Pareto Efficiency vs. the Ad Hoc Standard Monetary Objective

An Analysis of Inflation Targeting

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Abstract

The standard ad hoc monetary objective function creates a bias in favor of inflation targeting. Instead, this paper uses the Pareto criterion to assess inflation targeting (IT), price-level targeting (PLT), and nominal-income targeting (NIT). The effect that unanticipated inflation or deflation benefits one party to a nominal contract while hurting the other party is an effect that cannot be captured in a model with a representative consumer or identical consumers. To capture this effect, this paper analyses models with diverse consumers in a pure-exchange economy without storage. When nominal aggregate demand (NAD) is stochastic but real aggregate supply (RAS) is not, PLT Pareto dominates IT. This is because IT perpetuates price errors and hence nominal aggregate demand errors, while PLT tries to return to the original targeted price path. By perpetuating these errors, IT perpetuates the welfare losses, whereas PLT corrects so to help reduce these welfare losses in the future. When RAS is also stochastic, nominal contracts under NIT can lead to Pareto efficiency when consumers have average relative risk aversion, non-stochastic endowment-to-RAS ratios, and no utility shocks. Under the same assumptions IT and PLT lead to Pareto inefficiencies because they force the payers of nominal contracts to guarantee the real value of those payments to the receivers. In essence this transfers RAS risk from the receivers of the nominal obligations to payers of the nominal obligations. However, this transfer of risk would only be appropriate if all payers of nominal obligations had below average relative risk aversion and all receivers had above average relative risk aversion, a situation that rarely will hold.

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An Analysis of Inflation Targeting

Several leading monetary economists have expressed the importance of macro and monetary economic analysis being based on rigorous and consistent microfoundations, general equilibrium analysis, and welfare analysis using the Pareto criterion. For example, Sargent and Wallace (1975) described the IS-LM-NRPC model as an "ad hoc" model, which they considered "deplorable."

While some movement has taken place toward microfoundations and welfare analysis with the Pareto criterion and away from ad hoc models, many macro/monetary economists still continue to perform ad hoc analysis. This ad hoc analysis is most prevalent in the form of the ad hoc standard objective assumed for monetary policy.

The ad hoc standard objective is to minimize the following function:

$$\lambda \sum_{t=0}^{\infty} \beta^t \left(\pi_t - \pi_t^*\right)^2 + \left(1-\lambda\right) \sum_{t=0}^{\infty} \beta^t \left(Y_t - Y_t^P\right)^2$$

where π_t and Y_t are respectively the inflation rate and the level of real aggregate supply (RAS) at time t, π_t^* is the targeted inflation rate, Y_t^P is the potential level of aggregate output at time t, β is the time discount factor of the policy makers, and λ is the weight between 0 and 1 on the squared deviation of inflation; 1-λ is the weight on the squared output gap.

The problems with ad hoc objective functions are well known in microeconomics. An ad hoc objective function depends on the values of the researcher; they do not have a theoretical basis. They also bias the conclusions or policy recommendations. In particular, the monetary standard ad hoc objective function as applied to a model with flexible prices by assumption leads to the conclusion that inflation targeting (IT) is better than price-level targeting (PLT). However, if we replaced the actual and targeted inflation rates with the actual and targeted price levels, we then would conclude that price-level targeting is superior to inflation targeting. Actually, Svenson's (1999) definition of inflation targeting is in essence the use of the standard ad hoc objective function. Clearly, the use of the standard ad hoc objective function cannot be the basis for determining whether inflation targeting or price-level targeting is better.

Microeconomics provides Pareto efficiency as an alternative to ad hoc goals and objectives. A consumption allocation is Pareto efficient if there is no other feasible consumption allocation that makes at least one party better off without making anyone worse off. Today, most rigorous microeconomic welfare analysis uses the Pareto criterion.

Welfare analysis involving the Pareto criterion has been used in much macroeconomic analysis since the Rational Expectations revolution. Such analysis usually relies on microfoundations involving general equilibrium models. However, usually these models assume either a representative consumer or identical consumers. Nevertheless, most monetary economists consider the following to be one of the primary effects of monetary policy:

> Unanticipated inflation or deflation makes one party of a nominal bond or other nominal contract better off while making the other party worse off.

However, such an effect cannot be captured by a representative consumer model or in a model with identical consumers.[1]

By including diverse consumers, this paper's analysis does capture the different impacts of unanticipated inflation and deflation on different individuals. The *a priori* effect of this impact on individuals is what leads this paper to its unique conclusions. To simplify our analysis, we use flexible price models so that we need not worry about the tradeoff between

[1] See Chari, Christiano, and Kehoe (1991) for an example of the use of identical consumers.

inflation and output gap. We find that PLT is Pareto superior to IT when long-term nominal contracts exist. When consumers have the same relative risk aversion, we find that nominal income targeting (NIT) will Pareto dominate both PLT and IT.

The next section, section II, reviews the Fundamental Theorem of Welfare Economics and discusses the importance of complete markets and the need for contracts to deal with "contingencies." In section III, we review an Arrow-Debreu economy with state-contingent securities to see how Pareto-efficient consumption varies with NAD (nominal aggregate demand) and with RAS. Section IV studies IT and PLT when NAD is stochastic but RAS is not. Section V extends this analysis to when RAS is also stochastic and also considers NIT. In section VI, we discuss what happens when consumers differ in their relative risk aversion, have stochastic endowment-to-RAS ratios, and experience utility shocks. In section VII, we conclude and reflect upon this paper's findings.

II. Review of First Theorem of Welfare Economics and Complete Markets

The First Fundamental Theorem of Welfare Economics states that under rather general assumptions, any competitive equilibrium with complete markets is Pareto efficient. If money in the economy is nondistortionary, this theorem indicates that there are three ways that monetary policy can help an economy move to Pareto efficiency: (1) move the economy towards more competition, (2) move the economy towards equilibrium, and (3) move the economy towards complete markets. The analyses involving welfare improvements in economies with the Dixit-Stiglitz (1977) model can be looked at as the first, reducing the welfare loss resulting from monopolistic competition. Much of Keynesian economic recommendations to try to move the economy back to its full-employment level of output can be looked at as the second, moving the economy towards equilibrium.

However, many economists do their analysis using flexible price models with competitive markets. In competitive economies with flexible prices and nondistortionary money,[2] the First Fundamental Theorem of Welfare Economics indicates that the only way that monetary policy can help move the economy towards Pareto efficiency is to help complete markets. This paper takes the role of monetary policy to help complete markets very seriously.

To understand what complete markets means, we focus on the Arrow-Debreu economy, which is the economy that we most associate with complete markets. Markets are completed in the Arrow-Debreu economy by state-contingent securities. To help us understand how state-contingent securities help complete markets, Figure 1 plots the payoff function of one particular state-contingent security. Note that the state-contingent security pays iff one particular state-contingent security occurs; if any other state of nature occurs, the holder of the security will receive nothing. Figure 2 shows how we can combine many different state-contingent securities to obtain any payoff function one wishes to receive. Note that the payoff function is dynamic; it is not static. The payoff varies across state of nature; it does not remain constant. By combining state-contingent securities, an individual can obtain a payoff function that will vary based on contingencies. These state-contingent securities enable individuals to meet whatever contingency plan they decide is in their best interest; i.e., to maximize their utility functions.

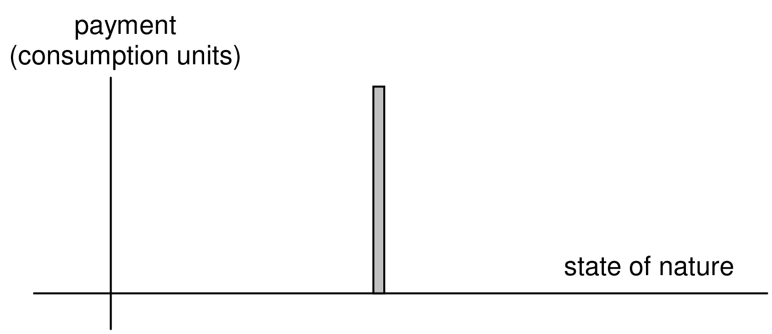

Figure 1: State-contingent security payoff function

[2] If money is distortionary, then monetary policy can effect Pareto improvements by reducing the distortionary effect of money.

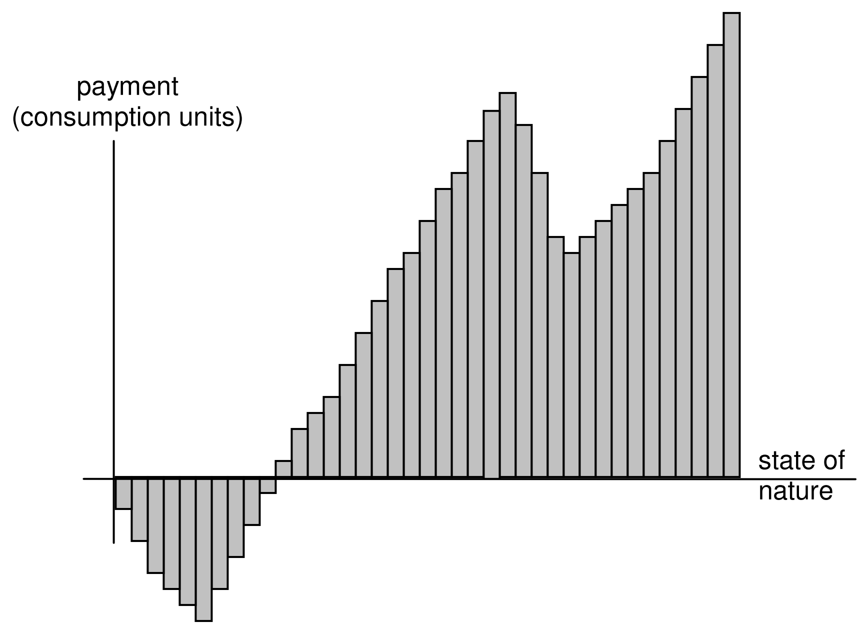

Figure 2: Dynamic Contract Receipts from State-contingent security payoff function

In a pure-exchange economy without storage, Eagle (2005a and 2005b) shows that there is only one type of risk that cannot be diversified away. This is the risk of RAS changing. Suppose we replace the state of nature on the horizontal axis in Figures 1 and 2 with RAS. Then we would see that individuals would be able to buy state contingent securities to produce any payoff function of RAS that they would need to maximize their utility. A major part of this payoff function should be concerned with how one's Pareto-efficient consumption should vary with RAS. Another factor relevant to macro and monetary economic theory is how Pareto-efficient consumption should vary with changes in NAD. The next section reviews Eagle's (2005a and 2005b) conclusions concerning how Pareto-efficient consumption should vary with both NAD and RAS.

III. How PE Consumption Varies with NAD and RAS

This section discusses how Pareto-efficient consumption should vary with NAD and RAS in a pure-exchange economy without storage. Since Eagle (2005a and 2005b) discusses these issues, this section just reviews his results.

In a pure exchange economy without storage but with risk-averse consumers, Eagle (2005b) proves the Consumption-Aggregate-Supply Invariance Property, which states that if there are no utility shocks and RAS stays the same, then an individual's Pareto-efficient consumption must be the same. He uses a proof by contradiction that is similar to proofs that insurance makes people better off. If there are two states of nature where RAS is the same but the Pareto-efficient consumption allocation differs between the two states, then replacing the consumption in each of the two states with the average of the two state consumption allocations makes the risk-averse consumers better off without making anyone worse off. This results in a Pareto Superior allocation, violating the assumption that the initial allocation was Pareto efficient.

The Consumption-Aggregate-Supply Invariance Property has profound implications concerning monetary economic theory. It implies that if NAD changes but RAS does not, then Pareto-efficient consumption should not change. As mentioned at the beginning of the paper, one of the most important effects of monetary policy can be summarized in the following statement:

> Unanticipated inflation or deflation makes one party of a nominal bond or other nominal contract better off while making the other party worse off.

Figure 3 explains how this effect is related to the need for PE consumption to be invariant to changes in NAD when RAS does not change. Let N_t^E be the expected level of NAD at time t when the two parties initially entered into their nominal contract. Let $c_{jt}(N_t^E)$ be individual j's consumption if NAD at time t is as expected. Figure 3 shows that if NAD exceeds N_t^E, then the receiver of a nominal obligation under the contract will be made worse off while the payer of the obligation will be made better off. Similarly, it shows that if NAD is less than expected, then the receiver of the nominal obligation is made better off, while the payer of the obligation is made worse off.

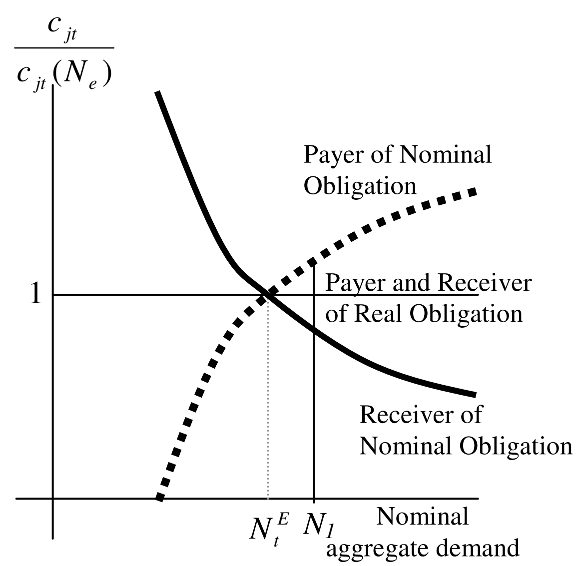

Figure 3: Effects of Unexpected Changes in Nominal Aggregate Demand on the Consumption of Parties to Nominal Obligations

To understand Figure 3, it is important to review the equation of exchange, which I would write as *MV=N=PY*. Note that I insert *N* for NAD in the middle of this equation. Note that while many economists describe the equation of exchange as a tautology, the right side of the equation is in fact an equilibrium condition, not a tautology.

Take the right side of the equation of exchange, *N=PY*, and solve for *P*; we get *P=N/Y*. Under the assumption that *Y* (RAS) does not change, *P* will change only if *N* changes. Again look at Figure 3. If *N* increases, the price level increases, causing the real value of the nominal obligation to decrease. This makes the payer better off and the receiver worse off.: On the other hand, if *N* decreases, the price level decreases, increasing the real value of the nominal obligation, which will make the payer worse off and the receiver better off.

Since both the payer and receiver are risk averse, both will be better off a prior if they were not exposed to this risk. Thus, as shown in Figure 3, any Pareto-efficient consumption allocation must have the property that both the payer's and receiver's consumption does not vary with NAD.

In the next section we use Figure 3 to show that PLT Pareto dominates IT when NAD is stochastic but RAS is not.

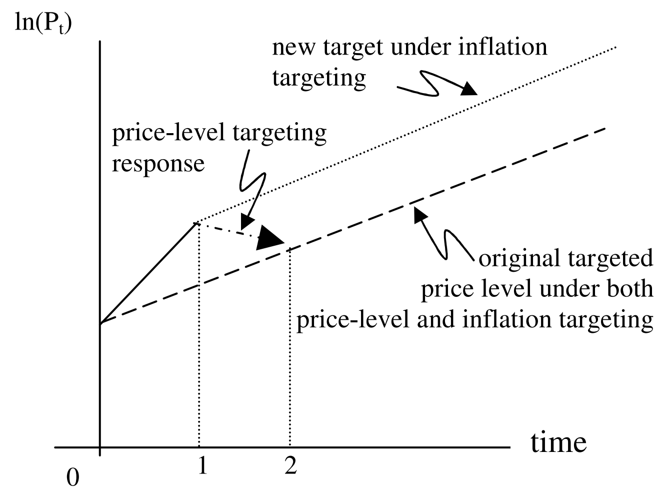

Figure 4: Changes in Implicit Price Targets Under Inflation Targeting

IV. IT and PLT when NAD is Stochastic, but RAS is Not

This section begins our discussion of IT and PLT. First, we explain the difference between IT and PLT. Second, we apply the Consumption-Aggregate-Supply Invariance Property to evaluate IT and PLT under the Pareto criterion.

Figure 4 shows an example that illustrates the difference of IT and PLT. On the vertical axis is the natural logarithm of the price level. The initial targeted paths of IT and PLT are the same. Assuming a constant desired inflation rate, this initial targeted path would be a straight line. The difference between IT and PLT occurs when IT or PLT miss their targets. Figure 4 shows the scenario where the price level and hence the inflation rate exceed the targeted price level and targeted inflation rate at time 1. Under PLT, the next period's targeted price level does not change. As a result, PLT tries to return to the original targeted path. Under IT, the central bank's response is to merely try to reach for its inflation targets for future periods. As a result, the implied targeted price level under IT will be parallel to that under PLT. In other words, under IT, the central bank changes its implied targeted price level, whereas with PLT, it does not.

An advantage of working with a flexible price model such as the competitive pure exchange model analyzed by Eagle (2005a and 2005b), is that it allows us to study IT and PLT without being concerned about output gap. This is because in a flexible price model, output gap always equals zero.

Let us define the inflation rate as $\pi_t \equiv P_t / P_{t-1}$. Then, the implied one-step-ahead price target under IT equals $\pi_t^* P_{t-1}$. In other words, monetary policy will be set so that $E_{t-1}[P_t] = \pi_t^* P_{t-1}$. By the (right side) of the equation of exchange, this implies that $E_{t-1}[N_t / Y_t] = \pi_t^* N_{t-1} / Y_{t-1}$. When RAS is known with certainty, we can rewrite this as:

$$E_{t-1}[N_t] = \pi_t^* N_{t-1} Y_t / Y_{t-1} \tag{1}$$

Taking the expectations at time 0 of both sides, we get:

$$E_0[N_t] = \pi_t^* N_{t-1} Y_t / Y_{t-1}$$

If we apply this recursively backwards to time 0, we get $E_0[N_t] = N_0 Y_t / Y_0 \prod_{s=1}^{t} \pi_s^*$. Let us assume that the nominal contract we are concerned about is entered into at time 0. Then, $N_t^E \equiv E_0[N_t] = N_0 Y_t / Y_0 \prod_{s=1}^{t} \pi_s^*$. Since $N_{t-1}^E \equiv E_0[N_{t-1}] = N_0 Y_{t-1} / Y_0 \prod_{s=1}^{t-1} \pi_s^*$, we conclude that $N_t^E = N_{t-1}^E Y_t / Y_{t-1}$. Subtracting this latter equation from (1) gives:

$$E_{t-1}[N_t - N_t^E] = \pi_t^* \left(N_{t-1} - N_{t-1}^E\right) Y_t / Y_{t-1}$$

Taking the expectation of both sides at some time s between 0 and t, we get:

$$E_s[N_t - N_t^E] = \pi_t^* E_s \left(N_{t-1} - N_{t-1}^E\right) Y_t / Y_{t-1}$$

Backwards recursion implies that:

$$E_s[N_t - N_t^E] = E_s \left(N_s - N_s^E\right) Y_t / Y_s \prod_{k=s+1}^{t} \pi_s^* \tag{2}$$

Equation (2) shows that IT perpetuates the difference between actual nominal aggregate demand and the level of nominal aggregate demand expected when the parties initially entered into the contract.

Now, let's look at PLT where the central bank will adjust its monetary policy so that $E_{t-1}[P_t] = P_t^*$. By the right side of the equation of exchange, *N=PY*, so *P=N/Y*. Substituting this in gives $E_t[N_t / Y_t] = P_t^*$. If RAS (Y_t) is known with certainty, we can then rewrite this as $E_{t-1}[N_t] = P_t^* Y_t$. By backwards recursion, we conclude that $N_t^E \equiv E_0[N_t] = P_t^* Y_t$. We therefore conclude that

$$E_{t-1}[N_t] = N_t^E \qquad (3)$$

Since equation (3) always applies under PLT when RAS is not stochastic, this implies that the expected NAD error under PLT is zero when RAS is not stochastic.

Let's now return to Figure 3 to evaluate IT and PLT under the Pareto criterion. Let N_t^E be the level of NAD the contract parties initially expected. Next assume that NAD in the first period equals $N_1 > N_1^E$. As shown in Figure 3, when RAS does not change, this would increase the price level, decreasing the real value of the nominal obligation, making the payer of the nominal obligation better off and the receiver of the nominal obligation worse off. By (2), IT would expect to maintain this NAD error. In other words, it would attempt to maintain the payer being better off and the receiver worse off for future periods as well as the current period. On the other hand, PLT would try to eliminate future NAD errors so that both the payer and the receiver will expect their real payments to be as they originally expected, that the expected future NAD would be as originally expected.

Similarly, if $N_1 < N_1^E$ and RAS does not change, then the price level decreases, causing the real value of the nominal obligation to increase. This makes the payer worse off and the receiver better off. Again, IT would attempt to maintain this NAD discrepancy, causing the payer to expect to be worse off and the receiver to expect to be better off relative to their initial expectation not only for the current period but also for future periods. With PLT, only in the current period would the payer be better off and the receiver worse off since PLT will attempt to return to NAD to its original targeted level.

A priori both the payer and the receiver will be worse off with IT compared to PLT. With IT, the present value of their welfare losses over time will be much greater than under PLT since IT perpetuates NAD errors, whereas PLT corrects for them.

The appendix presents a general equilibrium example to illustrate the welfare losses of IT compared to PLT. The per capital RAS is 300 consumption units and consumers have identical logarithmic utility functions with a common time preference discount factor. However, there are two types of consumers: Type-A consumers receive a positive endowment at time 0 and no endowments thereafter. Type-B consumers receive no endowment at time 0, but do receive a constant positive endowment thereafter.

The only security that exists in the model is a perpetual bond. We use a perpetual nominal bond because it is the extreme of a long-term nominal bond, but its mathematics are simpler than an annuity. The nominal payments on this perpetual bond grow over time to counteract the erosion on the real value of the payments caused by a positive expected inflation rate. Rather than disrupting the flow of the main body of this paper, most of the assumptions, model description, and analysis are put in the appendix. The particular example assumed does result in a Pareto-efficient consumption allocation under perfect certainty.

			Price Level Targeting				Inflation Targeting			
	prices on target		+ 2% Price Error		- 2% Price Error		+ 2% Price Error		- 2% Price Error	
Type A consumers:										
time	cons.	utility	cons.	utility	cons.	utility	cons.	utility	cons.	utility
0	300.00	5.70378	300.00	5.70378	300.00	5.70378	300.00	5.70378	300.00	5.7038
1	300.00	5.70378	294.12	5.68398	306.12	5.72399	294.12	5.68398	306.12	5.7240
t>1	300.00	5.70378	300.00	5.70378	300.00	5.70378	294.12	5.68398	306.12	5.7240
discounted sum	243.36139			243.34205		243.38111		242.53628		244.2032
					average discounted utility	243.36158			average discounted utility	243.3697
					expected utiltiy loss	-0.00020			expected utiltiy loss	-0.0083
Type B consumers:			Price Level Targeting				Inflation Targeting			
time	cons.	utility	cons.	utility	cons.	utility	cons.	utility	cons.	utility
0	300.00	5.70378	300.00	5.70378	300.00	5.70378	300.00	5.70378	300	5.70378
1	300.00	5.70378	300.14	5.70425	299.85	5.70329	300.14	5.70425	299.85	5.70329
t>1	300.00	5.70378	300.00	5.70378	300.00	5.70378	300.14	5.70425	299.85	5.70329
discounted sum	243.36139			243.36185		243.36091		243.38099		243.3410
					average discounted utility	243.36138				243.3610
					expected utiltiy loss	-0.00001				-0.00040
					aggregated disc. utility loss	-138.4749			aggregated disc. utility loss	-5908.26

Table I. Expected utility losses resulting from PLT and IT

Table I presents the results of the example in the appendix. If prices are always on target, then the Pareto-efficient consumption allocation is achieved where all individuals consume 300 units, resulting in utility each period of 5.70378 utils and a discounted sum of utility of 234.36139 utils for each individual. We then look at what happens if the price level at time 1 is 2% greater than expected or 2% less than expected. We assume that not only were these price errors unexpected, but the consumers did not anticipate even the possibility of these errors occurring.

If the price level is 2% greater than expected, the type-A consumers, who had purchased the perpetual bonds will consume less because the higher price level reduces the real value of their nominal payments. On the other hand, the type-B consumers will consume more since they had to pay less in real terms on the perpetual bonds they issued. Thus, the type-A consumers are hurt from this price-level increase, while the type-B consumers benefited. This is true in period 1 regardless whether the central bank targets the price level or inflation.

If the price level is 2% less than expected, the type-A consumers will consume more because the lower price level increases the real value of their nominal payments. On the other hand, the type-B consumers will consume less than they expected since they had to pay more in real terms on the perpetual bonds they issued. Thus, the type-A consumers benefit and the type-B consumers are hurt from this price-level decrease.

If the increase and decrease of the price levels at time 1 each occur with probability 0.5, then both the type-A and type-B consumers are made worse off in an expected sense as compared to the certainty of knowing the future prices.

Under Price-Level-Targeting (PLT), the central bank will return the price levels for periods t>1 to their original price targets. Thus, consumers only experience an expected utility loss in the first period. (In the example in the appendix, the only unintentional price error that occurs is at time t=1.) With IT, the central bank only tries to keep future inflation rates equal to the targeted inflation rates, which allows the price level to continue to either overshoot its target (for the +2% price error), or to continue to undershoot its target (for the –2% price error). Thus, IT perpetuates the utility losses beyond period 1. In a discounted sense the expected utility losses in this example are about 40 times greater with IT than with PLT.

This section's analysis assumed RAS did not change. In the next section, we study how stochastic RAS changes the analysis.

V. IT, PLT, and NIT when both NAD and RAS are Stochastic

In this section, we analyze what will happen when RAS is stochastic as well as NAD. In addition to studying IT and PLT, we also study NIT. We find that when all consumers have the same relative risk aversion, NIT should dominate IT and PLT.

Define $\bar{c}_t \equiv \frac{1}{m}\sum_{j=1}^{m} c_{jt}$ to be the average consumption across the consumption. In a pure exchange economy without storage, $\sum_{j=1}^{m} c_{jt} = Y_t$, which implies that $\bar{c}_t = Y_t / m$. The RAS elasticity of average consumption is defined as $\frac{\partial \bar{c}_t}{\partial Y_t}\frac{Y_t}{\bar{c}_t}$, which equals $\frac{\partial \bar{c}_t}{\partial Y_t}\frac{Y_t}{\bar{c}_t} = \frac{1}{m}\frac{Y_t m}{Y_t} = 1$. This means that an individual must decrease (increases) his/her consumption by 1% whenever RAS decreases (increases) by 1%. Eagle(2005a) goes beyond this basic truism to show that in a pure-exchange economy without storage, this same relationship needs to apply to any particular individual with average relative risk aversion. (By this we mean the average of relative risk aversion across the whole economy.) In other words, Eagle shows that the RAS elasticity of an individual's PE consumption must equal one if that individual has average relative risk aversion.

Now let's return to a nominal contract. Let C_t be the nominal payment on that contract due at time t. The real value of that payment will be C_t/P_t. Since $P_t=N_t/Y_t$ by the equation of exchange, the real value of this nominal payment is $c_t \equiv \frac{C_t}{N_t}Y_t$. Let's first evaluate the RAS elasticity of the real value of a nominal payment under successful NIT, which we define as when $N_t = N_t^*$. The RAS elasticity of $c_t \equiv \frac{C_t}{N_t}Y_t$ is $\frac{\partial c_t}{\partial Y_t}\frac{Y_t}{c_t} = \frac{C_t}{N_t}\frac{Y_t}{\frac{C_t}{N_t}Y_t} = 1$. In other words, under NIT, the RAS elasticity of the real value of a nominal payment under successful NIT is one, which is also what the RAS elasticity of PE consumption is for someone who has average relative risk aversion. Also, this is also the RAS elasticity of average consumption.

Now, let's consider the RAS elasticity of the real value of a nominal payment under successful IT or PLT. By successful PLT, we mean that the price level does end up equaling its target, i.e., $P_t = P_t^*$, for all t>0. For successful IT, we mean that the inflation rate does end up equaling its target, i.e., $\pi_t = \pi_t^*$, for all t>0. While such success is unlikely to be so perfect, analyzing how IT and PLT work under such perfect success is useful. Note that under such perfect success, the implied price levels under PLT and IT will be the same. Under such perfect success, the real value of nominal payments under IT and PLT will be nonstochastic, even if RAS is stochastic. Thus the RAS elasticity of the real value of a nominal payment under successful IT or PLT will equal zero.

In a pure-exchange economy without storage; Eagle (2005b) shows that when (i) consumers have the same relative risk aversion, (ii) no utility shocks occur, and (iii) consumers' endowments-to-RAS ratios are not stochastic; nominal bonds by themselves will complete markets when the central bank successfully targets nominal aggregate demand. A very specific example of this is the perpetual nominal bond example in the appendix. Even when RAS is stochastic in that example, successful NIT will result in a Pareto-efficient consumption allocation.

VI. Differences in Relative Risk Aversion and Endowment and Utility shocks.

While nominal bonds under successful NIT will complete markets when all consumers have the same relative risk aversion, non-stochastic endowment-to-RAS ratios, and no utility shocks can occur; changes in these assumption will mean that nominal bonds and NIT will need additional contracts to complete markets. Eagle (2005b) shows that in a pure-exchange economy without storage, four types of contracts can approximately complete the markets. Two of these contracts are insurance-like contracts that diversify away the risk individuals face when they face stochastic endowment-to-RAS ratios or utility shocks. A third contract is a RAS risk- transfer (RASRT) contract, which is used to transfer risk from people with below average relative risk aversion to people with above average relative risk aversion.

The fourth type contract Eagle (2005b) includes is what he calls a "normal contract," which he defines as a contract with a real payment that is unaffected by changes in nominal aggregate demand and whose RAS elasticity equals one. Two examples of such normal contracts are (i) nominal contracts under successful NIT, and (ii) Eagle and Domain's (1995, 2005a) quasi-real-indexed contracts.

Let's discuss IT, PLT, and NIT with regard to Eagle's (2005b) analysis. When there are no utility shocks and endowment-to-RAS ratios are not stochastic, then consumers with average relative risk aversion would only need normal contracts. For such consumers, nominal contracts under successful NIT would be normal contracts. However, under IT and PLT, nominal contracts would have a zero RAS elasticity and hence would not behave as normal contracts. Individuals could use RASRT contracts to combine with the nominal contracts to counter the effects of IT and PLT so to make the combination behave as normal contracts. However, doing so would require all consumers, even consumers with average relative aversion to buy or sell RASRT contracts. Under successful NIT, only consumers with above average or below average relative risk aversion need use RASRT contracts; consumers with average relative risk aversion would not need to use RASRT contracts. Since many average consumers may be financially unsophisticated, the average consumer may be unable to use RASRT contracts to counter the effects of IT and PLT.

VII. Conclusions and Reflections

This paper discusses IT, PLT, and NIT in the context of a pure-exchange economy without storage but with flexible prices. Doing so allows us ignore policy effects on output gap since no output gap exists in models with flexible prices. This paper shows that PLT dominates IT when NAD is stochastic but RAS is not stochastic. Because IT changes the targeted price levels as a result of missing its targets, IT perpetuates any utility losses. On the other hand, because PLT does maintain the original future price targets even if the current price misses its target, future expected utility losses should be unaffected when the current price misses its target. As a result, the welfare losses from the current price missing its target are many times greater under IT than PLT.

When RAS is also stochastic and consumers have the same relative risk aversion, then NIT should Pareto dominate IT and PLT when nominal contracts exist. This is because the RAS elasticity of the real payments on nominal contracts equals one under NIT. This RAS elasticity of one is exactly what is needed for Pareto-efficient consumption for individuals with average relative risk aversion. It is also what is needed for consumption on average over all consumers. However, if consumers face utility shocks or stochastic endowment-to-RAS ratios; they will also need insurance-like contracts to diversify away their risks concerning the endowment-to-RAS ratios and any utility shocks they may face.

Under NIT, consumers with average relative risk aversion will not need to use RASRT contracts, but under PLT or IT, they will need to use the RASRT contracts to undo the effects of

PLT or IT. If the average consumer is financially unsophisticated, they would be able to achieve their Pareto-efficient consumption easier under NIT than under PLT or IT.

By dealing with diverse consumers, this paper captured the effect that unanticipated inflation or deflation makes one party of a nominal contract better off while making the other part worse off. On an *a priori* basis, both parties are better off if they are not exposed to the risk of unanticipated inflation or deflation if that risk is caused by stochastic NAD. However, if that unanticipated inflation or deflation is caused by stochastic RAS, then the resulting effects on the real value of the nominal payments are just as is required for consumers with average relative risk aversion and is also what is required on average across the whole economy.

Under IT and PLT, the central bank is trying to make the price level so that it will not react to changes in RAS. However, that means the payer of a nominal payment is guaranteeing a constant real payment no matter what the level of RAS. If RAS drops, the issue of a nominal payment must then on average decrease his/her consumption more than proportional to the drop in RAS so that the receiver of the payment will be immune to the changes in RAS. In essence, IT and PLT make all payers of nominal obligations provide guarantees or insurance to the receivers of those nominal obligations. However, the only justification for doing this would be to assume that all payers of nominal obligations have less than average relative risk aversion and all receivers of nominal obligations have above average relative risk aversion. Nevertheless, there is no reason to assume that payers and receivers of nominal obligations have different levels of relative risk aversion. Under successful NIT, the real value of the nominal obligations have a RAS elasticity of one, which means that the payers and receivers of the obligations will proportionately share in the changes in RAS. Such sharing in the changes in RAS is appropriate when both parties have the same relative risk aversion.

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Appendix A

This appendix presents the assumptions and analysis of the example involving the perpetual bond discussed in the paper. Define $\pi_t^* \equiv P_t^* / P_{t-1}^*$. Assume that $\pi_t^* = \pi^*$ for t=1,2,…

. $g = \pi^* - 1$

Each consumer j has the following utility function:

$$\sum_{t=0}^{\infty} \beta^t \ln\left(c_{jt}\right) \tag{4}$$

where β is the consumers' time discount factor and c_{jt} is individual j's consumption at time t. The only security that exists in this economy is a perpetual bond, whose payment grows over time. Let z_j be the number of perpetual bonds that individual j demands at time 0. The payment on these bonds at time will equal $z_j(1+g)^t r$ where *g* is the growth rate of the payments each period and *r* is the after-growth return. If the growth rate equals the expected inflation rate, we can think about *r* as being a real interest rate. We will make this assumption. However, we will assume *g* is specified in the perpetuity contract, not indexed to the inflation rate. As a result, unanticipated inflation or deflation will still impact both parties to the perpetuity.

Let P_t be the price level at time t. Also, let k_{jt} be the portion of RAS that j receives at time t. Since the only available security is the growing perpetual bond, individual j's budget constraint at time 0 is:

$$P_0 c_{j0} + z_j = P_0 k_{j0} Y_0 \tag{5}$$

This states that the value of j's consumption at time 0 plus his/her demand for perpetual bonds equals the value of j's endowment at time 0.

Individual j's budget constraint at time t is:

$$P_t c_{jt} = P_t k_{jt} Y_t + z_{j0}(1+g)^t r \qquad (6)$$

This states that the value of j's consumption at time t equals the value of j's endowment plus the nominal payment j receives on his/her perpetual bonds.

Each individual j will maximize (4) subject to (5) and (6). The first order necessary conditions are that:

$$\frac{1}{P_0 c_{j0}} = \sum_{t=0}^{\infty} \frac{\beta^t (1+g)^t r}{P_t c_{jt}} \qquad (7)$$

We look at a special case where the resulting consumption allocation is Pareto efficient. We also assume that the time discount factor β=125/128 and the number of consumers m=10,000,000. Of these 10,000,000 consumers; 234,375 are type-A each of whom receive an endowment of 12,800 consumption units at time 0 and none therefore. The remaining 9,765,625 are type-B individuals, each of whom receives no endowment at time 0 but does receive an endowment of 307.2 consumption units for times t=1,2,… These assumptions imply that the RAS in each period is 3 billion units, and the per capita RAS is 300 units. We assume the targeted inflation rate is 2% each period, so that $\pi_t^* \equiv P_t^* / P_{t-1}^*$=1.02. We also assume that the growth rate g of the perpetual bond is also 2%. The perpetual bond's "real" interest rate that will clear markets under these assumptions equals 2.4%. Each type-A individual, who receives an endowment of 12,800 units at time 0 and no endowments thereafter, will buy 12,500 perpetual bonds. Each type-B individual, who receives no endowment at time 0, but an endowment of 307.2 units for each period thereafter, will issue 300 perpetual bonds. That the bond market clears is shown by 12,500(234,375)-300(9,765,625)=0.

If $P_t = P_0(\pi^*)^t$ and 1+g=π^*, then (7) simplifies to $1 = r\sum_{t=1}^{\infty}\beta^t$, which can be rewritten as $r = (1-\beta)/\beta$. Replacing β with its value of 125/128 confirms that r should equal 2.4%. Since consumption per capita equals RAS per capital, the goods market also clears. (Note: a spreadsheet is available from the author showing these calculations and the calculations that follow.)

This solution is valid under perfect certainty. The value of (5) for each consumer will be 243.3614 utils as shown in Table I in the main body of the paper. While we will continue to assume that consumers assumed perfect certainty when they maximized (5) subject to (6) and (7), let us now assume that in period 1, a mistake occurs. Not only are we assuming that this mistake was unexpected, but we are assuming consumers did not anticipate even the possibility of this error (otherwise the consumption optimization problem we specified would be much more complicated). Nevertheless, we can still discuss the welfare implications occurring as a result of this error.

We actually consider two errors: First that the price level at time 1 is 2% higher than expected and second that it is 2% lower than expected. We then average the two to determine the expected utility loss for each individual as a result of these errors occurring each with probability 0.5. These results are presented in Table I and discussed in the text.