# An Unobserved Components Model of the Monetary Transmission Mechanism in a Small Open Economy

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**Abstract**

This paper develops and estimates an unobserved components model for purposes of monetary policy analysis and inflation targeting in a small open economy. Cyclical components are modeled as a multivariate linear rational expectations model of the monetary transmission mechanism, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Full information maximum likelihood estimation of this unobserved components model, conditional on prior information concerning the values of trend components, provides a quantitative description of the monetary transmission mechanism in a small open economy, yields a mutually consistent set of indicators of inflationary pressure together with confidence intervals, and facilitates the generation of relatively accurate forecasts.

*JEL Classification:* E52; F41; F47

*Keywords:* Monetary policy analysis; Inflation targeting; Small open economy; Unobserved components model; Indicators of inflationary pressure; Monetary transmission mechanism; Forecast performance evaluation

## 1. Introduction

In recent years, the central banks of many economies have adopted inflation targeting monetary policy regimes. An inflation targeting monetary policy regime is characterized by three primary elements. First, there exists an explicit inflation target, which is typically quite low and is often specified as an interval. Second, achieving an inflation control objective, in the form of minimizing deviations of inflation from its target value, is emphasized relative to achieving an output stabilization objective. Third, the conduct of monetary policy is characterized by a high degree of transparency and accountability.

*Date:* February 3, 2006

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A stylized qualitative description of the monetary transmission mechanism in a small open economy distinguishes among instruments, indicators, and targets. Under an inflation targeting monetary policy regime, the central bank periodically adjusts a short term nominal interest rate in response to inflationary pressure. Provided that this response is sufficiently large, in the presence of short run nominal rigidities or imperfect information, an increase in the short term nominal interest rate causes an increase in the short term real interest rate, inducing intertemporal reductions in consumption and investment. In an open economy, an increase in the short term nominal interest rate causes a nominal appreciation, while an increase in the short term real interest rate causes a real appreciation. This adjustment of the real exchange rate induces an intratemporal reduction in exports together with an intratemporal increase in imports. In the presence of short run nominal rigidities or imperfect information, the resultant reduction in output is associated with a decline in output price inflation. In an open economy, the resultant reduction in consumption price inflation is accelerated and amplified by the adjustment of the real exchange rate.

Despite the remarkable success of many inflation targeting central banks at achieving low and stable inflation, the development of a mutually consistent set of accurate and precise indicators of inflationary pressure remains elusive. Theoretically prominent indicators of inflationary pressure such as the natural rate of interest and natural exchange rate are unobservable. As discussed in Woodford (2003), the natural rate of interest provides a measure of the neutral stance of monetary policy, with deviations of the real interest rate from the natural rate of interest generating inflationary pressure. Within the framework of an unobserved components model of selected elements of the monetary transmission mechanism in a closed economy, Laubach and Williams (2001, 2003) find that estimates of the natural rate of interest are relatively imprecise. Jointly estimating this and other indicators of inflationary pressure conditional on a larger information set may be expected to yield efficiency gains.

Definitions of indicators of inflationary pressure such as the natural rate of interest and natural exchange rate vary. Following Laubach and Williams (2001, 2003), we define the natural rate of interest as that short term real interest rate consistent with achieving inflation control and output stabilization objectives in the absence of shocks having temporary effects. In this long run equilibrium, there does not exist a cyclical stabilization role for monetary policy generated by nominal rigidities or imperfect information. In contrast, Woodford (2003) defines the natural rate of interest as that short term real interest rate consistent with achieving inflation control and output stabilization objectives in the absence of nominal rigidities. In this short run equilibrium, although there does not exist a cyclical stabilization role for monetary policy, the natural rate of interest varies in response to shocks having both temporary and permanent effects. Given an interest rate smoothing objective derived from a concern with financial market stability, it may be optimal for a central bank to adjust the short term nominal interest rate primarily in response to variation in the natural rate of interest caused by shocks having permanent effects.

Within the framework of a linear state space model, prior information concerning the values of unobserved state variables is often available in the form of deterministic or stochastic restrictions. Estimation of unobserved state variables with the filter due to Kalman (1960) does not exploit this prior information. Within the framework of an unobserved components model, prior information concerning the values of unobserved components is often available from alternative estimators. In the pursuit of efficiency gains in estimation of our unobserved components model of the monetary transmission mechanism in a small open economy, we extend the filter due to Kalman (1960) to incorporate prior information.

This paper develops and estimates an unobserved components model for purposes of monetary policy analysis and inflation targeting in a small open economy. In an extension of the empirical framework developed by Laubach and Williams (2001, 2003), cyclical components are modeled as a multivariate linear rational expectations model of the monetary transmission mechanism, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Although not derived from microeconomic foundations, this unobserved components model of the monetary transmission mechanism in a small open economy arguably provides a closer approximation to the data generating process than existing dynamic stochastic general equilibrium models. Full information maximum likelihood estimation of this unobserved components model, conditional on prior information concerning the values of trend components, provides a quantitative description of the monetary transmission mechanism in a small open economy, yields a mutually consistent set of indicators of inflationary pressure together with confidence intervals, and facilitates the generation of relatively accurate forecasts.

The organization of this paper is as follows. The next section develops an unobserved components model of the monetary transmission mechanism in a small open economy. In section three, unrestricted and restricted estimators of unobserved state variables are derived within the framework of a linear state space model. Estimation, inference and forecasting within the framework of a linear state space representation of our unobserved components model are the subjects of section four. Finally, section five offers conclusions and recommendations for further research.

## 2. The Unobserved Components Model

Consider two structurally isomorphic economies which are asymmetric in size. The domestic economy is of negligible size relative to the foreign economy, and hence takes the rational expectations equilibrium of the foreign economy as exogenous.

### 2.1. Cyclical Components

The cyclical component of output price inflation depends on a linear combination of past and expected future cyclical components of output price inflation driven by the contemporaneous cyclical component of output according to output price Phillips curve

$$\hat{\pi}_t^Y = \phi_{1,1}\hat{\pi}_{t-1}^Y + \phi_{1,2}\mathrm{E}_t\hat{\pi}_{t+1}^Y + \theta_{1,1}\ln\hat{Y}_t + \varepsilon_t^{\hat{P}^Y},\ \varepsilon_t^{\hat{P}^Y} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{P}^Y}^2), \tag{1}$$

where $\pi_t^Y = \Delta\ln P_t^Y$. The sensitivity of the cyclical component of output price inflation to changes in the cyclical component of output is increasing in $\theta_{1,1} > 0$.

The cyclical component of consumption price inflation depends on a linear combination of past and expected future cyclical components of consumption price inflation driven by the contemporaneous cyclical component of output according to consumption price Phillips curve

$$\begin{aligned}\hat{\pi}_t^C &= \phi_{2,1}\hat{\pi}_{t-1}^C + \phi_{2,2}\mathrm{E}_t\hat{\pi}_{t+1}^C + \theta_{2,1}\ln\hat{Y}_t \\ &\quad -\phi_{2,1}\theta_{2,2}\Delta\ln\hat{Q}_{t-1} + \theta_{2,2}\Delta\ln\hat{Q}_t - \phi_{2,2}\theta_{2,2}\mathrm{E}_t\Delta\ln\hat{Q}_{t+1} + \varepsilon_t^{\hat{P}^C},\ \varepsilon_t^{\hat{P}^C} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{P}^C}^2),\end{aligned} \tag{2}$$

where $\pi_t^C = \Delta\ln P_t^C$. The cyclical component of consumption price inflation also depends on past, contemporaneous, and expected future proportional changes in the cyclical component of the real exchange rate. The sensitivity of the cyclical component of consumption price inflation to changes in the cyclical component of output is increasing in $\theta_{2,1} > 0$, and to changes in the cyclical component of the real exchange rate is increasing in $0 < \theta_{2,2} < 1$.

The cyclical component of output follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of foreign output and a linear combination of the past cyclical components of the real interest rate and real exchange rate

$$\ln\hat{Y}_t = \phi_{3,1}\ln\hat{Y}_{t-1} + \phi_{3,2}\ln\hat{Y}_{t-2} + \theta_{3,1}\ln\hat{Y}_t^f + \theta_{3,2}\hat{r}_{t-1} + \theta_{3,3}\ln\hat{Q}_{t-1} + \varepsilon_t^{\hat{Y}},\ \varepsilon_t^{\hat{Y}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{Y}}^2), \tag{3}$$

where $r_t = i_t - \mathrm{E}_t \pi_{t+1}^C$ and $\ln Q_t = \ln S_t + \ln P_t^{Y,f} - \ln P_t^Y$. The sensitivity of the cyclical component of output to changes in the cyclical component of foreign output is increasing in $\theta_{3,1} > 0$, to changes in the cyclical component of the real interest rate is decreasing in $\theta_{3,2} < 0$, and to changes in the cyclical component of the real exchange rate is increasing in $\theta_{3,3} > 0$.

The cyclical component of consumption follows a stationary second order autoregressive process driven by the past cyclical component of the real interest rate:

$$\ln \hat{C}_t = \phi_{4,1} \ln \hat{C}_{t-1} + \phi_{4,2} \ln \hat{C}_{t-2} + \theta_{4,1} \hat{r}_{t-1} + \varepsilon_t^{\hat{C}},\ \varepsilon_t^{\hat{C}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{C}}^2). \tag{4}$$

The sensitivity of the cyclical component of consumption to changes in the cyclical component of the real interest rate is decreasing in $\theta_{4,1} < 0$.

The cyclical component of investment follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output:

$$\ln \hat{I}_t = \phi_{5,1} \ln \hat{I}_{t-1} + \phi_{5,2} \ln \hat{I}_{t-2} + \theta_{5,1} \ln \hat{Y}_t + \varepsilon_t^{\hat{I}},\ \varepsilon_t^{\hat{I}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{I}}^2). \tag{5}$$

The sensitivity of the cyclical component of investment to changes in the cyclical component of output is increasing in $\theta_{5,1} > 0$.

The cyclical component of exports follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of foreign output and the past cyclical component of the real exchange rate:

$$\ln \hat{X}_t = \phi_{6,1} \ln \hat{X}_{t-1} + \phi_{6,2} \ln \hat{X}_{t-2} + \theta_{6,1} \ln \hat{Y}_t^f + \theta_{6,2} \ln Q_{t-1} + \varepsilon_t^{\hat{X}},\ \varepsilon_t^{\hat{X}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{X}}^2). \tag{6}$$

The sensitivity of the cyclical component of exports to changes in the cyclical component of foreign output is increasing in $\theta_{6,1} > 0$, and to changes in the cyclical component of the real exchange rate is increasing in $\theta_{6,2} > 0$.

The cyclical component of imports follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output and the past cyclical component of the real exchange rate:

$$\ln \hat{M}_t = \phi_{7,1} \ln \hat{M}_{t-1} + \phi_{7,2} \ln \hat{M}_{t-2} + \theta_{7,1} \ln \hat{Y}_t + \theta_{7,2} \ln Q_{t-1} + \varepsilon_t^{\hat{M}},\ \varepsilon_t^{\hat{M}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{M}}^2). \tag{7}$$

The sensitivity of the cyclical component of imports to changes in the cyclical component of output is increasing in $\theta_{7,1} > 0$, and to changes in the cyclical component of the real exchange rate is decreasing in $\theta_{7,2} < 0$.

The cyclical component of wage inflation depends on a linear combination of past and expected future cyclical components of wage inflation driven by the contemporaneous cyclical component of the unemployment rate according to wage Phillips curve

$$\begin{aligned}\hat{\pi}_t^W &= \phi_{8,1}\hat{\pi}_{t-1}^W + \phi_{8,2}\mathrm{E}_t\hat{\pi}_{t+1}^W + \theta_{8,1}\hat{u}_t \\ &\quad -\phi_{8,1}\theta_{8,2}\hat{\pi}_{t-1}^C + \theta_{8,2}\hat{\pi}_t^C - \phi_{8,2}\theta_{8,2}\mathrm{E}_t\hat{\pi}_{t+1}^C + \varepsilon_t^{\hat{W}},\ \varepsilon_t^{\hat{W}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{W}}^2),\end{aligned} \tag{8}$$

where $\pi_t^W = \Delta \ln W_t$. The cyclical component of wage inflation also depends on past, contemporaneous, and expected future cyclical components of consumption price inflation. The sensitivity of the cyclical component of wage inflation to changes in the cyclical component of the unemployment rate is decreasing in $\theta_{8,1} < 0$, and to changes in the cyclical component of consumption price inflation is increasing in $0 < \theta_{8,2} < 1$.

The cyclical component of employment follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output:

$$\ln \hat{L}_t = \phi_{9,1} \ln \hat{L}_{t-1} + \phi_{9,2} \ln \hat{L}_{t-2} + \theta_{9,1} \ln \hat{Y}_t + \varepsilon_t^{\hat{L}},\ \varepsilon_t^{\hat{L}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{L}}^2). \tag{9}$$

The sensitivity of the cyclical component of employment to changes in the cyclical component of output is increasing in $\theta_{9,1} > 0$.

The cyclical component of the unemployment rate follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output:

$$\hat{u}_t = \phi_{10,1}\hat{u}_{t-1} + \phi_{10,2}\hat{u}_{t-2} + \theta_{10,1} \ln \hat{Y}_t + \varepsilon_t^{\hat{u}},\ \varepsilon_t^{\hat{u}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{u}}^2). \tag{10}$$

The sensitivity of the cyclical component of the unemployment rate to changes in the cyclical component of output is decreasing in $\theta_{10,1} < 0$.

The cyclical component of the nominal interest rate follows a stationary first order autoregressive process driven by the contemporaneous cyclical components of consumption price inflation and output:

$$\hat{i}_t = \phi_{11,1}\hat{i}_{t-1} + \theta_{11,1}\hat{\pi}_t^C + \theta_{11,2} \ln \hat{Y}_t + \varepsilon_t^{\hat{i}},\ \varepsilon_t^{\hat{i}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{i}}^2). \tag{11}$$

The sensitivity of the cyclical component of the nominal interest rate to changes in the cyclical component of consumption price inflation is increasing in $\theta_{11,1} > 0$, and to changes in the cyclical component of output is increasing in $\theta_{11,2} > 0$.

The cyclical component of the nominal exchange rate depends on a linear combination of the past and expected future cyclical components of the nominal exchange rate driven by the contemporaneous cyclical component of the nominal interest rate differential:

$$\ln \hat{S}_t = \phi_{12,1} \ln \hat{S}_{t-1} + \phi_{12,2} \mathrm{E}_t \ln \hat{S}_{t+1} + \theta_{12,1}(\hat{i}_t - \hat{i}_t^f) + \varepsilon_t^{\hat{S}},\ \varepsilon_t^{\hat{S}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{S}}^2). \tag{12}$$

The sensitivity of the cyclical component of the nominal exchange rate to changes in the cyclical component of the nominal interest rate differential is decreasing in $\theta_{12,1} < 0$.

## *2.2. Trend Components*

The trend components of the prices of output and consumption follow random walks with time varying drift $\pi_t$:

$$\ln \overline{P}_t^Y = \pi_t + \ln \overline{P}_{t-1}^Y + \varepsilon_t^{\overline{P}^Y},\ \varepsilon_t^{\overline{P}^Y} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^Y}^2), \tag{13}$$

$$\ln \overline{P}_t^C = \pi_t + \ln \overline{P}_{t-1}^C + \varepsilon_t^{\overline{P}^C},\ \varepsilon_t^{\overline{P}^C} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^C}^2). \tag{14}$$

It follows that the trend component of the relative price of consumption follows a random walk without drift. This implies that along a balanced growth path, this relative price is constant but state dependent.

The trend components of output, consumption, investment, exports, and imports follow random walks with time varying drift $g_t + n_t$:

$$\ln \overline{Y}_t = g_t + n_t + \ln \overline{Y}_{t-1} + \varepsilon_t^{\overline{Y}},\ \varepsilon_t^{\overline{Y}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{Y}}^2), \tag{15}$$

$$\ln \overline{C}_t = g_t + n_t + \ln \overline{C}_{t-1} + \varepsilon_t^{\overline{C}},\ \varepsilon_t^{\overline{C}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{C}}^2), \tag{16}$$

$$\ln \overline{I}_t = g_t + n_t + \ln \overline{I}_{t-1} + \varepsilon_t^{\overline{I}},\ \varepsilon_t^{\overline{I}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{I}}^2), \tag{17}$$

$$\ln \overline{X}_t = g_t + n_t + \ln \overline{X}_{t-1} + \varepsilon_t^{\overline{X}},\ \varepsilon_t^{\overline{X}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{X}}^2), \tag{18}$$

$$\ln \overline{M}_t = g_t + n_t + \ln \overline{M}_{t-1} + \varepsilon_t^{\overline{M}}, \ \varepsilon_t^{\overline{M}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{M}}^2). \tag{19}$$

It follows that the trend components of the ratios of consumption, investment, exports, and imports to output follow random walks without drift. This implies that along a balanced growth path, these great ratios are constant but state dependent.

The trend component of the nominal wage follows a random walk with time varying drift $\pi_t + g_t$, while the trend component of employment follows a random walk with time varying drift $n_t$:

$$\ln \overline{W}_t = \pi_t + g_t + \ln \overline{W}_{t-1} + \varepsilon_t^{\overline{W}}, \ \varepsilon_t^{\overline{W}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{W}}^2), \tag{20}$$

$$\ln \overline{L}_t = n_t + \ln \overline{L}_{t-1} + \varepsilon_t^{\overline{L}}, \ \varepsilon_t^{\overline{L}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{L}}^2). \tag{21}$$

It follows that the trend component of the income share of labour follows a random walk without drift. This implies that along a balanced growth path, the income share of labour is constant but state dependent.

The trend components of the unemployment rate, nominal interest rate, and nominal exchange rate follow random walks without drift:

$$\overline{u}_t = \overline{u}_{t-1} + \varepsilon_t^{\overline{u}}, \ \varepsilon_t^{\overline{u}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{u}}^2), \tag{22}$$

$$\overline{i}_t = \overline{i}_{t-1} + \varepsilon_t^{\overline{i}}, \ \varepsilon_t^{\overline{i}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{i}}^2), \tag{23}$$

$$\ln \overline{S}_t = \ln \overline{S}_{t-1} + \varepsilon_t^{\overline{S}}, \ \varepsilon_t^{\overline{S}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{S}}^2). \tag{24}$$

It follows that along a balanced growth path, the unemployment rate, nominal interest rate, and nominal exchange rate are constant but state dependent. The trend component of the real interest rate satisfies $\overline{r}_t = \overline{i}_t - \mathrm{E}_t \overline{\pi}_{t+1}^C$, while the trend component of the real exchange rate satisfies $\ln \overline{Q}_t = \ln \overline{S}_t + \ln \overline{P}_t^{Y,f} - \ln \overline{P}_t^Y$.

Long run nominal growth is driven by three common stochastic trends. Trend inflation, productivity growth, and population growth follow random walks without drift:

$$\pi_t = \pi_{t-1} + \varepsilon_t^{\pi}, \ \varepsilon_t^{\pi} \sim \text{iid } \mathcal{N}(0, \sigma_{\pi}^2), \tag{25}$$

$$g_t = g_{t-1} + \varepsilon_t^g,\ \varepsilon_t^g \sim \text{iid } \mathcal{N}(0, \sigma_g^2), \tag{26}$$

$$n_t = n_{t-1} + \varepsilon_t^n,\ \varepsilon_t^n \sim \text{iid } \mathcal{N}(0, \sigma_n^2). \tag{27}$$

As an identifying restriction, all innovations are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

## 3. Estimation of Unobserved State Variables

Linear state space models consist of signal and state equations. The signal equation expresses a vector of observed nonpredetermined endogenous variables as a static deterministic or stochastic linear function of a vector of contemporaneous observed exogenous or predetermined endogenous variables, and a vector of contemporaneous unobserved state variables. The state equation expresses a vector of unobserved state variables as a dynamic deterministic or stochastic linear function of a vector of contemporaneous observed exogenous or predetermined endogenous variables, and a vector of lagged unobserved state variables.

Within the framework of a linear state space model, if the signal and state innovation vectors are multivariate normally distributed and contemporaneously uncorrelated, then conditional on the parameters associated with the signal and state equations, mean squared error optimal estimates of the unobserved state vector may be calculated with the filter due to Kalman (1960). If the signal and state innovation vectors are not multivariate normally distributed, then these state vector estimates retain minimum mean squared error status among the class of linear estimators. Estimation, inference and forecasting within the framework of a linear state space model is discussed in Hamilton (1994), Kim and Nelson (1999), and Durbin and Koopman (2001).

Within the framework of a linear state space model, prior information concerning the values of unobserved state variables is often available in the form of deterministic or stochastic restrictions. Estimation of unobserved state variables with the filter due to Kalman (1960) does not exploit this prior information. This section derives unrestricted and restricted estimators of unobserved state variables within the framework of a linear state space model. The former approach is standard, while the latter is a contribution of this paper. Exploiting prior information concerning the values of unobserved state variables may be expected to yield efficiency gains in estimation.

*3.1. Unrestricted Estimation of Unobserved State Variables*

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of $N$ observed nonpredetermined endogenous variables, let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of $M$ observed exogenous or predetermined endogenous variables, and let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of $K$ unobserved state variables. Suppose that these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{x}_t + \boldsymbol{A}_2\boldsymbol{z}_t + \boldsymbol{A}_3\boldsymbol{\varepsilon}_{1,t}, \tag{28}$$

$$\boldsymbol{z}_t = \boldsymbol{B}_1\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{z}_{t-1} + \boldsymbol{B}_3\boldsymbol{\varepsilon}_{2,t}, \tag{29}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_1)$ and $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_2)$. The signal and state innovation vectors are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

Within the framework of this linear state space model, define $\boldsymbol{z}_{t|t-1} = \text{E}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{P}_{t|t-1} = \text{Var}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{y}_{t|t-1} = \text{E}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$ and $\boldsymbol{Q}_{t|t-1} = \text{Var}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$, where $\mathcal{I}_{t-1} = \{\{\boldsymbol{y}_s\}_{s=1}^{t-1}, \{\boldsymbol{x}_s\}_{s=1}^{t}\}$. Conditional on the parameters associated with the signal and state equations, these conditional means and variances satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{B}_1\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{z}_{t-1|t-1}, \tag{30}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{B}_2\boldsymbol{P}_{t-1|t-1}\boldsymbol{B}_2^\mathsf{T} + \boldsymbol{B}_3\boldsymbol{\Sigma}_2\boldsymbol{B}_3^\mathsf{T}, \tag{31}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{A}_1\boldsymbol{x}_t + \boldsymbol{A}_2\boldsymbol{z}_{t|t-1}, \tag{32}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{A}_2\boldsymbol{P}_{t|t-1}\boldsymbol{A}_2^\mathsf{T} + \boldsymbol{A}_3\boldsymbol{\Sigma}_1\boldsymbol{A}_3^\mathsf{T}. \tag{33}$$

These predicted estimates of the means and variances of the signal and state vectors are conditional on past information.

Given these predicted estimates, estimates of the state vector conditional on past and present information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|t}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \mathcal{I}_{t-1}) = \frac{f(\boldsymbol{y}_t \mid \boldsymbol{z}_t, \mathcal{I}_{t-1}) f(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})}{f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})}. \tag{34}$$

Under the assumption of multivariate normally distributed signal and state innovation vectors, $\boldsymbol{z}_{t|t}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1})^{\mathsf{T}} \boldsymbol{P}_{t|t-1}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1}) - (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}), \tag{35}$$

subject to signal equation (28). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_t (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}), \tag{36}$$

where $\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1} \boldsymbol{A}_2^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t-1}^{-1} - \boldsymbol{A}_2^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} \boldsymbol{A}_2$ is positive definite. Define $\boldsymbol{P}_{t|t}$ as the mean squared error of $\boldsymbol{z}_{t|t}$, conditional on $\mathcal{I}_{t-1}$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_t \boldsymbol{A}_2 \boldsymbol{P}_{t|t-1}. \tag{37}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|t}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \mathcal{I}_{t-1})$, and is therefore mean squared error optimal. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, recursive evaluation of equations (30), (31), (32), (33), (36) and (37) yields predicted and filtered estimates of the state vector.

Given these predicted and filtered estimates, estimates of the state vector conditional on past, present and future information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|T}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t) = \frac{f(\boldsymbol{z}_{t+1} \mid \boldsymbol{z}_t, \mathcal{I}_t) f(\boldsymbol{z}_t \mid \mathcal{I}_t)}{f(\boldsymbol{z}_{t+1} \mid \mathcal{I}_t)}. \tag{38}$$

Under the assumption of a multivariate normally distributed state innovation vector, $\boldsymbol{z}_{t|T}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t})^{\mathsf{T}} \boldsymbol{P}_{t|t}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t}) - (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t})^{\mathsf{T}} \boldsymbol{P}_{t+1|t}^{-1} (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t}), \tag{39}$$

subject to state equation (29). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|T} = \boldsymbol{z}_{t|t} + \boldsymbol{J}_t(\boldsymbol{z}_{t+1|T} - \boldsymbol{z}_{t+1|t}), \tag{40}$$

where $\boldsymbol{J}_t = \boldsymbol{P}_{t|t}\boldsymbol{B}_2^{\mathsf{T}}\boldsymbol{P}_{t+1|t}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t}^{-1} - \boldsymbol{B}_2^{\mathsf{T}}\boldsymbol{P}_{t+1|t}^{-1}\boldsymbol{B}_2$ is positive definite. Define $\boldsymbol{P}_{t|T}$ as the mean squared error of $\boldsymbol{z}_{t|T}$, conditional on $\mathcal{I}_t$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|T} = \boldsymbol{P}_{t|t} + \boldsymbol{J}_t(\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t})\boldsymbol{J}_t^{\mathsf{T}}. \tag{41}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|T}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t)$, and is therefore mean squared error optimal. Given terminal conditions $\boldsymbol{z}_{T|T}$ and $\boldsymbol{P}_{T|T}$ obtained from the final evaluation of the prediction and updating equations, recursive evaluation of equations (40) and (41) yields smoothed estimates of the state vector.

### *3.2. Restricted Estimation of Unobserved State Variables*

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of $N$ observed nonpredetermined endogenous variables, let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of $M$ observed exogenous or predetermined endogenous variables, and let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of $K$ unobserved state variables. Suppose that these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{x}_t + \boldsymbol{A}_2\boldsymbol{z}_t + \boldsymbol{A}_3\boldsymbol{\varepsilon}_{1,t}, \tag{42}$$

$$\boldsymbol{z}_t = \boldsymbol{B}_1\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{z}_{t-1} + \boldsymbol{B}_3\boldsymbol{\varepsilon}_{2,t}, \tag{43}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_1)$ and $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_2)$. Let $\boldsymbol{w}_t$ denote a vector stochastic process consisting of $J$ observed synthetic variables. Suppose that this vector stochastic process satisfies

$$\boldsymbol{w}_t = \boldsymbol{C}_1\boldsymbol{z}_t + \boldsymbol{C}_2\boldsymbol{\varepsilon}_{3,t}, \tag{44}$$

where $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_3)$. Conditional on given parameter values, this signal equation defines a set of deterministic or stochastic restrictions on linear combinations of unobserved state variables. The signal and state innovation vectors are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

Within the framework of this linear state space model, define $\boldsymbol{z}_{t|t-1} = \mathrm{E}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{P}_{t|t-1} = \mathrm{Var}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{y}_{t|t-1} = \mathrm{E}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{Q}_{t|t-1} = \mathrm{Var}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{w}_{t|t-1} = \mathrm{E}(\boldsymbol{w}_t \mid \mathcal{I}_{t-1})$ and $\boldsymbol{R}_{t|t-1} = \mathrm{Var}(\boldsymbol{w}_t \mid \mathcal{I}_{t-1})$, where $\mathcal{I}_{t-1} = \{\{\boldsymbol{y}_s\}_{s=1}^{t-1}, \{\boldsymbol{w}_s\}_{s=1}^{t-1}, \{\boldsymbol{x}_s\}_{s=1}^{t}\}$. Conditional on the parameters associated with the signal and state equations, these conditional means and variances satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{B}_1 \boldsymbol{x}_t + \boldsymbol{B}_2 \boldsymbol{z}_{t-1|t-1}, \tag{45}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{B}_2 \boldsymbol{P}_{t-1|t-1} \boldsymbol{B}_2^{\mathsf{T}} + \boldsymbol{B}_3 \boldsymbol{\Sigma}_2 \boldsymbol{B}_3^{\mathsf{T}}, \tag{46}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{A}_1 \boldsymbol{x}_t + \boldsymbol{A}_2 \boldsymbol{z}_{t|t-1}, \tag{47}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{A}_2 \boldsymbol{P}_{t|t-1} \boldsymbol{A}_2^{\mathsf{T}} + \boldsymbol{A}_3 \boldsymbol{\Sigma}_1 \boldsymbol{A}_3^{\mathsf{T}}, \tag{48}$$

$$\boldsymbol{w}_{t|t-1} = \boldsymbol{C}_1 \boldsymbol{z}_{t|t-1}, \tag{49}$$

$$\boldsymbol{R}_{t|t-1} = \boldsymbol{C}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{C}_1^{\mathsf{T}} + \boldsymbol{C}_2 \boldsymbol{\Sigma}_3 \boldsymbol{C}_2^{\mathsf{T}}. \tag{50}$$

These predicted estimates of the means and variances of the signal and state vectors are conditional on past information.

Given these predicted estimates, estimates of the state vector conditional on past and present information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|t}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \boldsymbol{w}_t, \mathcal{I}_{t-1}) = \frac{f(\boldsymbol{y}_t \mid \boldsymbol{z}_t, \boldsymbol{w}_t, \mathcal{I}_{t-1}) f(\boldsymbol{w}_t \mid \boldsymbol{z}_t, \mathcal{I}_{t-1}) f(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})}{f(\boldsymbol{y}_t \mid \boldsymbol{w}_t, \mathcal{I}_{t-1}) f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1})}. \tag{51}$$

Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, $\boldsymbol{z}_{t|t}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1})^{\mathsf{T}} \boldsymbol{P}_{t|t-1}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1}) \\ -(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) - (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1})^{\mathsf{T}} \boldsymbol{R}_{t|t-1}^{-1} (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{52}$$

subject to signal equations (42) and (44). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_{\boldsymbol{y}_t} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) + \boldsymbol{K}_{\boldsymbol{w}_t} (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{53}$$

where $\boldsymbol{K}_{\boldsymbol{y}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{A}_2^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1}$ and $\boldsymbol{K}_{\boldsymbol{w}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{C}_1^{\mathsf{T}} \boldsymbol{R}_{t|t-1}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t-1}^{-1} - \boldsymbol{A}_2^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} \boldsymbol{A}_2 - \boldsymbol{C}_1^{\mathsf{T}} \boldsymbol{R}_{t|t-1}^{-1} \boldsymbol{C}_1$ is positive definite. Define $\boldsymbol{P}_{t|t}$ as the mean squared error of $\boldsymbol{z}_{t|t}$, conditional on $\mathcal{I}_{t-1}$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{y}_t} \boldsymbol{A}_2 \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{w}_t} \boldsymbol{C}_1 \boldsymbol{P}_{t|t-1}. \tag{54}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|t}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \boldsymbol{w}_t, \mathcal{I}_{t-1})$, and is therefore mean squared error optimal. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, recursive evaluation of equations (45), (46), (47), (48), (49), (50), (53) and (54) yields predicted and filtered estimates of the state vector.

Given these predicted and filtered estimates, estimates of the state vector conditional on past, present and future information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|T}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t) = \frac{f(\boldsymbol{z}_{t+1} \mid \boldsymbol{z}_t, \mathcal{I}_t) f(\boldsymbol{z}_t \mid \mathcal{I}_t)}{f(\boldsymbol{z}_{t+1} \mid \mathcal{I}_t)}. \tag{55}$$

Under the assumption of a multivariate normally distributed state innovation vector, $\boldsymbol{z}_{t|T}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t})^{\mathsf{T}} \boldsymbol{P}_{t|t}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t}) - (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t})^{\mathsf{T}} \boldsymbol{P}_{t+1|t}^{-1} (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t}), \tag{56}$$

subject to state equation (43). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|T} = \boldsymbol{z}_{t|t} + \boldsymbol{J}_t (\boldsymbol{z}_{t+1|T} - \boldsymbol{z}_{t+1|t}), \tag{57}$$

where $J_t = P_{t|t} B_2^{\mathsf{T}} P_{t+1|t}^{-1}$. This necessary first order condition is sufficient if $P_{t|t}^{-1} - B_2^{\mathsf{T}} P_{t+1|t}^{-1} B_2$ is positive definite. Define $P_{t|T}$ as the mean squared error of $z_{t|T}$, conditional on $\mathcal{I}_t$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$P_{t|T} = P_{t|t} + J_t (P_{t+1|T} - P_{t+1|t}) J_t^{\mathsf{T}}. \tag{58}$$

Under our distributional assumptions, $z_{t|T}$ equals the mean of posterior distribution $f(z_t \mid z_{t+1}, \mathcal{I}_t)$, and is therefore mean squared error optimal. Given terminal conditions $z_{T|T}$ and $P_{T|T}$ obtained from the final evaluation of the prediction and updating equations, recursive evaluation of equations (57) and (58) yields smoothed estimates of the state vector.

## 4. Estimation, Inference and Forecasting

Although unobserved components models feature prominently in the empirical macroeconomics literature, an unobserved components model of the monetary transmission mechanism has yet to be developed and estimated. Given that the monetary transmission mechanism is a cyclical phenomenon, it seems natural to model it within the framework of an unobserved components model.

### *4.1. Estimation*

The traditional econometric interpretation of macroeconometric models regards them as representations of the joint probability distribution of the data. Adopting this traditional econometric interpretation, the parameters and trend components of our unobserved components model of the monetary transmission mechanism in a small open economy are jointly estimated by full information maximum likelihood, conditional on prior information concerning the values of trend components.

*4.1.1. Estimation Methodology*

Let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of the levels of $N$ nonpredetermined endogenous variables, of which $M$ are observed. The cyclical components of this vector stochastic process satisfy third order stochastic linear difference equation

$$\boldsymbol{A}_0\hat{\boldsymbol{x}}_t = \boldsymbol{A}_1\hat{\boldsymbol{x}}_{t-1} + \boldsymbol{A}_2\hat{\boldsymbol{x}}_{t-2} + \boldsymbol{A}_3\mathrm{E}_t\hat{\boldsymbol{x}}_{t+1} + \boldsymbol{\varepsilon}_{1,t}, \tag{59}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_1)$. If there exists a unique stationary solution to this multivariate linear rational expectations model, then it may be expressed as:

$$\hat{\boldsymbol{x}}_t = \boldsymbol{B}_1\hat{\boldsymbol{x}}_{t-1} + \boldsymbol{B}_2\hat{\boldsymbol{x}}_{t-2} + \boldsymbol{B}_3\boldsymbol{\varepsilon}_{1,t}. \tag{60}$$

The trend components of vector stochastic process $\boldsymbol{x}_t$ satisfy first order stochastic linear difference equation

$$\boldsymbol{C}_0\bar{\boldsymbol{x}}_t = \boldsymbol{C}_1\boldsymbol{v}_t + \boldsymbol{C}_2\bar{\boldsymbol{x}}_{t-1} + \boldsymbol{\varepsilon}_{2,t}, \tag{61}$$

where $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_2)$. Vector stochastic process $\boldsymbol{v}_t$ consists of the levels of $L$ common stochastic trends, and satisfies first order stochastic linear difference equation

$$\boldsymbol{v}_t = \boldsymbol{v}_{t-1} + \boldsymbol{\varepsilon}_{3,t}, \tag{62}$$

where $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_3)$. Cyclical and trend components are additively separable, which implies that $\boldsymbol{x}_t = \hat{\boldsymbol{x}}_t + \bar{\boldsymbol{x}}_t$.

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of the levels of $M$ observed nonpredetermined endogenous variables. Also, let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of the contemporaneous levels of $N-M$ unobserved nonpredetermined endogenous variables, the contemporaneous and lagged cyclical components of $N$ nonpredetermined endogenous variables, the contemporaneous trend components of $N$ nonpredetermined endogenous variables, and the levels of $L$ common stochastic trends. Given unique stationary solution (60), these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{F}_1\boldsymbol{z}_t, \tag{63}$$

$$\boldsymbol{z}_t = \boldsymbol{G}_1 \boldsymbol{z}_{t-1} + \boldsymbol{G}_2 \boldsymbol{\varepsilon}_{4,t}, \tag{64}$$

where $\boldsymbol{\varepsilon}_{4,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_4)$. Let $\boldsymbol{w}_t$ denote a vector stochastic process consisting of preliminary estimates of the trend components of $M$ observed nonpredetermined endogenous variables. Suppose that this vector stochastic process satisfies

$$\boldsymbol{w}_t = \boldsymbol{H}_1 \boldsymbol{z}_t + \boldsymbol{\varepsilon}_{5,t}, \tag{65}$$

where $\boldsymbol{\varepsilon}_{5,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_5)$. Conditional on given parameter values, this signal equation defines a set of deterministic or stochastic restrictions on selected unobserved state variables. The signal and state innovation vectors are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

Conditional on the parameters associated with these signal and state equations, estimates of unobserved state vector $\boldsymbol{z}_t$ and its mean squared error matrix $\boldsymbol{P}_t$ may be calculated with the filter derived previously. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, estimates conditional on information available at time $t-1$ satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{G}_1 \boldsymbol{z}_{t-1|t-1}, \tag{66}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{G}_1 \boldsymbol{P}_{t-1|t-1} \boldsymbol{G}_1^\top + \boldsymbol{G}_2 \boldsymbol{\Sigma}_4 \boldsymbol{G}_2^\top, \tag{67}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{F}_1 \boldsymbol{z}_{t|t-1}, \tag{68}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{F}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{F}_1^\top, \tag{69}$$

$$\boldsymbol{w}_{t|t-1} = \boldsymbol{H}_1 \boldsymbol{z}_{t|t-1}, \tag{70}$$

$$\boldsymbol{R}_{t|t-1} = \boldsymbol{H}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{H}_1^\top + \boldsymbol{\Sigma}_5. \tag{71}$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, estimates conditional on information available at time $t$ satisfy updating equations

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_{y_t}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) + \boldsymbol{K}_{w_t}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{72}$$

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{y}_t} \boldsymbol{F}_1 \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{w}_t} \boldsymbol{H}_1 \boldsymbol{P}_{t|t-1}, \tag{73}$$

where $\boldsymbol{K}_{\boldsymbol{y}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{F}_1^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1}$ and $\boldsymbol{K}_{\boldsymbol{w}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{H}_1^{\mathsf{T}} \boldsymbol{R}_{t|t-1}^{-1}$. Given terminal conditions $\boldsymbol{z}_{T|T}$ and $\boldsymbol{P}_{T|T}$ obtained from the final evaluation of these prediction and updating equations, estimates conditional on information available at time $T$ satisfy smoothing equations

$$\boldsymbol{z}_{t|T} = \boldsymbol{z}_{t|t} + \boldsymbol{J}_t (\boldsymbol{z}_{t+1|T} - \boldsymbol{z}_{t+1|t}), \tag{74}$$

$$\boldsymbol{P}_{t|T} = \boldsymbol{P}_{t|t} + \boldsymbol{J}_t (\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t}) \boldsymbol{J}_t^{\mathsf{T}}, \tag{75}$$

where $\boldsymbol{J}_t = \boldsymbol{P}_{t|t} \boldsymbol{G}_1^{\mathsf{T}} \boldsymbol{P}_{t+1|t}^{-1}$. Under our distributional assumptions, these estimators of the unobserved state vector are mean squared error optimal.

Let $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^K$ denote a $K$ dimensional vector containing the parameters associated with the signal and state equations of this linear state space model. The maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ of this parameter vector maximizes conditional loglikelihood function:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ell_t(\boldsymbol{\theta}). \tag{76}$$

Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, the contributions to this conditional loglikelihood function satisfy $\ell_t(\boldsymbol{\theta}) = \ell_{\boldsymbol{y}_t}(\boldsymbol{\theta}) + \ell_{\boldsymbol{w}_t}(\boldsymbol{\theta})$, where:

$$\ell_{\boldsymbol{y}_t}(\boldsymbol{\theta}) = -\frac{M}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{Q}_{t|t-1}| - \frac{1}{2}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}), \tag{77}$$

$$\ell_{\boldsymbol{w}_t}(\boldsymbol{\theta}) = -\frac{M}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{R}_{t|t-1}| - \frac{1}{2}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1})^{\mathsf{T}} \boldsymbol{R}_{t|t-1}^{-1} (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}). \tag{78}$$

Under regularity conditions stated in Watson (1989), maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ is consistent and asymptotically normal,

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, \boldsymbol{\mathcal{A}}_0^{-1} \boldsymbol{\mathcal{B}}_0 \boldsymbol{\mathcal{A}}_0^{-1}), \tag{79}$$

where $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$ denotes the true parameter vector. Following Engle and Watson (1981), consistent estimators of $\boldsymbol{\mathcal{A}}_0$ and $\boldsymbol{\mathcal{B}}_0$ are given by

$$\hat{\mathcal{A}}_T = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T), \tag{80}$$

$$\hat{\mathcal{B}}_T = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}, \tag{81}$$

where $\boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T) = \boldsymbol{a}_{y_t}(\hat{\boldsymbol{\theta}}_T) + \boldsymbol{a}_{w_t}(\hat{\boldsymbol{\theta}}_T)$ and $\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T) = \boldsymbol{b}_{y_t}(\hat{\boldsymbol{\theta}}_T) + \boldsymbol{b}_{w_t}(\hat{\boldsymbol{\theta}}_T)$. Under our distributional assumptions,

$$\boldsymbol{a}_{y_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\boldsymbol{y}_{t|t-1}^{\mathsf{T}}\boldsymbol{Q}_{t|t-1}^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{y}_{t|t-1} + \frac{1}{2}\nabla_{\boldsymbol{\theta}}\boldsymbol{Q}_{t|t-1}^{\mathsf{T}}(\boldsymbol{Q}_{t|t-1}^{-1}\otimes\boldsymbol{Q}_{t|t-1}^{-1})\nabla_{\boldsymbol{\theta}}\boldsymbol{Q}_{t|t-1}, \tag{82}$$

$$\boldsymbol{a}_{w_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\boldsymbol{w}_{t|t-1}^{\mathsf{T}}\boldsymbol{R}_{t|t-1}^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{w}_{t|t-1} + \frac{1}{2}\nabla_{\boldsymbol{\theta}}\boldsymbol{R}_{t|t-1}^{\mathsf{T}}(\boldsymbol{R}_{t|t-1}^{-1}\otimes\boldsymbol{R}_{t|t-1}^{-1})\nabla_{\boldsymbol{\theta}}\boldsymbol{R}_{t|t-1}, \tag{83}$$

$\boldsymbol{b}_{y_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\ell_{y_t}(\hat{\boldsymbol{\theta}}_T)$ and $\boldsymbol{b}_{w_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\ell_{w_t}(\hat{\boldsymbol{\theta}}_T)$. If the signal innovation vectors are multivariate normally distributed, then the conditional information matrix equality holds, and $\mathcal{A}_0 = \mathcal{B}_0$.

*4.1.2. Estimation Results*

Our unobserved components model of the monetary transmission mechanism in a small open economy is estimated by full information maximum likelihood, conditional on prior information concerning the values of trend components. The data set consists of the levels of twenty observed nonpredetermined endogenous variables for Canada and the United States described in Appendix A. The conditional loglikelihood function is maximized numerically using a modified steepest ascent algorithm. Estimation results pertaining to the period 1972Q1 through 2005Q1 appear in Appendix B, with robust $t$ ratios reported in parentheses. The necessary and sufficient condition for the existence of a unique stationary rational expectations equilibrium due to Blanchard and Kahn (1980) is satisfied in a neighbourhood around the full information maximum likelihood estimate, while the analytical Hessian is nonsingular at the full information maximum likelihood estimate, suggesting that the linear state space representation of this unobserved components model is locally identified.

Prior information concerning the values of trend components is generated by fitting third order deterministic polynomial functions to the levels of observed nonpredetermined endogenous variables by ordinary least squares. Stochastic restrictions on the trend components of observed nonpredetermined endogenous variables are derived from the fitted values associated with these ordinary least squares regressions, with innovation variances set proportional to estimated prediction variances assuming known parameters. All stochastic restrictions are independent, represented by a diagonal covariance matrix, and are harmonized, represented by a common factor of proportionality. Reflecting little confidence in these preliminary trend component estimates, this common factor of proportionality is set equal to one.

The signs of all parameter estimates are consistent with our priors, while most are statistically significant at conventional levels. Estimates of the variances of innovations associated with both cyclical and trend components are often statistically significant at conventional levels, suggesting that the levels of the observed nonpredetermined endogenous variables under consideration are subject to shocks having both temporary and permanent effects.

Predicted, filtered and smoothed estimates of the cyclical and trend components of observed nonpredetermined endogenous variables are plotted together with confidence intervals in Appendix B. These confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. The predicted estimates are conditional on past information, the filtered estimates are conditional on past and present information, and the smoothed estimates are conditional on past, present and future information. Visual inspection reveals close agreement with the conventional dating of business cycle expansions and recessions.

In order to examine whether our unobserved components model of the monetary transmission mechanism in a small open economy is dynamically complete in mean and variance, we subject the levels and squares of the predicted standardized residuals to the autocorrelation test of Ljung and Box (1978). We also examine whether there exist significant departures from conditional normality with the test of Jarque and Bera (1980). The predicted standardized residual vector $\boldsymbol{\zeta}_{t|t-1}$ is related to the predicted ordinary residual vector $\boldsymbol{\xi}_{t|t-1}$ by $\boldsymbol{\zeta}_{t|t-1} = \boldsymbol{Q}_{t|t-1}^{-1/2}\boldsymbol{\xi}_{t|t-1}$, where $\boldsymbol{\xi}_{t|t-1} = \boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}$. The inverse square root of predicted conditional covariance matrix $\boldsymbol{Q}_{t|t-1}$ is calculated with a spectral decomposition as $\boldsymbol{Q}_{t|t-1}^{-1/2} = \boldsymbol{X}_{t|t-1}\boldsymbol{\Lambda}_{t|t-1}^{-1/2}\boldsymbol{X}_{t|t-1}^{\top}$, where $\boldsymbol{X}_{t|t-1}$ denotes a square matrix containing distinct orthonormal eigenvectors, while $\boldsymbol{\Lambda}_{t|t-1}$ denotes a diagonal matrix containing the corresponding positive eigenvalues.

We find moderate evidence of autocorrelation in the predicted standardized residuals, suggesting that the conditional mean function is dynamically incomplete. Furthermore, we find strong evidence of autoregressive conditional heteroskedasticity in the predicted standardized residuals, suggesting that the conditional variance function is dynamically incomplete. Finally, we find strong evidence of departures from normality in the predicted standardized residuals, in part attributable to the existence of excess kurtosis. These residual diagnostic test results suggest that our full information maximum likelihood estimation results are consistent and asymptotically normal, but are asymptotically inefficient.

### *4.2. Inference*

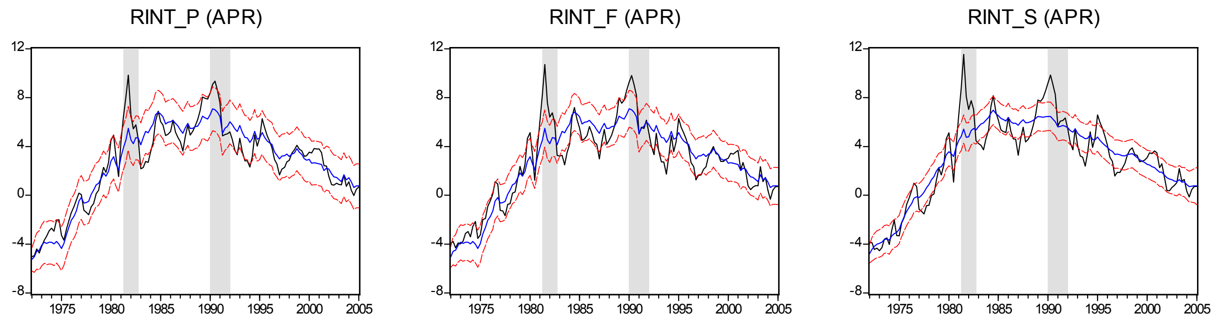
Achieving low and stable inflation calls for accurate and precise indicators of inflationary pressure, together with an accurate and precise quantitative description of the monetary transmission mechanism. Our unobserved components model of the monetary transmission mechanism in a small open economy addresses both of these challenges within a unified empirical framework.

#### *4.2.1. Quantifying Inflationary Pressure*

Theoretically prominent indicators of inflationary pressure such as the natural rate of interest and natural exchange rate are unobservable. As discussed in Woodford (2003), the natural rate of interest provides a measure of the neutral stance of monetary policy, with deviations of the real interest rate from the natural rate of interest generating inflationary pressure.

Predicted, filtered and smoothed estimates of the natural rate of interest are plotted together with confidence intervals versus corresponding estimates of the real interest rate in Figure 1. This concept of the natural rate of interest represents that short term real interest rate consistent with achieving inflation control and output stabilization objectives in the absence of shocks having temporary effects. Visual inspection reveals that our estimates of the natural rate of interest exhibit persistent low frequency variation and are relatively precise. Deviations of the estimated real interest rate from the estimated natural rate of interest are in close agreement with the conventional dating of business cycle expansions and recessions.

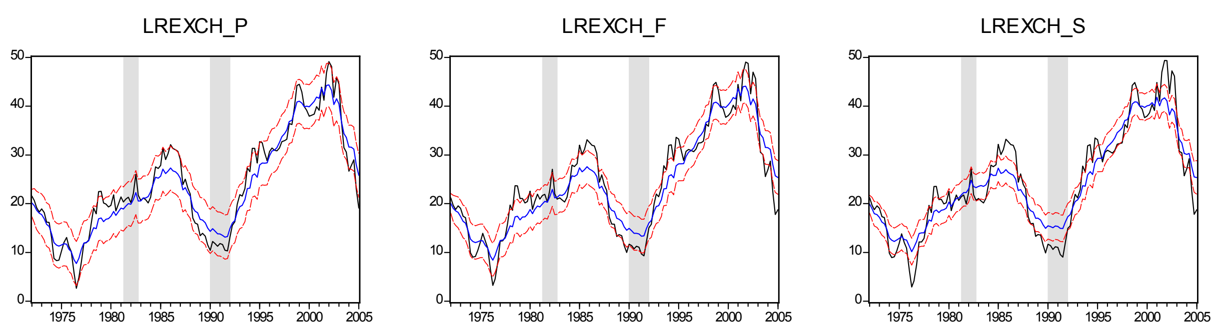
Figure 1. Predicted, filtered and smoothed estimates of the natural rate of interest




*Note:* Estimated levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Predicted, filtered and smoothed estimates of the natural exchange rate are plotted together with confidence intervals versus the observed real exchange rate in Figure 2. This concept of the natural exchange rate represents that real exchange rate consistent with achieving inflation control and output stabilization objectives in the absence of shocks having temporary effects. Visual inspection reveals that our estimates of the natural exchange rate exhibit persistent low frequency variation and are relatively precise.

Figure 2. Predicted, filtered and smoothed estimates of the natural exchange rate




*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

### *4.2.2. Quantifying the Monetary Transmission Mechanism*

The monetary transmission mechanism describes the dynamic effects of unsystematic variation in the instrument of monetary policy on indicators and targets. In a small open economy, the monetary transmission mechanism features both interest rate and exchange rate channels, while an inflation targeting central bank must react to shocks originating both domestically and abroad. Estimated impulse responses to domestic and foreign monetary policy shocks are plotted in Appendix B, providing a quantitative description of the monetary transmission mechanism in a small open economy.

In response to a domestic monetary policy shock, the domestic nominal and real interest rates exhibit immediate increases followed by gradual declines. The domestic currency appreciates in nominal and real terms, with the nominal exchange rate exhibiting delayed overshooting. These real interest rate and real exchange rate dynamics induce persistent hump shaped reductions in domestic output, consumption, investment, exports and imports, together with persistent hump shaped declines in domestic output price inflation and consumption price inflation, with peak effects realized after one to two years. These output dynamics are associated with a persistent hump shaped reduction in domestic employment, together with a persistent hump shaped increase in the domestic unemployment rate, inducing a persistent hump shaped decline in domestic wage inflation, with peak effects realized after one to two years. These results are qualitatively consistent with those of structural vector autoregressive analyses of the monetary transmission mechanism in open economies such as Eichenbaum and Evans (1995), Clarida and Gertler (1997), Kim and Roubini (1995), and Cushman and Zha (1997).

In response to a foreign monetary policy shock, the foreign nominal and real interest rates exhibit immediate increases followed by gradual declines. These real interest rate dynamics induce persistent hump shaped reductions in foreign output, consumption and investment, together with a persistent hump shaped decline in foreign inflation, with peak effects realized after one to two years. These output dynamics are associated with a persistent hump shaped reduction in foreign employment, together with a persistent hump shaped increase in the foreign unemployment rate, inducing a persistent hump shaped decline in foreign wage inflation, with peak effects realized after one to two years. The domestic currency depreciates in nominal and real terms, with the nominal exchange rate exhibiting delayed overshooting. These real interest rate and real exchange rate dynamics induce persistent hump shaped reductions in domestic output, exports and imports, together with persistent hump shaped declines in domestic output price inflation and consumption price inflation. These output dynamics are associated with a persistent hump shaped reduction in domestic employment, together with a persistent hump shaped increase in the domestic unemployment rate, inducing a persistent hump shaped decline in domestic wage inflation. These results are qualitatively consistent with those of structural vector autoregressive analyses of the monetary transmission mechanism in closed economies such as Sims and Zha (1995), Gordon and Leeper (1994), Leeper, Sims and Zha (1996), and Christiano, Eichenbaum and Evans (1998, 2005).

*4.3. Forecasting*

While it is desirable that forecasts be unbiased and efficient, the practical value of any forecasting model depends on its relative predictive accuracy. As a benchmark against which to evaluate the predictive accuracy of our unobserved components model of the monetary transmission mechanism in a small open economy, we consider the autoregressive integrated moving average or ARIMA class of models. In particular, we consider ARIMA models for scalar stochastic process $\{y_t\}_{t=1}^{T}$ of the form

$$\Delta^d y_t = \mu + \sum_{i=1}^{p} \phi_i \Delta^d y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \tag{84}$$

where $\varepsilon_t \sim \text{iid}\ \mathcal{N}(0,\sigma^2)$. Theoretical support for this univariate forecasting framework is provided by the decomposition theorem due to Wold (1938), which states that any covariance stationary purely linearly indeterministic scalar stochastic process has an infinite order moving average representation. As discussed in Clements and Hendry (1998), any infinite order moving average process can be approximated to any required degree of accuracy by an autoregressive moving average process, with the required autoregressive and moving average orders typically being relatively low.

The ARIMA models are estimated by maximum likelihood over the period 1972Q3 through 2005Q1. The autoregressive, ordinary difference, and moving average orders are jointly selected to minimize the model selection criterion function proposed by Schwarz (1978). Those ARIMA model specifications deemed optimal are reported in Table 1.

Table 1. Optimal ARIMA model specifications

| $y_t$ | $p$ | $d$ | $q$ |
|---|---|---|---|
| $\ln P_t^Y$ | 2 | 0 | 0 |
| $\ln P_t^C$ | 2 | 0 | 1 |
| $\ln Y_t$ | 4 | 1 | 2 |
| $\ln C_t$ | 4 | 1 | 2 |
| $\ln I_t$ | 1 | 2 | 1 |
| $\ln X_t$ | 1 | 1 | 1 |
| $\ln M_t$ | 0 | 1 | 2 |
| $\ln W_t$ | 2 | 0 | 0 |
| $\ln L_t$ | 3 | 1 | 2 |
| $u_t$ | 4 | 0 | 2 |
| $i_t$ | 3 | 1 | 2 |
| $\ln S_t$ | 2 | 0 | 1 |
| $\ln P_t^{Y,f}$ | 2 | 0 | 1 |
| $\ln Y_t^f$ | 1 | 1 | 0 |
| $\ln C_t^f$ | 2 | 1 | 2 |
| $\ln I_t^f$ | 1 | 1 | 0 |
| $\ln W_t^f$ | 4 | 0 | 2 |
| $\ln L_t^f$ | 1 | 1 | 0 |
| $u_t^f$ | 3 | 0 | 1 |
| $i_t^f$ | 1 | 2 | 2 |

*Note:* The autoregressive order $p$, ordinary difference order $d$, and moving average order $q$ are jointly selected subject to upper bounds of four, two and two, respectively.

In the absence of a well defined mapping between forecast errors and their costs, relative predictive accuracy is generally assessed with mean squared prediction error based measures. As discussed in Clements and Hendry (1998), mean squared prediction error based measures are noninvariant to nonsingular, scale preserving linear transformations, even though linear models are. It follows that mean squared prediction error based comparisons may yield conflicting rankings across models, depending on the variable transformations examined.

To evaluate the dynamic out of sample forecasting performance of our unobserved components model of the monetary transmission mechanism in a small open economy, we retain forty quarters of observations to evaluate forecasts one through eight quarters ahead, generated conditional on parameters estimated using information available at the forecast origin. The models are compared on the basis of mean squared prediction errors in levels, ordinary differences, and seasonal differences. The unobserved components model is not recursively estimated as the forecast origin rolls forward due to the high computational cost of such a procedure, while the ARIMA models are. Presumably, recursively estimating the unobserved components model would increase its predictive accuracy.

Mean squared prediction error differentials are plotted together with confidence intervals accounting for contemporaneous and serial correlation of forecast errors in Appendix B. If these mean squared prediction error differentials are negative then the forecasting performance of the unobserved components model dominates that of the ARIMA models, while if positive then the unobserved components model is dominated by the ARIMA models in terms of predictive accuracy. The null hypothesis of equal squared prediction errors is rejected by the predictive accuracy test of Diebold and Mariano (1995) if and only if these confidence intervals exclude zero. The asymptotic variance of the average loss differential is estimated by a weighted sum of the autocovariances of the loss differential, employing the weighting function proposed by Newey and West (1987). Visual inspection reveals that these mean squared prediction error differentials are of variable sign, suggesting that the unobserved components model matches the ARIMA models in terms of predictive accuracy, in spite of a considerable informational disadvantage. However, these mean squared prediction error differentials are rarely statistically significant at conventional levels, indicating that considerable uncertainty surrounds these forecast accuracy comparisons.

Dynamic out of sample forecasts of levels, ordinary differences, and seasonal differences are plotted together with confidence intervals versus realized outcomes in Appendix B. These confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Visual inspection reveals that the realized outcomes generally lie within their associated confidence intervals, suggesting that forecast failure is absent. However, these confidence intervals are rather wide, indicating that considerable uncertainty surrounds the point forecasts.

## 5. Conclusion

This paper develops and estimates an unobserved components model of the monetary transmission mechanism in a small open economy for purposes of monetary policy analysis and inflation targeting. This estimated unobserved components model provides a quantitative description of the monetary transmission mechanism in a small open economy, yields a mutually consistent set of indicators of inflationary pressure together with confidence intervals, and facilitates the generation of relatively accurate forecasts.

Definitions of indicators of inflationary pressure such as the natural rate of interest and natural exchange rate vary, while estimates are typically sensitive to identifying restrictions. It follows that combinations of estimates of indicators of inflationary pressure derived under alternative definitions from dissimilar models may be more useful for purposes of monetary policy analysis and inflation targeting in a small open economy than any of the constituents. An examination of the inflation control and output stabilization benefits conferred by combining alternative estimates remains an objective for future research.

## Acknowledgements

The author gratefully acknowledges advice provided by Paul Beaudry. The author thanks the Social Sciences and Humanities Research Council of Canada for financial support.

## Appendix A. Description of the Data Set

The data set consists of quarterly seasonally adjusted observations on several macroeconomic variables for Canada and the United States over the period 1971Q1 through 2005Q1. Price indexes are derived from observed nominal and real output and consumption. Employment is derived from observed nominal labour income and a nominal wage index, while the unemployment rate is quoted as a period average. The nominal interest rate is measured by the three month Treasury bill rate quoted as a period average, while the nominal exchange rate is quoted as an end of period value. National accounts data for Canada was retrieved from the CANSIM database maintained by Statistics Canada, national accounts data for the United States was obtained from the FRED database maintained by the Federal Reserve Bank of Saint Louis, and other data was extracted from the IFS database maintained by the International Monetary Fund.

## Appendix B. Tables and Figures

Table 2. Full information maximum likelihood estimation results, domestic economy

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_{1,1}$ | $\phi_{1,2}$ | $\phi_{2,1}$ | $\phi_{2,2}$ | $\phi_{3,1}$ | $\phi_{3,2}$ | $\phi_{4,1}$ | $\phi_{4,2}$ | $\phi_{5,1}$ | $\phi_{5,2}$ | $\phi_{6,1}$ | $\phi_{6,2}$ |
| 0.387 | 0.279 | 0.416 | 0.208 | 0.979 | –0.190 | 0.884 | 0.011 | 0.956 | –0.321 | 0.567 | 0.051 |
| (2.782) | (0.388) | (2.129) | (0.177) | (6.795) | (–1.324) | (5.812) | (0.077) | (7.272) | (–3.007) | (4.285) | (0.466) |
| $\phi_{7,1}$ | $\phi_{7,2}$ | $\phi_{8,1}$ | $\phi_{8,2}$ | $\phi_{9,1}$ | $\phi_{9,2}$ | $\phi_{10,1}$ | $\phi_{10,2}$ | $\phi_{11,1}$ | $\phi_{12,1}$ | $\phi_{12,2}$ | |
| 0.634 | –0.311 | 0.471 | 0.182 | 0.564 | 0.096 | 0.815 | –0.228 | 0.723 | 0.568 | 0.378 | |
| (3.708) | (–2.763) | (0.933) | (0.082) | (2.944) | (0.601) | (5.590) | (–2.063) | (9.716) | (0.262) | (0.118) | |
| $\theta_{1,1}$ | $\theta_{2,1}$ | $\theta_{2,2}$ | $\theta_{3,1}$ | $\theta_{3,2}$ | $\theta_{3,3}$ | $\theta_{4,1}$ | $\theta_{5,1}$ | $\theta_{6,1}$ | $\theta_{6,2}$ | $\theta_{7,1}$ | $\theta_{7,2}$ |
| 0.049 | 0.032 | 0.009 | 0.200 | –0.199 | 0.007 | –0.382 | 0.920 | 1.248 | 0.107 | 1.796 | –0.051 |
| (0.617) | (0.435) | (0.462) | (4.681) | (–0.997) | (0.630) | (–2.131) | (4.509) | (5.489) | (1.727) | (4.934) | (–0.786) |
| $\theta_{8,1}$ | $\theta_{8,2}$ | $\theta_{9,1}$ | $\theta_{10,1}$ | $\theta_{11,1}$ | $\theta_{11,2}$ | $\theta_{12,1}$ | | | | | |
| –0.048 | 0.172 | 0.581 | –0.243 | 0.123 | 0.082 | –0.049 | | | | | |
| (–0.224) | (0.769) | (5.905) | (–5.332) | (1.569) | (5.666) | (–0.070) | | | | | |
| $\sigma^2_{\hat{P}^Y}$ | $\sigma^2_{\hat{P}^C}$ | $\sigma^2_{\hat{Y}}$ | $\sigma^2_{\hat{C}}$ | $\sigma^2_{\hat{I}}$ | $\sigma^2_{\hat{X}}$ | $\sigma^2_{\hat{M}}$ | $\sigma^2_{\hat{W}}$ | $\sigma^2_{\hat{L}}$ | $\sigma^2_{\hat{u}}$ | $\sigma^2_{\hat{i}}$ | $\sigma^2_{\hat{S}}$ |
| 0.132 | 0.067 | 0.223 | 0.283 | 2.451 | 3.580 | 1.969 | 0.120 | 0.186 | 0.023 | 0.036 | 1.718 |
| (1.082) | (0.672) | (4.624) | (4.532) | (5.255) | (4.865) | (3.373) | (0.340) | (1.644) | (2.324) | (4.945) | (0.130) |
| $\sigma^2_{\bar{P}^Y}$ | $\sigma^2_{\bar{P}^C}$ | $\sigma^2_{\bar{Y}}$ | $\sigma^2_{\bar{C}}$ | $\sigma^2_{\bar{I}}$ | $\sigma^2_{\bar{X}}$ | $\sigma^2_{\bar{M}}$ | $\sigma^2_{\bar{W}}$ | $\sigma^2_{\bar{L}}$ | $\sigma^2_{\bar{u}}$ | $\sigma^2_{\bar{i}}$ | $\sigma^2_{\bar{S}}$ |
| 0.106 | 0.088 | 0.196 | 0.269 | 2.080 | 3.601 | 3.086 | 0.160 | 0.363 | 0.043 | 0.011 | 1.945 |
| (2.602) | (3.133) | (4.518) | (4.360) | (3.989) | (4.154) | (4.106) | (3.596) | (3.150) | (3.342) | (3.844) | (4.371) |
| $\sigma^2_{\pi}$ | $\sigma^2_{g}$ | $\sigma^2_{n}$ | | | | | | | | | |
| $4.24\times10^{-3}$ | $5.77\times10^{-6}$ | $6.51\times10^{-5}$ | | | | | | | | | |
| (2.200) | (0.666) | (1.345) | | | | | | | | | |

| | $\hat{\zeta}^{P^Y}_{t\mid t-1}$ | $\hat{\zeta}^{P^C}_{t\mid t-1}$ | $\hat{\zeta}^{Y}_{t\mid t-1}$ | $\hat{\zeta}^{C}_{t\mid t-1}$ | $\hat{\zeta}^{I}_{t\mid t-1}$ | $\hat{\zeta}^{X}_{t\mid t-1}$ | $\hat{\zeta}^{M}_{t\mid t-1}$ | $\hat{\zeta}^{W}_{t\mid t-1}$ | $\hat{\zeta}^{L}_{t\mid t-1}$ | $\hat{\zeta}^{u}_{t\mid t-1}$ | $\hat{\zeta}^{i}_{t\mid t-1}$ | $\hat{\zeta}^{S}_{t\mid t-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q(2)$ | 0.005 | 1.515 | 1.908 | 4.085 | 4.001 | 0.159 | 11.847*** | 1.742 | 0.802 | 0.366 | 5.407* | 1.279 |
| $Q(4)$ | 5.535 | 2.326 | 12.904** | 9.590** | 4.868 | 4.420 | 12.216** | 1.917 | 2.628 | 1.255 | 8.632* | 8.443* |
| $Q^2(2)$ | 38.412*** | 26.761*** | 48.637*** | 20.354*** | 47.537*** | 38.687*** | 49.381*** | 20.434*** | 44.281*** | 46.589*** | 70.018*** | 34.746*** |
| $Q^2(4)$ | 78.916*** | 56.878*** | 84.273*** | 59.024*** | 75.979*** | 69.106*** | 115.630*** | 43.255*** | 62.165*** | 67.326*** | 153.501*** | 69.865*** |
| Skewness | –0.268 | –0.149 | –0.180 | –0.453** | 0.157 | 0.018 | 0.452** | 0.288 | 0.018 | 0.595*** | 0.365* | –0.103 |
| Kurtosis | 3.293 | 3.362 | 3.236 | 4.227*** | 2.933 | 3.105 | 2.901 | 3.917** | 4.735*** | 3.700* | 4.702*** | 3.669 |
| *JB* | 2.071 | 1.219 | 1.025 | 12.879*** | 0.572 | 0.068 | 4.589 | 6.503** | 16.680*** | 10.552*** | 18.995*** | 2.716 |

*Note:* Rejection of the null hypothesis at the 1%, 5% and 10% levels is indicated by ***, ** and *, respectively.

Table 3. Full information maximum likelihood estimation results, foreign economy

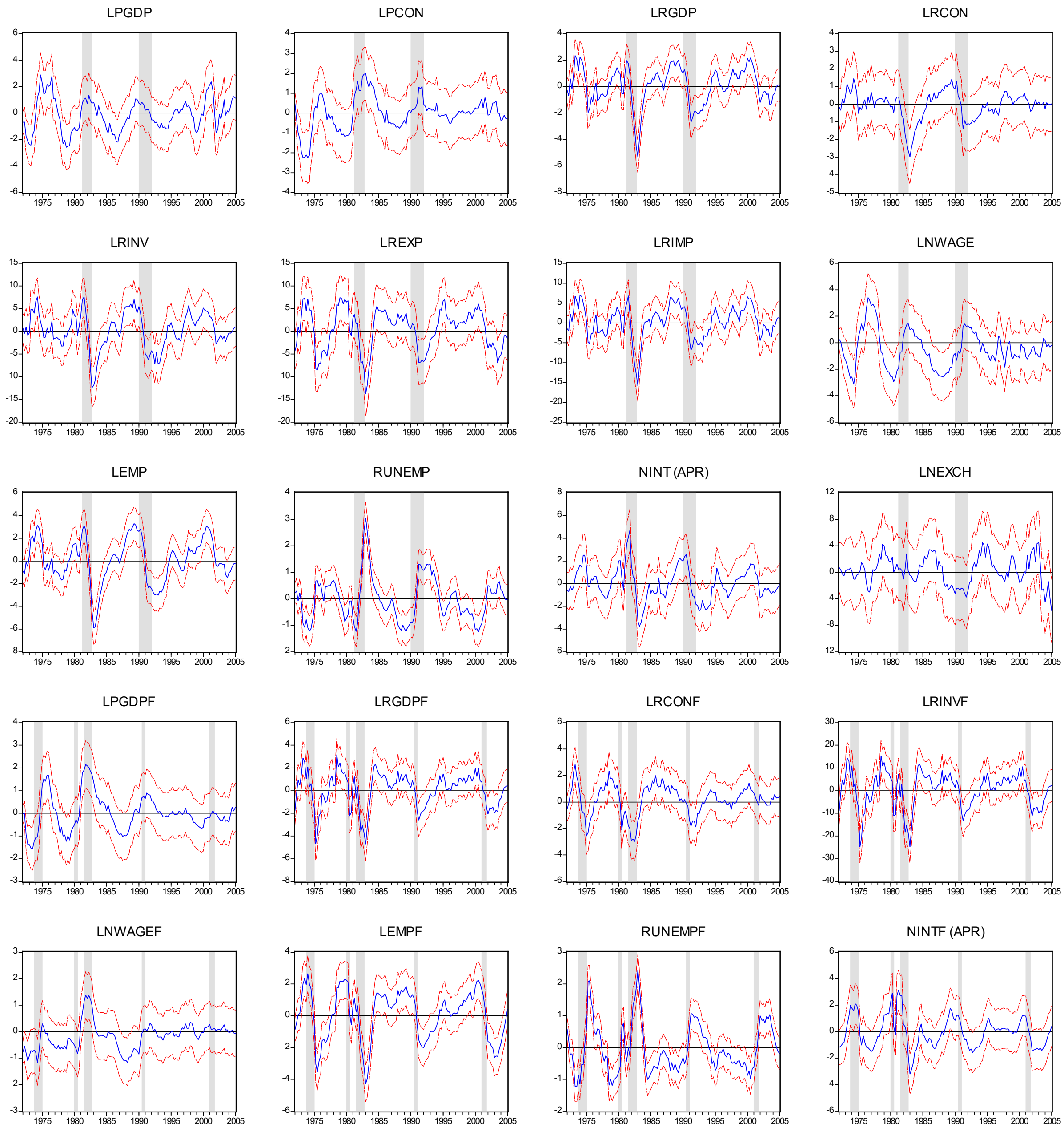
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\phi_{1,1}$ | $\phi_{1,2}$ | $\phi_{3,1}$ | $\phi_{3,2}$ | $\phi_{4,1}$ | $\phi_{4,2}$ | $\phi_{5,1}$ | $\phi_{5,2}$ |
| 0.500 | 0.289 | 1.223 | –0.300 | 1.072 | –0.185 | 0.289 | –0.113 |
| (0.563) | (0.114) | (12.908) | (–2.876) | (9.326) | (–1.713) | (3.267) | (–1.567) |
| $\phi_{8,1}$ | $\phi_{8,2}$ | $\phi_{9,1}$ | $\phi_{9,2}$ | $\phi_{10,1}$ | $\phi_{10,2}$ | $\phi_{11,1}$ | |
| 0.102 | 0.140 | 0.929 | –0.187 | 0.731 | –0.179 | 0.737 | |
| (0.376) | (0.075) | (5.983) | (–1.434) | (8.004) | (–2.493) | (8.803) | |
| $\theta_{1,1}$ | $\theta_{3,2}$ | $\theta_{4,1}$ | $\theta_{5,1}$ | $\theta_{8,1}$ | $\theta_{8,2}$ | $\theta_{9,1}$ | $\theta_{10,1}$ |
| 0.007 | –0.739 | –0.858 | 4.078 | –0.039 | 0.429 | 0.314 | –0.228 |
| (0.153) | (–1.809) | (–3.218) | (8.747) | (–0.410) | (3.595) | (5.846) | (–6.902) |
| $\theta_{11,1}$ | $\theta_{11,2}$ | | | | | | |
| 0.226 | 0.056 | | | | | | |
| (1.789) | (5.151) | | | | | | |
| $\sigma^2_{\hat{P}^Y}$ | $\sigma^2_{\hat{Y}}$ | $\sigma^2_{\hat{C}}$ | $\sigma^2_{\hat{I}}$ | $\sigma^2_{\hat{W}}$ | $\sigma^2_{\hat{L}}$ | $\sigma^2_{\hat{u}}$ | $\sigma^2_{\hat{i}}$ |
| 0.024 | 0.398 | 0.225 | 2.640 | 0.035 | 0.097 | 0.011 | 0.024 |
| (0.251) | (5.180) | (5.697) | (3.090) | (1.472) | (2.475) | (1.906) | (3.217) |
| $\sigma^2_{\bar{P}^Y}$ | $\sigma^2_{\bar{Y}}$ | $\sigma^2_{\bar{C}}$ | $\sigma^2_{\bar{I}}$ | $\sigma^2_{\bar{W}}$ | $\sigma^2_{\bar{L}}$ | $\sigma^2_{\bar{u}}$ | $\sigma^2_{\bar{i}}$ |
| 0.034 | 0.096 | 0.084 | 2.098 | 0.076 | 0.214 | 0.023 | 0.007 |
| (3.004) | (2.982) | (3.365) | (3.036) | (4.070) | (3.972) | (3.685) | (2.872) |
| $\sigma^2_{\pi}$ | $\sigma^2_{g}$ | $\sigma^2_{n}$ | | | | | |
| $2.77\times10^{-3}$ | $7.65\times10^{-5}$ | $1.66\times10^{-4}$ | | | | | |
| (2.493) | (2.084) | (1.707) | | | | | |

| | $\hat{\zeta}^{P^Y}_{t\vert t-1}$ | $\hat{\zeta}^{Y}_{t\vert t-1}$ | $\hat{\zeta}^{C}_{t\vert t-1}$ | $\hat{\zeta}^{I}_{t\vert t-1}$ | $\hat{\zeta}^{W}_{t\vert t-1}$ | $\hat{\zeta}^{L}_{t\vert t-1}$ | $\hat{\zeta}^{u}_{t\vert t-1}$ | $\hat{\zeta}^{i}_{t\vert t-1}$ |
|---|---|---|---|---|---|---|---|---|
| $Q(2)$ | 2.182 | 5.068* | 1.868 | 8.125** | 8.390** | 6.621** | 1.426 | 9.304*** |
| $Q(4)$ | 13.504*** | 5.536 | 8.708* | 11.484** | 12.300** | 11.267** | 1.568 | 20.355*** |
| $Q^2(2)$ | 28.097*** | 10.374*** | 13.304*** | 11.573*** | 65.814*** | 54.277*** | 24.548*** | 29.186*** |
| $Q^2(4)$ | 45.424*** | 16.953*** | 24.303*** | 25.069*** | 116.061*** | 105.171*** | 39.542*** | 57.803*** |
| Skewness | 0.189 | 0.644*** | –0.275 | –0.287 | –0.391* | 0.039 | 0.971*** | 1.164*** |
| Kurtosis | 4.009** | 6.362*** | 4.551*** | 6.153*** | 2.495 | 3.241 | 5.003*** | 10.638*** |
| *JB* | 6.431** | 71.833*** | 15.006*** | 56.924*** | 4.799* | 0.355 | 43.122*** | 353.305*** |

$\mathcal{L}(\hat{\boldsymbol{\theta}}_T) = -6648.950$

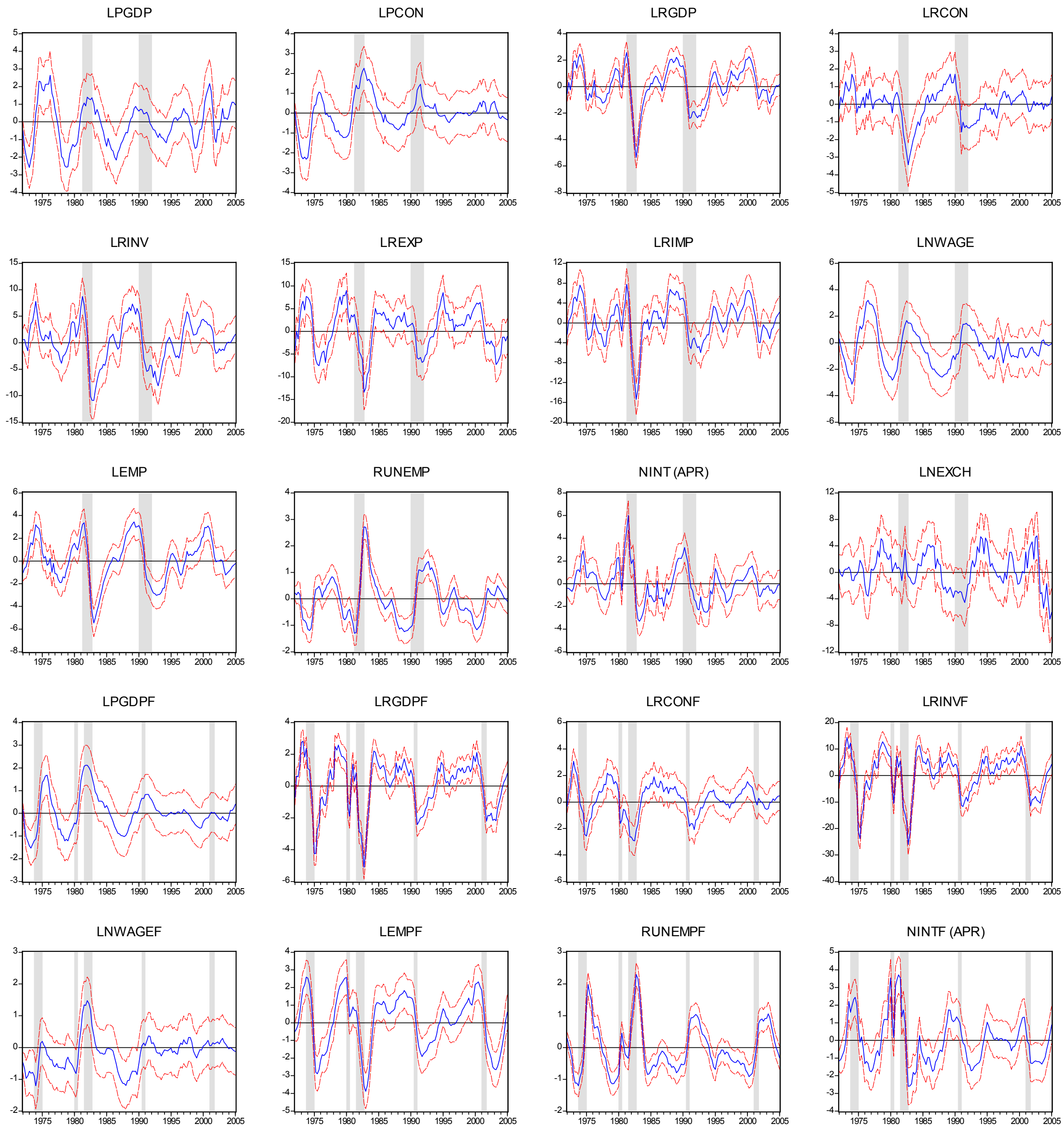
*Note:* Rejection of the null hypothesis at the 1%, 5% and 10% levels is indicated by ***, ** and *, respectively.

Figure 3. Predicted cyclical components of observed nonpredetermined endogenous variables



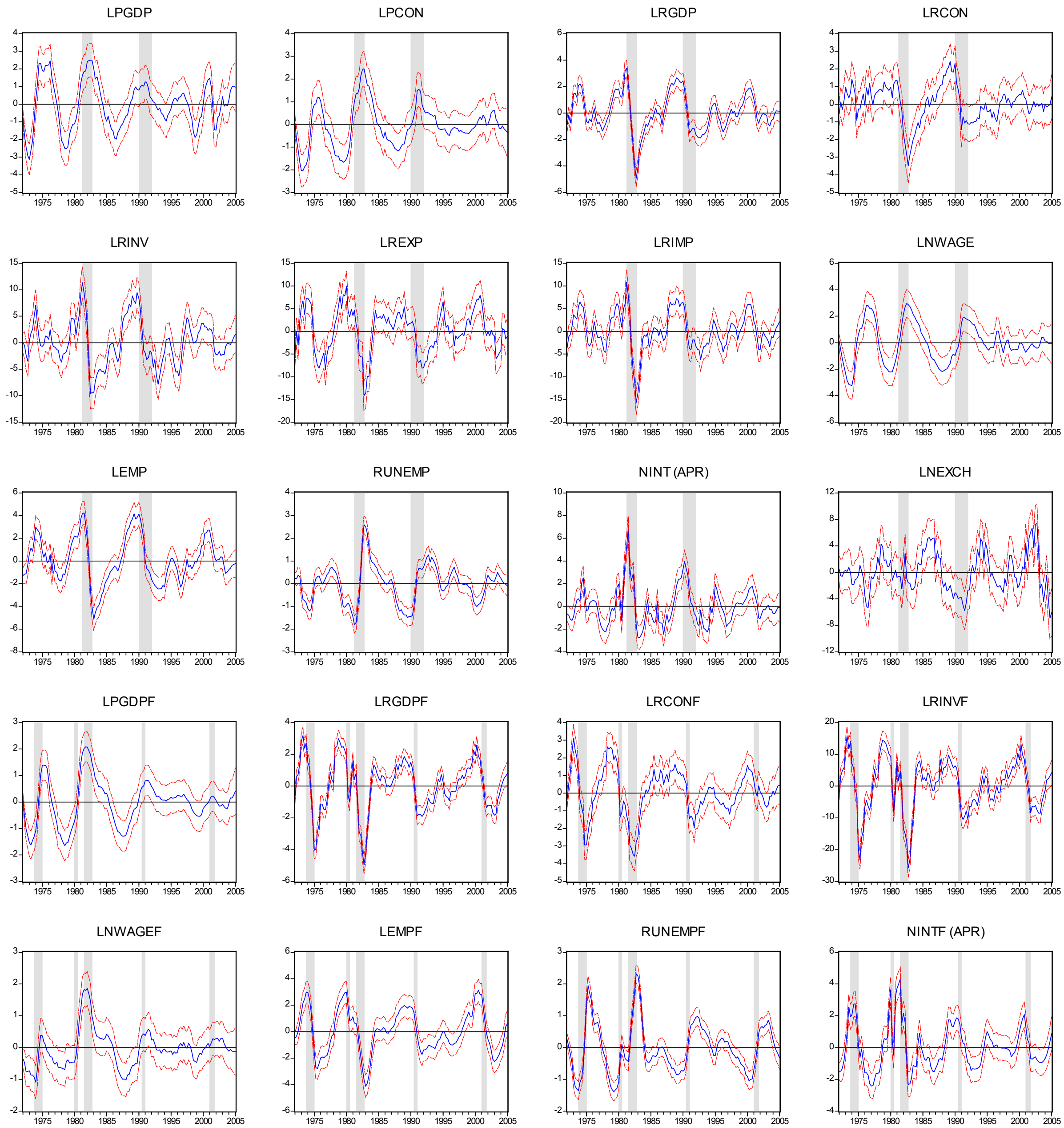

*Note:* Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 4.  Filtered cyclical components of observed nonpredetermined endogenous variables



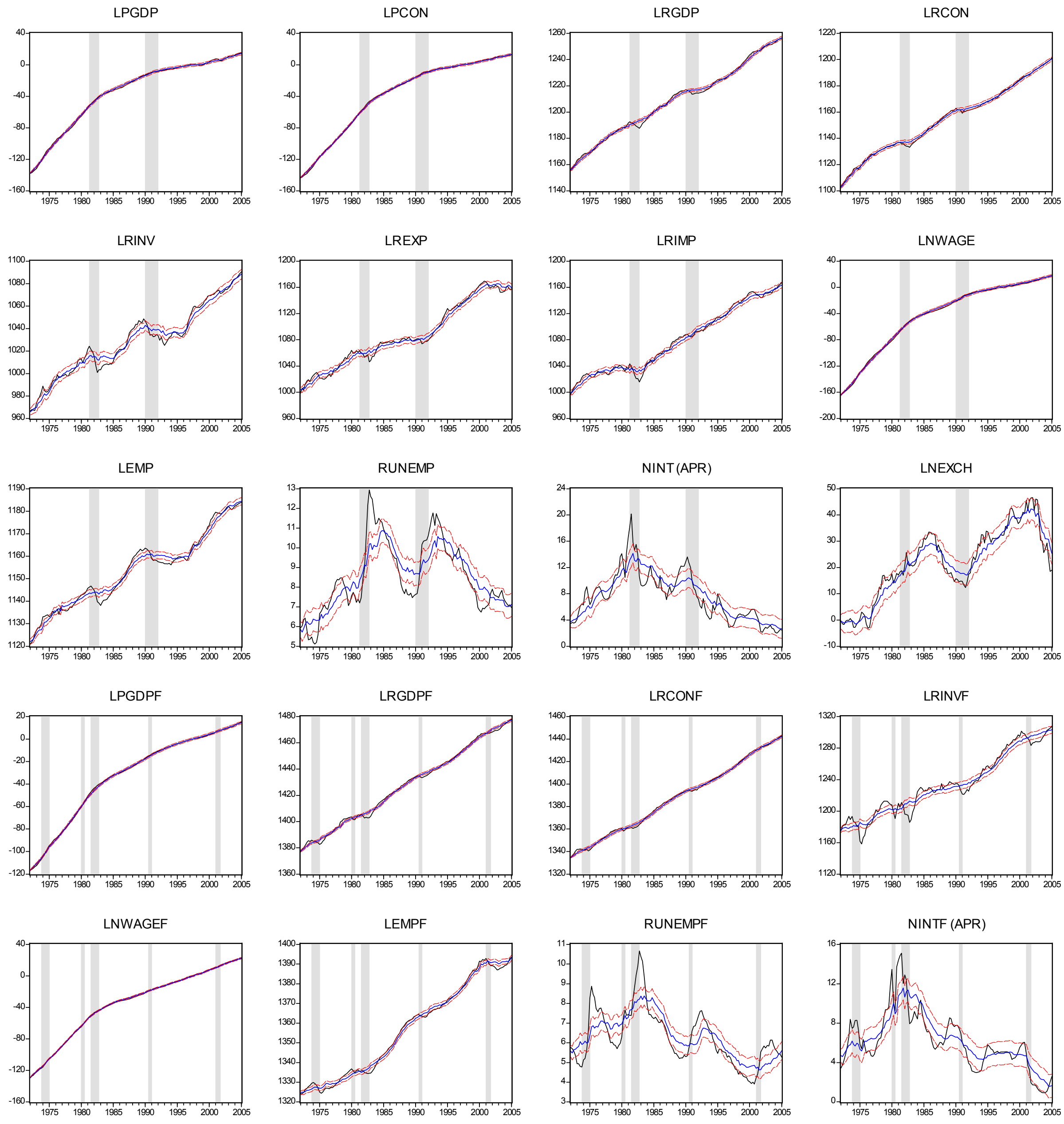

*Note:* Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 5. Smoothed cyclical components of observed nonpredetermined endogenous variables



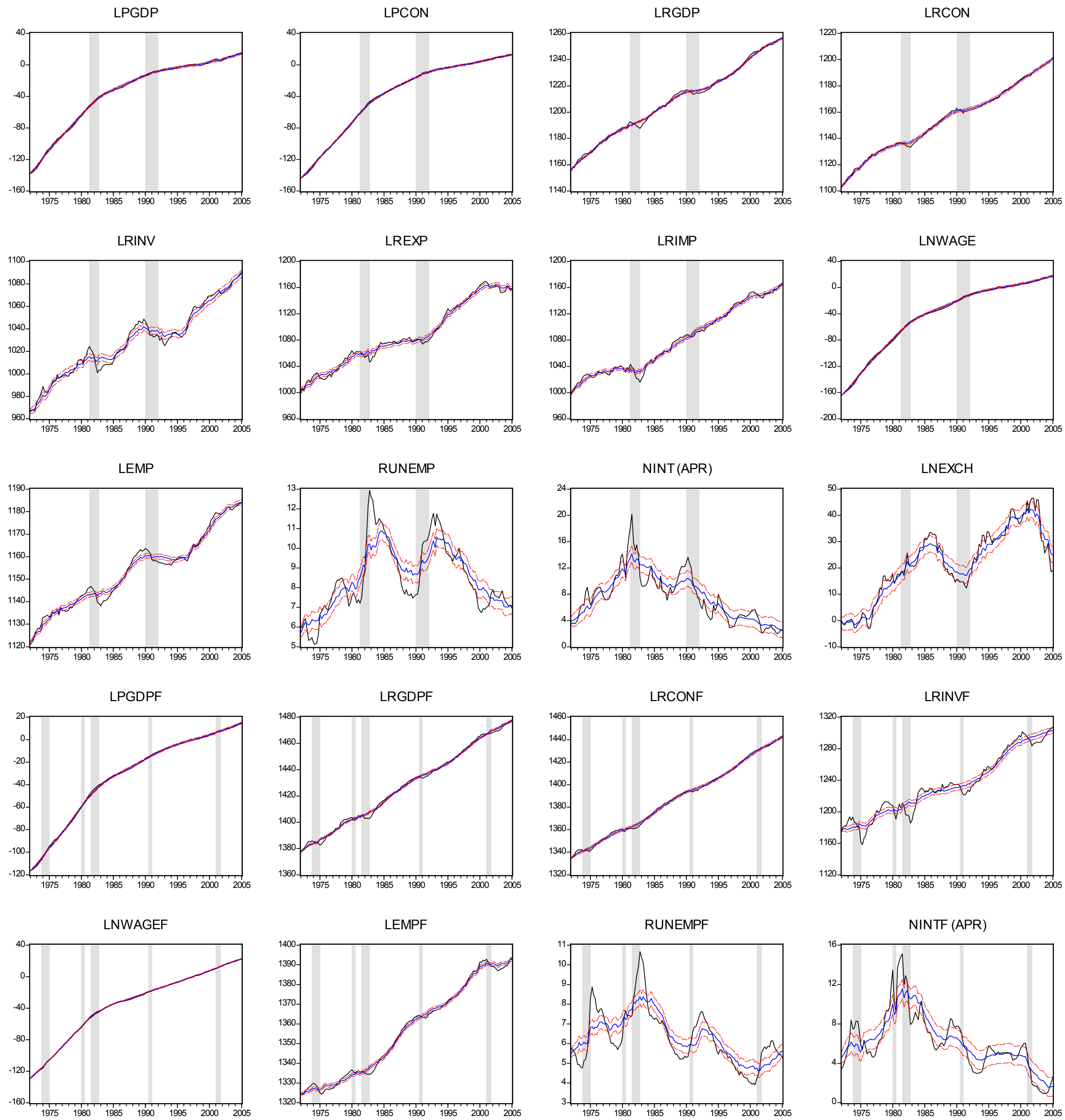

*Note:* Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 6.  Predicted trend components of observed nonpredetermined endogenous variables



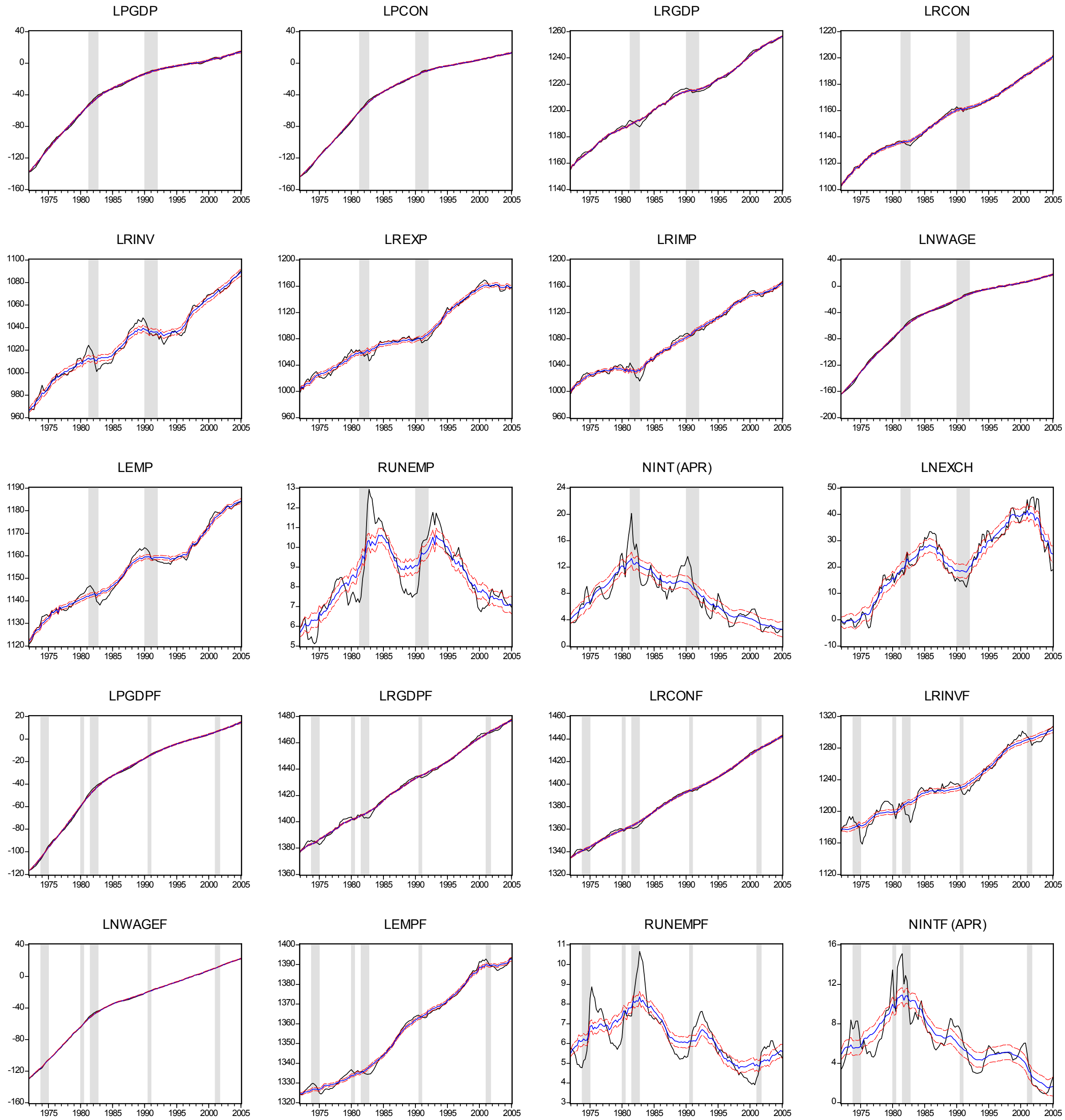

*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components.  Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.  Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 7. Filtered trend components of observed nonpredetermined endogenous variables



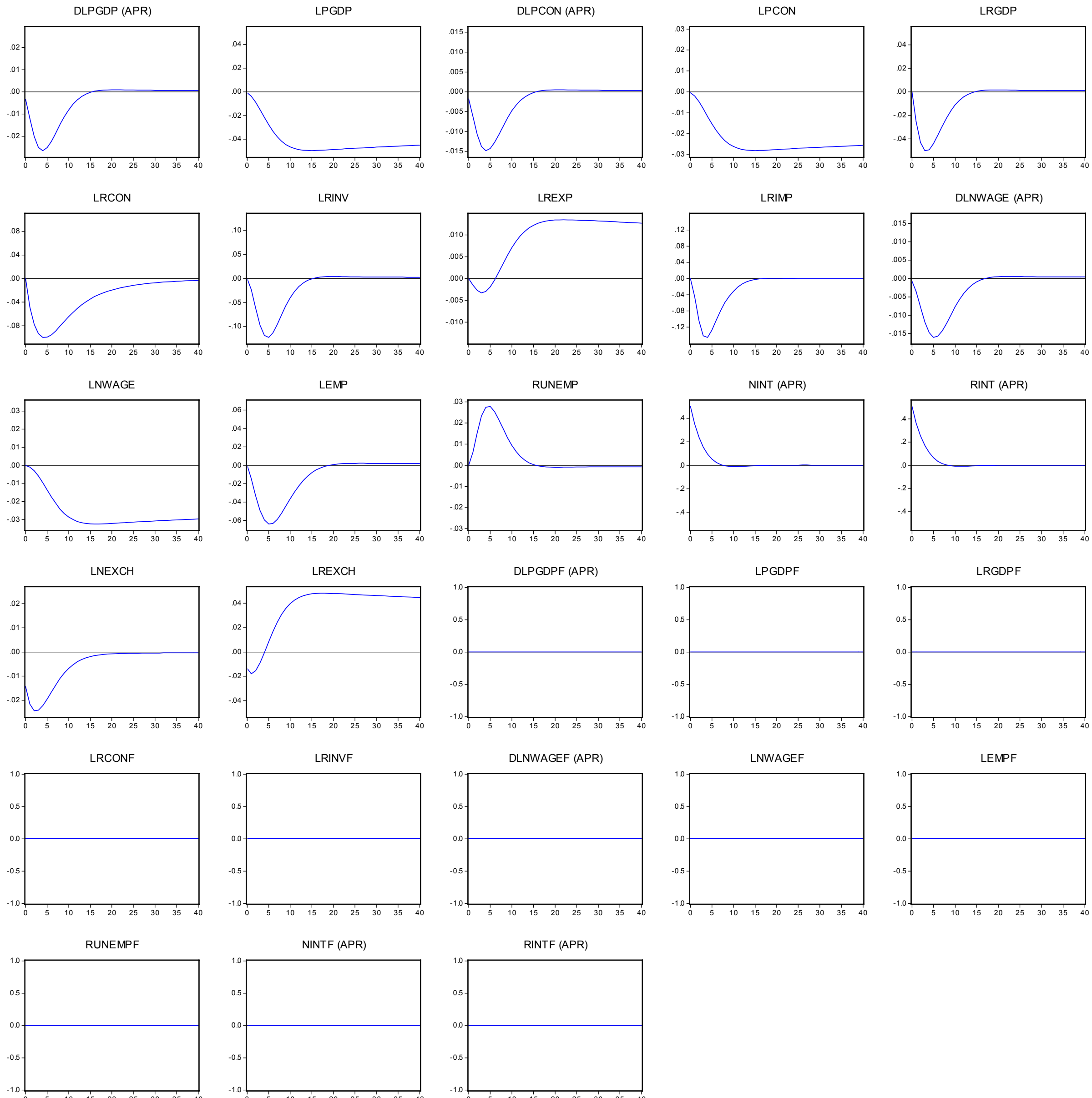

*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 8. Smoothed trend components of observed nonpredetermined endogenous variables



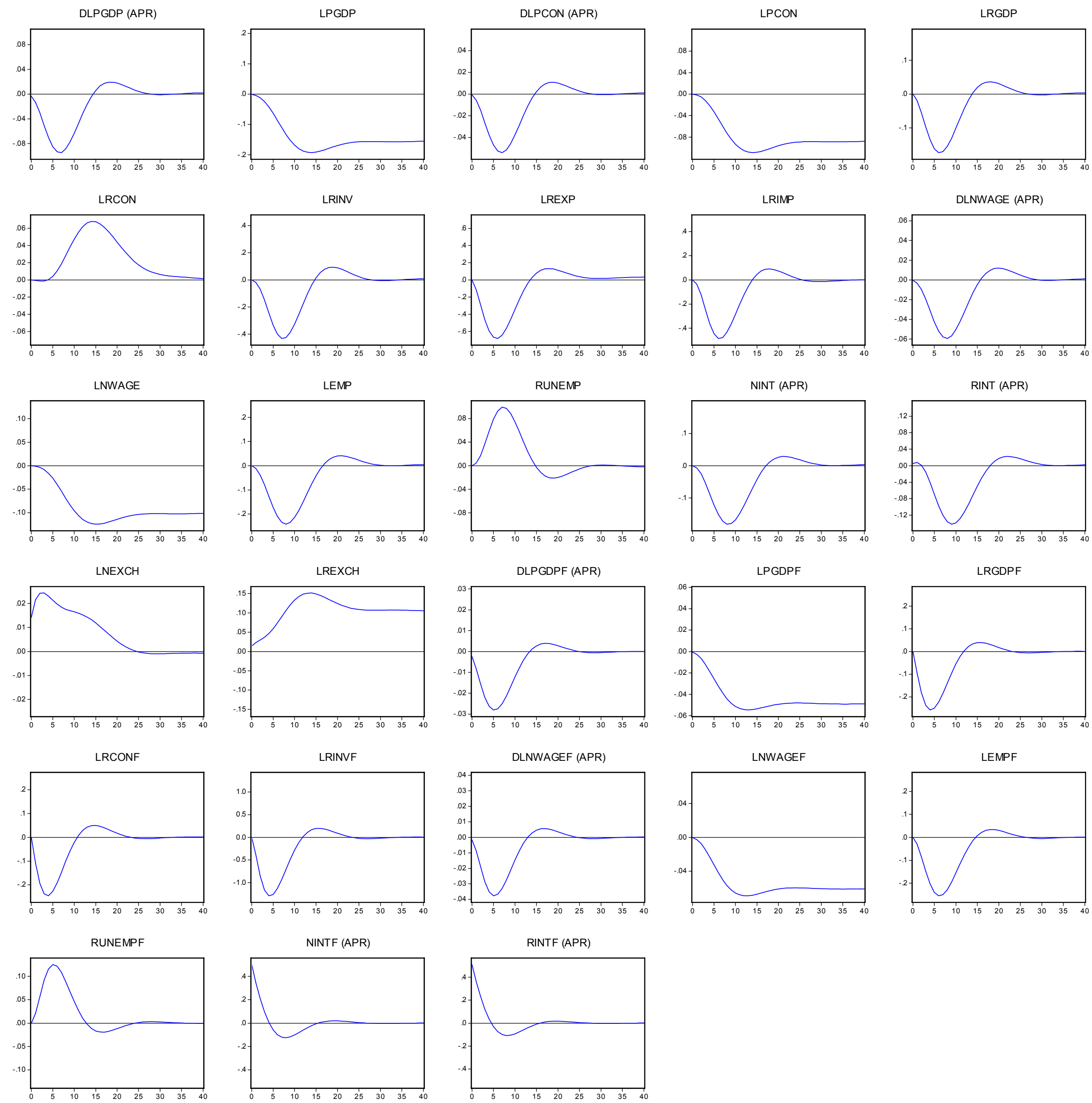

*Note:* Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 9. Estimated impulse responses to a domestic monetary policy shock



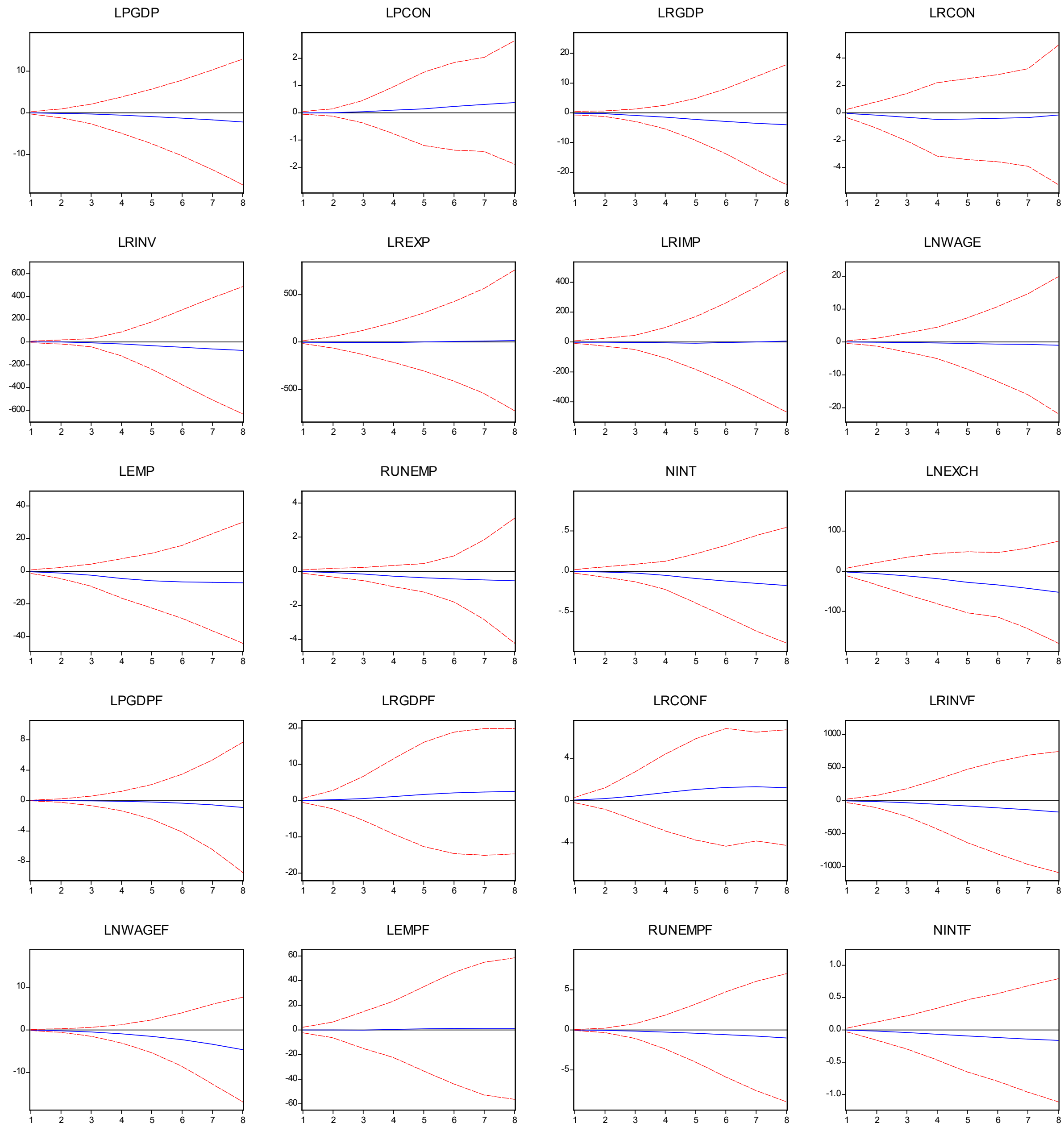

*Note:* Estimated impulse responses to a 50 basis point monetary policy shock are depicted.

Figure 10.  Estimated impulse responses to a foreign monetary policy shock



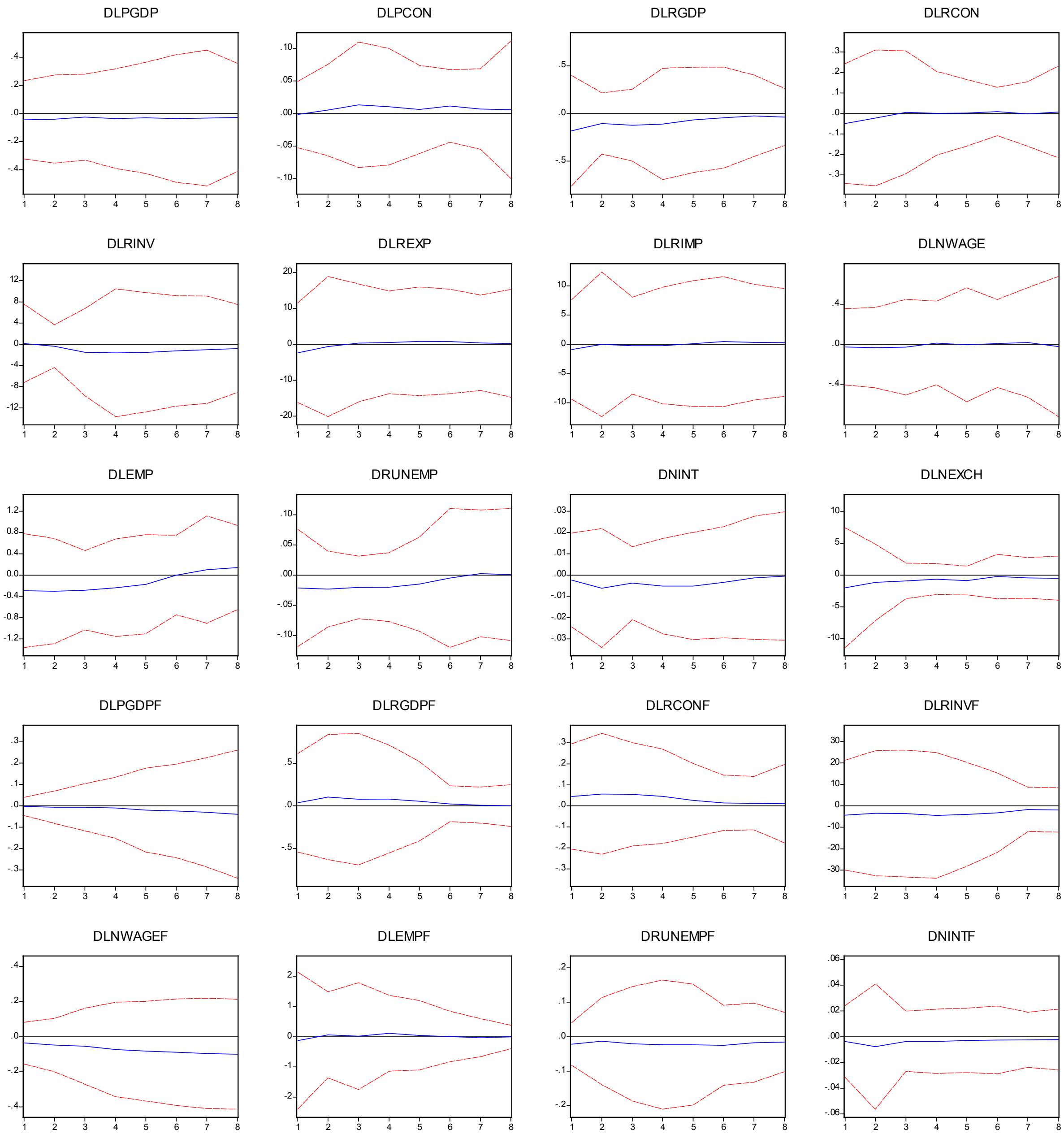

*Note:* Estimated impulse responses to a 50 basis point monetary policy shock are depicted.

Figure 11. Mean squared prediction error differentials for levels



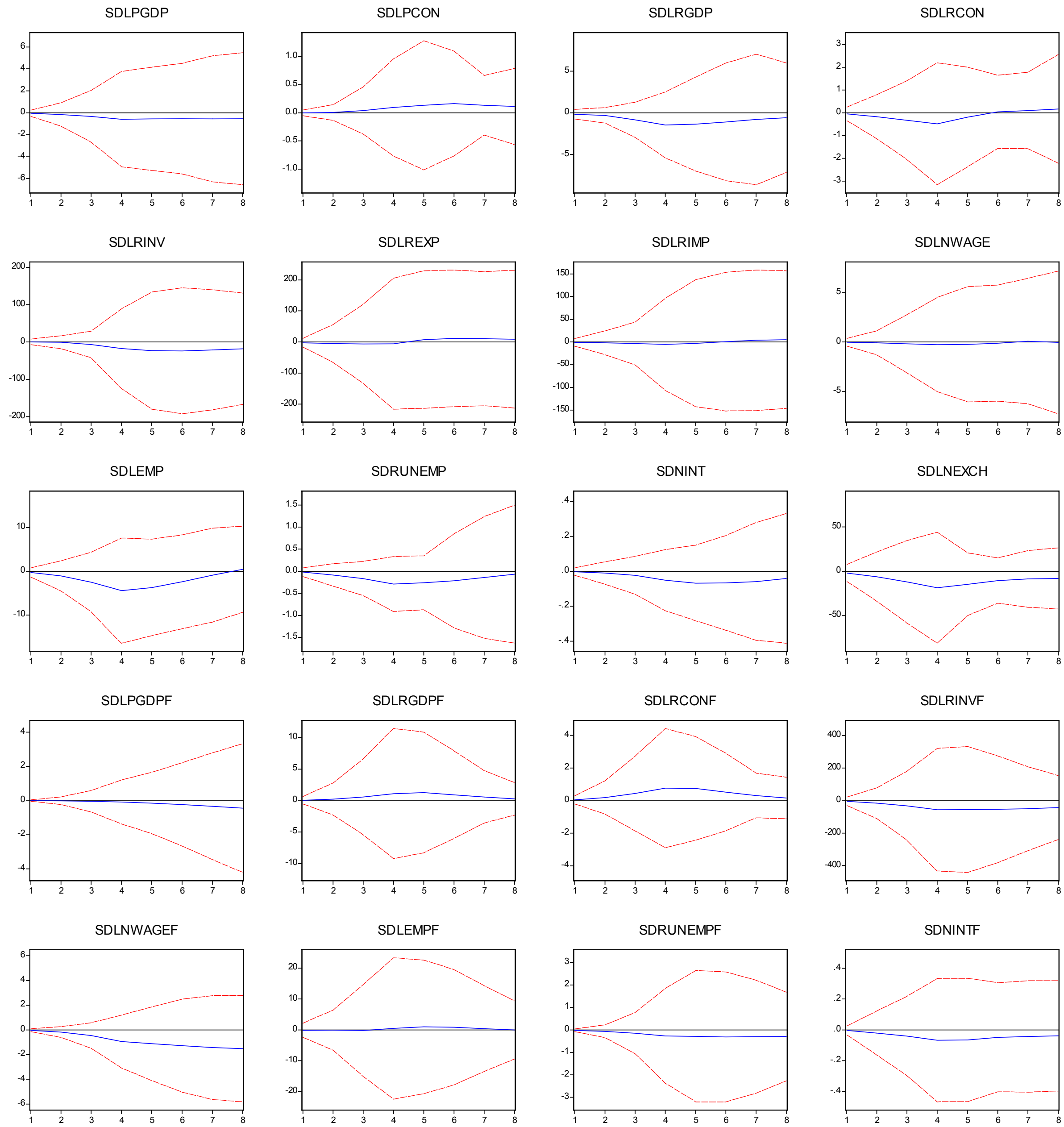

*Note:* Mean squared prediction error differentials are defined as the mean squared prediction error for the unobserved components model less that for the ARIMA model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 12. Mean squared prediction error differentials for ordinary differences



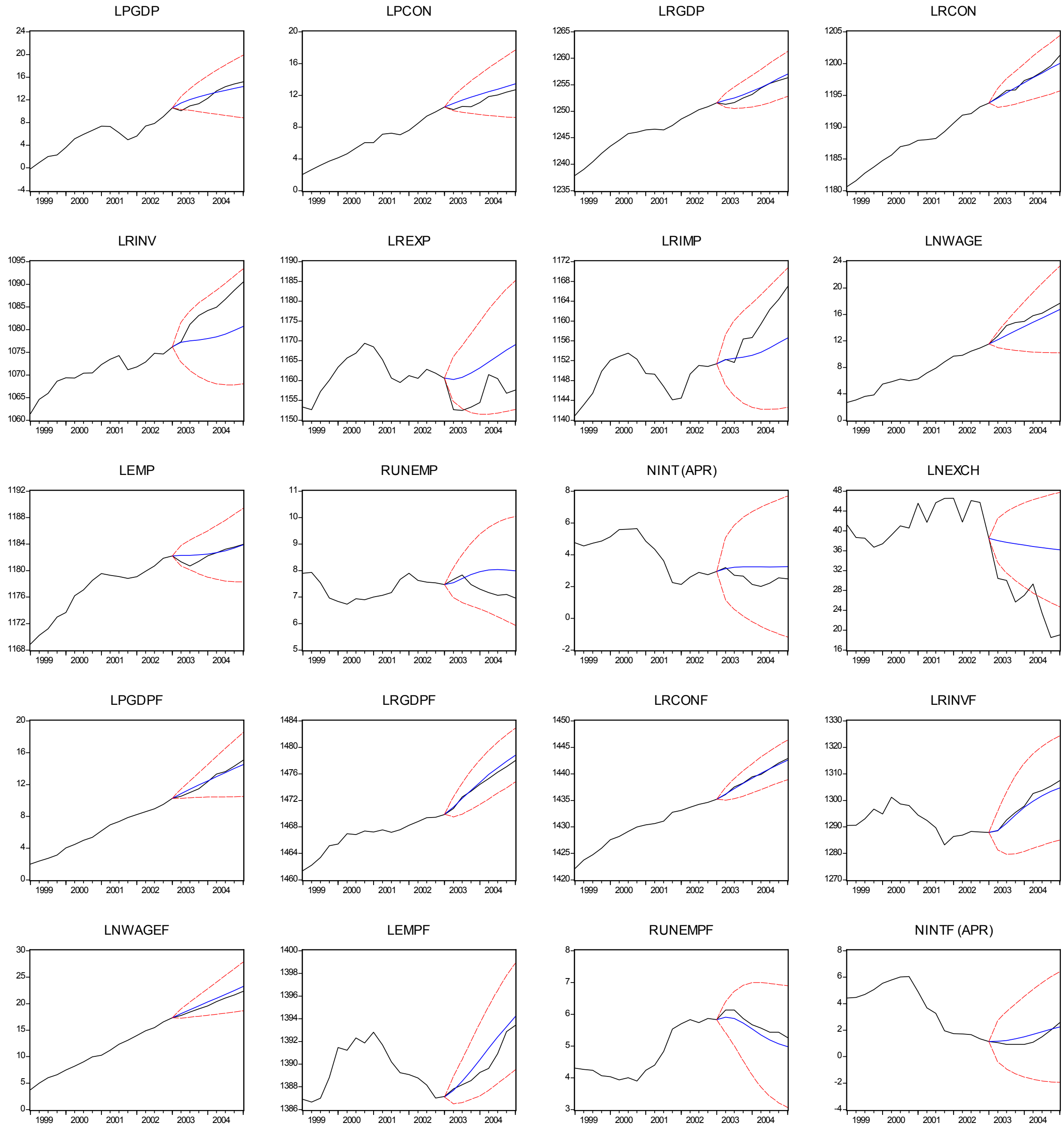

*Note:* Mean squared prediction error differentials are defined as the mean squared prediction error for the unobserved components model less that for the ARIMA model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 13. Mean squared prediction error differentials for seasonal differences



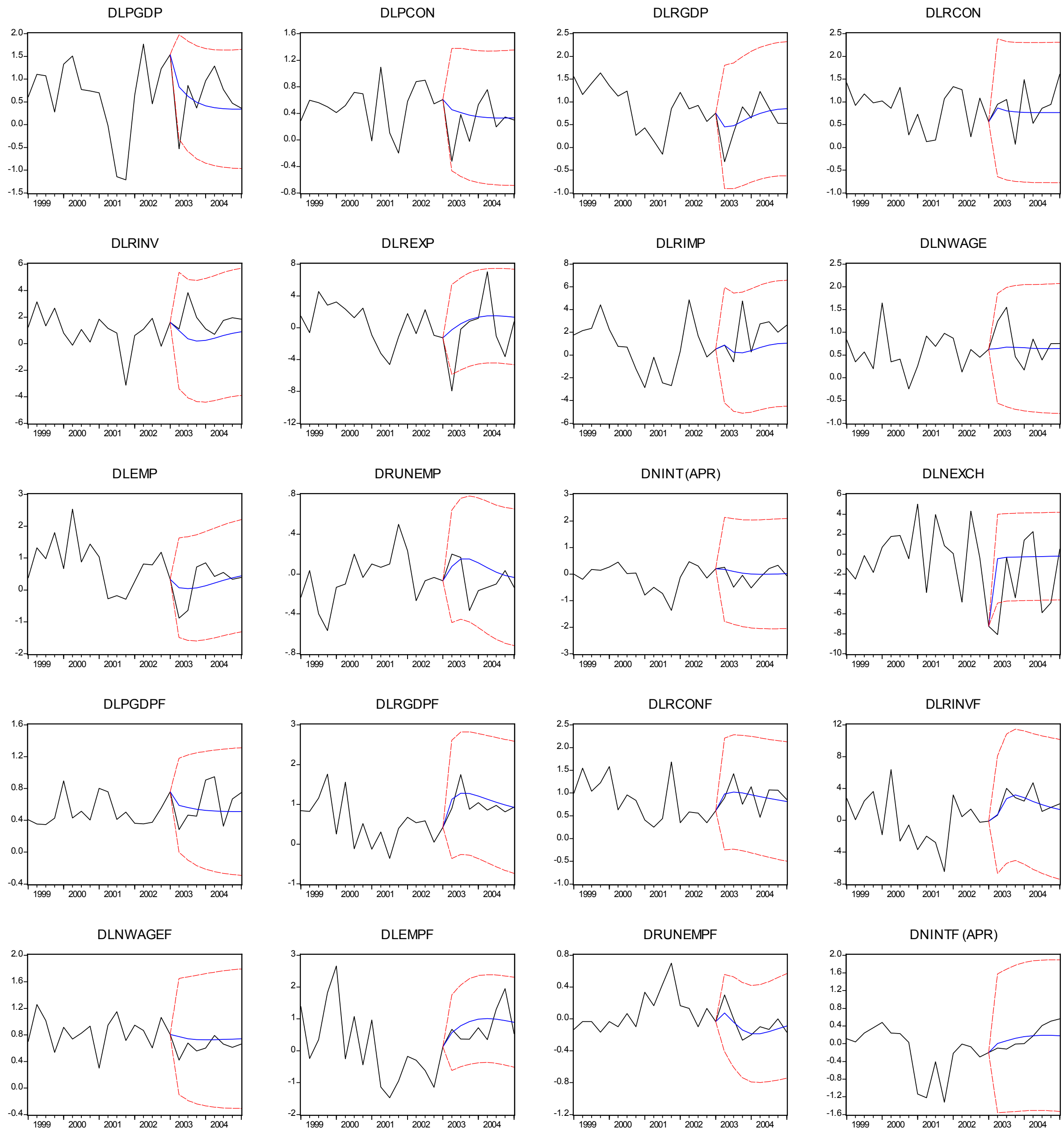

*Note:* Mean squared prediction error differentials are defined as the mean squared prediction error for the unobserved components model less that for the ARIMA model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 14. Dynamic forecasts of levels of observed nonpredetermined endogenous variables



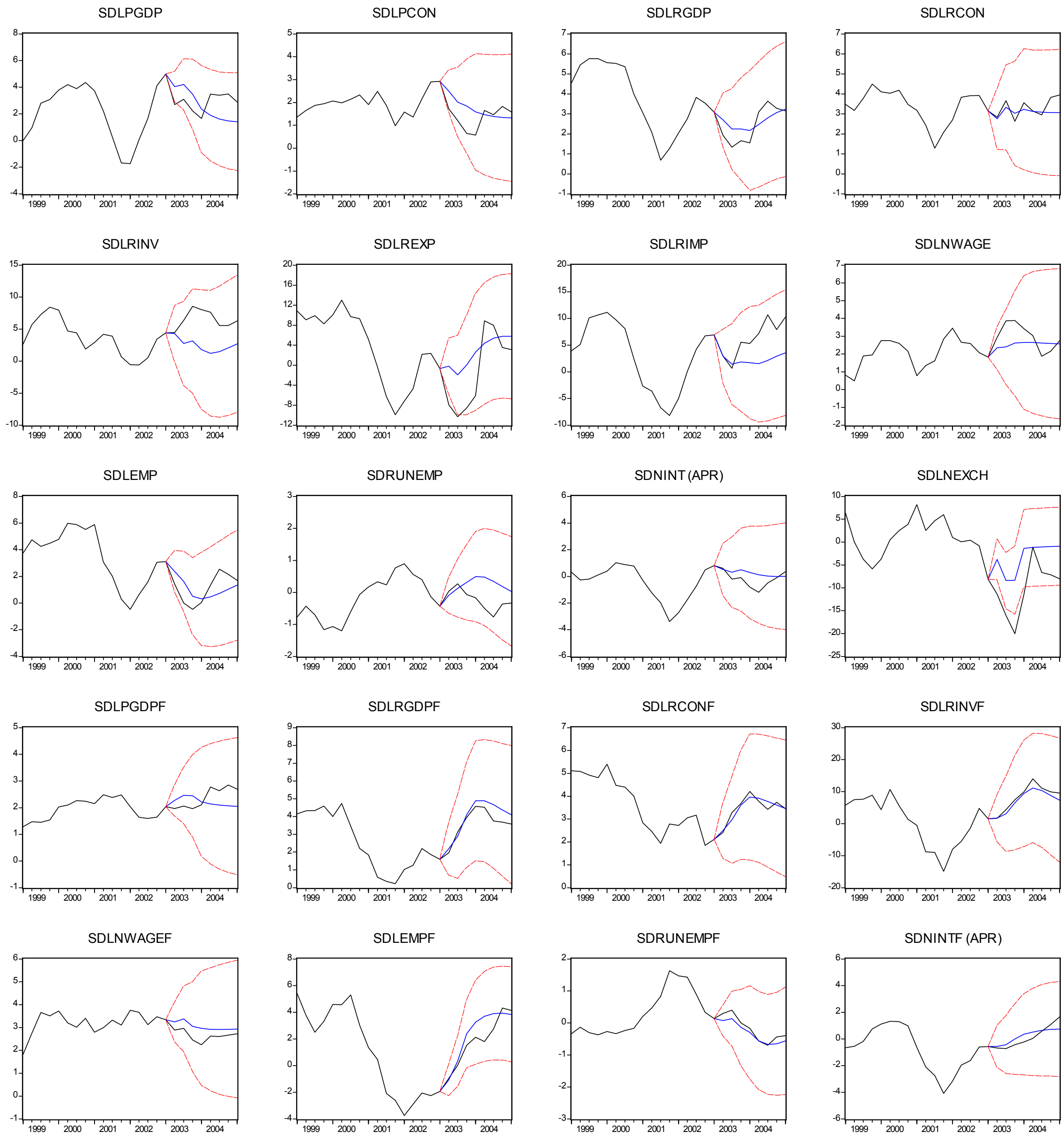

*Note:* Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.

Figure 15. Dynamic forecasts of ordinary differences of observed nonpredetermined endogenous variables




*Note:* Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.

Figure 16. Dynamic forecasts of seasonal differences of observed nonpredetermined endogenous variables




*Note:* Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.

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