Stock market returns and economic activity: evidence from wavelet analysis

Marco Gallegati*

December 27, 2005

In this paper we investigate the relationship between stock market returns and economic activity by using signal decomposition techniques based on wavelet analysis. In particular, we apply the maximum overlap discrete wavelet transform (MODWT) to the DJIA stock price index and the industrial production index for US over the period 1961:1-2005:3 and using the definitions of wavelet variance, wavelet correlation and cross-correlations analyze the association as well as the lead/lag relationship between stock prices and industrial production at the different time scales. Our results show that stock market returns tends to lead the level of economic activity but only at the highest scales (lowest frequencies), corresponding to periods of 16 months and longer, and that the periods by which stock returns lead output increase as the wavelet time scale increases.

KEYWORDS: Stock markets, Industrial production, Wavelet analysis

JEL CLASSIFICATION: C32, E44.

*DEA and SIEC, Universit Politecnica delle Marche, Ancona, Italy

1 Introduction

According to the discounted cash flow valuation model firms' stock prices reflect investors' expectations about future corporate earnings. Thus, at aggregate level, if expectations are correct on average, stock prices should provide information about future economic conditions, as corporate profits are generally correlated with the level of economic activity. Many empirical studies (Fama, 1981, 1990, Schwert, 1990, Choi *et al.*, 1999, among the others) have analyzed the relationship between stock returns and production growth rates finding that stock returns are significantly correlated with overall economic activity, with the degree of correlation that increases with the lenght of the time period (*i.e.* monthly, quarterly or annual). In particular, the existing evidence, mainly coming from univariate or multivariate time series analyses, suggests that stock returns have a predictive power over real economic activity, that is they tend to lead real output).[1]

These studies analyze the interations between the stock market and aggregate economic activity by examining either their short run or long run relationships, as the time series methodologies employed (usually cointegration analysis) may separate out just two time periods (or time scales) in economic time series, *i.e.* the short run and the long run. But the stock market provides an example of a market in which the agents involved consist of heterogeneous investors making decisions over different time horizons (from minutes to years) and operating at each moment on different time scales (from speculative to investment activity). In this way, the nature of the relationship between stock returns and production growth rates may well vary across time scales according to the investment horizon of the traders, as the small time scales may be related to speculative activity and the coarsest scales to investment activity. Thus, for example, if we think that big institutional investors have long term horizons and, consequently, follow macroeconomic fundamentals, we should expect the relationship between stock returns and economic activity to be stronger at the intermediate and coarsest time scales than at the finest ones.

In such a context, where both the time horizons of economic decisions and the strenght and direction of economic relationships between variables may differ according to the time scale of the analysis (Ramsey and Lampart, 1998a), a useful analytical tool may be represented by wavelet analysis. Wavelets are particular types of function $f(x)$ that are localized both in time and frequency domain and used to decompose a function $f(x)$, *i.e.* a surface, a series, etc.., in more elementary functions which include informations about the same $f(x)$. The main advantage of wavelet analysis is its ability to decompose macroeconomic time series, and data in general, into their

[1]The explanation for such relation refers mainly to the notion of stock prices are a leading indicator of the real economic activity, as they reflect investors' expectations about future economic conditions.

time scale components. Several applications of wavelet analysis in economics and finance have been recently provided by Ramsey and Lampart (1998a, 1999b), Ramsey (2002), and Kim and In (2003) among the others, but no attempts has been made to apply this methodology to the analysis of the relationship between stock returns and overall economic activity.

The main of this paper is to reconsider the positive relationship between stock returns and real activity by using signal decomposition techniques based on wavelet analysis. In particular, we apply the maximum overlap discrete wavelet transform (MODWT) to the DJIA stock price index and the industrial production index for US over the period 1961:1-2005:3 and analyze the correlation between stock prices and industrial production at the different time scales stemming from the multiresolution decomposition of wavelet analysis. The structure of the paper is as follows. Section 2 describes briefly the methodology employed, *i.e.* wavelet analysis, and the data set used. Section 3 presents the empirical results from wavelet variance, wavelet correlation and wavelet cross-correlation analyis, while Section 4 concludes the paper.

2 Wavelet analysis

The series were filtered using wavelet analysis that is a relatively new (at least for economists) statistical tool that, roughly speaking, decomposes a given series in orthogonal components, as in the Fourier approach, but according to scale (time components) instead of frequencies. The comparison with the Fourier analysis is useful first because wavelets use a similar strategy: find some orthogonal objects (wavelets functions instead of sines and cosines) and use them to decompose the series. Second, since the Fourier analysis is a common tool in economics, it may be useful in understanding the methodology and also in the interpretation of results. Saying that, we have to stress the main difference between the two tools. Wavelet analysis does not need stationary assumption in order to decompose the series. This is because the Fourier approach decomposes in frequencies space that may be interpreted as events of time-period T (where T is the number of observations). Put differently, spectral decomposition methods perform a global analysis whereas, on the other hand, wavelets methods act locally in time and so do not need stationary cyclical components. Recently, to relax the stationary frequencies assumption a windowing Fourier decomposition that essentially use, for frequencies estimation, a time-period M (the window) event less than the number of observations T. The problem with this approach is the right choice of the window and, more important, its constancy over time.

Coming back to wavelets and going into some mathematical detail we may note that there are two basic wavelet functions: the father-wavelet and the mother-wavelet. The formal definition of the father wavelets is the function

$$\phi_{J,k} = 2^{-(J/2)}\phi((t - 2^J k)/(2^J)) \tag{1}$$

defined as non-zero over a finite time length support that corresponds to given mother wavelets

$$\psi_{J,k} = 2^{-(J/2)}\psi((t - 2^J k)/(2^J)) \tag{2}$$

with j=1,,J in a J-level wavelets decomposition. The former integrates to 1 and reconstructs the longest time-scale component of the series (trend), while the latter integrates to 0 (similarly to sine and cosine) and is used to describe all deviations from trend. The mother wavelets, as said above, play a role similar to sins and cosines in the Fourier decomposition. They are compressed or dilated, in time domain, to generate cycles fitting actual data.

For a discrete signal or function $f_1, f_2, ..., f_n$, the wavelet representation of the signal or function $f(t)$ in $L^2(R)$ can be given by

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + ... + \sum_k d_{j,k}\psi_{j,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \tag{3}$$

where J is the number of multiresolution components or scales, and k ranges from 1 to the number of coefficients in the specified components. The coefficients d_{jk} and s_{Jk} of the wavelet series approximations in (3) are the details and smooth wavelet transform coefficients representing, respectively, the projections of the time series onto the basic functions generated by the chosen family of wavelets, that is

$$d_{j,k} = \phi_{j,k} f(t)dt \tag{4}$$

$$s_{J,k} = \psi_{J,k} f(t)dt \tag{5}$$

for j=1,2,....,J. The smooth coefficients s_{Jk} mainly capture the underlying smooth behaviour of the data at the coarsest scale, while the details coefficients $d_{1k}, .., d_{jk}, .., d_{Jk}$, representing deviations from the smooth behaviour, provide progressively finer scale deviations. Each of the sets of the coefficients $s_J, d_J, d_{J-1}, ..., d_1$ is called a crystal.

The multiresolution decomposition of the original signal $f(t)$ is given by the following expression

$$f(t) = S_J + D_J + D_{J-1} + ... + D_j + ... + D_1 \tag{6}$$

where $S_J = \sum_k s_{J,k}\phi_{J,k}(t)$ and $D_j = \sum_k d_{J,k}\psi_{J,k}(t)$ with j=1,,J. The sequence of terms $S_J, D_J, .., D_j, ..., D_1$ in (4) represent a set of signals components that provide representations of the signal at the different resolution levels 1 to J, and the detail signals D_j provide the increments at each individual scale, or resolution, level.

In addition to the features stated above Whitcher *et al.* (1999, 2000) have extended the notion of wavelet covariance for the maximal overlap DWT (MODWT), where the maximal overlap DWT (MODWT) may be regarded as a modified version of the discrete wavelet transform (DWT), and defined the wavelet covariance and correlation between two processes.

For a signal $f(t)$ the MODWT applying the Daubechies compactly supported wavelet produces J vectors of wavelet coefficients $w_1, w_2, ..., w_J$ and one vector of scaling coefficients, v_J. The wavelet variance for a signal $f(t)$ is defined as the variance of the wavelet coefficients at scale 2^{j-1} and an unbiased estimator using the MODWT after removing all coefficients affected by the periodic boundary conditions[2] through

$$v_{f(t)}(2^{j-1}) = (1/(N_j)) \sum_{t=L_{J-1}} w_{j,t} \tag{7}$$

where $N_j = (N/(2^j - L_j))$ with $L_j = [(L-2)(1-2^j)]$ being the lenght of the scale 2^{j-1} wavelet filter,[3] and the vector w are n-dimension vectors containing the coefficients $v_J, w_J, ..., w_1$ of the wavelet series approximations. Thus, level j wavelet variance is simply the variance of the wavelet coefficients at that level (Gencay et al., 2002). Similarly, the covariance is defined to be the covariance between the scale wavelet coefficients of $f(t)$ and $g(t)$. Again, after removing all wavelet coefficietns affected by the boundary conditions, an unbiased estimator of the wavelet covariance using the MODWT may be given by:

$$Cov_{f(t)g(t)}(2^{j-1}) = (1/(N_j)) \sum_{t=L_{J-1}} w^{f(t)}_{j,t} w^{g(t)}_{j,t} \tag{8}$$

Analogously to the usual unconditional correlation coefficients, the MODWT estimator of the wavelet cross correlation coefficients may then be obtained making use of the wavelet covariance $Cov_{f(t)g(t)}$ and the square root of their wavelet variances $v_{f(t)}$ and $v_{g(t)}$ as follows:

$$\rho_{f(t)g(t)}(2^{j-1}) = ((Cov_{f(t)g(t)}(2^{j-1}))/(v_{f(t)}(2^{j-1})v_{g(t)}(2^{j-1})) \tag{9}$$

[2]As MODWT employs circular convolution the coefficients generated by both beginning and ending data could be spurious

[3]If the lenght of the filter is L, there are $(2^j-1)(L-1)$ coefficients affected for 2^{j-1}-scale wavelet and scaling coefficients, while $(2^j-1)(L-1)-1$ beginning and $(2^j-1)(L-1)$ ending components in 2^{j-1}-scale details and smooth would be affected (Percival and Walden, 2000).

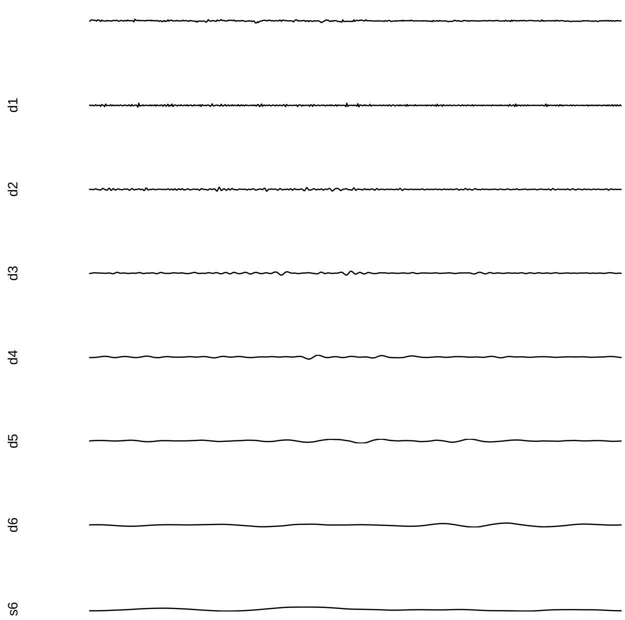

Figure 1: Industrial production index - Crystals

3 Wavelet variance, correlation and cross-correlations

The analysis was conducted using monthly data for the DJIA stock market index and the industrial production index for US between 1961:1-2005:3 (source: yahoo.finance.it and DREDII). Continously compounded one-month returns and industrial production growth rates are calculated as the log difference of the stock price index and of the industrial production index, respectively. We decompose the two transformed series into their time-scale components using the maximum overlap discrete wavelet transform (MODWT) which is a non-orthogonal variant of the classical discrete wavelet transform that, unlike the orthogonal discrete wavelet transform, is translation invariant, as shifts in the signal do not change the pattern of coefficients. The wavelet filter used in the decomposition is the Daubechies least asymmetric (LA) wavelet filter of lenght L=8, that is LA(8), based on eight non-zero coefficients (Daubechies, 1992), with periodic boundary conditions. The application of the translation invariant wavelet transform with a number of scales $J = 6$ produces six wavelet and scaling filter coefficients $v_5, w_5, w_4, w_3, w_2, w_1$. Since we use monthly data scale 1 represents 2-4 monthly period dynamics, while scales 2, 3, 4, 5 and 6 correspond to

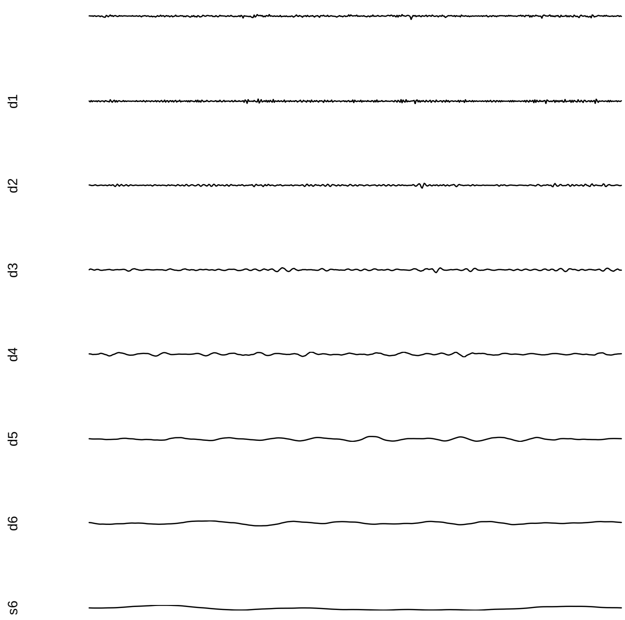

Figure 2: Dow Jones Industrial Average - Crystals

4-8, 8-16, 16-32, 32-64 and 64-128 monthly period dynamics, respectively.[4]

With wavelet analysis we may disentangle the variance, correlation and cross-correlations on a scale by scale basis and determine which scales contributes to the overall relationship between two series. Figure 3 illustrates the MODWT-based variance of the two series at the different scales where the straight lines indicate the variance and the dotted lines the 95 percent confidence interval. An approximate linear relationship emerges between the wavelet variance and the wavelet scale with the wavelet variance decreasing as the wavelet scale increases and the DJIA stock returns being, as expected, more volatile than the IPI growth rate.

A common finding in the analysis of the relationship between stock market and the level of economic activity is that the degree of correlation tends to increases with the lenght of the time period (*i.e.* monthly, quarterly or annual). Thus, we analyze both the contemporaneous and the lead/lag association between stock market returns and industrial production growth rate at the different time scales looking at wavelet correlations and cross-correlation coefficients.[5] In Figure 5 and 6 we report the MODWT-based

[4]There are many possible choices for wavelet functions (see Gencay et al., 2001), like the nearly symmetric wavelets or symlet with 8 vanishing moments meaning that polynomial of an order less than 8 will be passed through by the mother wavelets.

[5]In a time series context the most commonly used measure to analyze co-movements in

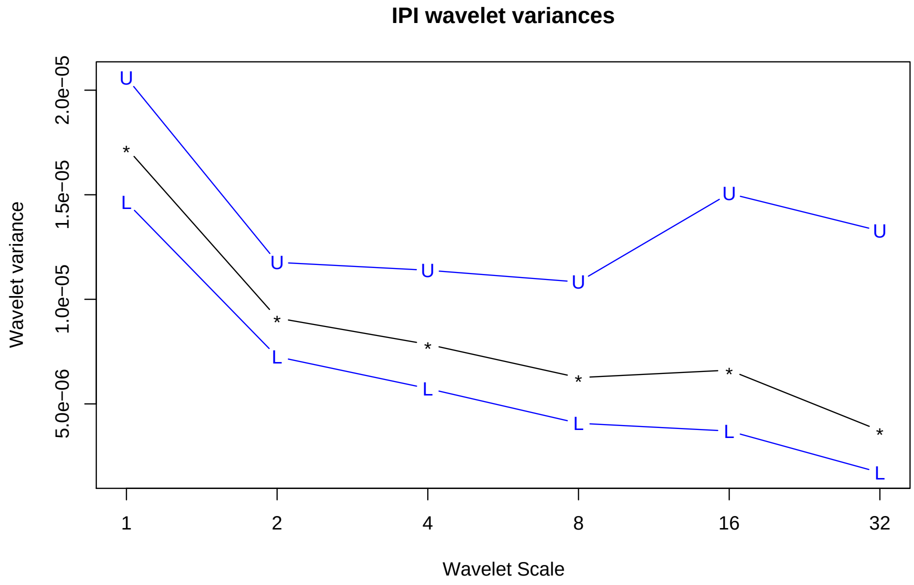

Figure 3: Industrial production index

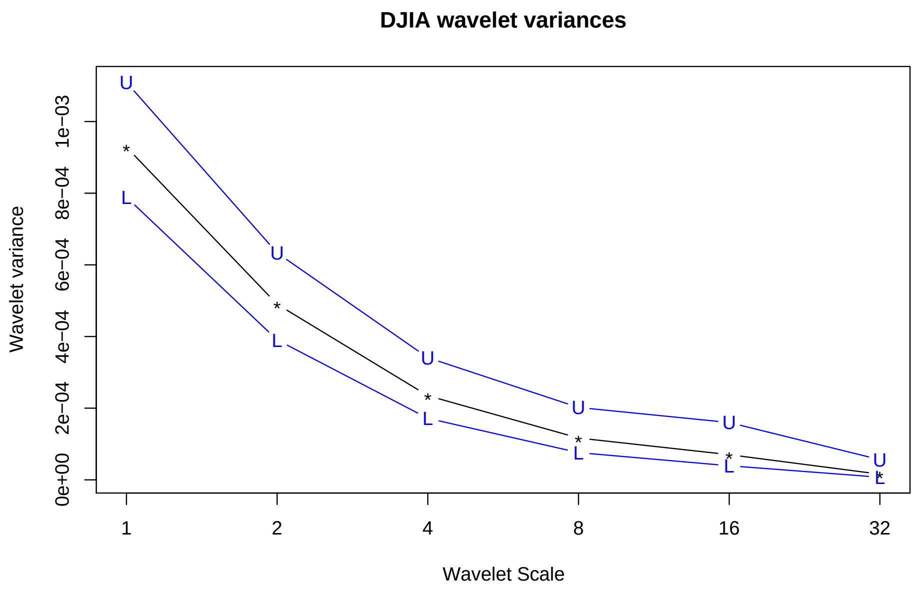

Figure 4: Dow Jones Industrial Average

the level of economic activity across countries and over time is, undoubtedly, correlation analysis. Cross-correlations have been widely used to obtain a static estimate of comovements across variables (among the others Backus and Kehoe, 1992, Christodoulakis et al., 1995, and Comin and Gertler, 2003).

wavelet correlations and cross-correlation coefficients against various time scales, respectively, where each scale is associated with a particular time period.[6] The wavelet correlation coefficients reported in Figure 5 indicates that the magnitude of the association between the two series is low at all time scales, except at the longest wavelet scale, *i.e.* at scale 6, where a positive signficant relationship can be observed.

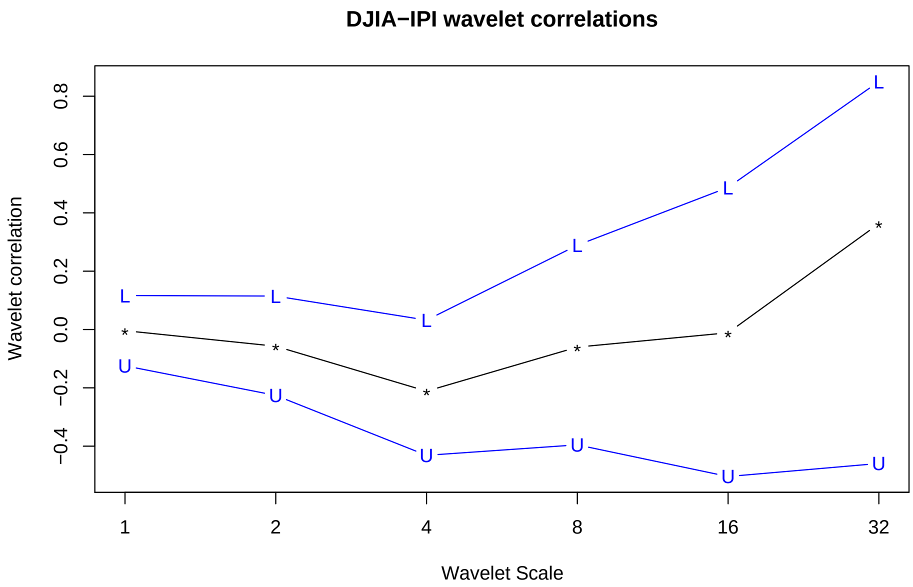

Figure 5: Wavelet correlation between DJIA stock returns and US Industrial production index

Since our focus is on the leading/lagging relationship between the stock market and the level of economic activity wavelet cross-correlation for various leads and lags may be useful in providing a picture of the scale by scale timing relationship. Thus, in Figure 6 we report the wavelet cross-correlations and the corresponding approximate confidence intervals against time leads and lags for all scales. At the shortest scales, *i.e.* scales 1-3, the relationships between the two series is generally not signficantly different from zero at all leads and lags (slightly significant only at scale 3). On the other hand, at the coarsest scales, *i.e.* scales 4-6, a significant positive relationship between stock market returns at time $t-k$ and production growth rate at time t seems to emerge clearly, with a maximum correlation coefficient value of about 0.5 and with the number of lags k at which the maximum value

[6]For example the first scale is 2 months, the second 4 months, the third scale 8 months, and so on, with the last scale corresponds to approximately 128 months

of cross-correlation occurs that tends to increase as the wavelet time scale increases. Indeed, the largest coefficient value occurs at lag -6 for wavelet scale 4, -10 for wavelet scale 5, and -15 for wavelet scale 6. Therefore, the underlying positive leading relationship between stock returns and economic activity seems to emerge only over long investment horizons (periods of 16 months and longer).

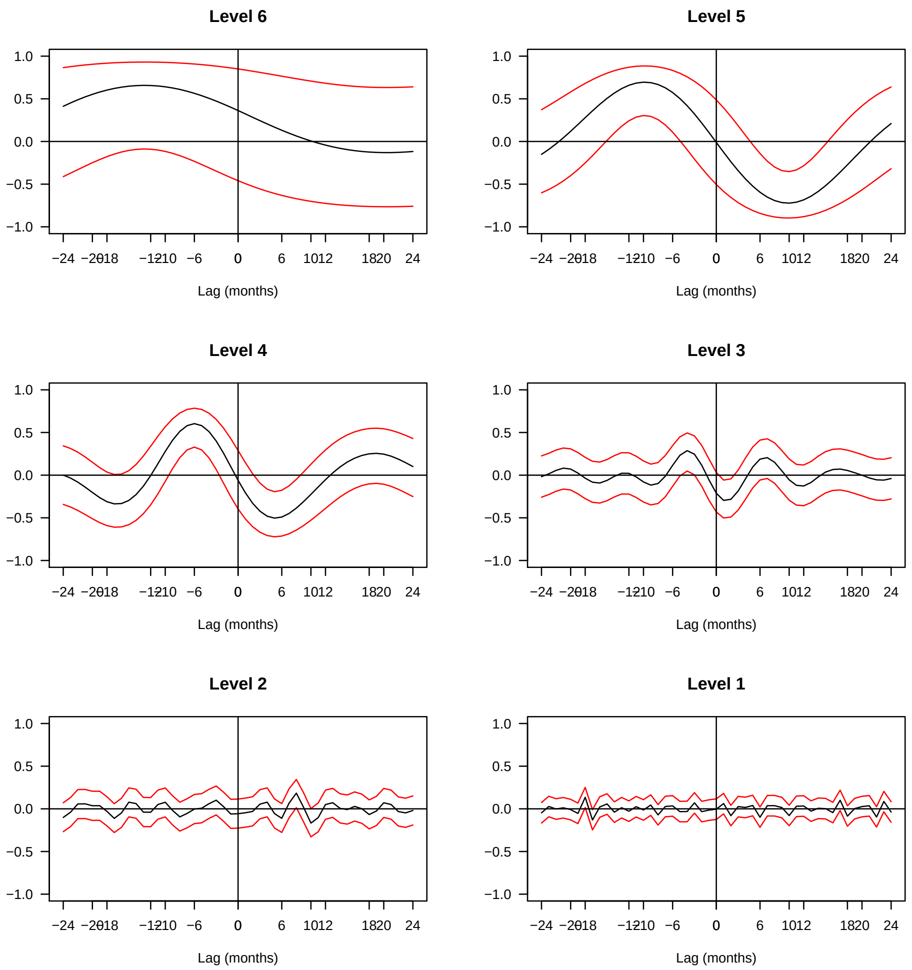

Figure 6: Wavelet cross-correlations between DJIA stock returns and US Industrial production index

4 Conclusions

In this paper we apply a wavelet multi-scaling approach based on a non-decimated discrete wavelet transform to investigate the relationship between stock returns and economic activity over different time scales. Through a scale by scale decomposition of the variance of the series, and of the correlation and cross-correlation between two time series we try to shed some light on the scaling properties of stock returns and production growth rates and on their relationship at different time horizons.

The main results may be summarized as follows:

1. the wavelet variance of the two transformed time series tends to decrease as the wavelet scale increases;

2. the wavelet correlation between the two series does not differ significantly from zero, with the exception of the longest scales, *i.e.* scale 6, where it is slightly positive;

3. the wavelet cross-correlation analysis provides evidence about the findings that stock returns are leading economic activity at the coarsest scales, *i.e.* scales 4-6, which correspond to longer investment horizons.

Therefore, our results show that stock market returns tends to lead the level of economic activity but only at the highest scales (lowest frequencies), which correspond to periods of 16 months and longer, and that the periods by which stock returns lead output increase as the wavelet time scale increases. Such a finding seems to be consistent with a leading relationship between stock market returns and overall economic activity determined by the behaviour of big institutional investors that, having long term horizons, refer mainly to macroeconomic fundamentals in their investment activity.

5 References

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