The Theory of
Monetary Aggregation

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Preface to
The Theory of Monetary Aggregation

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W. T. Foster wrote the following lines as a start to his preface of Irving Fisher's classic study, *The Making of Index Numbers*:

> "To determine the pressure of steam, we do not take a popular vote; we consult a gauge. Concerning a patient's temperature, we do not ask for opinions: we read a thermometer. In economics, however, as in education, though the need for measurement is as great as in physics or in medicine, we have been guided in the past largely by opinions. In the future, we must substitute measurement. Toward this end, we must agree upon instruments of measurement. That is the subject of this book."

The above lines are also an appropriate introduction to the present book, edited by William Barnett and Apostolos Serletis. The present book is a collection of papers by Barnett and his co-authors (E. Offenbacher, P. Spindt, A. Serletis, M. Hinich, P. Yue, Y. Liu, M. Jensen, H. Xu, G. Zhou, D. Fisher, W. E. Weber, J. H. Hahm, M. Kirova, and M. Pasupathy). Each paper has better measurement as its central theme and hence this book follows in the tradition of Irving Fisher, who also tried to improve economic measurement. In what follows, when I refer to Barnett, this should be understood as a shorthand notation for Barnett and his co-authors, when appropriate.

Barnett's basic research program has been to integrate monetary theory into macroeconomics starting with microeconomic theory and then using index number and aggregation theory to go from microeconomics to macroeconomics. Barnett has also used modern econometric techniques to estimate demand and supply functions for money and test for the existence of various monetary aggregates. More specifically, some of the major theoretical contributions of Barnett, which appear in this book, are: (i) producer and consumer user costs for money are rigorously derived and used as the appropriate prices for monetary components; (ii) the insertion of real balances into neoclassical utility and production functions is rigorously justified using the work of Fischer, Feenstra, and others; (iii) when aggregating commodities,

superlative index number formulae are used; (iv) flexible functional forms for utility and production functions are consistently used throughout the book; (v) modern developments in testing for the existence of weakly separable aggregates are used to test for the existence of various monetary aggregates; and (vi) the usual consumer and producer models are extended to include *risk* in a fundamental way. I would also like to note the contribution made in chapters 3 and 19 where Barnett points out that the existence of bank reserve requirements creates a regulatory wedge in the user cost of money. That is, the reserve requirement acts like a capital tax on the bank and thus the user cost of money will be different on the supply (or bank) side of the market compared to the demand side of the market. This point creates a tremendous difficulty for macro models or applied general equilibrium models: there is no unique price that can equilibrate the demand and supply of money!

In addition to the above theoretical contributions, Barnett compares the performance of his superlative indexes, which use monetary user costs, with simple sum monetary aggregates, which do not use user costs. In chapter 24, he notes that Milton Friedman predicted that a resurgence of inflation would inevitably follow the explosion that occurred in the simple sum aggregates for the U. S. from late 1982 to mid 1983. Friedman also predicted that once the inevitable inflation began, the Federal Reserve would tighten monetary policy in a manner that would produce a recession. However, on the very same day that Friedman made his prediction, Barnett went on the record with a dramatically different forecast based on his superlative Divisia monetary indexes (which showed no monetary explosion). In fact, Friedman's predicted inflation and subsequent recession did not occur.

It is also interesting to observe what happened during the immediately preceding period. The following quotation, taken from pages 581-582 of chapter 24, explains how the different measurement techniques led to very different numerical estimates of money supply growth and to the mistakes in policy between 1979 and 1982 that produced the recession of 1982:

> "As I reported in Barnett (1984), the growth rate of simple sum M2 during the period of the 'monetarist experiment' averaged 9.3%, while the growth rate of Divisia M2 during the period averaged 4.5%. Similarly, the growth rate of simple sum M3 during the period averaged 10%, while the growth rate of Divisia M3 during the period averaged 4.8%. This period followed double digit growth rates of all simple sum and Divisia monetary aggregates. In short, believers in simple sum monetary aggregation, who had been the advocates of the 'monetarist experiment,' were put in the embarrassing position of witnessing an outcome (the subsequent recession) that was inconsistent with the intent

of the prescribed policy and with the behavior of the simple sum aggregates during the period. This unwelcome and unexpected outcome rendered vulnerable those economists who advocated a policy based upon the assumption of a stable simple sum demand for money function.

Friedman's very visible forecast error on 26 September 1983 followed closely on the heels of the end of the monetarist experiment in August 1982 and the recession that it produced. The road buckled and collapsed below the monetarists and those who believed in stable simple sum demand for money functions. Those two associated groups have never recovered.

But the recession that followed the monetarist experiment was no surprise to anyone who had followed the Divisia monetary aggregates, since those aggregates indicated that a severe deflationary shock had occurred. To those who were using data based upon valid index number and aggregation theory, rather than the obsolete simple sum monetary aggregates, the road remained smooth — no bumps, no breaks. Nothing unexpected had happened."

The above quotation shows that measurement matters! It is a topic that is dear to my heart, having labored in the measurement field for some 25 years. Thus it is perhaps no surprise that I am very enthusiastic about the basic Barnett research program: there is a substantial overlap in our research agendas. I too have worked with user costs, aggregation theory, flexible functional forms, tests for separability, and superlative index numbers. In Diewert (1974c), I derived a very simple user cost formula for non-interest bearing money, but I did not deal with interest bearing monetary assets and I did not deal adequately with the problem of converting nominal balances into real balances. The path breaking works of Fischer (1974), Samuelson and Sato (1984), and Feenstra (1986) on this tough problem were not yet available at that time. After this early attempt to integrate money into consumer theory, I never wrote another paper on this topic, although my former students — Donovan, Epstein, Feenstra, Hancock, and Kohli — have all made important contributions in this area of research. To further differentiate the research products of Barnett and Diewert, I note that, in addition to being the master of monetary user cost theory, Barnett has very substantial skills as an econometrician and macroeconomist — skills that I lack!

Barnett is very generous in this book about giving me credit for unifying the statistical (or test) approach to index number theory with the economic approach based on weakly separable aggregator functions. I would like to take this opportunity to point out that I was not the first to note the link of statistical agency index number formulae with functional forms for aggre-

gator (utility or production) functions. In Diewert (1976, p. 116), I referred
to Byushgens, Konüs, Frisch, Wald, Afriat, and Pollak as early pioneers in
making this connection. However, these early researchers did not have the
concept of a flexible functional form at their disposal, so they could not deter-
mine which exact index number formula might be "best" from the viewpoint
of the approximation properties of the corresponding aggregator function.
Barnett is well aware of this point, but I do not want others to be confused
about the nature of my contribution to the literature.

What is a possible future research agenda that might flow out of this
book? It seems to me that there are a number of basic problems that need
additional research.

- There is a need to examine more closely the problem of deriving the
  "right" price deflator for monetary balances. The "right" deflator de-
  pends on one's theory of how money enters the constraints of the con-
  sumer's and producer's constrained maximization problems. Moreover,
  the producer model of Fischer (1974) and the consumer model of Feen-
  stra (1986) are both highly aggregated, and there is a need to generalize
  their deflator results to higher dimensionality models.

- Chapters 10, 11, 12, and 21 all deal with the extension of riskless con-
  sumer and producer models to situations where the consumer or pro-
  ducer make decisions under uncertainty. This is very innovative work,
  which I applaud, but these chapters use an expected utility approach.
  Starting with Allais (1953), various researchers, including for example,
  Machina (1982), Mehra and Prescott (1985), and Chew and Epstein
  (1989), have noted various paradoxes associated with the use of the
  expected utility approach. Using the state contingent commodity ap-
  proach to choice under risk that was pioneered by Blackorby, Davidson,
  and Donaldson (1977), Diewert (1993) tried to show that the expected
  utility framework led to a relatively inflexible class of functional forms
  to model preferences over uncertain alternatives. Diewert showed that a
  much more flexible class of functional forms can be obtained by moving
  to nonexpected utility models that are counterparts to the choice over
  lotteries models of the type pioneered by Dekel (1986), Chew (1989),
  Epstein and Zin (1990, 1991), and Gul (1991). Epstein and Zin (1990),
  Epstein (1992), and Diewert (1993, 1995b) showed that these more flexi-
  ble models can explain many of the choice under uncertainty paradoxes,
  including the equity premium puzzle of Mehra and Prescott (1985).
  Thus there is a need for the Barnett research agenda to be extended
  to a nonexpected utility approach. A related problem in this uncer-
  tainty area that needs further research is the problem of determining

the firm's preference for risk utility function, given that the owners of the firm might have rather diverse risk preferences.

- There is a need to solve the problem raised in chapter 19 where the price of money on the supply side of the market is not equal to the corresponding price on the demand side. Actually, this problem is a special case of a wider problem, which may not have a satisfactory solution. The wider problem is this: if our macro model or applied general equilibrium model of the economy distinguishes more than one class of consumer or more than one class of producer (e.g., industries or firms are distinguished), then the index number commodity aggregates for the household and production sectors constructed by statistical agencies *will never match up*. In other words, the composition of aggregate "food" consumption by say, the elderly, will never be *precisely* equal to the composition of aggregate "food" consumption by say, single person working households. This means that the aggregate "food" equation for the economy will never add up precisely; i.e., the physical balancing of commodity supply and demand that input-output analysis attempts to do cannot be done *precisely*.

Before closing, I would like to discuss a few additional points that struck me as I read the manuscript.

- At times Barnett is somewhat critical of the monetary authorities for not adopting a user cost approach to the price of monetary services while he praises statistical agencies like the Bureau of Labor Statistics for producing consumer price indexes that are closer to the ideal indexes that economic theorists might prefer. However, while some statistical agencies may be willing to construct user costs for housing (or use a rental equivalence approach as the BLS does), most statistical agencies are just as opposed to constructing user costs for other consumer durables as the monetary authorities are opposed to constructing user costs for monetary components. Why is this? It is because statistical agencies feel that user costs are not *objective* or *reproducible*. In constructing a user cost, various choices have to be made about the appropriate depreciation rate, the appropriate interest rate, whether expected or ex post capital gains should be included, whether tax considerations should be included and so on. Since there is usually no single unambiguous choice for all of these components of a user cost, the agency is open to a charge of being nonobjective, and of course different statisticians will make different choices, and so the resulting user cost will not be reproducible. Of course, as an economic theorist, I am not as worried about this lack of objectivity problem as the statistician since

I believe that reasonably objective procedures could be worked out. In addition, it is worth observing that the greatest problems in measuring depreciation rates — the dependency upon usage rates, maintenance, and wear-and-tear — are not relevant to financial and monetary assets. However, it is important for theorists to recognize the concerns of the practitioners.

- This leads us to Barnett's interesting discussion on page 401 below on why government statistical agencies shy away from using econometrics in their procedures. Barnett points out that there are many possible econometric specifications (both of functional forms and of stochastic specifications) that could be used to address a particular problem, and there are many methods of statistical estimation (and of model selection). Thus statistical agencies will have difficulty in justifying an econometric model to persons untrained in econometrics. In other words, the use of econometrics these days is inherently *nonreproducible*: different econometricians will come up with different models (including functional forms, stochastic specification, model selection criterion, and method of estimation) and possibly, very different results. I believe that this *nonreproducibility* problem is even worse today than it was two decades ago due to the widespread use of the Generalized Method of Moments (GMM) method of estimation, which requires the researcher to choose a set of instrumental variables. As far as I can determine, there is no *objective* way for researchers to choose these instruments. In many cases, the choice of instruments will affect the results obtained, so GMM has just added to the *nonreproducibility* problems associated with the use of econometric techniques. Let me add here that I am not advocating throwing out econometrics; I am just pointing out that there is a problem out there (the lack of reproducibility problem) that the econometric literature has not adequately addressed.

- On page 566 and elsewhere, Barnett refers to the statistical or test approach to index number theory that was pioneered by Irving Fisher (1911, 1922). Readers who might be interested in more recent work on the test or axiomatic approach to index number theory could refer to Diewert (1992b) and Balk (1995).

To conclude, I note that Barnett and Serletis have nice introductions to each major section of the book, which will give the reader an overview of each section's content. For the reader who is not familiar with the Barnett approach, I recommend reading chapters 18, 23, 24, and 25 first. These chapters lay out much of the practical importance of the Barnett research philosophy and will serve to motivate further reading of the book.

Editors' Introduction to Volume

William A. Barnett and Apostolos Serletis

The fields of aggregation theory and index number theory are vast and have been growing. Certain landmark publications have been critical to the current state of the art. Of particular note are:

- Irving Fisher's (1922) famous book on index numbers,

- A. Konüs's (1924) derivation of the true cost of living index,

- Francois Divisia's (1925) derivation of the Divisia index,

- S. Malmquist's (1953) derivation of the Malmquist index,

- Dale Jorgenson's (1967) derivation of the user cost (rental price) of capital, and

- Erwin Diewert's (1976) unification of index number theory and aggregation theory.

In recent years, there has been a resurgence of interest in index number theory and aggregation theory, since the two previously divergent fields have been successfully unified. The underlying aggregator functions are the building blocks of economic theory. Those fundamental aggregator functions are weakly separable subfunctions of utility, cost, distance, and production functions. The derivation of index numbers based upon their ability to track those aggregator functions is now called the "economic theory of index numbers."

Modern economic index number theory was introduced into monetary and financial economics by William Barnett (1980a), who is a coeditor of this volume. He merged the economic theory of index numbers with monetary theory, and argued for a new microeconomic and aggregation-theoretic approach to monetary economics. The new approach involves use of the aggregator functions of neoclassical monetary theory and the construction of "superlative" non-parametric approximations to those functions. The result is aggregated data and models such that the aggregation theory that produced the data is consistent with the theory that produced the models within which the data is used. Without internally consistent nesting of aggregator functions within models, inferences become incoherent. In addition, the index number approximations to those aggregator functions must track those functions accurately.

Clearly these new developments in the field of monetary and financial economics would not have been possible without the earlier results in general index number theory, aggregation theory, and durable goods theory.

This book comprises a focussed and unified collection of the most important publications in monetary and financial aggregation by Barnett and his co-authors. The two coeditors of this volume have organized the papers into logical sections, with unifying introductions and overviews. The result is a systematic development of the state of the art in monetary and financial aggregation theory, covering:

- derivation of the user cost price of monetary services,

- exact aggregation of monetary assets on the demand and supply sides,

- general equilibrium of all economic agents' demands and supplies,

- dynamic solution of the exact system, and

- extension to monetary aggregation under risk.

Fisher's (1922, p. 29) book had already concluded over 75 years ago that: "The simple arithmetic [index] should not be used under any circumstances. The simple arithmetic average produces one of the very worst of index numbers, and if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose."

Clearly by that criterion, Fisher's book was successful in all areas other than monetary and financial aggregation. But disillusionment is now widespread with the simple sum monetary aggregates and their arithmetic average interest rate and opportunity cost aggregates. This book demonstrates that this disillusionment is well founded.

In some ways, the developments contained in this book were objectives of early research by Milton Friedman and his workshop participants at the University of Chicago. In fact, Friedman and Anna Schwartz (1970, pp. 151-152) criticized simple-sum monetary aggregation and discussed the possibility of generalizing the conventional monetary aggregates to index numbers: "This (summation) procedure is a very special case of the more general approach. In brief, the general approach consists of regarding each asset as a joint product having different degrees of 'moneyness,' and defining the quantity of money as the weighted sum of the aggregate value of all assets."

With monetary and financial assets yielding interest, we shall see in this book that the components of monetary aggregates are indeed joint products and that application of modern aggregation and index number theory requires aggregation over imperfect substitutes to be nonlinear. But a weighted sum

of component levels is a linear aggregator and implies perfect substitutability. When components are imperfect substitutes, aggregator functions are strictly concave, and index numbers must be able to track those aggregator functions.

Research by Friedman and his associates preceded developments in index number and aggregation theory that have been critical to the derivation of monetary and financial index number and aggregation theory. Nevertheless, it is interesting to see what Friedman and his students attempted to do in this area. See Friedman and Schwartz (1970, pp. 151-154) for a list of dissertations and related research produced by that group.

As illustrations of the economic theory of monetary aggregation, this book includes relevant empirical articles applying the theory of monetary aggregation to:

- problems in monetary policy,

- econometric modeling of money demand and supply,

- modeling and estimation of Euler equations,

- measurement of regulatory wedges in financial markets, and

- testing for stability of the economy's structure.

The included applied papers demonstrate that many of the empirical and policy puzzles in the area of monetary and financial economics disappear when simple sum monetary aggregates are replaced by index numbers that are coherent with the relevant theory

This book's results are heavily dependent upon the literature on microeconomic theory, index number theory, aggregation theory, and durables demand and supply, but not at all dependent upon any macroeconomic school of thought (e.g., monetarist, real business cycle, or Keynesian). Aggregation theory and index number theory are logically prior to any and all macroeconomic theories and are equally as relevant to all traditions in macroeconomics.

The following table provides an overview of the structure of the book. Prior to each section, there is an introduction highlighting some of the more important contributions of that section and briefly summarizing each chapter. The table identifies the organization of the book, including the clustering of chapters into sections and subsections, and locates the pages on which the section introductions can be found.

TABLE 1

Section and Subsection Structure, and Page Location of Editors' Introduction to Each Section

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Section and Subsection Structure   *Introduction Location*

PART 1: MONETARY INDEX NUMBER THEORY AND THE PRICE OF MONEY

Section 1.1: Editors' Overview of Part 1 . . . . .p. 3

Section 1.2: Derivation of the User Cost of Monetary Services

- Chapter 1

Section 1.3: The Price of Monetary Services and its Use in
Monetary Index Number Theory

- Chapters 2, 3, 4

PART 2: INDEX NUMBER THEORY

Section 2.1: Editors' Overview of Part 2 . . . . .p. 111

Section 2.2: General Index Number Theory

- Chapters 5, 6

Section 2.3: Monetary Index Number Theory

- Chapters 7, 8

PART 3: EXTENSIONS OF INDEX NUMBER THEORY

Section 3.1: Editors' Overview of Part 3 . . . . **p. 163**

Section 3.2: Extensions to Second Moments

- Chapter 9

Section 3.3: Extensions to Risk

> **Section 3.3.1:** Monetary Aggregation Theory under Risk
>
> - Chapters 10, 11
>
> **Section 3.3.2:** Monetary Index Number Theory under Risk
>
> - Chapters 12, 13

Section 3.4: Extension to Capitalized Money Stock Aggregation

- Chapters 14, 15