiat money versus private intermediation through corporate shares, assuming both assets are protected from threat of entry. Understanding whether or not a private earnings-driven intermediary can even *in principle* enhance the efficiency of a monetarized economy would seem to be a necessary prerequisite for determining the extent to which social welfare might be improved by the regulation of private intermediaries.

## **5. EQUILIBRIUM IN THE CORPORATE ECONOMY**

This section sets out a definition of equilibrium for the Corporate Economy. It is assumed that the private intermediation sector consists of a single corporation issuing unsecured debt, with no threat of entry. As will be clarified below (Propositions 6.5 and 6.8), the share prices announced by the optimizing private intermediary will be positive whenever the initial real money supply is

lower than the particular level needed to achieve the golden rule Pareto efficient outcome in the absence of the intermediary. For expositional simplicity, our definition of a Corporate Economy equilibrium rules out the special case in which the initial real money supply is set “just right” and focuses on those outcomes in which the optimizing private intermediary plays a nontrivial role in the economy through positive share price announcements.

**Definition 5.1.** Given an initial level of money balances  $M_0 > 0$  and an initial level of share holdings  $\theta_0 > 0$ , a vector

$$e(M_0, \theta_0) = (\mathbf{c}, \mathbf{M}, \boldsymbol{\theta}, \boldsymbol{\theta}^e, \mathbf{I}, \mathbf{P}) \quad (14)$$

consisting of a consumption allocation  $\mathbf{c} = (c_1^o, (c_1^y, c_2^o), (c_1^y, c_2^o), \dots)$ , a nominal money demand sequence  $\mathbf{M} = (M_1, M_2, \dots)$ , a share demand sequence  $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots)$ , a corporate expected share demand sequence  $\boldsymbol{\theta}^e = (\theta_1^e, \theta_2^e, \dots)$ , a corporate plan  $\mathbf{I} = (\mathbf{v}, \mathbf{d}^e)$  with  $\mathbf{v} > 0$ , and a nominal goods price sequence  $\mathbf{P} = (P_1, P_2, \dots)$  is an *equilibrium* for the Corporate Economy, conditional on  $M_0$  and  $\theta_0$ , if it satisfies the following five conditions:

- **Positive Nominal Goods Prices:**  $0 < P_t$  for each  $t \geq 1$ .
- **Consumer Optimization:** Consumer demands for goods, money, and shares constitute finite-valued solutions to the generation 0 consumer’s budget constraint (8) and the lifetime utility maximization problem (2) for each  $t \geq 1$ .
- **Corporate Optimization:** The corporate plan  $I$  is an element of the corporation’s optimal choice set  $\mathcal{I}(\mathbf{P})$ .
- **Market Clearing for Goods and Money:** For each  $t \geq 1$ ,

$$w^y + w^o \geq c_t^y + c_t^o ; \quad (15)$$

$$M_{t-1} \geq M_t . \quad (16)$$

- **Fulfilled Share and Dividend Expectations:** For each  $t \geq 1$ ,  $\theta_t^e = \theta_t$  and  $d_t^e = v_t[\theta_t - \theta_{t-1}]/\theta_{t-1}$ .

While the first condition is a non-primitive restriction that excludes equilibria with zero nominal goods prices, it does permit the price of money,  $1/P_t$ , to be zero in finite time. In the latter case, only shares can be used to store value. The consumer optimization condition ensures that each consumer is maximizing his utility, conditional on expected prices and dividends, and the corporate optimization condition ensures that the corporation is acting in accordance with the interests of its shareholders.

The goods market clearing condition (15) ensures the feasibility of the equilibrium consumption allocation. The money market clearing condition (16) ensures that the demand for money in period  $t$  does not exceed the supply, which is given by the amount of money held by the generation  $t - 1$  old agent. Recall that government only issues money in period 1. The absence of a market clearing condition for shares implies that the corporation is free to issue new shares in each period  $t$ . Consequently, the number of shares  $\theta_{t-1}$  which the generation  $t - 1$  old consumer redeems in period  $t$  places no direct restriction on the number of shares  $\theta_t$  which can be issued to the generation  $t$  young consumer.

The final condition ensures that the corporation's share demand and dividend expectations are correct.

## 6. EFFICIENCY AND DYNAMIC PROPERTIES

If the corporate intermediary is eliminated from the Corporate Economy, i.e., if share prices and expected dividend distributions are simply constrained to be zero, then the Corporate Economy reduces to the basic monetary OG economy. The no-trade outcome in which money is without value and each consumer in each period  $t$  simply consumes his own endowment can be supported as a competitive equilibrium for the latter economy; and, given the regularity condition (1), this

no-trade outcome is *not* Pareto efficient [Gale (1973)]. Does the presence of an earnings-driven corporate intermediary engaging in debt issue and rollover necessarily eliminate the possibility of Pareto inefficient equilibria for the Corporate Economy? Can such an economy exhibit an endogenously generated “business cycle”?

To begin the analysis of these questions, consider the cross-sectional summation of the young age budget constraint of the generation  $t$  consumer and the old age budget constraint of the generation  $t - 1$  consumer. Assuming that dividend expectations are fulfilled, one obtains

$$0 = [w^y + w^o - c_t^y - c_t^o] + [M_{t-1} - M_t]/P_t, \quad t \geq 1. \quad (17)$$

The market clearing conditions (15) and (16) for goods and money then imply that

$$0 = w^y + w^o - c_t^y - c_t^o, \quad t \geq 1; \quad (18)$$

$$0 = [M_{t-1} - M_t]/P_t, \quad t \geq 1. \quad (19)$$

Recalling that the generation  $t$  savings level is defined by  $s_t \equiv w^y - c_t^y$  for each  $t \geq 1$ , it follows from the budget constraint (6) and the goods market clearing condition (18) that  $c_t^o = w^o + s_t$  and

$$s_{t+1} = q_t s_t, \quad t \geq 1. \quad (20)$$

Moreover, the first order condition (3) can be expressed as

$$MRS(w^y - s_t, w^o + q_t s_t) = q_t, \quad t \geq 1. \quad (21)$$

Given a value for the initial share rate of return  $q_1$ , relations (20) and (21) determine all possible equilibrium paths for  $s_t$  and  $q_t$ .

In the following subsections the efficiency and dynamic properties of Corporate Economy equilibria are examined, first under the assumption that consumer preferences exhibit gross substitutability, and then without this assumption. The next two propositions will be useful for this purpose. The first proposition establishes a uniform positive lower bound on the share rates of return in any Corporate Economy equilibrium.

**PROPOSITION 6.1.** *In any Corporate Economy equilibrium, the share rates of return  $q_t$  satisfy  $q_t \geq MRS(w^y, w^o)$  for all  $t \geq 1$ .*

Recalling Proposition 2.1, it follows from Proposition 6.1 that all young consumers in a Corporate Economy equilibrium choose nonnegative savings levels  $s_t$ . The next proposition relies on the well-known Cass-Balasko-Shell “transversality condition” elaborated in Balasko and Shell (1980, Prop. 5.6, p. 296) to obtain a general characterization of Pareto inefficient consumption allocations for the Corporate Economy in terms of the long-run behavior of these savings levels.<sup>2</sup>

**PROPOSITION 6.2.** *Let  $e(M_0, \theta_0) = (c, M, \theta, \theta^e, I, P)$  denote an equilibrium for the Corporate Economy. Suppose the equilibrium consumption profiles  $(c_t^y, c_{t+1}^o)$  are uniformly bounded above and below by strictly positive vectors. Then  $c$  is a Pareto inefficient consumption allocation if and only if  $\lim_{t \rightarrow \infty} s_t = 0$ , where  $s_t \equiv [w^y - c_t^y]$ .*

### 6.A Efficiency and Dynamic Properties Under Gross Substitutability

As established in Proposition 2.1, the optimal savings level of each consumer is uniquely determined as a function  $s(q)$  of the share rate of return  $q$  for each  $q > 0$ . Suppose the preferences of consumers in the Corporate Economy satisfy gross substitutability, in the sense that  $s(q)$  is a strictly increasing<sup>3</sup> function of  $q$ . In this case each optimal savings level is supported by a unique rate of return, and at most one equilibrium path for  $q_t$  and  $s_t$  is associated with each initial rate of return  $q_1$ . As shown in Figure 1, the offer curve for the generation  $t$  consumer is everywhere negatively sloped.

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<sup>2</sup>Balasko and Shell impose certain technical regularity conditions (referred to as Properties C, C', and G) on the curvature of the utility function  $U(\cdot)$  to rule out the possibility that indifference surfaces come arbitrarily close to being kinked, linear, or infinitely steep or flat along the equilibrium path, respectively. The uniform positive lower bound on the rates of return  $q_t$  established in Proposition 6.1 ensures that Property G holds. Hereafter it will be assumed that Properties C and C' hold for  $U(\cdot)$  as well, as additional strengthenings of our previous restrictions on consumer preferences. The reader is referred to Balasko and Shell (1980) for a detailed discussion of these regularity conditions.

<sup>3</sup>If the nonnegative bordered Hessian determinant associated with the strictly quasi-concave utility function  $U(\cdot)$  is strictly positive, it follows from a comparative static analysis of relation (21) that  $\text{sign}(ds(q)/dq) = \text{sign}(U_2 + s(q)[qU_{22} - U_{12}])$ , where all partials of  $U(\cdot)$  are evaluated at  $(w^y - s(q), w^o + qs(q))$ .

In any stationary-structured OG economy such as the Corporate Economy, the “golden rule” rate of return is defined to be the stationary rate of return  $\bar{q}$  that supports the highest possible lifetime utility for a representative consumer. As shown by Samuelson (1958), this rate of return (in gross terms) coincides with 1 plus the population growth rate  $g$ , where  $g$  is assumed to be zero for the Corporate Economy. The golden rule consumption profile and savings level supported by the Corporate Economy golden rule rate of return  $\bar{q} = 1$  are depicted in Figure 1 as  $\bar{c} = (\bar{c}^y, \bar{c}^o)$  and  $\bar{s} \equiv w^y - \bar{c}^y$ , respectively. Given the regularity conditions imposed on preferences in section 2,  $\bar{q} = 1$  is strictly greater than  $MRS(w^y, w^o)$ , and  $\bar{c}^y$ ,  $\bar{c}^o$ , and  $\bar{s}$  are necessarily positive. Consequently, it follows from Proposition 6.2 that any Corporate Economy equilibrium supported by the stationary golden rule rate of return  $\bar{q} = 1$  is Pareto efficient.

Do any such equilibria exist? The next proposition provides a first step toward answering this question.

**PROPOSITION 6.3.** *Given gross substitutability, in any Corporate Economy equilibrium the optimizing corporation must set  $q_1 = 1$ .*

As seen in the appendix proof of this proposition,  $q_1 > 1$  violates the assumption that the corporation chooses a corporate plan it perceives to be viable, and  $q_1 < 1$  violates the assumption that the corporation acts in the best interests of its shareholders. But this raises two new questions. Do any Corporate Economy equilibria exist that support the initial share rate of return  $q_1 = 1$ ? If so, what are their efficiency properties? The following proposition answers both of these questions.

**PROPOSITION 6.4.** *Given gross substitutability, the only possible Corporate Economy equilibria are the Pareto efficient golden rule equilibria supported by  $q_t = 1$  for all  $t \geq 1$ ; and the set of such equilibria is not empty.*

Proposition 6.4 shows that the corporation has a dramatic impact on the efficiency and dynamic properties of the Corporate Economy. Assuming gross substitutability, the economy has a unique equilibrium consumption allocation: namely, the Pareto efficient golden rule consumption alloca-

tion characterized by the stationary consumption profile  $\bar{c}$ , as depicted in Figure 1. As seen in the appendix proof of Proposition 6.4, this unique equilibrium consumption allocation can be supported by Corporate Economy equilibria entailing different corporate plans  $\mathbf{I}$ , money demand sequences  $\mathbf{M}$ , share demand sequences  $\theta$ , expected share demand sequences  $\theta^e$ , and price sequences  $\mathbf{P}$ .

In contrast, Gale (1973) establishes that the basic monetary OG economy with preferences satisfying gross substitutability and regularity condition (1) has *infinitely* many equilibrium consumption allocations. Only one of these consumption allocations—the golden rule consumption allocation supported by the stationary rate of return  $\bar{q} = 1$ —is Pareto efficient. All other equilibrium consumption allocations are Pareto inefficient, and are associated with initial rates of return that are less than one. In the Corporate Economy an initial rate of return less than one on both money and shares gives the corporation an opportunity to increase the welfare of each of its shareholders by raising the initial rate of return on shares and by increasing its issue of unsecured debt. But active private financial intermediaries are missing in the basic monetary OG economy, hence the potential profit opportunities arising from intermediation can remain unexploited.

Proposition 6.4 also raises another interesting point: the fundamental trade-off between efficiency and stability in both the Corporate Economy and the basic monetary OG economy. Assuming gross substitutability, the setting of an initial rate of return exceeding one ultimately results in insolvency for either economy. For example, making repeated use of relation (20) and gross substitutability, a simple induction argument establishes that the optimal savings sequence  $(s_t)$  for the Corporate Economy must diverge to  $+\infty$  if  $q_1 > 1$ , implying that the finite resources  $w^y + w^o$  available in the economy in each period  $t$  are exceeded in finite time. The drive of an active earnings-driven corporation to enhance the welfare of its shareholders pushes the initial rate of return to  $q_1 = 1$ , and hence increases efficiency; but it also pushes the economy to the brink of collapse. Similarly, for the basic monetary OG economy, a government monetary policy designed to achieve efficiency would (if possible) set the initial rate of return on money to one, and hence bring the economy to the brink of collapse as well.

Although the set of possible Corporate Economy equilibria exhibits a considerable degree of indeterminacy with regard to the setting of nominal variables, some interesting inferences can be drawn from the finding in Proposition 6.4 that the equilibrium wealth windfall of the generation 0 old consumer,  $M_0/P_1 + [v_1 + d_1]\theta_0$ , must equal the golden rule savings level,  $\bar{s}$ . Since  $[v_1 + d_1]\theta_0 \geq 0$ , this relation implies that the initial real money supply,  $M_0/P_1$ , cannot exceed  $\bar{s}$  in equilibrium. As the next proposition shows, the corporation has a viable way to enter the economy if and only if  $M_0/P_1 < \bar{s}$ .

**PROPOSITION 6.5.** *Let  $M_0 > 0$  and  $\theta_0 > 0$  be given, and suppose consumer preferences satisfy gross substitutability. Then in order for a Corporate Economy equilibrium  $e(M_0, \theta_0)$  to exist, the nominal goods price sequence  $\mathbf{P} = (P_1, P_2, \dots)$  must satisfy  $M_0/P_1 < \bar{s}$  and  $P_t = P_{t+1}$  for all  $t \geq 1$ . Conversely, given any such price sequence, there exists a Corporate Economy equilibrium  $e(M_0, \theta_0)$  that generates this price sequence; and the market value of the corporation in this equilibrium is given by  $v_t\theta_t = [\bar{s} - M_0/P_1] > 0$  in each period  $t \geq 1$ .*

Consequently, if preferences exhibit gross substitutability, then government can prevent the viable entry of a private corporate intermediary issuing unsecured debt if it can set the initial real money supply equal to the golden rule level  $M_0/P_1 = \bar{s}$ . In this special case the Pareto efficient golden rule outcome is achieved without the need of a private intermediary. On the other hand, if the initial real money supply is below this golden rule level for any reason, then a profit opportunity exists in the economy that can be exploited by the entry of a private intermediary willing to issue and forever roll over unsecured debt.

### 6.B Efficiency and Dynamic Properties Without Gross Substitutability

Suppose, instead, that consumer preferences fail to satisfy gross substitutability. The optimal consumption and savings levels of each consumer are still uniquely determined as functions  $(c^y(q), c^o(q), s(q))$  of the rate of return  $q$  for each  $q > 0$ . However, the absence of gross substitutabil-

ity implies that there exist savings levels  $s_t$  for which the first order condition (21) determines at least two possible supporting rates of return  $q_t$ . Consequently, the mapping from  $q$  to  $s(q)$  is not one-to-one and the offer curve of each consumer bends back upon itself at least once, as illustrated in Figure 2. It follows that more than one equilibrium path can be associated with an initial rate of return  $q_1$ .

In view of the potential complications caused by this indeterminacy of equilibrium paths, most researchers using the OG framework have assumed gross substitutability.<sup>4</sup> Nevertheless, some efficiency properties can still be established in its absence.

By assumption (1), the offer curve of each consumer has a slope  $-MRS(w^y, w^o) > -1$  at the endowment point  $(w^y, w^o)$ , implying that the optimal savings level  $s(q)$  is a positive and increasing function of  $q$  for sufficiently small  $q > MRS(w^y, w^o)$ . Suppose that the offer curve of each consumer is as depicted in Figure 2. More precisely, suppose that the offer curve bends back upon itself only once, that the bend point is supported by a rate of return  $q^*$  that is strictly less than one and strictly greater than  $MRS(w^y, w^o)$ ,<sup>5</sup> and that the optimal old-age consumption level  $c^o(q)$  implied by this offer curve is a strictly increasing function of  $q$  for all  $q \geq MRS(w^y, w^o)$  and satisfies  $\lim_{q \rightarrow +\infty} c^o(q) = +\infty$ .<sup>6</sup> Given these restrictions on the offer curve of each consumer, the Corporate Economy in the *absence* of the corporation essentially reduces to the basic monetary OG economy studied by Grandmont (1985).

To understand more clearly what is depicted in Figure 2, suppose  $\mathbf{q} = (q_1, q_2, \dots)$  is an equilibrium sequence of return rates. Given any point  $A$  on the offer curve in Figure 2 associated with

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<sup>4</sup>An important exception is Grandmont (1985), who relies on the absence of gross substitutability to obtain his endogenous competitive business cycle.

<sup>5</sup>Another alternative would be for the offer curve shown in Figure 2 to bend back upon itself at a supporting rate of return  $q^* \geq 1$ . However, using the geometric analysis described below, one can deduce for this case that no equilibrium could exist with  $q_t > 1$  in any period  $t \geq 1$ . Thus, this case is essentially the same as the case examined in section 6.A for gross substitutability.

<sup>6</sup>Recalling that  $c^o(q) = w^o + qs(q)$ , it can be shown using footnote 3 that a sufficient condition for  $dc^o(q)/dq > 0$  for all  $q > 0$  is that the utility function be separable; i.e.,  $U_{12} \equiv 0$ . In addition, separability of the utility function also guarantees that  $\lim_{q \rightarrow +\infty} c^o(q) = +\infty$ ; for it follows from  $c^o(q) = w^o + qs(q)$  that  $s(q) = [w^y - c^y(q)]$  must tend to 0 as  $q$  approaches  $+\infty$  if  $c^o(q)$  remains bounded. But it then follows from separability that  $U_1(c^y(q), c^o(q))$  remains bounded while  $qU_2(c^y(q), c^o(q))$  diverges to  $+\infty$ , a violation of the first-order condition (3).

the equilibrium savings level  $s(q_t) = w^y - c^y(q_t)$  for the young consumer in period  $t$ , note that the projection of  $A$  onto the vertical axis gives the equilibrium consumption level  $c_{t+1}^o = c^o(q_t) = w^o + q_t s(q_t)$  for the old consumer in period  $t + 1$ . Thus, moving horizontally from  $A$  to the point  $B$  on the budget constraint associated with the rate of return  $\bar{q} = 1$ , it follows from the goods market clearing condition (18) that the projection of  $B$  onto the horizontal axis must yield the equilibrium consumption level  $c_{t+1}^y = c^y(q_{t+1}) = w^y + w^o - c^o(q_t)$  for the young consumer in period  $t + 1$ . The equilibrium savings level  $s(q_{t+1})$  for the young consumer in period  $t + 1$  is then given by the distance  $w^y - c^y(q_{t+1})$ , which by construction equals  $q_t s(q_t)$ . This is simply a geometric illustration of condition (20), shown earlier to be a necessary condition for equilibrium.

Let  $s_{max}$  denote the maximum possible level for the consumer's optimal savings, achieved at the first and only bend point in the offer curve. By assumption, this maximum savings level is supported by the rate of return  $q^*$ , i.e.,  $s_{max} = s(q^*)$ . Note that  $s_{max}$  in Figure 2 is associated with a rate of return  $q_{max}$  through the relation  $s_{max} = q_{max} s(q_{max})$ . Recalling (20), this relation has the following interpretation: given the rate of return  $q_{max}$  in some period  $t$ , the equilibrium savings level for period  $t + 1$  is  $s_{max}$ . For later purposes, various properties of  $s_{max}$  and  $q_{max}$  depicted in Figure 2 will now be established analytically.

**PROPOSITION 6.6.** *Suppose the offer curve of the consumer bends back upon itself exactly once, at a point supported by a rate of return  $q^*$  satisfying  $MRS(w^y, w^o) < q^* < 1$ , and that the optimal old-age consumption level  $c^o(q)$  is a strictly increasing function of  $q$  for all  $q \geq MRS(w^y, w^o)$  and satisfies  $\lim_{q \rightarrow +\infty} c^o(q) = +\infty$ . Then  $\bar{s} < s_{max} < w^y$ , and there exists a unique solution  $q_{max}$  to the equation  $s_{max} = qs(q)$ , where  $q_{max} > 1$ . Moreover, in any Corporate Economy equilibrium the share rates of return  $q_t$  lie in the interval  $[MRS(w^y, w^o), q_{max}]$ , and the consumption profiles  $(c_t^y, c_{t+1}^o)$  are uniformly bounded above and below by strictly positive vectors.*

Propositions 6.2 and 6.6 can now be used to establish a first welfare theorem for the Corporate Economy in the absence of gross substitutability. The proof of this result, given in the appendix,

proceeds by showing that the existence of a Pareto inefficient equilibrium contradicts the assumption that the corporation acts in the best interests of its shareholders.

**PROPOSITION 6.7.** *Under the assumptions of Proposition 6.6, all Corporate Economy equilibria are Pareto efficient.*

Several important questions still need to be addressed: Do equilibria necessarily exist for the Corporate Economy in the absence of gross substitutability? And, if so, what dynamic properties do they exhibit? The next proposition establishes that equilibria do exist, given the assumptions of Proposition 6.6 and an additional restriction on the curvature of the upper branch of the offer curve. Interestingly, in the absence of the latter curvature restriction the optimizing corporation may exhibit time inconsistent behavior—i.e., a desire in some period  $t > 1$  to veer away from the corporate plan announced in period 1 in order to improve the welfare of its shareholders. We shall return to this point below, after a discussion of the implications of Proposition 6.8.

**PROPOSITION 6.8.** *Suppose the assumptions of Proposition 6.6 hold. In addition, suppose the curvature of the upper branch of the offer curve is such that*

$$q_{max} \geq 1/q^* . \quad (22)$$

*Then there exist infinitely many Corporate Economy equilibria; but all of these equilibria entail the same real equilibrium solution values for consumption levels, savings levels, and share rates of return. In particular, in any Corporate Economy equilibrium the optimizing corporation sets  $q_1 = q^*$ , and all equilibrium share rates of return  $q_t$  lie in the interval  $[q^*, q_{max}]$ .*

It is informative to sketch here a heuristic proof for Proposition 6.8. A detailed proof can be found in the appendix.

Let  $M_0$  and  $\theta_0$  be any given positive values for the money and share holdings of the generation 0 consumer. In any Corporate Economy equilibrium the windfall return of the generation 0 consumer,  $M_0/P_1 + [v_1 + d_1^e]\theta_0$ , must equal the savings level  $s_1$  of the generation 1 consumer. Given the assumptions of Proposition 6.6, the maximum possible value for  $s_1$  is  $s_{max}$ .

Let  $P_1^*$  denote any positive (possibly infinite) nominal price level for period 1 that satisfies  $0 \leq M_0/P_1^* < s_{max}$ . Then, taking  $M_0/P_1^*$  and  $\theta_0$  as given, it follows from the definition of a Corporate Economy equilibrium that the optimizing corporation sets  $[v_1 + d_1^e] > 0$  in period 1 so that the windfall return of the generation 0 consumer is equal to  $s_{max}$ . But, given such a promised windfall to the generation 0 consumer, the highest feasible rate of return that the corporation can offer to the generation 1 consumer is  $q_1^* \equiv q^*$ , resulting in the savings level  $s_1^* = s(q^*) \equiv s_{max}$ ; for any higher rate of return would result in a lower savings level, and the corporation would not be able to fulfill its contractual obligations to the generation 0 consumer.

By relation (20), the equilibrium savings level for period 2 must then be given by  $s_2^* = q_1^* s(q_1^*) < s_{max}$ . The restriction (22) implies that  $s_2^* \geq s(q_{max})$ . Consequently, as depicted in Figure 2, two possible rates of return  $q_2$  support the savings level  $s_2^*$ —that is, two possible rates of return  $q_2$  satisfy relation (21) for the given  $s_2^*$ —and neither of these rates of return exceeds  $q_{max}$ . But it follows from the definition of a Corporate Economy equilibrium that the optimizing corporation always chooses the highest possible rate of return for each successive generation. The economy thus moves along the upper branch of the offer curve, the branch that passes through the golden rule consumption point  $\bar{c}$ .

Continuing this line of reasoning, the equilibrium savings levels  $s_t^*$  and rates of return  $q_t^*$ , and hence also the equilibrium consumption levels  $c_t^{y*}$  and  $c_{t+1}^{o*}$ , are uniquely determined by successive applications of relations (20) and (21), together with the “selection principle” that the highest possible rate of return is always to be chosen. The regularity condition (22) guarantees that the rates of return  $q_t^*$  all lie in the interval  $[q^*, q_{max}]$ , hence the economy remains on the upper branch of the offer curve for all periods  $t \geq 1$ .

As shown in the detailed appendix proof of Proposition 6.8, infinitely many Corporate Economy equilibria can be constructed to support this unique real equilibrium solution for consumption, savings, and share rates of return. One source of indeterminacy is the degree of freedom the corporation has to realize the rates of return  $q_t^*$  either in the form of share price appreciation or in

the form of dividend distributions. Another source of indeterminacy is the setting of the initial real money balances  $M_0/P_1^*$ . In analogy to the case of gross substitutability, the corporation is viable as long as these real balances are strictly less than  $s_{max}$ , the maximum possible level for consumer savings; for this permits the corporation to issue and roll over positively valued shares in each period  $t \geq 1$ .

What dynamic properties are exhibited by the unique real equilibrium solution for the Corporate Economy? The next proposition establishes that only three relatively simple types of dynamic behavior are possible.

**PROPOSITION 6.9.** *Under the hypotheses of Proposition 6.8, only three types of dynamic behavior are possible for the rate of return sequence  $(q_t^*)$  in any Corporate Economy equilibrium: either (a) condition (22) holds as an equality and the sequence  $(q_t^*)$  cycles back and forth between  $q^*$  and  $q_{max}$  in a period 2 orbit, starting with  $q_1^* = q^*$ ; or (b) condition (22) holds as a strict inequality and the sequence  $(q_t^*)$  converges to a limit cycle supported by a period 2 orbit  $\{q_L, q_U\}$  with  $q^* < q_L < 1 < q_U < q_{max}$  and  $q_L q_U = 1$ ; or (c) condition (22) holds as a strict inequality and the sequence  $(q_t^*)$  converges cyclically to the golden rule rate of return  $\bar{q} = 1$ .*

In summary, under the hypotheses of Proposition 6.8 the Corporate Economy in equilibrium either exhibits a deterministic and endogenously generated period 2 “business cycle” or converges cyclically to the golden rule consumption allocation. In either case, however, it follows from Proposition 6.7 that the equilibrium is Pareto efficient.

If condition (22) does not hold, implying that  $q^* s(q^*) < s(q_{max})$ , then no equilibrium exists for the Corporate Economy. To understand why, note that it is always in the best interest of the generation 0 shareholder if the corporation sets  $q_1 = q^*$  in period 1 to ensure that the generation 1 consumer saves the maximum amount  $s(q^*) = s_{max}$ . The consumer budget constraints and the product market clearing condition embodied in condition (20) then imply that the savings level  $s_2$  of the generation 2 consumer must equal  $q^* s(q^*)$ .

From the consumer's perspective, two different rates of return  $q_2'$  and  $q_2''$  support the savings level  $s_2 = q^*s(q^*)$  in period 2, with  $q_2' < 1 < q_2''$ . Given  $q^*s(q^*) < s(q_{max})$ , however, neither represents a feasible equilibrium rate of return. The higher rate of return  $q_2''$  exceeds  $q_{max}$ , and is thus not feasible by Proposition 6.6. The lower rate of return  $q_2'$  leads the corporation into a time inconsistency problem. The corporation in period 2 would desire to deviate from the corporate plan it announced in period 1 by giving the generation 1 old consumer an unexpected windfall return, but such reoptimization is not allowed under the current definition of a Corporate Economy equilibrium.

More precisely, the following scenario might occur when condition (22) fails to hold. To attract and satisfy its potential shareholder in period 1, the corporation in period 1 announces a corporate plan with  $q_1 = q^*$  and  $q_2 = q_2'$ . The period 1 rate of return ensures that the generation 1 consumer chooses the maximum possible savings level and hence that the generation 0 old consumer receives the maximum possible windfall return. Once period 2 is actually reached, however, the rate of return  $q_2'$  is not in the best interest of either the generation 1 old shareholder or the generation 2 young shareholder. Rather, these shareholders would best be served by the corporation changing its corporate plan and setting  $q_2 = q^*$  to generate the maximum savings level from the generation 2 young consumer and hence an unexpected windfall return for the generation 1 old consumer.

The difficulty is that if the generation 1 young consumer perceives a positive probability that he will receive a windfall return from the corporation in period 2—i.e., a return above and beyond the return generated by the dividend and/or share price appreciation promised in the original corporate plan—then he might lower his savings level in period 1 below  $s_{max}$ , making the generation 0 old consumer worse off. Consequently, to obtain a deterministic outcome for this case, more detail would have to be added to the model concerning how the consumers and the corporation behave in the uncertain environment created by the possibility of time inconsistent reoptimization.

## 7. CONCLUSION

Trade and credit arrangements in modern market economies are primarily accomplished through earnings-driven private intermediaries such as retail stores, banks, and brokerage firms. Understanding how private intermediaries affect the allocation of resources is therefore of considerable importance. The findings of this paper suggest that the inclusion of private intermediaries is essential for the study of efficiency in OG economies, even in the absence of transactions costs and asymmetric information problems. In particular, it is shown that a first welfare theorem can be recovered for the basic monetary OG economy if the economy is generalized to encompass a corporate intermediary that maximizes its market value in direct accordance with the interests of its successive shareholders.

Further work is of course needed to check the robustness of these findings. One important issue is the degree of market power exercised by private intermediaries. In this paper it is assumed that the corporate intermediary exercises control over both its dividend distributions and its share prices, which in turn determine the equilibrium interest rates for the economy. Because of product differentiation, chartering restrictions, and so forth, actual financial intermediaries often do exercise some market power in setting rates of return for local deposits and loans [Hannan and Berger (1992)], but the intermediary may be too small relative to the securities market to influence its marginal funding costs and its marginal earnings on other financial assets such as large-denomination CD's [Fama (1985)].

The issue of market power is closely tied to the issue of entry. Empirical findings suggest that chartering and other restrictions currently in force do reduce initial entry into financial markets [Amel and Liang (1992)], but the special features of financial intermediaries that might warrant such particular forms of supervision are still under debate. Nearly all arguments for the regulation of private financial intermediaries to date have been based on the belief that information problems are particularly severe in financial markets [Gertler (1988), Williamson (1992)]. This paper suggests that some form of initial entry deterrence may be necessary to ensure the long-run viability of private financial intermediaries, even in the absence of information problems; but complete

deterrence resulting in a sole reliance on financial assets passively supplied by government to coordinate trade and credit will generally be inefficient.

Another interesting area for future research concerns the relationship between efficiency and private intermediation in economies with capital accumulation. In Pingle and Tesfatsion (1991c) it is shown that the inclusion of a private earnings-driven corporate intermediary can eliminate the inefficiency that arises in the one-sector growth model studied by Diamond (1965) and Tirole (1985). Still unresolved, however, is whether private intermediation alleviates the “inadequate distribution of capital among firms” that can arise in multi-sector growth models when the production decisions of individual firms result in aggregate capital overaccumulation [Malinvaud (1953)].

Finally, Grandmont (1985) stresses the importance of taking due account of learning when formulating the dynamics of an economy. In particular, he establishes for the basic monetary OG economy that the stability properties of equilibria depend on the processes that agents use to form their price expectations; and this point is surely relevant for the Corporate Economy as well. Interestingly, as seen in section 6, another type of learning problem also arises for the Corporate Economy: namely, in the absence of gross substitutability, it can happen that the corporate intermediary will face a time inconsistency problem. That is, in some period  $t > 1$  the corporate intermediary might desire to deviate from the corporate plan it announced in period 1 in order to increase the welfare of its current and future shareholders. In this case the dynamic path of the economy cannot be determined without a more fully articulated modelling of consumer decision making in the face of uncertain corporate behavior.

## REFERENCES

- AMEL, D. F., and LIANG, J. N. (1992), "A Dynamic Model of Entry and Performance in the U.S. Banking Industry" (Federal Reserve Board, Washington D. C., Discussion Paper 210, Finance and Economics Discussion Series)
- AZARIADIS, C. (1993), *Intertemporal Macroeconomics* (Cambridge, MA: Blackwell).
- BALASKO, Y., and SHELL, K. (1980), "The Overlapping Generations Model, I: The Case of Pure Exchange Without Money," *Journal of Economic Theory*, 23, 281-306.
- BALASKO, Y. and SHELL, K. (1981), "The Overlapping Generations Model, II: The Case of Pure Exchange with Money," *Journal of Economic Theory*, 24, 112-142.
- BRYANT, J. (1981), "Bank Collapse and Depression," *Journal of Money, Credit, and Banking*, 13, 454-464.
- CASS, D., and YAARI, M. (1966), "A Re-examination of the Pure Consumption Loan Model," *Journal of Political Economy*, 74, 353-367.
- CHAMP, B. and FREEMAN, S. (1994), *Modelling Monetary Economies* (New York: John Wiley and Sons, Inc.).
- DIAMOND, P. (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126-1150.
- FAMA, E. (1985), "What's Different About Banks?," *Journal of Monetary Economics*, 15, 29-39.
- GALE, D. (1973), "Pure Exchange Equilibrium of Dynamic Economic Models," *Journal of Economic Theory*, 6, 12-36.
- GERTLER, M. (1988), "Financial Structure and Aggregate Economic Activity: An Overview," *Journal of Money, Credit, and Banking*, 20, 559-596.
- GRANDMONT, J.M., and LAROQUE, G. (1973), "Money in the Pure Consumption Loan Model," *Journal of Economic Theory*, 6, 382-395.
- GRANDMONT, J.M. (1985), "On Endogenous Competitive Business Cycles," *Econometrica*, 53, 995-1045.
- HANNAN, T. H., and BERGER, A. N. (1991), "The Rigidity of Prices: Evidence from the Banking Industry," *American Economic Review*, 81, 938-945.

- KANE, E. (1989), "The High Cost of Incompletely Funding the FSLIC Shortage of Explicit Capital," *Journal of Economic Perspectives*, 3, 31-47.
- LERNER, A. (1959), "Consumption-Loan Interest and Money," *Journal of Political Economy*, 67, 512-518, plus "Rejoinder," *Ibid.*:523-525.
- MALINVAUD, E. (1953), "Capital Accumulation and the Efficient Allocation of Resources," *Econometrica*, 21, 233-268, plus "Efficient Capital Production: A Corrigendum," *Ibid.*, 30 (1962), 570-573.
- PINGLE, M., and TEFATSION, L. (1991a), "Overlapping Generations, Intermediation, and the First Welfare Theorem," *Journal of Economic Behavior and Organization*, 15, 325-345.
- \_\_\_\_\_ (1991b), "Intermediation in Overlapping Generations Economies" (Iowa State University, Economic Report No. 29).
- \_\_\_\_\_ (1991c), "Intermediation, Bubbles, and Pareto Efficiency in Economies with Production" (Iowa State University, Economic Report No. 24).
- SAMUELSON, P. (1958), "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy*, 66, 467-482.
- TIROLE, J. (1985), "Asset Bubbles and Overlapping Generations," *Econometrica*, 53, 1499-1528.
- WILLIAMSON, S. (1992), "Laissez-Faire Banking and Circulating Media of Exchange" (University of Western Ontario, Department of Economics Working Paper).
- THOMPSON, E. (1967), "Debt Instruments in both Macroeconomic Theory and Capital Theory," *American Economic Review*, 57, 1196-1210.

## APPENDIX: PROPOSITION PROOFS

**PROOF OF PROPOSITION 2.1.** Suppose  $P_t < +\infty$ . The budget constraints for problem (2) can then be written as

$$c_t^y = w^y - s_t; \quad (23)$$

$$c_{t+1}^o = w^o + q_t s_t + \frac{M_t}{P_t} [r_t - q_t], \quad (24)$$

where  $r_t \equiv [P_t/P_{t+1}]$  denotes the gross rate of return on holding money from period  $t$  to  $t+1$ . If  $r_t > q_t$ , it is apparent from (24) that the generation  $t$  consumer would desire to sell shares short without bound in his youth in order to finance the acquisition of an arbitrarily large holding of money. Consequently, in order for problem (2) to have a finite solution when  $P_t < +\infty$ , it is necessary that  $r_t \leq q_t$ .

If  $P_t < +\infty$  and  $r_t < q_t$ , the young consumer in generation  $t$  will choose not to hold money (i.e.,  $M_t = 0$ ), and the optimal consumption and savings levels of the generation  $t$  consumer will be uniquely determined as functions of  $q_t$  by the relations (3), (5), and (6). If  $P_t < +\infty$  and  $r_t = q_t$ , the generation  $t$  young consumer will be indifferent between holding shares and holding money. In this case the particular levels of  $M_t$  and  $\theta_t$  will be indeterminate, but the optimal consumption and savings levels of the generation  $t$  consumer will again be uniquely determined as functions of  $q_t$  by relations (3), (5), and (6).

In summary, if  $P_t < \infty$ , then a finite solution exists for problem (2) if and only if either  $r_t < q_t$  (implying  $M_t = 0$ ) or  $r_t = q_t$ . In either case the budget constraints (23) and (24) reduce to (5) and (6) and the optimal consumption and savings levels are uniquely determined as functions of  $q_t$ . Moreover, it follows from the regularity condition (1) that  $s(q_t) \geq 0$  if and only if  $q_t \geq MRS(w^y, w^o)$ .

Suppose, instead, that  $P_t$  and  $P_{t+1}$  are both infinite. Then the real money balances of the generation  $t$  consumer are zero-valued regardless of his choice of nominal money holdings  $M_t$ . Consequently, the budget constraints (23) and (24) once again reduce to (5) and (6), and the remaining assertions of Proposition 2.1 follow as for the case  $P_t < \infty$ . Finally, if  $P_t = +\infty$  and  $P_{t+1} < +\infty$ , then problem (2) has no solution since the generation  $t$  consumer will desire to hold an arbitrarily large amount of (costless) money in period  $t$ . Q.E.D.

**PROOF OF PROPOSITION 6.1.** By Proposition 2.1,  $s(q) \geq 0$  if and only if  $q \geq MRS(w^y, w^o)$ . Suppose there exists a Corporate Economy equilibrium in which the share rate of return  $q_t$  first falls below  $MRS(w^y, w^o)$  in some period  $t^* \geq 1$ . The generation  $t^*$  young consumer would then plan to borrow rather than to save, i.e.,  $c_{t^*}^y > w^y$ . But the planned optimal savings of the generation  $t^* - 1$  old consumer are nonnegative, implying that  $c_{t^*}^o \geq w^o$ , since either this consumer is the initial old consumer or he faced a share rate of return  $q_{t^*-1} \geq MRS(w^y, w^o)$  in period  $t^* - 1$ . The goods market clearing condition (18) thus fails for period  $t^*$ . It must therefore hold that  $q_t \geq MRS(w^y, w^o)$  for all  $t \geq 1$ . Q.E.D.

**PROOF OF PROPOSITION 6.2.** Let  $p_t \equiv [q_{t-1} \cdots q_1]^{-1} > 0$  denote the real price of good 1 measured in units of good  $t$ ,  $t \geq 2$ , with  $p_1 \equiv 1$ . Then  $p_{t+1} = [1/q_t]p_t$ , where it follows from

Proposition 6.1 that  $q_t$  is uniformly bounded from below by the positive constant  $MRS(w^y, w^o)$  for all  $t \geq 1$ . It is then straightforward to show that Property G in the Balasko-Shell Proposition 5.6 is satisfied. Applying the latter proposition to the Corporate Economy—a special case of the Balasko-Shell pure exchange model in terms of the structural specifications for preferences, real endowments, and technology (nonstorable resource)—the equilibrium consumption allocation  $c$  is Pareto inefficient if and only if  $\sum_{t=1}^{t=\infty} 1/p_t < +\infty$ .

Since the optimal savings levels  $s_t$  are bounded between 0 and  $w^y$  for  $q_t \geq MRS(w^y, w^o)$ , either  $\limsup s_t = 0$  or  $\limsup s_t > 0$ . Suppose  $\limsup s_t > 0$ . From condition (20),  $s_{t+1} = [p_t/p_{t+1}]s_t$  for all  $t \geq 1$ , implying that  $s_t = [p_1/p_t]s_1$  for all  $t \geq 1$ . If  $s_1 = 0$ , then  $s_t = 0$  for all  $t \geq 1$ , contradicting the supposition. Therefore,  $s_1 > 0$ . If  $\lim_{t \rightarrow +\infty} [1/p_t] = 0$ , then  $\lim_{t \rightarrow +\infty} s_t = 0$  again contradicting the supposition. Therefore  $\sum_{t=1}^{t=T} [1/p_t]$  does not converge as  $T \rightarrow +\infty$ . Because  $p_t > 0$  for all  $t \geq 1$ , it follows that  $\sum_{t=1}^{t=\infty} [1/p_t] = +\infty$ .

Conversely, suppose  $\limsup s_t = 0$ , which in turn implies that  $\limsup c_t^y = w^y$  and  $\limsup c_{t+1}^o = w^o$ . It follows from the regularity condition (1) that, for sufficiently small  $\epsilon$  in  $(0, 1)$ , there exists a period  $k$  such that  $p^t/p^{t+1} = MRS(c_t^y, c_{t+1}^o) \leq [1 - \epsilon]$  for all  $t \geq k$ . Defining  $\rho_t \equiv 1/p_t = [\frac{1}{p_2} \frac{p_2}{p_3} \dots \frac{p_{t-1}}{p_t}]$ , one has  $\rho_t \leq \rho_k [1 - \epsilon]^{t-k}$  for all  $t \geq k$ . Thus,  $\sum_{t=k}^{t=\infty} 1/p_t \leq \rho_k/\epsilon < +\infty$ . Finally, since  $\sum_{t=1}^{t=k} 1/p_t < +\infty$ , it follows that  $\sum_{t=1}^{t=\infty} 1/p_t < +\infty$ .

In summary, the equilibrium consumption allocation  $c$  is Pareto inefficient if and only if  $\limsup s_t = 0$ . However,  $s_t \geq 0$  for all  $t \geq 0$  implies that  $\limsup s_t = 0$  if and only if  $\lim_{t \rightarrow \infty} s_t = 0$ . Q.E.D.

**PROOF OF PROPOSITION 6.3.** Let a Corporate Economy equilibrium be given, characterized by a rate of return sequence  $(q_t)$ . By Propositions 2.1 and 6.1,  $q_t \geq MRS(w^y, w^o)$  and  $s_t \geq 0$  for each  $t \geq 1$ .

Suppose  $q_1 \geq k$  for some  $k > 1$ . Then gross substitutability implies  $s_1 > \bar{s}$  and relation (20) implies  $s_2 > s_1$ . Knowing  $s_2 > s_1$ , gross substitutability implies  $q_2 > q_1$ . Making repeated use of relation (20) and gross substitutability, a simple induction argument then establishes that  $q_{t+1} > q_t$  for all  $t \geq 1$ , hence  $q_t \geq k$  for all  $t \geq 1$ . It follows from the expression (13) for  $q_t$  that  $v_{t+1}\theta_{t+1}^e \geq kv_t\theta_t^e$  for all  $t \geq 1$ , which implies that the sequence of expected real share demands,  $v_t\theta_t^e$ , diverges to  $+\infty$ . But this violates the section 3 assumption that the optimizing corporation only chooses from among those corporate plans it perceives to be viable—implying, in particular, that the corporate plan must entail a bounded sequence of expected real share demands. Consequently, in any Corporate Economy equilibrium, it cannot be true that  $q_1 > 1$ .

Suppose, instead, that  $MRS(w^y, w^o) \leq q_1 < 1$ . It will be shown that the corporation could then increase the welfare of each of its shareholders by making a viable change in its corporate plan, a violation of the assumption that the corporation in any Corporate Economy equilibrium chooses a viable corporate plan in accordance with the best interests of its shareholders.

Suppose the corporation increases the share rate of return  $q_1$  to  $\hat{q}_1 \leq 1$  by increasing the share price  $v_2$ . The gross substitutability assumption implies that this increase in  $q_1$  leads the generation 1 consumer to increase his savings level; i.e.,  $\hat{s}_1 > s_1$ . Also, taking the price sequence  $(P_t)$  as given, the increase in  $q_1$  would lead the generation  $t$  consumer to reduce his money demand to zero (if it

were not already zero); i.e.,  $\hat{M}_1 = 0$ . This implies that the real demand for shares would become  $v_1 \hat{\theta}_1 = \hat{s}_1$ . Since  $\hat{s}_1 > s_1$  and  $\hat{M}_1/P_1 \leq M_1/P_1$ , it follows from the generation 1 consumer's young age budget constraint that  $\hat{\theta}_1 > \theta_1$ . This implies that the dividend of the generation 0 consumer would increase from  $d_1 = v_1[\theta_1 - \theta_0]/\theta_0$  to  $\hat{d}_1 = v_1[\hat{\theta}_1 - \theta_0]/\theta_0$ . Thus, the welfare of the generation 0 consumer is enhanced due to a "windfall" dividend.

Because  $\hat{q}_1 > q_1$  and  $\hat{s}_1 > s_1$  as determined above, condition (20) implies that  $\hat{s}_2 > s_2$  would also have to hold in order for the sequences  $(\hat{q}_t)$  and  $(\hat{s}_t)$  to be part of an equilibrium. By gross substitutability,  $\hat{s}_2 > s_2$  implies  $\hat{q}_2 > q_2$ . Continuing this line of reasoning, it is seen that for all  $t \geq 1$ ,  $\hat{q}_t > q_t \geq MRS(w^y, w^o)$ . Suppose, then, that the corporation adjusts its corporate plan to generate these higher share rates of return  $\hat{q}_t$ ,  $t \geq 1$ . Since the optimal savings level of each consumer in the original equilibrium was nonnegative, it follows from the budget constraints (5) and (6) that each consumer is strictly better off. Q.E.D.

**PROOF OF PROPOSITION 6.4.** Recall that  $\bar{q} = 1$  is the golden-rule rate of return for the Corporate Economy. By Proposition 6.3,  $q_1$  must equal 1 in any Corporate Economy equilibrium. Given  $q_1 = 1$ , it follows by gross substitutability that  $s_1$  must equal the golden-rule savings level  $\bar{s}$ . Also, relation (20) implies that  $s_2 = s_1$ . Hence, applying gross substitutability once more,  $q_2 = q_1 = 1$ . A simple induction argument then gives  $q_t = 1$  and  $s_t = \bar{s}$  for all  $t \geq 1$ . Consequently, in any Corporate Economy equilibrium, the only possible real outcome is the Pareto efficient golden-rule outcome characterized by the stationary share rate of return  $\bar{q} = 1$ , the stationary savings level  $\bar{s}$  for each young consumer, and the stationary consumption levels  $\bar{c}^y$  and  $\bar{c}^o$  for each young and old consumer, respectively.

It will now be shown that there exists at least one Corporate Economy equilibrium that supports this golden-rule outcome. Suppose that the sequence  $\mathbf{P} = (P_1, P_2, \dots)$  of nominal goods prices is a positive sequence satisfying

$$0 \leq M_0/P_1 < \bar{s}; \quad P_t = P_{t+1}, \quad t \geq 1. \quad (25)$$

Taking this sequence of nominal goods prices as given, suppose the corporation selects the corporate plan  $\mathbf{I}^*$  defined as follows: the corporation sets  $d_t^e = 0$  for all  $t \geq 1$ , sets  $v_1 > 0$  so that  $v_1 \theta_0 = \bar{s} - M_0/P_1$ , and sets  $v_t = v_1$  for all  $t \geq 2$ . Note that the share rates of return implied by this corporate plan satisfy  $q_t = 1$  for all  $t \geq 1$  by definition (4).

Consider, first, the perceived viability of the plan  $\mathbf{I}^*$ . Suppose the share demands expected by the corporation under this plan are  $\theta_t^e = \theta_0$  for all  $t \geq 1$ . The real share demands expected by the corporation over time then satisfy  $v_t \theta_t^e = v_1 \theta_0$  for all  $t \geq 1$ , a bounded sequence. Also, by relation (11), the net receipts expected by the corporation in any period  $t$  are zero. Finally, zero expected net receipts in any period  $t$  implies a zero expected dividend distribution, hence the plan  $\mathbf{I}^*$  exhibits dividend consistency. It follows that there is a set of corporate expectations for share demands under which the corporation will perceive the plan  $\mathbf{I}^*$  to be viable.

What about the optimality of the plan  $\mathbf{I}^*$  from the viewpoint of the corporation? Under this plan,  $q_t = 1$  for all  $t \geq 1$ . If the corporation were to attempt to increase the share rate of return  $q_t$  above 1 in any period  $t \geq 1$ , relation (13) implies that the sequence of real share demands expected

by the corporation would become unbounded, a violation of the viability of the corporate plan. Consequently, by gross substitutability, the corporation cannot increase the savings level  $s_1$  of the generation 1 young consumer above  $\bar{s}$ , implying that the corporation is maximizing the windfall return it can provide to the generation 0 old consumer in period 1. In addition, the corporation is maximizing the share rate of return  $q_t$  it can provide to the generation  $t$  young consumer in each period  $t \geq 1$ . Hence, the corporation perceives the plan  $\mathbf{I}^*$  to be optimal.

What about consumer optimization? Given  $q_t = 1$  for all  $t \geq 1$ , it follows from (5) and (6) that each young consumer will choose to consume the golden-rule consumption profile  $(c_t^y, c_{t+1}^o) = (w^y - \bar{s}, w^o + \bar{s}) = (\bar{c}^y, \bar{c}^o)$ . Moreover, under the plan  $\mathbf{I}^*$ , the value of the shares held by the generation 0 old consumer is  $v_1\theta_0 = \bar{s} - M_0/P_1$ , and his expected dividend is 0. It follows from the budget constraint (8) that the generation 0 old consumer will choose to consume  $c_1^o = w^o + \bar{s} = \bar{c}^o$ .

The above discussion establishes that, for positive nominal goods prices satisfying (25), the consumer and corporate optimization conditions in the definition of a Corporate Economy equilibrium are satisfied if the corporation sets the plan  $\mathbf{I}^*$  and has share demand expectations  $\theta_t^e = \theta_0$  for all  $t \geq 1$ . But what about the remaining conditions requiring market clearing for goods and money and fulfilled share and dividend expectations?

Given  $\mathbf{I}^*$ , real outcomes must coincide with the golden-rule solution, and the goods market clearing condition (15) holds by construction for this golden-rule solution. If  $P_1 < +\infty$ , then it follows from (25) that the money rate of return  $r_t$  equals the share rate of return  $q_t = 1$  in each period  $t$ , implying that the consumer is indifferent between holding his optimal real savings  $\bar{s}$  in the form of money or shares. Without loss of generality, it can then be assumed that  $M_t = M_0$  for all  $t \geq 1$ , implying that the money market clearing condition (16) is satisfied. Alternatively, if  $P_1 = +\infty$ , then it follows from (25) that  $P_t = +\infty$  for all  $t \geq 1$ . In this case money has no value and  $M_t = 0$  for all  $t \geq 1$ ; but the money market clearing condition (16) is clearly satisfied for all  $t \geq 1$ . In either case,  $v_t\theta_t = v_1\theta_0 = \bar{s} - M_0/P_1$  for all  $t \geq 1$ . But  $v_t = v_1 > 0$  for all  $t \geq 2$  under plan  $\mathbf{I}^*$ , which forces  $\theta_t = \theta_0$  for all  $t \geq 1$ . It follows that the actual net receipts (dividend payout) of the corporation in each period  $t$  are zero, exactly what the corporation expects under plan  $\mathbf{I}^*$ . Also, the corporation's expected share demands are fulfilled.

In summary, given gross substitutability, the only possible Corporate Economy equilibria are the Pareto efficient golden-rule equilibria supported by  $q_t = 1$  for all  $t \geq 1$ . And it has been shown that at least one Corporate Economy equilibrium exists for each nominal goods price sequence satisfying (25). Q.E.D.

**PROOF OF PROPOSITION 6.5.** The necessity of having  $M_0/P_1 \leq \bar{s}$  in equilibrium follows from the fact that  $M_0/P_1 + [v_1 + d_1]/\theta = \bar{s}$  in equilibrium, as established in Proposition 6.4. But, given  $\theta_0 > 0$ ,  $M_0/P_1 = \bar{s}$  would force  $[v_1 + d_1^e] = 0$ , a violation of the requirement that  $v_1 > 0$  and  $d_1^e \geq 0$ . If  $P_1 < +\infty$ , it follows from the money market clearing condition (19) that  $M_1 = M_0 > 0$ ; but the period 1 young consumer will only be willing to hold a positive amount of money in lieu of shares if  $r_1 = q_1 = 1$ , implying  $P_1 = P_2$ . A simple induction argument then yields the necessity of having  $P_t = P_{t+1}$  for every  $t \geq 1$  in any equilibrium. Conversely, if  $P_1 = +\infty$ , no solution exists for the utility maximization problem (2) in period 1 unless  $P_2 = +\infty$ ; and a simple induction argument

again yields the necessity of having  $P_t = +\infty$  for all  $t \geq 1$  in any equilibrium.

Given a nominal goods price sequence satisfying  $M_0/P_1 < \bar{s}$  and  $P_t = P_{t+1}$  for each  $t \geq 1$ , together with gross substitutability, the proof of Proposition 6.4 establishes the existence of a Corporate Economy equilibrium with the market value of the firm given by  $v_t \theta_t = \bar{s} - M_0/P_1 > 0$  for each  $t \geq 1$ . Q.E.D.

**PROOF OF PROPOSITION 6.6.** The maximum optimal savings level  $s_{max}$  must be strictly less than  $w^y$ ; for otherwise a consumer would choose to consume nothing when young when faced with the rate of return  $q^*$ , an impossibility given the restrictions on  $U(\cdot)$  imposed in section 2. Moreover, the regularity condition (1), together with the assumption of a unique backward bend in the offer curve at  $q^* < 1$ , implies that  $s_{max} \equiv s(q^*)$  is greater than the golden rule savings level  $\bar{s} = s(1)$ ; see Figure 2.

Since  $qs(q) = c^o(q) - w^o$  is a strictly increasing function of  $q$  that takes on the value 0 at  $q = MRS(w^y, w^o)$  and diverges to  $+\infty$  as  $q \rightarrow +\infty$ , the equation  $s_{max} = qs(q)$  has a unique solution  $q_{max} > 0$  for every  $s_{max} > 0$ . By definition of  $s_{max}$ , the rate of return  $q_{max}$  must be greater than 1; for  $q_{max} < 1$  would imply that  $0 < s_{max} = q_{max}s(q_{max}) < s(q_{max}) \leq s_{max}$ , a contradiction, and  $q_{max} = 1$  would imply that  $s_{max} = \bar{s}$ , another contradiction. Also, no share rate of return  $q_t > q_{max}$  can occur in any Corporate Economy equilibrium; for this would imply that the equilibrium savings rate  $s_{t+1} = q_t s(q_t)$  for period  $t + 1$  exceeds  $s_{max} = q_{max}s(q_{max})$ , a contradiction of the definition of  $s_{max}$ .

Since by assumption  $c^o(q)$  is an increasing function of  $q$ , the finding  $q_t \leq q_{max}$ , together with the goods market clearing condition (18) and Proposition 6.1, imply that  $0 < w^o = c^o(MRS(w^y, w^o)) \leq c^o(q_t) \leq c^o(q_{max}) \leq w^o + w^y$ ,  $t \geq 1$ , in any Corporate Economy equilibrium; that is, the equilibrium old-age consumption levels are uniformly bounded above and below by positive finite quantities. Finally, it follows from the goods market clearing condition (18) and the finding  $s_{max} < w^y$  that  $c^y(q_t)$  must satisfy  $0 < w^y - s_{max} \leq w^y - s(q_t) = c^y(q_t) \leq w^y + w^o$ ,  $t \geq 1$ ; that is, the equilibrium young-age consumption levels are also uniformly bounded above and below by positive finite quantities. Q.E.D.

**PROOF OF PROPOSITION 6.7.** Suppose there exists a Corporate Economy equilibrium for which the consumption allocation is not Pareto efficient. By Propositions 6.2 and 6.6, the sequence  $(s_t : t = 1, 2, \dots)$  of (nonnegative) equilibrium savings levels  $s_t$  must then converge to zero. In particular, given any  $\epsilon > 0$ , there exist at most finitely many periods  $t$  such that  $s_t > \epsilon$ .

Let  $\epsilon \equiv s(q_{max})/2 > 0$ . Then for each  $b \leq \epsilon$  the equation  $s(q) = b$  has only one solution  $q(b)$  satisfying  $q(b) \leq q_{max}$ , and this solution satisfies  $0 < q(b) \leq q(\epsilon) < q^* < 1$ ; see Figure 2. Let  $k \geq 1$  be such that the equilibrium savings levels satisfy  $s_t = s(q_t) \leq \epsilon/2$  for all  $t \geq k$ . By Proposition 6.6, the equilibrium rates of return are uniformly bounded above by  $q_{max}$  and below by  $MRS(w^y, w^o)$ . Hence, by choice of  $\epsilon$ , the equilibrium rate of return  $q_t$  for any period  $t \geq k$  must satisfy  $MRS(w^y, w^o) \leq q_t < q(\epsilon) < q^* < 1$ . It follows from (20) that the (nonnegative) equilibrium savings rates  $s_t$  converge to 0 and the equilibrium rates of return  $q_t$  converge to  $MRS(w^y, w^o)$  as  $t \rightarrow \infty$ .

Suppose the corporation increases the rate of return from  $q_t$  to  $q(\epsilon) < q^*$  in each period  $t \geq k$  by an announcement of new higher share prices  $(\hat{v}_{t+1} : t \geq k)$  satisfying  $\hat{v}_{t+1} = q(\epsilon)\hat{v}_t$  for all  $t \geq k$ ,

with  $\hat{v}_k \equiv v_k$ , and an announcement of an increased expected dividend  $\hat{d}_k^e$  for period  $k$  (derived below) and zero expected dividends for all periods  $t + 1$  with  $t \geq k$ . It will now be shown that these changes in its corporate plan would be perceived by the corporation to be viable changes that would benefit each of its shareholders, contradicting the assumption that the economy was originally in a Corporate Economy equilibrium.

By assumption, the optimal savings level  $s(q)$  is a strictly increasing function of  $q$  for all  $q < q^*$ . The increase from  $q_k$  to  $q(\epsilon) < q^*$  in period  $k$  thus leads the generation  $k$  young consumer to increase his savings level; i.e.,  $\hat{s}_k > s_k$ . In particular, since the price sequence  $\mathbf{P}$  is taken as given, the increase in  $q_k$  leads the generation  $k$  young consumer in period  $k$  to reduce his money demand to zero (if it is not already zero) and to increase his real demand for shares. Recalling that  $\hat{v}_k \equiv v_k$ , this implies that  $\hat{s}_k = v_k \hat{\theta}_k > s_k = v_k \theta_k + M_k/P_k \geq v_k \theta_k$ , hence  $\hat{\theta}_k > \theta_k$ . It follows that the expected dividend of the generation  $k - 1$  consumer increases from  $d_k^e = v_k[\theta_k - \theta_{k-1}]/\theta_{k-1}$  to  $\hat{d}_k^e = v_k[\hat{\theta}_k - \theta_{k-1}]/\theta_{k-1}$ . Thus, the welfare of the generation  $k - 1$  consumer is enhanced due to a “windfall” dividend. Moreover, the optimized lifetime utility of each generation  $t$  consumer is an increasing function of the share rate of return  $q_t$  over the range  $q_t \geq MRS(w^y, w^o)$  where the consumer desires to save, hence the change in corporate plan benefits all consumers in generations  $t \geq k$  as well. The welfare of shareholders in any generations previous to  $k - 1$  is unaffected.

Setting the share rate of return equal to the constant level  $q(\epsilon) < q^* < 1$  for all periods  $t \geq k$  ensures by (13) that the real share demands expected by the corporation under its modified corporate plan converge to zero over time, and hence constitute a bounded sequence. Moreover, expected profits  $\hat{\pi}_t^e$  for periods  $t \geq k$  are nonnegative. More precisely, as explained above, the expected dividend (hence the expected amount of profit) for period  $k$  is strictly increased from its original (nonnegative) equilibrium level; and, for any period  $t + 1$  with  $t \geq k$ , it follows from (11), (13), and the specification of the new share prices that expected profits take the form

$$\hat{\pi}_{t+1}^e = \hat{v}_{t+1}[\theta_{t+1}^e - \theta_t^e] = \theta_t^e[\hat{v}_t q(\epsilon) - \hat{v}_{t+1}] = 0. \quad (26)$$

Finally, it follows from (12) that dividend consistency holds as well.

In summary, the assumption that there exists a Corporate Economy equilibrium that supports a Pareto inefficient consumption allocation has led to the contradictory conclusion that the criterion for corporate optimization appearing in the definition of a Corporate Economy equilibrium—namely,  $\mathbf{I} \in \mathcal{I}(\mathbf{P})$ —is not satisfied by this equilibrium. It follows that any Corporate Economy equilibrium must support a Pareto efficient consumption allocation. Q.E.D.

**PROOF OF PROPOSITION 6.8.** Consider the sequential generation of savings and rate of return sequences  $(s_t^*)$  and  $(q_t^*)$  by the relations (20) and (21), starting from the initial condition  $q_1^* = q^*$ , under the “selection principle” that the highest rate of return is always to be chosen when multiple solutions are possible. It will first be shown (Lemma 2, below) that, under the regularity condition (22), such sequences can be generated because the rates of return  $q_t^*$  all lie in the interval  $[q^*, q_{max}]$ , implying in turn that the savings levels  $s_t^*$  all lie in the feasibly supported interval  $[s(q_{max}), s(q^*)]$ , where  $s(q^*) \equiv s_{max}$ .

**LEMMA 1.** *Condition (22) holds if and only if  $q^*s(q^*) \geq s(q_{max})$ .*

**PROOF OF LEMMA 1.** By construction,  $s_{max} \equiv s(q^*) = q_{max}s(q_{max})$ , where  $s(q^*)$  and  $s(q_{max})$  are both strictly positive. Multiplying each side by  $q^*$ , the proof is immediate. Q.E.D.

**LEMMA 2.** *Suppose condition (22) holds as a strict inequality. Then, for all periods  $m \geq 1$  and  $k \geq 1$ , one has: (a) if  $q^* \leq q_m^* < q_k^* < 1$ , then  $1 < q_{k+1}^* < q_{m+1}^* < q_{max}$ ; and (b) if  $1 < q_k^* < q_m^* < q_{max}$ , then  $q^* < q_{m+1}^* < q_{k+1}^* < 1$ .*

**PROOF OF LEMMA 2.** By assumption, the optimal old-age consumption level  $c^o(q) = w^o + qs(q)$  is a strictly increasing function of  $q$  over the interval  $q \geq q^*$ , and the optimal savings level  $s(q)$  is a strictly decreasing function of  $q$  over  $q \geq q^*$ . Moreover, by (20),  $s(q_{t+1}^*) = q_t^*s(q_t^*)$  for all  $t \geq 1$ . Hence the conditional statement in (a) implies that  $c^o(q^*) \leq c^o(q_m^*) < c^o(q_k^*) < c^o(1)$ , which in turn implies—using (20), (22), and Lemma 1—that  $s(q_{max}) < s(q_{m+1}^*) < s(q_{k+1}^*) < s(1)$ . The selection principle then guarantees that the desired conclusion in part (a) holds. Similarly, the conditional statement in part (b) implies that  $s(1) < s(q_{k+1}^*) < s(q_{m+1}^*) < s_{max}$ , thus the selection principle guarantees that the desired conclusion in part (b) holds. Q.E.D.

**LEMMA 3.** *The rate of return  $q_t^*$  lies in the interval  $[q^*, q_{max}]$  for each  $t \geq 1$ . In particular, if (22) holds as an equality, then  $q_t^* \in \{q^*, q_{max}\}$  for all  $t \geq 1$ ; and if (22) holds as a strict inequality, then  $q^* \leq q_{2t-1}^* < q_{2t+1}^* < 1 < q_{2t+2}^* < q_{2t}^* < q_{max}$  for all  $t \geq 1$ .*

**PROOF OF LEMMA 3.** If  $q^*q_{max} = 1$ , implying that  $q^*s(q^*) = s(q_{max})$ , then the sequence  $(q_t^*)$  cycles back and forth between  $q^*$  and  $q_{max}$  in a simple period 2 orbit starting with  $q_1^* = q^*$ . To see this, note by (20) that, if  $q_t = q_{max}$  in some period  $t$ , then the equilibrium savings level for period  $t + 1$  is  $q_{max}s(q_{max}) = s_{max}$ , a savings level that is uniquely supported by the share rate of return  $q^*$ . Conversely, if  $q_t^* = q^*$  in some period  $t$ , then the equilibrium savings level for period  $t + 1$  is  $q^*s(q^*) = s(q_{max})$ . The rates of return  $q_t$  thus cycle back and forth between  $q^*$  and  $q_{max}$ .

Suppose, instead, that  $q^*q_{max} > 1$ , implying that  $q^*s(q^*) > s(q_{max})$ . By construction,  $c^o(q_1^*) < \bar{c}^o$ ; and all savings levels greater than  $s(q)$  are supported by rates of return that are strictly smaller than  $q$ , for all  $s(q) \geq s(q_{max})$ . Also, from (20), one has the goods market clearing condition  $c^o(q_t^*) + c^y(q_{t+1}^*) = w^y + w^o$  for every  $t \geq 1$ . In particular, the golden rule consumption profile satisfies this condition, implying that  $\bar{c}^y + \bar{c}^o = w^y + w^o$ . It then follows from  $s(q_{max}) < q_1^*s(q_1^*) = s(q_2^*)$  that  $q_2^* < q_{max}$ . And  $c^o(q_1^*) + c^y(q_2^*) = w^y + w^o$ , with  $c^o(q_1^*) < \bar{c}^o$ , implies that  $c^y(q_2^*) > \bar{c}^y = c^y(1)$ ; hence  $q_2^* > 1$ , for otherwise the selection principle used in the construction of  $(q_t^*)$  is violated. Thus,  $q_1^* < 1 < q_2^* < q_{max}$ . To complete the proof, it suffices to show that  $q_1^* < q_3^* < 1$ ; for then, using Lemma 2, it follows by a simple induction proof that  $q^* \leq q_{2t-1}^* < q_{2t+1}^* < 1 < q_{2t+2}^* < q_{2t}^* < q_{max}$  for all  $t \geq 1$ .

By assumption,  $c^o(q) = w^o + qs(q)$  is a strictly increasing function of  $q$  for all  $q \geq MRS(w^y, w^o)$ . Together with relation (20), and the earlier finding that  $q_2^* > 1$ , this implies that  $s(q_3^*) = q_2^*s(q_2^*) > s(1)$ . But, by the restrictions on the offer curve, all savings levels greater than  $s(1)$  are supported by rates of return less than 1; cf. Figure 2. In particular, then,  $q_3^* < 1$ . Moreover, the selection principle guarantees that  $q_3^* \geq q_1^* \equiv q^*$ . If  $q_3^* = q_1^*$ , then  $s(q_3^*) = s(q^*) = s_{max} = q_{max}s(q_{max})$ . But this leads to a contradiction; for it holds by (20) that  $s(q_3^*) = q_2^*s(q_2^*)$ , and it has been established above that  $q_2^* < q_{max}$ , hence  $s(q_3^*) < s_{max}$ . It follows that  $q_3^*$  must be strictly greater than  $q_1^*$ . Q.E.D.

By Lemma 3, the savings and rate of return sequences  $(s_t^*)$  and  $(q_t^*)$  constitute a feasible real outcome for the Corporate Economy. For the reasons explained in the main text, under the hypotheses of Proposition 6.6 any Corporate Economy equilibrium must support these sequences. It will now be shown that there exist infinitely many such equilibria.

Let  $M_0$  and  $\theta_0$  be any given positive values for the money and share holdings of the generation 0 consumer, and let  $P_1^*$  denote any positive (possibly infinite) nominal price level for period 1 that satisfies  $0 \leq M_0/P_1^* < s_1^*$ , where by construction  $s_1^* = s(q_1^*) = s_{max}$ . It follows from (19) that the nominal price sequence must then be given by

$$P_t^* = \left[ \frac{1}{q_{t-1}^* \cdots q_1^*} \right] P_1^*, \quad t \geq 2, \quad (27)$$

in order for the money market to clear in each period  $t \geq 1$ .

Taking the price sequence  $(P_t^*)$  as given, suppose the corporation announces a corporate plan  $I = (v, d^e)$  that is consistent with the following requirements. The corporation announces a period 1 share price  $v_1 > 0$  and expected dividend  $d_1^e \geq 0$  that satisfy

$$[v_1 + d_1^e]\theta_0 = [s_1^* - M_0/P_1^*] > 0. \quad (28)$$

This setting supports the maximum possible windfall return for the generation 0 consumer in period 1. All remaining share prices  $v_{t+1} > 0$  and expected dividends  $d_{t+1}^e \geq 0$  are then set so that they are consistent with the requirement

$$\frac{v_{t+1} + d_{t+1}^e}{v_t} = q_t^*, \quad t \geq 1. \quad (29)$$

By construction, these settings yield the highest feasible share rate of return  $q_t^*$  in each period  $t \geq 1$ . Finally, the real share demand  $\theta_t^e$  expected by the corporation in each period  $t$  is determined by

$$s_t^* = M_0/P_t^* + v_t\theta_t^e, \quad t \geq 1. \quad (30)$$

By construction, given  $(P_t^*)$ , these real share demand expectations are fulfilled along any path for which the savings sequence  $(s_t^*)$  is fulfilled.

Finally, what about dividend consistency, nonnegative net receipts, and fulfilled dividend expectations in each period  $t$ ? Combining (28) with (30) for  $t = 1$ , one obtains

$$v_1[\theta_1^e - \theta_0] = d_1^e\theta_0. \quad (31)$$

And noting that (27) implies that  $M_0/P_t^* = q_{t-1}^*M_0/P_{t-1}^*$  for all  $t \geq 2$ , it follows from (20), (29), and (30) that

$$v_t\theta_t^e = q_{t-1}^*v_{t-1}\theta_{t-1}^e = [v_t + d_t^e]\theta_{t-1}^e, \quad t \geq 2. \quad (32)$$

Consequently, expected net receipts  $\pi_t^e$  in each period  $t \geq 1$  satisfy

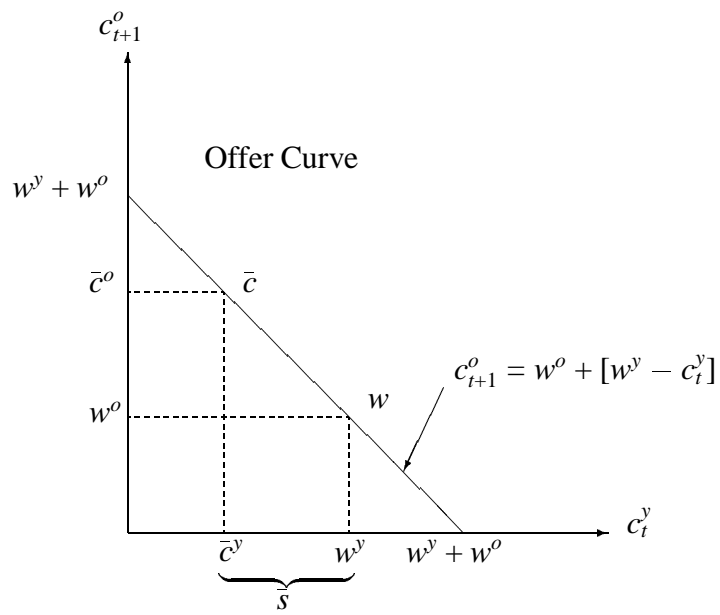
$$\pi_t^e \equiv v_t[\theta_t^e - \theta_{t-1}^e] = d_t^e\theta_{t-1}^e, \quad (33)$$

where  $\theta_0^e \equiv \theta_0 > 0$ . Thus, in period 1, dividend consistency holds and expected net receipts are nonnegative. A simple induction argument then establishes that  $\theta_t^e \geq \theta_{t-1}^e > 0$  for all  $t \geq 1$ , hence dividend consistency holds and expected net earnings are nonnegative in each period  $t \geq 1$ . Finally, given the fulfillment of the real share demand expectations in each period  $t \geq 1$ , it follows from (33) that all dividend expectations are fulfilled as well.

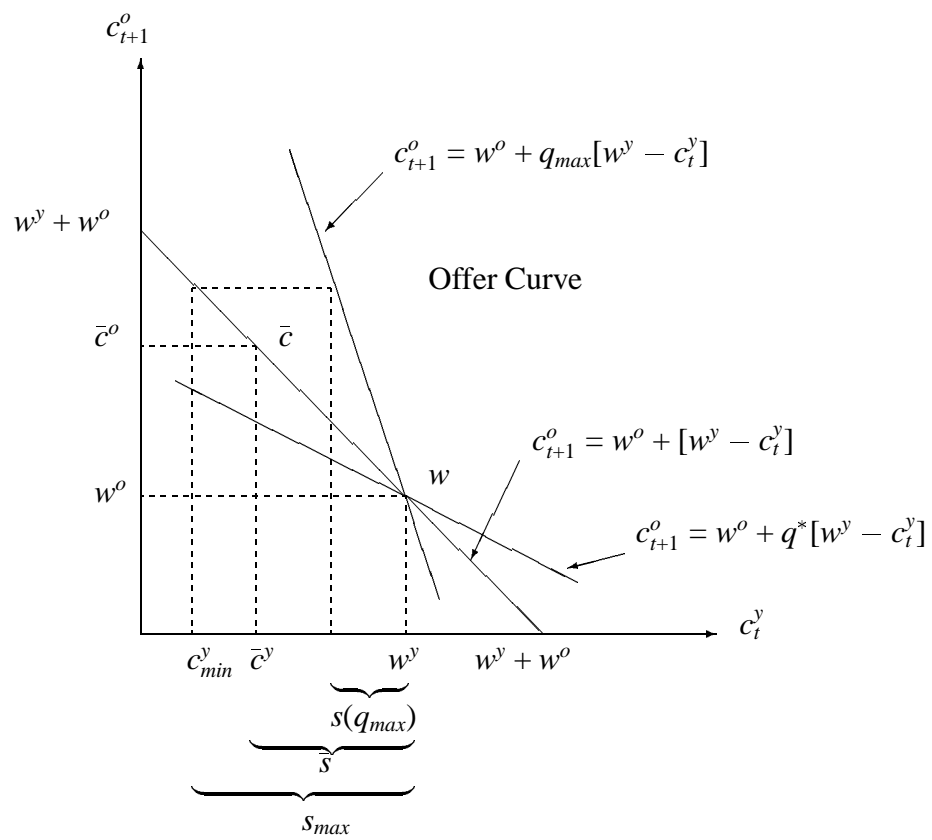
In summary, infinitely many Corporate Economy equilibria can be constructed to support the savings and rate of return sequences  $(s_t^*)$  and  $(q_t^*)$ . Q.E.D.

**PROOF OF PROPOSITION 6.9.** The proof of case (a) follows from Lemma 2 and Lemma 3, established in the course of proving Proposition 6.8.

Suppose (22) holds as a strict inequality. It then follows from Lemma 3 that the subsequence  $(q_{2t-1}^* : t \geq 1)$  is strictly increasing in  $t$  over the subinterval  $[q^*, 1]$  and the subsequence  $(q_{2t}^* : t \geq 1)$  is strictly decreasing in  $t$  over the subinterval  $[1, q_{max}]$ . It follows by the monotone convergence theorem that these subsequences must converge to elements in these subintervals, say  $q_L \in [q^*, 1]$  and  $q_U \in [1, q_{max}]$ , respectively. If  $q_L = q_U = 1$ , then  $q_t^*$  converges cyclically to the golden rule rate of return  $\bar{q} = 1$ . Suppose  $q_L < q_U$ , implying that the full sequence  $(q_t^*)$  converges to a limit cycle supported by the period 2 orbit  $\{q_L, q_U\}$  with either  $q_L < 1$  or  $1 < q_U$ . In order for such a limit cycle to exist, it must hold by (20) that  $s(q_L) = q_U s(q_U)$  and  $s(q_U) = q_L s(q_L)$ . But this implies that  $s(q_L) = q_L q_U s(q_L)$ , which is only possible if  $q_L q_U = 1$  with  $q_L < 1 < q_U$ . Q.E.D.



**Figure 1**



**Figure 2**