What Happens After A Technology Shock? A Bayesian Perspective

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Abstract

This paper investigates the effect of a positive technology shock on per capita hours worked within the class of Bayesian Vector Auto-Regressive [BVAR] models. Such a framework avoids the current debate regarding the specification issue of per capita hours [level versus first-difference stationary]. Six priors are considered and for each, we examine the impulse responses of per capita hours following a positive technology shock. The marginal posteriors of the VAR parameters are generated using the Markov Chain Monte Carlo (MCMC) Gibbs sampler. We find that the estimation of the VAR yields significantly different estimates under competing priors. Using the Francis and Ramey (2004, UCSD working paper) new measure for per capita hours, and after imposing the identifying restrictions (i.e., Blanchard-Quah and sign restrictions), the results show that per capita hours worked rise following a positive technology shock - if one [objectively] assumes a non-informative prior.

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1 Introduction

For more than two decades, relentless and rigid criticisms to the basic real business cycle (RBC) model persisted with the expectation of a final conclusion that will end its reign. RBC endured, withstood, adapted, evolved to accommodate and provided answers. From the literature, one can draw a taxonomy of criticisms to RBC models that register as: ideological, methodological, end-result and goodness-of-fit.[1]

Ideologically: Critics attack the built-in Walrasian market clearing foundation as a way of describing markets behavior, specifically that of the labor market.[2] Methodologically: The criticisms frequently pointed to the objectivity/subjectivity of the calibrating exercise (Economic Journal 1995, vol. 105). The end-result opponents of RBC address the models' inability to reproduce certain business cycles empirical regularities. The Goodness-of-fit criticisms highlight and strongly condemn the ad-hoc method(s) of judging the merits of each model. The absence of a metric and a formal class of test statistics, whereby one can measure and infer how good the model is as an approximation to the business-cycle data, is still an open debate. On the latter issue, Eichenbaum (1995, p.1609) reiterated that "We do not need high power econometrics to tell us that models are false. We know that. What we need are interesting diagnostic tools to help us understand the dimensions along which misspecified models do well and the dimensions along which they do poorly".

Seldom found in the literature a criticism that has the potential of being fatal to the premise

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[1] Also, see Stadler 1994, p. 1766.

[2] The objections focus and emphasize the notion of agents' decisions to generate an intertemporal substitution of labor supply. An example that has been used to explain the Great Depression, is that the agents anticipated WWII a decade prior to its start and decided to hold off their supply for labor until the increase for demand generated by WWII.

and promise of RBC as a vehicle to comprehend and advance business-cycle analysis, Gali (1999) articulated and provided the spark.

Central to the vitality of the basic RBC model is that a permanent positive technology shock generates an increase in hours worked. Using a basic identifying assumption, - that innovations to technology are the only shocks that have an effect on the long-run level of labor productivity - Gali reported that hours worked fall after a positive technology shock. Faced with the empirical regularity that hours worked are strongly procyclical, Gali concluded that other shocks must be in command, with a small role allotted to technology shocks in driving the business cycle.

Gali's findings did set off a serious debate regarding the viability of the RBC model (Basu, Kimball and Fernald 2004, Chang and Hong 2003, Chari, Kehoe and McGrattan 2004, Christiano, Eichenbaum and Vigfusson 2003, Christiano, Vigfusson and Eichenbaum 2004, Fisher 2003, Francis, Owyang, and Theodorou 2003, Francis and Ramey 2004 and 2001, Gali 2005, Gali and Rabanal 2004, Liu and Phaneuf 2004, Shea 1998, Sims 2005, Vigfusson 2002).

Viewed as a "... potential paradigm shifter" (Francis and Ramey 2001, p. 2), Gali's findings gained impetus and threatened the core and base of RBC. However, and using the same identifying assumptions and the level specification of per capita hours, Christiano, Eichenbaum and Vigfusson (2003) [CEV, henceforward], concluded and reported that hours worked rise after a positive technology shock. Gali's findings were upheld when using the first difference specification (growth rate of hours). Mainly and briefly, the CEV conclusion sorted the debate - and Gali's findings - into a specification issue rather than a paradigm issue. The specification issue focused on answering the following question. Is per capita hours level or difference stationary?

In this paper, we use the new measure of per capita hours, as proposed and developed by Francis and Ramey (2004) [FR, henceforward], to investigate and seek an answer to what happens to per capita hours after a technology shock from a Bayesian perspective. The reason for using the FR data is that their proposed [new] per-capita hours worked declined in the short run following a positive technology shock, *regardless* of the specification used in the analysis [level or first-difference stationary].

We investigate the effect of a positive technology shock on per capita hours worked within the class of Bayesian VAR. As derived and reported by Kadiyala and Karlsson (1997), a total of six priors are investigated and generate the posterior densities for the VAR parameters with the Markov Chain Monte Carlo (MCMC) Gibbs sampler. Finally, we impose the identifying restrictions (i.e., Blanchard-Quah and sign restrictions) and then generate the impulse responses for per capita hours following a positive technology shock, as well their respective standard errors.

The paper is organized as follows. Section 2 provides a discussion of alternative identification methodologies and explains the rational for using a VAR approach. Section 3 presents the Bayesian VAR and the priors used. Section 4 reports and interprets the results. Section 5 discusses the convergence diagnostics of the Markov Chain Monte Carlo Gibbs sampler. And finally, Section 6 concludes.

2 Identification Methodologies

This section briefly introduces the identification[3] problem and provide the argument for using a VAR approach. To understand the purpose and the development of identification, refer to Favero (2001) wherein the traditional 'Cowles Commission'[4], the LSE and the VAR approaches to identification are well described. The traditional approach broke down in the 1970s after the well-known critiques[5] of Lucas (1976) and Sims (1980). The LSE and VAR approaches reject the 'Cowles Commission' identifying restrictions as 'incredible' (Favero 2001, p. 164).

Developed by Denis Sargan and advocated by David Hendry to correct the failures of the traditional approach, the LSE approach is based on the theory of reduction (i.e., simplification process). It interprets the econometric model as a simplified representation of the unobserved data generating process (DGP). For the representation to be 'congruent' [valid]

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[3] A model is identifiable if all its possible structures are identifiable, i.e., each structure is associated with a different distribution. A simple dynamic multi-equation provides a statistical distribution for the variables involved. The problem is that many (unknown) models could have been the true data generating process of these variables of interest.

[4] The traditional approach to macroeconometric modelling - referred to as the 'Cowles Commission' approach - aims at the quantitative evaluation of the impact of changes in the exogenous variables in the system on the endogenous ones. Policy-controlled variables are considered as exogenous while final goals variables are portrayed as endogenous. The policy experiment consists of assessing the impact on final goal variables by modifying the exogenous ones. Identification in these models is achieved by imposing coefficient restrictions (e.g., zero value for the coefficient) on the structural equation to ensure that the rank condition is satisfied. Note that the identified structure is estimated without testing if the implied probability structure of the model properly describe the data.

[5] Lucas' critique emphasized that the coefficients of the structural equations that describe the impact of a policy, depend on the policy regime under which they were estimated. Lucas' critique pointed out that the traditional 'Cowles Commission' approach does not take explicit account for expectations, so that these models are unstable across different policy regimes. Sims focused on the ad hoc exogeneity of some variables in the traditional model to achieve identification. In a forward-looking world, agents' behavior depends on the solution of an intertemporal optimization problem and therefore, no variable is exogenous. By incorrectly assuming exogeneity, Sims argued that these models induce spurious effects.

the information lost in the specification process must be irrelevant to the problem at hand, e.g., omission of relevant variables. Therefore, one can test the model adequacy by analyzing the reduced form.

In the traditional case, the statistical baseline model describes structural relationships and the reduced form is then derived. Here, one starts by specifying and identifying a general reduced form model. The reduced form model should be sufficiently general to produce a congruent representation of the underlying unknown DGP. The LSE approach emphasizes the lack of validation of the reduced form that existed within the traditional approach. This lack of validation is interpreted as a lack of credibility in the structural model estimates. Finally, the system is validated by applying an extensive number of tests. A series of diagnostic tests are undertaken to verify the congruency of the baseline model. The absence of misspecification symptoms are viewed as success, e.g., in rejecting residuals non-normality and autocorrelation. The general criterion for assessment is that congruent models should feature true random residuals and any departure from this criterion is viewed as a sign of misspecification. Once the baseline is validated, one reduces the dimensionality of the reduced form by eliminating the equations for those variables for which the null hypothesis of exogeneity is not rejected. Another stage in the simplification process is to impose rank reduction restrictions based on cointegration vectors (for example, the Blanchard-Quah identification). The final product of this simplification process is a statistical model for the data and a structural model that is identified and estimated. Here, the long-run structure is discussed in relation to the Blanchard-Quah identification.

Sims' (1980) critique led to the development and estimation of VAR models. This VAR approach emphasizes a new role for empirical analysis, that is to provide stylized evidence to include in the theoretical model adopted for policy analysis (CEV 2003 and FR 2004).

Using a VAR approach, the estimates provide empirical evidence on the response of macroeconomic variables to impulses in order to discriminate between alternative theoretical models of the economy (Christiano, Eichenbaum and Evans 1996a, 1996b). Briefly, using theory-free restrictions and taking into account the potential endogeneity of the variables in the system, VAR models concentrate on shocks, and provide a theory-independent dynamics that serve as criteria for general equilibrium model evaluation (Favero 2001, p. 266). Corroboration of the theoretical general equilibrium model is achieved when the responses of variables to shocks in the theoretical model match the stylized facts derived from the empirical VAR.

Given that the technology debate was reduced to a specification issue [per capita hours: level or first-difference stationary] within a VAR approach, we estimate the Bayesian VAR models in levels. The Bayesian approach is very flexible: 1) it allows different lags for different equations, 2) there is neither a restriction on lags, nor specification restrictions, 3) the presence of trending variables does not cause any particular problems in this framework and inference is based on the likelihood principle, and 4) the approach only requires 'good' priors, and it is invariant to the size of the dominant root of the system.

An advantage - among many - of using a Bayesian VAR is the absence of the specification issue, in a sense that one can set the own-lag prior to equal one (i.e., non-stationary) and then let the data decide on the best specification. A strategy that we adhered to.

3 Bayesian VAR

Within a Bayesian VAR, we attempt to filter as much information from the data prior to the model specification, and then let the data decide on the specification in the system. Using the AIC and SIC as a guiding criteria for the lag length, we decided to use four lags in the VAR. Let's write the bi-variate VAR as,

$$y_t = \mu + \sum_{p=1}^{4} y_{t-p}.\Phi_p + \varepsilon_t \tag{1}$$

where,

$$\Phi_p = \begin{pmatrix} \phi_{11}^{(p)} & \phi_{12}^{(p)} \\ \\ \phi_{21}^{(p)} & \phi_{22}^{(p)} \end{pmatrix}_{2\times 2} \qquad\qquad p = 1, ..., 4 \tag{2}$$

$y_t \equiv [y_{1t}\ y_{2t}]_{1\times 2}$ where y_{1t} and y_{2t} denote labor productivity and per capita hours, respectively. $y_{t-p} \equiv [y_{1t-p}\ y_{2t-p}]_{1\times 2}$ and $\varepsilon_t \equiv [\varepsilon_{1t}\ \varepsilon_{2t}]_{1\times 2}$ where ε_{1t} and ε_{2t} refer to demand shocks and technology shocks, respectively. The constant vector is $\mu \equiv [\mu_1\ \mu_2]$. Let $z_t \equiv (1\ y_{1t-1}\ y_{2t-1} ... y_{1t-4}\ y_{2t-4})$ and $\Phi' \equiv (1\ \Phi_1'\ \Phi_2'\ \Phi_3'\ \Phi_4')$. y_t, z_t and Φ are of dimensions $1\times m$, $1\times(1+mp)$ and $(1+mp)\times m$, respectively, wherein $m = 2$ and $p = 4$. Rewrite the row stacked system as $y_{2\times 1} = (I \otimes Z)_{2\times 18}\phi_{18\times 1} + \varepsilon_{2\times 1}$ where $(I \otimes Z)$ is of dimension $m \times m(mp+1)$ and $\phi \equiv vec(\Phi)$ is of dimension $m(mp+1)\times 1$. Finally, assume that the ε_t are *i.i.d.* $N(0, \Sigma)$.

This latter formulation of the system is useful to derive the posterior distributions of the parameters. The following notation will be used (Kadiyala and Karlsson 1997). Let $\tilde{}$ (tilde) and $\bar{}$ (bar) denote the parameters of the prior and the posterior distribution, respectively. The OLS estimates of Φ and ϕ are denoted by $\widehat{\Phi}$ and $\widehat{\phi}$. The likelihood function is given[6]

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[6] With some modifications to Kadiyala and Karlsson (1997, p. 101), Bauwens et al. (1999, p. 266, see also theorem A.19, p. 307-308), and Zellner (1987, p. 22).

by,

$$
\begin{aligned}
L(\phi, \not\Sigma) &\propto \mid \not\Sigma \mid^{-T+m+1/2} \exp\left\{\frac{-tr\left[(Y-Z\Phi)^T \not\Sigma^{-1}(Y-Z\Phi)\right]}{2}\right\} && (3)\\
&\propto \mid \not\Sigma \mid^{-T+m+1/2} \exp\left\{\begin{array}{c} -\frac{1}{2}(\phi-\widehat{\phi})^T(\not\Sigma^{-1}\otimes Z^T Z)(\phi-\widehat{\phi}) \\ -\frac{1}{2}tr\left[\not\Sigma^{-1}(Y-Z\widehat{\Phi})^T(Y-Z\widehat{\Phi})\right] \end{array}\right\} && (4)\\
&\propto \mid \not\Sigma \mid^{-k/2} \left\{\begin{array}{c} \exp\left\{-\frac{1}{2}(\phi-\widehat{\phi})^T(\not\Sigma^{-1}\otimes Z^T Z)(\phi-\widehat{\phi})\right\} \\ \times \mid \not\Sigma \mid^{-(T-k)/2} \exp\left\{-\frac{1}{2}tr\left[\not\Sigma^{-1}(Y-Z\widehat{\Phi})^T(Y-Z\widehat{\Phi})\right]\right\} \end{array}\right\} && (5)\\
&\propto \left\{\begin{array}{c} MN\left(\phi|\widehat{\phi}, \not\Sigma\otimes\left(Z^T Z\right)^{-1}\right) \\ \times iW\left(\not\Sigma|(Y-Z\widehat{\Phi})^T(Y-Z\widehat{\Phi}), T-k-m-1\right) \end{array}\right\} && (6)
\end{aligned}
$$

Therefore, the likelihood function is proportional to the product of an inverse Wishart density for $\not\Sigma$ and a normal density[7] for ϕ conditional on $\not\Sigma$.

Basic to the discussion of the priors and their advantages, a note about the variance of ϕ is in order. The relative tightness of the prior variances for the parameters is set by the hyperparameters π_1 and π_2. Specifically, define the variance of the VAR coefficients as follows,

$$
var(\phi_{ij}^{(p)}) = \begin{cases} \dfrac{\pi_1}{p} & i = j \\ \dfrac{\pi_2 \sigma_i^2}{p \sigma_j^2} & i \neq j \end{cases} \tag{7}
$$

where p denotes the lag length. σ_i is set equal to the residual standard error of a p-lag univariate autoregression for the variable i. In short, the variance decreases as a function of the lag length.

In the Bayesian VAR literature, and to circumvent the "incredible identifying assumptions"

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[7] MN denotes a matricvariate normal distribution as defined in Bauwens et al. (1999, p. 301) and outlined in the Appendix.

made by the 'Cowles Commission' approach, the Minnesota prior[8] is widely used. It is informative on all the diagonal coefficients of the Φ_i matrices, and non-informative on the other parameters. The prior covariance matrix for all the parameters in Φ_i is diagonal and the residual variance-covariance matrix Σ, is taken to be *fixed and diagonal*. This amounts to assuming that each equation in the system is a-priori uncorrelated with any other equation.

Here, we explore a spectrum of priors that generalize the Minnesota prior. Candidly viewed as too restrictive, the Minnesota prior can be generalized by allowing for a non-diagonal Σ and/or unknown Σ. First, let's briefly discuss the Minnesota prior.

To adopt this prior, the procedure is implemented by placing a normal prior with mean zero on the coefficients of the lags [except the own-first lag], and allowing for a smaller standard deviation the longer is the lag, i.e., the importance of lagged variables decreases with the lag length. A mean of one is placed on the first own lag, and means of zero on all other coefficients. This centers the prior around a random walk process[9]. Formally, $\phi_{ii}^{(1)} = 1$ and all the other $\phi_{ij}^{(p)} = 0 \quad (i \neq j,\, p \neq 1)$, which is equivalent to assuming that for each variable in the VAR, $y_t = y_{t-1} + \varepsilon_t$. Specifically, $\phi_{ii}^{(1)} \sim N(1, \gamma^2)$, where γ is the degree of overall tightness and represents the confidence in the prior information. For the other coefficients, the standard deviation of the prior decays with respect to the lag p, i.e., $\phi_{ij}^{(p)} \sim N(0, S^2(i,j,p)) \quad for \quad i \neq j$. The standard deviation of the prior distribution for lag p of the variable j in equation i is defined as $S(i,j,p) = \{\gamma . g(p) . f(i,j)\} \; \frac{s_i}{s_j}$, and $f(i,i) = g(1) = 1.0$. s_i denotes the standard

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[8] The label 'Minnesota Prior' is used to identify the specific prior proposed by Litterman (Amisano et al. 1997, p. 9 and Bauwens et al. 1999, p. 269).

[9] The Minnesota prior assumes that the variables are $I(1)$ but not cointegrated. However, this prior and all extensions from it do not rule out cointegration. For a discussion of the Bayesian analysis of cointegrated VAR, see Bauwens and Lubrano (1996), Dorfman (1995, p. 49) and Koop (1992b, p. 105).

deviation of the residuals of a univariate autoregression on the dependent variable of equation i (OLS of y_{it} on a constant and own p lags). s_i/s_j represents a correction for different scales of the variables. In other words, it is an adjustment for the units in which the data are measured. In equation i, $f(i, j)$ is the tightness on variable j relative to variable i, while γ represents the overall tightness, and $f(i, j) = w \quad i \neq j$, where w is a weight parameter, and represents the relative tightness applied to all off-diagonal variables in the system. As w goes to zero, the system reduces to a set of univariate autoregressions. In other words, it forces coefficients on other than own lags toward zero. $g(p)$ is the tightness on lag p relative to lag 1. It captures how the standard deviation changes with increasing lags. The $g(p)$ lag decay function could be harmonic as $g(p) = p^{-d}$ or geometric as $g(p) = d^{p-1}$, where d is the lag decay parameter. A large (small) value for d reflects a tighter (looser) prior.

Many criteria for choosing the hyperparameters of the prior were proposed in the literature. Among others, one can use the log determinant of the covariance matrix of out-of-sample forecast errors or use a forecast performance statistic such as the Theil U statistic. Here, we selected the values of the hyperparameters which minimizes the sum of the MSE of the forecasts for 20 periods ahead.

The aforementioned Minnesota prior is too restrictive. Therefore, we investigated a spectrum of alternative priors. The following table summarizes the different priors and their posterior distributions, respectively.

[Insert Table 1 here]

We let the data speak in the sense that we investigate the possibility of a non-diagonal residual variance-covariance matrix, i.e., the structural shocks are [possibly and probably slightly] correlated. Such an empirical proposition could be theoretically integrated in a standard real business cycle whenever the technology shock induces an increase in consumption demand via an externality parameter that is technology-state dependent. A positive technology shock can induce an agent's consumption to keep up with the current technology and consequently, spread throughout the economy via the 'keeping up with the Joneses' channel, for example. Basically and, by investigating these hosts of priors, we are putting to rest the strong assumption of 'orthogonal' shocks (Sims 2005, p. 488).

The Minnesota prior forces posterior independence between equations and a fixed residual var-cov matrix. These restrictions are absent with the Diffuse and Normal-Wishart priors. The non-informative diffuse prior (using Jeffrey's invariance principle to reflect the principle of insufficient reason) reflects ignorance. The Normal-Wishart relaxes the assumption of a fixed and a diagonal residual variance-covariance matrix. However, the coefficients prior variances are treated symmetrically when specifying the prior (i.e., $\pi_1 = \pi_2$). Therefore, the Normal-Wishart prior is inappropriate to be combined with the Blanchard-Quah identification and sign restrictions. We only report its results for comparative purposes. The Normal-Diffuse prior is a combination of the Diffuse and the Minnesota priors. It combines the multivariate normal prior on the regression parameters from the Minnesota prior with the diffuse prior on the residual variance-covariance matrix [i.e., non-diagonal]. This combination results in prior independence between ϕ and $\not\Sigma$.

The Extended Natural Conjugate (ENC) circumvent the restrictions on the $\operatorname{var}\left(\phi_{ij}\right)$, as emphasized by the Normal-Wishart prior. Let Δ be a diagonal $(1+pm)m \times m$ matrix with

ϕ_i on the diagonal.

$$\Delta \equiv \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix}_{18\times 2} \tag{8}$$

Rewrite the system as $Y = [[1\ 1] \otimes Z]_{1\times 18}\, \Delta + \varepsilon_{1\times 2}$. This prior generalizes the Minnesota prior to an unknown and a non-diagonal Σ. If $E(\Sigma|\Delta = \widetilde{\Delta})$ is matched to the fixed residual of the var-cov matrix of the Minnesota prior, then the ENC is said to be conditional. If $E(\Sigma) = \widetilde{\Sigma}$, then the ENC is said to be unconditional.

Numerical algorithms are needed to evaluate few of these posteriors, for few of the set have no closed form solutions. We use the Gibbs sampler and cycle through the conditional distributions (Bauwens, Luc, Michel Lubrano, and Jean-François Richard 1999, p. 83, Casella and Geroge 1992, Chen, Shao, and Ibrahim, 2000, p. 20). For example, the Gibbs sampler for the Normal-Diffuse is as follows, $\phi|\Sigma, y \sim N(\overline{\phi}(\Sigma^{-1} + \Sigma^{-1} \otimes Z'Z)^{-1})$, and

$$\Sigma^{-1}|\phi, y \sim W\left(\left[\left(Y - Z\widehat{\Phi}\right)\left(Y - Z\widehat{\Phi}\right) + \left(\Phi - \widehat{\Phi}\right)' Z'Z\left(\Phi - \widehat{\Phi}\right)\right]^{-1}, T\right).$$

We generated 20,000 iterations with a burn-in of 200 observations. The sampler converged in distribution and the convergence diagnostics are reported in Section 5. Once generated, the marginal posteriors for the parameters ϕ_i are then used to produce impulse responses of the VAR system combined with the Blanchard-Quah identification and sign restrictions[10] . Following Chari, Kehoe and McGrattan (2004) and Taylor (2004), we set the same two overidentifying sign restrictions that are consistent with an underlying economic model. We drew 1000 impulse responses, and then used the whole sample to compute the impulse

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[10] We modified the Kadiyala and Karlsson FORTRAN code and use it to generate the marginal posteriors. Following which, we imposed the identifying restrictions and generated the impulse responses using MATLAB.

numerical standards errors. The posterior density function of the impulse response function is defined as follows (Koop 1992a, p. 398),

$$p(I_i(y_k,\varepsilon_{y_k})) = \left\{ \begin{matrix} p(y_{k,T+i}|y, \varepsilon_{y_k,T+1}=1, \varepsilon_{y_k,T+2}=0, ..., \varepsilon_{y_k,T+j}=0) \\ -p(y_{k,T+i}|y, \varepsilon_{y_k,T+j}=0, \varepsilon_{y_k,T+j}=0) \end{matrix} \right\} \qquad (j=1,...,i) \quad (9)$$

where k denotes the variable of interest. $I_i(y_k,\varepsilon_{y_k})$ is the response, i periods later, of the variable y_k to a shock ε_{y_k}. The shock occurs at period $T+1$. Therefore, the probability density function is conditioned on observed data and the shock. The impulse responses were computed using the companion matrix approach as outlined in L*ü*tkepohl (1991).

4 Data and Results

We estimated the VAR and used the same identification restrictions as outlined in CEV (2003) and Gali (1999). The bivariate y_t includes y_{1t} and y_{2t} wherein they refer to the log of productivity and the log of per capita hours, respectively. The VAR lag is set equal to 4. The identifying assumption on the structural VAR residuals are: 1) the demand shocks (ε_{2t}) have no permanent effect on the level of productivity while technology shocks (ε_{1t}) do, and 2) the demand and technology shocks are orthogonal. The latter assumption is relaxed with the Diffuse, Normal-Diffuse, Normal-Wishart and [both] the ENC priors.

The data on productivity and per capita hours is from FR[11] (2004). We also experimented with the data set from CEV[12] (2003) to compare the impulse results across the two data sets.

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[11] FR (2004) constructed a new - and adjusted for institutional and demographic changes on the population available for work - per capita hours that eliminates all low-frequency movements.

[12] The CEV (2003) data is from the DRI Economic Database. The mnemonic for labor productivity is LBOUT. Per capita hours is LBMN divided by the civilian population over the age of 16 (P16).

Figures 1 and 2 display the FR (2004) data. Visual inspection reveals the presence of a trend in labor productivity and a quasi-segemented trend in per capita hours.

Figure 3 illustrates the sum of the mean square error forecast as function of the own lag π_1 and other lag π_2 precision priors for the Minnesota prior. The observations from 1989:3 to 1994:2 were used to compute the forecasts and their respective MSE. For each one of the priors, the sum of the MSE of the eight periods ahead forecasts were evaluated and the prior hyperparameters were chosen to insure a minimum. The following Table reports the values at which the MSE was minimized.

[Insert Table 2 here]

Figure 4 illustrates all impulse responses of hours under different prior specifications. The Minnesota prior coincides with the Normal-Diffuse. The Normal-Wishart is inappropriate when combined with the Blanchard-Quah identification. When the residual variance-covariance matrix is allowed be non-informative and non-diagonal (as articulated by the Diffuse prior), per capita hours worked rise following a positive technology shock.

The estimation of a the VAR yields significantly different estimates under competing priors (Ni and Sun 2005). The identifying restriction of orthogonal shock precludes the remote possibility of correlated shocks [even at a very small magnitude]. Used in the literature, this assumption is too restrictive (Hamilton 1994, pp. 335-336). Conditional on the Diffuse prior and the data set, whenever the shocks are allowed to display a relatively small correlation of the magnitude of 0.03, the impulse responses show that per capita hours worked rise following a positive technology shock.

To compare the effects of the level versus difference specifications, Figure 5 illustrates the impulse responses of hours using the Minnesota prior for both data sets: CEV (2003) and FR (2004) and using two different own-lag priors to reflect both specifications. The own-lag prior ϕ_{11} was set equal to 1 and equal to 0.5. The former is to be interpreted as the first-difference specification, while the latter reflects the level specification. This specification issue is absent in the Bayesian VAR, provided that the precision is large enough to let the own-lag parameter free to move. For example, if $\gamma = 0.5$ and the own-lag is set equal to 0.8, then the posterior [mean] estimate is free between $[0.55, 1.05]$ with a confidence of 95%. It is interesting to observe the difference in magnitude that is implied by the two data sets DRI Economic Database and the FR (2004).

Figure 5 illustrates that the difference in the impulse results is not specification related. The difference stem from the data set used. Regardless of the specification of hours (i.e., forcing the per capita hours own-lag prior to display stationarity $\phi_{11} = 0.5$ or non-stationarity $\phi_{11} = 1$), and using the Minnesota prior, the FR data consistently show that per capita hours decline following a technology shock.

Under the spectrum of priors, Figures 6 to 11 illustrate the impulse responses of hours following a positive technology shock as well as their numerical standard errors. All are generated using the FR (2004) data.

Figures 12 and 13, illustrate the marginal posterior densities of the VAR coefficients under all priors. The left hand side column refers to the coefficients from the productivity equation (PRD) and the right hand side column refers to the coefficients from the per capita hours equation (Hours). The non-stationarity of hours is evident across all priors. The mean of the posterior density for the coefficient of Hours_{t-1} in the Hours equation is larger than one. The

Diffuse prior displays evidence of a stationary productivity (Figure 12, first cell on top left hand side corner).

To compare the models, we computed the posterior odds ratio across the following priors: Diffuse, ENC, ENC conditional, Normal-Diffuse and Normal-Wishart. The Minnesota prior imposes the diagonal restriction on the residual variance-covariance matrix, and consequently increases its likelihood value. Therefore, we decided to compute the posterior odds for all priors - with the exception of the Minnesota prior. Assuming an equal probability for each prior model [one-fifth], the diffuse prior shows the highest posterior odds at 0.207, wherein hours worked rise after a technology shock.

5 Convergence and Diagnostics

We have used the Gibbs sampler to generate the marginal posterior of the VAR parameters. A simulator among the class of Markov Chain Monte Carlo (MCMC) algorithms, the Gibbs sampler draws a parameter θ^s that is conditioned on the previous draw (θ^{s-1}) to produce a sequence that displays the Markov chain properties. Associated with the sampler, there exists a host of MCMC diagnostics tools to test for convergence of the sequence. The fact of increasing the number of draws does not guarantee convergence - in our case, the number of draws was set to 20,000. For example, if one starts the sampler initial point far away from the region of the parameter space where the most posterior probabilities lies, then numerous draws are in order and a larger burn-in sample is required to be able to reach convergence and then building up a sample from the posterior. Fortunately, there are convergence diagnostics that can be used to assess the merit and the accuracy of the convergence. Here, we will use the autocorrelation estimates and the Geweke (1992) diagnostics (J. LeSage 1999, pp. 163-167).

Tables 3 to 8 report the convergence diagnostics for the VAR parameters under different priors. The autocorrelation estimates provide evidence of in-sample independence, if any. The autocorrelation ρ estimate is computed at lags 1, 5, 10 and 50. For all priors, the autocorrelation estimates are approximately of the order 0.5 for the first lag and of marginal magnitude for the subsequent lags. Based on the autocorrelation estimates, the draws represent an independent and identically distributed (iid) process. Also, and using time series spectral analysis, the variance of the parameter $\widehat{\phi}$, is given by, $var(\widehat{\phi}) = S(0)/k$, where $S(0)$ refers to the spectral density[13] evaluated at the frequency $\omega = 0$. For each parameter, the numerical standard error (NSE) is small and the relative numerical efficiency (RNE) is close to one. The RNE is indicative of the i.i.d. nature of the sample. It provides an indication of the number of draws that would be required to produce the same numerical accuracy if the draws have been made from an iid sample drawn directly from the posterior. I also computed the Geweke Chi-squared test[14] for each parameter chain and concluded that the means are equal, i.e., convergence in distribution was reached.

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[13] An alternative tapering of the spectral window was explored - wherein the numerical standard errors (NSE) and the relative numerical efficiency (RNE) are based on 4%, 8% and 15% tapering [truncation] of the periodgram window - and similar results were concluded.

[14] These are not reported for space consideration, and available upon request.

6 Conclusions

We did start with the view of giving per capita hours every possible opportunity to display a decline following a technology shock. To do so, we forced the own-lag prior to equal one and used the FR (2004) data, whereby it was shown that per capita hours worked decline following a technology shock, regardless of the stationarity specification [level or first-difference stationary]. Then, we imposed the strict identifying restrictions (Blanchard-Quah and sign restrictions). Such a strategy generated a result wherein per capita hours rise after a technology shock - if one [objectively] assumes a non-informative prior.

Allowing for a positive [and small] covariance between the demand and supply shocks, reveals that hours worked rise following a positive technology shock. This conclusion shifts the debate away from the specification issue regarding the stationarity of per capita hours towards the possibility of formalizing a propagation mechanism in real business cycle models, whereby technology shocks induce demand shocks, or vice versa. Here, what is concluded is that a correlation of 0.03 between the technology and demand shocks, when combined with a non-informative prior, induces a rise of per capita hours following a technology shock.

References

[1] Amisano, Gianni. and Carlo Giannini (1997) *Topics in Structural VAR Econometrics* Second Edition, Springer-Verlag, New York.

[2] Basu, Susanto, John G. Fernald, and Miles S. Kimball (2004) "Are Technology Improvements Contractionary?" International Finance Discussion Paper no 625, September 1998, Federal Reserve Board of Governors; and National Bureau of Economic Research (Cambridge, MA) Working Paper No. 10592, June 2004.

[3] Bauwens, Luc. and Michel Lubrano. (1996) "Identification Restrictions and Posterior Densities in Cointegrated Gaussian VAR systems." In: Thomas B. Fomby and R. Carter Hill (Ed.) *Advances in Econometrics*, vol. 11, Part B, pp. 3-28. JAI Press; London.

[4] Bauwens, Luc, Michel Lubrano, and Jean-François Richard, (1999) *Bayesian Inference in Dynamic Econometric Models,* Advanced Texts in Econometrics, (Ed.) Granger, C.W.J. and Mizon, G.E. New York: Oxford University Press.

[5] Casella, Geroge and Edward I. geroge (1992) "Explaining the Gibbs Sampler" The American Statistician, 46, 167-174.

[6] Chang, Yongsung and Jay H. Hong (2003) "On the employment effect of technology : evidence from U.S. manufacturing for 1958-1996" Working Paper 03-06, Federal Reserve Bank of Richmond.

[7] Chari, V. V. Patrick J. Kehoe, and Ellen R. McGrattan (2004) "A Critique of Structural VARs Using Real Business Cycle Theory." Federal Reserve Bank of Minneapolis, Research Department, Working Paper 631.

[8] Chen, Ming-Hui, Qi-Man Shao, and Joseph G. Ibrahim, (2000) *Monte Carlo Methods in Bayesian Computation*, New York: Springer Series in Statistics, Springer.

[9] Christiano, Lawrence J., Martin S. Eichenbaum, and Charles L. Evans (1996a) "The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds." *Review of Economic and Statistics*, vol. 78, pp. 16-34.

[10] Christiano, Lawrence J., Martin S. Eichenbaum, and Charles L. Evans (1996b) "Monetary Policy Shocks and their Consequences: Theory and Evidence." *Mimeo*, Dept. of Economics, Northwestern University.

[11] Christiano, Lawrence J., Martin S. Eichenbaum, and Robert Vigfusson (2003) "What Happens After A Technology Shock?" Federal Reserve Board, International Finance Discussion Papers, Paper 2003-768.

[12] Christiano, Lawrence J., Robert Vigfusson and Martin S. Eichenbaum (2004) "The Response of Hours to a Technology Shock: Evidence Based on Direct Measures of Technology" Journal of the European Economic Association 2 (2), 381-395.

[13] Dorfman, Jeffrey H. (1995) "A Numerical Bayesian Test for Cointegration of AR Processes." *Journal of Econometrics* 66, 289-324.

[14] Eichenbaum, Martin. (1995) "Some Comments on the Role of Econometrics in Economic Theory." *The Economic Journal*, vol. 105, pp. 1609-1621.

[15] Favero, A. Carlo. (2001) *Applied Macroeconometrics.* Oxford University Press.

[16] Fisher, Jonas D. M. (2003) "Technology Shocks Matter" Federal Reserve Bank of Chicago, WP 2002-14, revised version from 2002.

[17] Francis, Neville R. Michael T. Owyang, and Athena T. Theodorou (2003) "The Use of Long-Run Restrictions for the Identification of Technology Shocks" *The Federal Reserve Bank of St. Louis Review* 85 (6), 53-66.

[18] Francis, Neville R. and Valerie A. Ramey (2004) "A New Measure of Hours Per Capita with Implications for the Technology-Hours Debate" *Working Paper*, Department of Economics, University of California, San Diego.

[19] Francis, Neville R. and Valerie A. Ramey (2001) "Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited." *Working Paper*, Department of Economics, University of California, San Diego.

[20] Gali, Jordi (2005) "Trends in Hours, Balanced Growth, and the Role of Technology in the Business Cycle" *The Federal Reserve Bank of St. Louis Review* 87 (4), 459-486.

[21] Gali, Jordi (1999) "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations." American Economic Review, 249-271.

[22] Gali, Jordi and Pau Rabanal (2004) "Technology Shocks and Aggregate Fluctuations: How Well Does the RBS Model Fit Postwar U.S. Data?" National Bureau of Economic Research, NBER Working Paper W10636.

[23] Geweke, John (1992) "Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments." in J.O. Berger, J.M. Bernardo, A.P. Dawid, and A.F.M. Smith (eds.), *Bayesian Statistics* 4, 169-194. Oxford: Oxford University Press.

[24] Hamilton, James D. (1994) *Time Series Analysis.* Princeton: Princeton University Press.

[25] Kadiyala, K.R., and S. Karlsson (1997), "Numerical Methods for Estimation and Inference in Bayesian VAR-models." Journal of Applied Econometrics, 12, 99-132.

[26] Koop, Garry (1992a) "Aggregate Shocks and Macroeconomic Fluctuations: A Bayesian Approach." Journal of Applied Econometrics, 7, 395-411.

[27] Koop, Gary. (1992b) "Cointegration Tests in Present Value Relationships: A Bayesian Look at the Bivariate Properties of the Stock Prices and Dividends." *Journal of Econometrics*, vol. 49, pp. 105-139.

[28] LeSage, James P. (1999) *Applied Econometrics using Matlab.* Department of Economics, University of Toledo.

[29] Liu, Zheng and Louis Phaneuf (2004) "What Explains the Effects of Technology Shocks on Labor Market Dynamics?" Working Paper 2004-10, Department of Economics, Emory University.

[30] Lucas, Robert E. (1976) "Econometric Policy Evaluation: a Critique." *In the Phillips Curve and Labor Markets* (Ed. Karl Brunner and Alan H. Meltzer) Amsterdam North-Holland, 19-46.

[31] Lütkepohl, Helmut (1991) *Introduction to Multiple Time Series Analysis.* Springer Verlag.

[32] Ni, Shawn and Dongchu Sun (2005) "Bayesian Estimates for Vector-Autoregressive Models" *Journal of Business & Economic Statistics* 23 (1), 105-117.

[33] Shea, John (1998) "What Do Technology Shocks Do?" National Bureau of Economic Research, NBER Working Paper 6632.

[34] Sims, Christopher A. (2005) Commentary on "Trends in Hours, Balanced Growth, and the Role of Technology in the Business Cycle" *The Federal Reserve Bank of St. Louis Review* 87 (4), 487-492.

[35] Sims, Christopher A. (1980) "Macroeconomics and Reality." *Econometrica*, 48 (1), 1-48.

[36] Stadler, George W. (1994) "Real Business Cycles." *Journal of Economic Literature*, vol. XXXII, 1750-1783.

[37] Taylor, Mark P. (2004) "Estimating structural macroeconomic shocks through long-run recursive restrictions on vector autoregressive models: the problem of identification." International Journal of Finance & Economics 9 (3), 229-244.

[38] Vigfusson, Robert (2002) "Why Does Employment Fall After A Positive Technology Shock." Board of Governors, Federal Reserve Bank, mimeo.

[39] Zellner, Arnold (1987) *An Introduction to Bayesian Inference in Econometrics,* Original Edition 1971, New York: John Wiley & Sons Inc.

7 Appendix (NOT FOR PUBLICATION)

7.1 The Matricvariate Normal Distribution

See Zellner (1971) and Bauwens et al. (1999) for complete details. Let X and $vecX$ denote a $p \times q$ random matrix and its pq-dimensional column expansion respectively. X is said to have a matricvariate normal distribution with parameters $M \in \Re^{p\times q}$, $P \in C_p$, i.e., $X \sim MN_{p\times q}(vecM,\ Q \otimes P)$ if and only if $vecX \sim N_{pq}\left\{vecM,\ Q \otimes P\right\}$. As derived in Bauwens et al. (1999, p. 301), its density function is given by,

$$f_{MN}^{p\times q}(X|M, Q\otimes P) = \left\{ \begin{array}{c} C_{MN}^{-1}(P,Q;p,q) \\ \times \exp\left\{-\frac{1}{2}\left[vec(X-M)^T(Q\otimes P)^{-1}vec(X-M)\right]\right\} \end{array} \right\} \tag{10}$$

$$= \left\{ \begin{array}{c} C_{MN}^{-1}(P,Q;p,q) \\ \times \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{q}\sum_{j=1}^{q} q^{ij}(x_i-m_i)^T P^{-1}(x_j-m_j)\right]\right\} \end{array} \right\} \tag{11}$$

$$= \left\{ \begin{array}{c} C_{MN}^{-1}(P,Q;p,q) \\ \times \exp\left\{-\frac{1}{2}tr\left[Q^{-1}(X-M)^T P^{-1}(X-M)\right]\right\} \end{array} \right\} \tag{12}$$

$$C_{MN}(P,Q;p,q) = \left\{(2\pi)^{pq} \quad | P |^{q} \quad | Q |^{p}\right\}^{1/2} \tag{13}$$

All the properties of the multivariate normal distribution apply to the matricvariate normal distribution through the vec operator. See Bauwens et al. (1999, pp. 301-302) for details.

7.2 The Inverted Wishart Distribution

A random matrix $\Sigma \in C_q$ has an inverted Wishart distribution with parameters $S \in C_q$ and $v > q-1$, i.e., $\Sigma \sim IW_q(S, v)$, if its density function is given by,

$$f_{IW}^{q}(\Sigma|S,v) = C_{IW}^{-1}(S,v;q) \mid \Sigma \mid^{-\frac{(v+q+1)}{2}} \exp\left[-\frac{1}{2}tr\left(\Sigma^{-1}S\right)\right] \tag{14}$$

$$C_{IW}^{-1}(S,v;q) = 2^{-\frac{vq}{2}}\pi^{\frac{q(q-1)}{4}}\prod_{i=1}^{p}\Gamma\left(\frac{v+1-i}{2}\right) \mid S \mid^{-\frac{1}{2}v} \tag{15}$$

The recursion on the dimension q is the key device for deriving many of this density properties.

8 Tables

Table 1: Priors and Posteriors

	Prior	Posterior
Minnesota	$\phi_i \sim N(\tilde{\phi}_i, \tilde{\not\Sigma}_i)$	$\phi_i \vert y \sim N(\bar{\phi}_i, \bar{\not\Sigma}_i)$
	$\not\Sigma$ fixed and diagonal	with $\bar{\not\Sigma}_i = (\tilde{\not\Sigma}_i^{-1} + \sigma_{ii}^{-1} Z^T Z)^{-1}$
Diffuse (Jeffrey's)	$p(\phi, \not\Sigma) \propto \vert \not\Sigma \vert^{-(m+1)/2}$	$\Phi \vert y \sim MT(Z^T Z, (Y - Z\hat{\Phi})^T)$
		$\times(Y - Z\hat{\Phi}), \hat{\Phi}, T - k)$
Normal-Wishart	$\phi \vert \not\Sigma \sim N(\tilde{\phi}, \not\Sigma \otimes \tilde{\Omega}),$	$\Phi \vert y \sim MT(\bar{\Omega}^{-1}, \bar{\not\Sigma}, \bar{\Phi}, T + \alpha)$
	$\not\Sigma \sim iW(\tilde{\not\Sigma}, \alpha)$	
Normal-Diffuse	$\phi \sim N(\widetilde{\phi}, \widetilde{\Sigma}),$	$p(\phi \vert y) \propto \exp\{-(\phi - \widetilde{\phi})' \widetilde{\Sigma}^{-1} \times (\phi - \widetilde{\phi})/2\}$
	$p(\not\Sigma) \propto \vert \not\Sigma \vert^{-(m+1)/2}$	$\times \vert (Y - Z\hat{\Phi})'(Y - Z\hat{\Phi}) +$
		$(\Phi - \hat{\Phi})' Z' Z (\Phi - \hat{\Phi}) \vert^{-T/2}$
Extended Natural	$p(\Delta) \propto \vert \widetilde{\Sigma} + (\Delta - \widetilde{\Delta})' \vert$	$p(\Delta \vert y) \propto \vert \widetilde{\Sigma} + (\Delta - \widetilde{\Delta})' \vert$
Conjugate	$\times \widetilde{M}(\Delta - \widetilde{\Delta}) \vert^{-\alpha/2}$	$\times \widetilde{M}(\Delta - \widetilde{\Delta}) \vert^{-(T+\alpha)/2}$
	$\Sigma \vert \Delta \sim iW(\widetilde{\Sigma} + (\Delta - \widetilde{\Delta})'$	$\Sigma \vert \Delta, y \sim iW((\Delta - \widetilde{\Delta})' M (\Delta - \widetilde{\Delta}), T - \alpha)$
	$\times \widetilde{M}(\Delta - \widetilde{\Delta}), \alpha)$	
	$\not\Sigma$ unknown and non-diagonal	

Source: Kadiyala and Karlsson (1997, p. 103).

Table 2: Prior Hyperparameters

	π_1	π_2
Minnesota	0.029	0.00035
Normal-Wishart	0.008	0.00800
Normal-Diffuse	0.033	0.00044
ENC, unconditional mean of $\not\Sigma$	0.029	0.00041
ENC, conditional mean of $\not\Sigma$	0.076	0.00093

The following variable abbreviations are used in Tables 3 to 8.

- pc: the constant coefficient estimate of the PRODUCTIVITY equation.
- ppi: the coefficient estimate of PRODUCTIVITY on its i^{th} lag in the the PRODUCTIVITY equation.
- phi: the coefficient estimate of PRODUCTIVITY on HOURS i^{th} lag in the PRODUCTIVITY equation.
- hc: the constant coefficient estimate of the HOURS equation.
- hpi: the coefficient estimate of HOURS on PRODUCTIVITY i^{th} lag in the HOURS equation.
- hhi: the coefficient estimate of HOURS on its i^{th} lag in the HOURS equation.

```
TABLE 3: DIFFUSE PRIOR - MCMC CONVERGENCE diagnostics
Autocorrelations within each parameter chain
Variable          Lag 1          Lag 5         Lag 10         Lag 50
pc               -0.507          0.006          0.003          0.002
pp1              -0.502          0.001         -0.004         -0.013
pp2              -0.498         -0.001         -0.003         -0.004
pp3              -0.504         -0.001         -0.004          0.014
pp4              -0.499          0.004         -0.014         -0.008
ph1              -0.494          0.003         -0.006         -0.003
ph2              -0.494         -0.003         -0.007          0.008
ph3              -0.494          0.003         -0.005          0.008
ph4              -0.498          0.010         -0.000         -0.012
hc               -0.498          0.007         -0.006         -0.010
hp1              -0.510          0.014         -0.009         -0.005
hp2              -0.505          0.007         -0.005          0.005
hp3              -0.493         -0.009          0.018         -0.010
hp4              -0.502         -0.007          0.020         -0.009
hh1              -0.500          0.005         -0.003          0.000
hh2              -0.498          0.001         -0.010         -0.010
hh3              -0.498         -0.009         -0.002         -0.003
hh4              -0.499         -0.007          0.008         -0.001

Geweke Diagnostics for each parameter chain
Variable           Mean        std dev        NSE iid        RNE iid
pc            -0.058166       0.155658       0.001101       1.000000
pp1            0.882974       0.069140       0.000489       1.000000
pp2            0.193971       0.092293       0.000653       1.000000
pp3           -0.154013       0.089862       0.000635       1.000000
pp4            0.073696       0.068163       0.000482       1.000000
ph1            0.092796       0.077995       0.000552       1.000000
ph2           -0.298128       0.139319       0.000985       1.000000
ph3            0.041932       0.140104       0.000991       1.000000
ph4            0.152270       0.078314       0.000554       1.000000
hc            -0.502770       0.135604       0.000959       1.000000
hp1            0.166914       0.061056       0.000432       1.000000
hp2            0.018753       0.082010       0.000580       1.000000
hp3           -0.102302       0.080180       0.000567       1.000000
hp4           -0.076325       0.061096       0.000432       1.000000
hh1            1.465461       0.069496       0.000491       1.000000
hh2           -0.521070       0.123700       0.000875       1.000000
hh3            0.060511       0.125221       0.000885       1.000000
hh4           -0.070965       0.069878       0.000494       1.000000
```

```
TABLE 4: MINNESOTA PRIOR - MCMC CONVERGENCE diagnostics
Autocorrelations within each parameter chain
Variable        Lag 1        Lag 5       Lag 10       Lag 50
pc             -0.496        0.002       -0.000       -0.001
pp1            -0.505        0.002        0.005       -0.004
pp2            -0.499       -0.003        0.001       -0.009
pp3            -0.496       -0.000        0.006        0.010
pp4            -0.499       -0.008        0.013       -0.003
ph1            -0.502       -0.001        0.006        0.014
ph2            -0.499        0.006       -0.013       -0.021
ph3            -0.498        0.001       -0.000        0.009
ph4            -0.503        0.011       -0.009       -0.004
hc             -0.507        0.007       -0.003        0.007
hp1            -0.498       -0.002       -0.001       -0.008
hp2            -0.498        0.006       -0.000       -0.014
hp3            -0.504        0.001       -0.013       -0.011
hp4            -0.494       -0.000       -0.020       -0.002
hh1            -0.496        0.008       -0.004        0.009
hh2            -0.499        0.000       -0.001        0.024
hh3            -0.508        0.007        0.005        0.003
hh4            -0.515        0.018        0.000       -0.007

Geweke Diagnostics for each parameter chain
Variable         Mean      std dev      NSE iid      RNE iid
pc          -0.144420     0.127492     0.000902     1.000000
pp1          0.957346     0.058027     0.000410     1.000000
pp2          0.089026     0.068993     0.000488     1.000000
pp3         -0.080347     0.062336     0.000441     1.000000
pp4          0.032068     0.048877     0.000346     1.000000
ph1         -0.022108     0.016405     0.000116     1.000000
ph2         -0.008759     0.013492     0.000095     1.000000
ph3          0.001801     0.011338     0.000080     1.000000
ph4          0.006780     0.009707     0.000069     1.000000
hc          -0.638971     0.138650     0.000980     1.000000
hp1          0.013296     0.011672     0.000083     1.000000
hp2          0.001461     0.010202     0.000072     1.000000
hp3         -0.003275     0.008765     0.000062     1.000000
hp4         -0.004041     0.007773     0.000055     1.000000
hh1          1.339618     0.052839     0.000374     1.000000
hh2         -0.285554     0.076151     0.000538     1.000000
hh3         -0.110210     0.066637     0.000471     1.000000
hh4         -0.029138     0.044515     0.000315     1.000000
```

```
TABLE 5: NORMAL-DIFFUSE - MCMC CONVERGENCE diagnostics
Autocorrelations within each parameter chain
Variable        Lag 1       Lag 5      Lag 10      Lag 50
pc             -0.499      -0.002       0.008       0.000
pp1            -0.470      -0.001       0.008       0.006
pp2            -0.499      -0.000       0.018      -0.003
pp3            -0.501       0.001      -0.008       0.009
pp4            -0.499       0.007       0.003       0.006
ph1            -0.490      -0.009      -0.004       0.008
ph2            -0.504       0.002       0.009       0.018
ph3            -0.495      -0.003      -0.009       0.013
ph4            -0.498       0.003       0.014      -0.002
hc             -0.488      -0.002      -0.002       0.019
hp1            -0.492       0.004      -0.008      -0.003
hp2            -0.503      -0.002      -0.001      -0.001
hp3            -0.500       0.002      -0.001      -0.011
hp4            -0.498      -0.006       0.013      -0.008
hh1            -0.483       0.001       0.001      -0.032
hh2            -0.482       0.012      -0.009      -0.010
hh3            -0.507       0.006       0.024       0.017
hh4            -0.491      -0.003       0.009       0.007

Geweke Diagnostics for each parameter chain
Variable         Mean     std dev     NSE iid     RNE iid
pc          -0.156024    0.131037    0.000927    1.000000
pp1          0.950958    0.059330    0.000420    1.000000
pp2          0.095812    0.070877    0.000501    1.000000
pp3         -0.084709    0.064752    0.000458    1.000000
pp4          0.036137    0.050524    0.000357    1.000000
ph1         -0.025340    0.017845    0.000126    1.000000
ph2         -0.010019    0.014883    0.000105    1.000000
ph3          0.002713    0.012377    0.000088    1.000000
ph4          0.008784    0.010834    0.000077    1.000000
hc          -0.633425    0.137240    0.000970    1.000000
hp1          0.015744    0.013088    0.000093    1.000000
hp2          0.001319    0.011291    0.000080    1.000000
hp3         -0.004401    0.009808    0.000069    1.000000
hp4         -0.005261    0.008552    0.000060    1.000000
hh1          1.351492    0.053391    0.000378    1.000000
hh2         -0.300028    0.079825    0.000564    1.000000
hh3         -0.108049    0.070017    0.000495    1.000000
hh4         -0.027939    0.046460    0.000329    1.000000
```

```
TABLE 6: NORMAL-WISHART - MCMC CONVERGENCE diagnostics
Autocorrelations within each parameter chain
Variable        Lag 1        Lag 5       Lag 10       Lag 50
pc             -0.501       -0.003        0.009        0.001
pp1            -0.505        0.007       -0.001       -0.018
pp2            -0.510       -0.009       -0.006       -0.009
pp3            -0.500        0.013       -0.003        0.006
pp4            -0.501       -0.005       -0.001        0.012
ph1            -0.510       -0.006        0.009       -0.019
ph2            -0.501       -0.001       -0.002       -0.024
ph3            -0.509        0.005       -0.006       -0.005
ph4            -0.505       -0.001        0.017        0.009
hc             -0.511       -0.016        0.005        0.008
hp1            -0.500        0.003        0.005       -0.004
hp2            -0.494        0.008        0.017       -0.015
hp3            -0.505       -0.004       -0.005       -0.009
hp4            -0.503        0.004        0.013       -0.000
hh1            -0.493       -0.010       -0.013        0.019
hh2            -0.491       -0.002       -0.002        0.007
hh3            -0.498       -0.005        0.007        0.008
hh4            -0.495        0.010       -0.014       -0.008

Geweke Diagnostics for each parameter chain
Variable         Mean      std dev      NSE iid      RNE iid
pc          -0.148904     0.150954     0.001067     1.000000
pp1          0.967063     0.044451     0.000314     1.000000
pp2          0.047073     0.046182     0.000327     1.000000
pp3         -0.025755     0.039828     0.000282     1.000000
pp4          0.009695     0.033216     0.000235     1.000000
ph1         -0.040047     0.043544     0.000308     1.000000
ph2         -0.071344     0.053559     0.000379     1.000000
ph3          0.021474     0.044544     0.000315     1.000000
ph4          0.066985     0.033290     0.000235     1.000000
hc          -0.612428     0.139583     0.000987     1.000000
hp1          0.110171     0.041126     0.000291     1.000000
hp2          0.010749     0.042914     0.000303     1.000000
hp3         -0.050933     0.036828     0.000260     1.000000
hp4         -0.062061     0.030588     0.000216     1.000000
hh1          1.208659     0.040088     0.000283     1.000000
hh2         -0.131740     0.049716     0.000352     1.000000
hh3         -0.092842     0.041546     0.000294     1.000000
hh4         -0.065126     0.031024     0.000219     1.000000
```

```
TABLE 7: ENC UNCONDITIONAL - MCMC CONVERGENCE diagnostics
Autocorrelations within each parameter chain
Variable        Lag 1        Lag 5       Lag 10       Lag 50
pc             -0.500       -0.005       -0.004       -0.012
pp1            -0.494        0.004       -0.007        0.005
pp2            -0.494       -0.003       -0.008       -0.000
pp3            -0.494       -0.012        0.002        0.013
pp4            -0.492       -0.009        0.005        0.001
ph1            -0.501        0.003        0.007        0.005
ph2            -0.495       -0.001       -0.012       -0.005
ph3            -0.502        0.005       -0.000       -0.007
ph4            -0.505        0.002       -0.015       -0.005
hc             -0.492        0.002        0.001        0.008
hp1            -0.501       -0.000        0.006       -0.011
hp2            -0.497        0.013       -0.010       -0.002
hp3            -0.504        0.002        0.005        0.002
hp4            -0.511        0.003        0.002        0.002
hh1            -0.505        0.002        0.001       -0.003
hh2            -0.495        0.003       -0.011        0.009
hh3            -0.497       -0.005       -0.009        0.006
hh4            -0.501        0.001       -0.009       -0.007

Geweke Diagnostics for each parameter chain
Variable         Mean      std dev      NSE iid      RNE iid
pc          -0.153223     0.128592     0.000909     1.000000
pp1          0.953786     0.058259     0.000412     1.000000
pp2          0.090277     0.068011     0.000481     1.000000
pp3         -0.079378     0.061892     0.000438     1.000000
pp4          0.033489     0.049052     0.000347     1.000000
ph1         -0.024445     0.017268     0.000122     1.000000
ph2         -0.009647     0.014417     0.000102     1.000000
ph3          0.002441     0.012086     0.000085     1.000000
ph4          0.008171     0.010512     0.000074     1.000000
hc          -0.638408     0.139024     0.000983     1.000000
hp1          0.015066     0.012754     0.000090     1.000000
hp2          0.001384     0.011210     0.000079     1.000000
hp3         -0.004068     0.009546     0.000067     1.000000
hp4         -0.004928     0.008575     0.000061     1.000000
hh1          1.340575     0.053209     0.000376     1.000000
hh2         -0.285357     0.076934     0.000544     1.000000
hh3         -0.109912     0.066275     0.000469     1.000000
hh4         -0.030500     0.045204     0.000320     1.000000
```

```
TABLE 8: ENC CONDITIONAL - MCMC CONVERGENCE diagnostics
Autocorrelations within each parameter chain
Variable        Lag 1        Lag 5       Lag 10       Lag 50
pc             -0.507       -0.007       -0.003       -0.000
pp1            -0.500       -0.001        0.009       -0.001
pp2            -0.507        0.011       -0.004       -0.006
pp3            -0.503        0.002       -0.008        0.012
pp4            -0.513       -0.007       -0.005        0.015
ph1            -0.500        0.004        0.001        0.003
ph2            -0.493       -0.012       -0.006        0.021
ph3            -0.504        0.002        0.007       -0.010
ph4            -0.501       -0.007        0.009        0.004
hc             -0.500       -0.009       -0.002        0.002
hp1            -0.497       -0.007       -0.012        0.002
hp2            -0.494       -0.000        0.017        0.019
hp3            -0.493        0.002        0.008        0.003
hp4            -0.497        0.001        0.010       -0.000
hh1            -0.497        0.007        0.003       -0.011
hh2            -0.500        0.001       -0.013       -0.011
hh3            -0.502       -0.008       -0.020       -0.004
hh4            -0.502       -0.004        0.006       -0.009

Geweke Diagnostics for each parameter chain
Variable         Mean      std dev      NSE iid      RNE iid
pc          -0.171582     0.140363     0.000993     1.000000
pp1          0.941190     0.063722     0.000451     1.000000
pp2          0.124243     0.079010     0.000559     1.000000
pp3         -0.113036     0.073838     0.000522     1.000000
pp4          0.045975     0.056887     0.000402     1.000000
ph1         -0.032813     0.021704     0.000153     1.000000
ph2         -0.013582     0.019339     0.000137     1.000000
ph3          0.005545     0.016210     0.000115     1.000000
ph4          0.014905     0.014001     0.000099     1.000000
hc          -0.607581     0.140234     0.000992     1.000000
hp1          0.022644     0.017349     0.000123     1.000000
hp2          0.000897     0.015155     0.000107     1.000000
hp3         -0.007728     0.012858     0.000091     1.000000
hp4         -0.008666     0.011349     0.000080     1.000000
hh1          1.400615     0.059564     0.000421     1.000000
hh2         -0.372966     0.093332     0.000660     1.000000
hh3         -0.091929     0.086735     0.000613     1.000000
hh4         -0.016753     0.054314     0.000384     1.000000
```

9 Figures

Figure 1: Labor Productivity

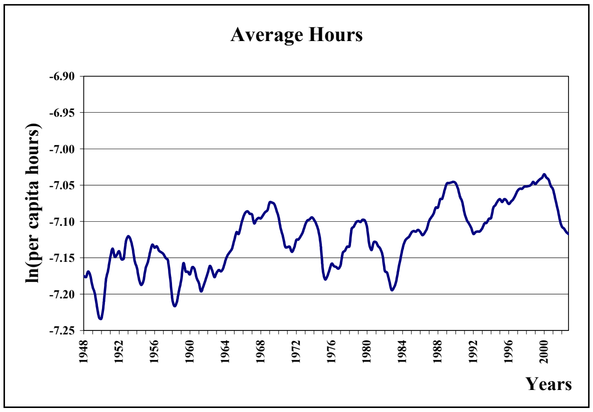

Figure 2: Average Hours: FR Data.

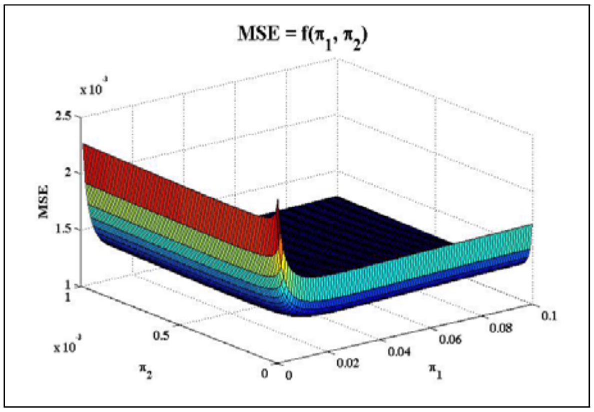

Figure 3: MSE as function of π_1 and π_2.

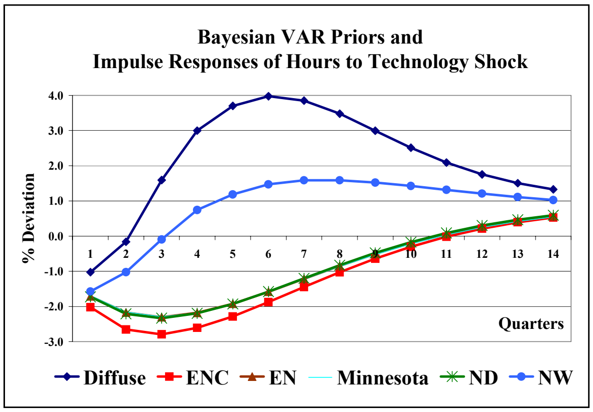

Figure 4: Impulse Responses - All Priors.

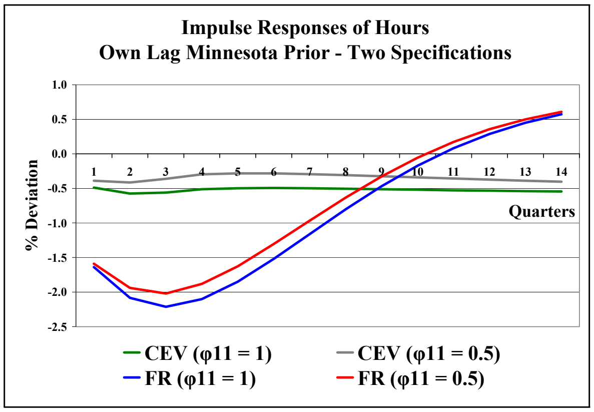

Figure 5: Own Lag Priors $\phi_{11} \in \{0.5, 1\}$

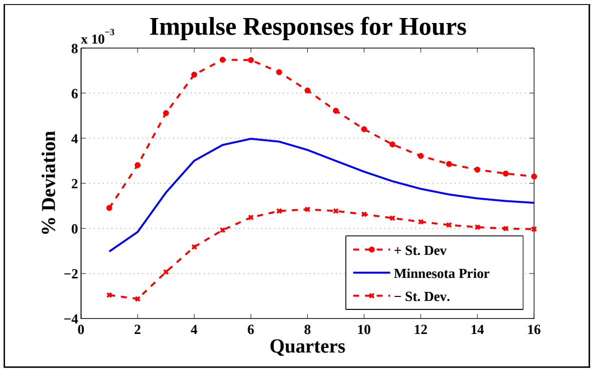

Figure 6: Diffuse Prior

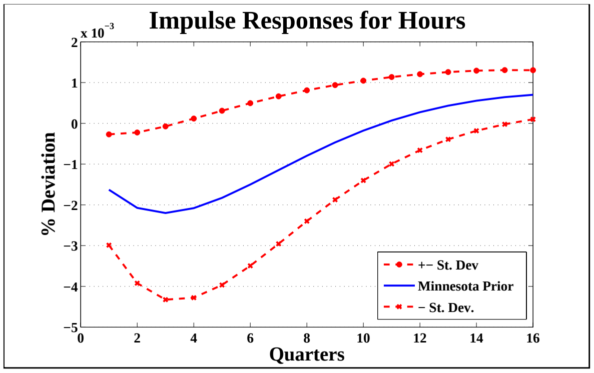

Figure 7: Minnesota Prior

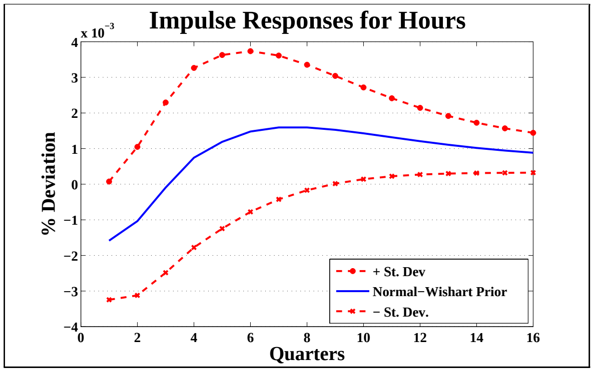

Figure 8: Normal-Wishart Prior

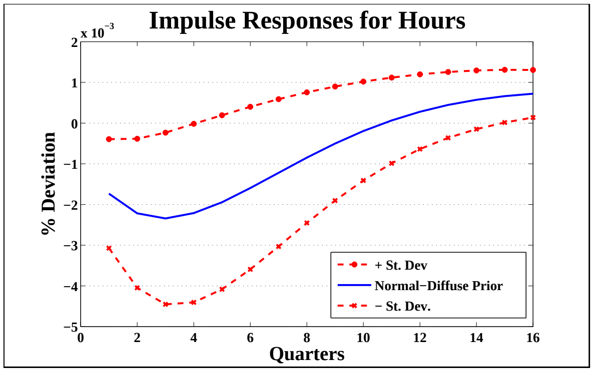

Figure 9: Normal-Diffuse Prior

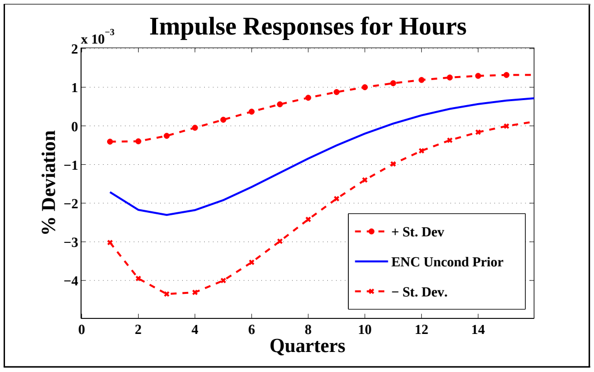

Figure 10: ENC UnCond Prior

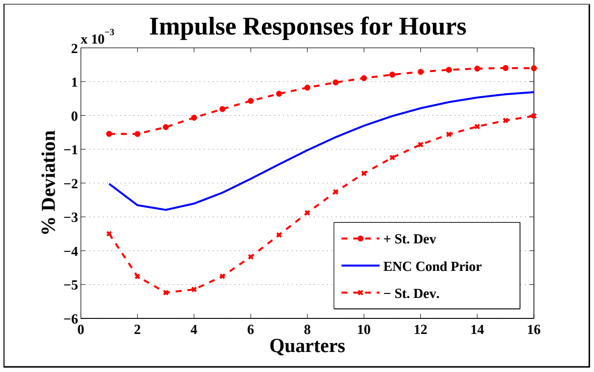

Figure 11: ENC Conditional Prior

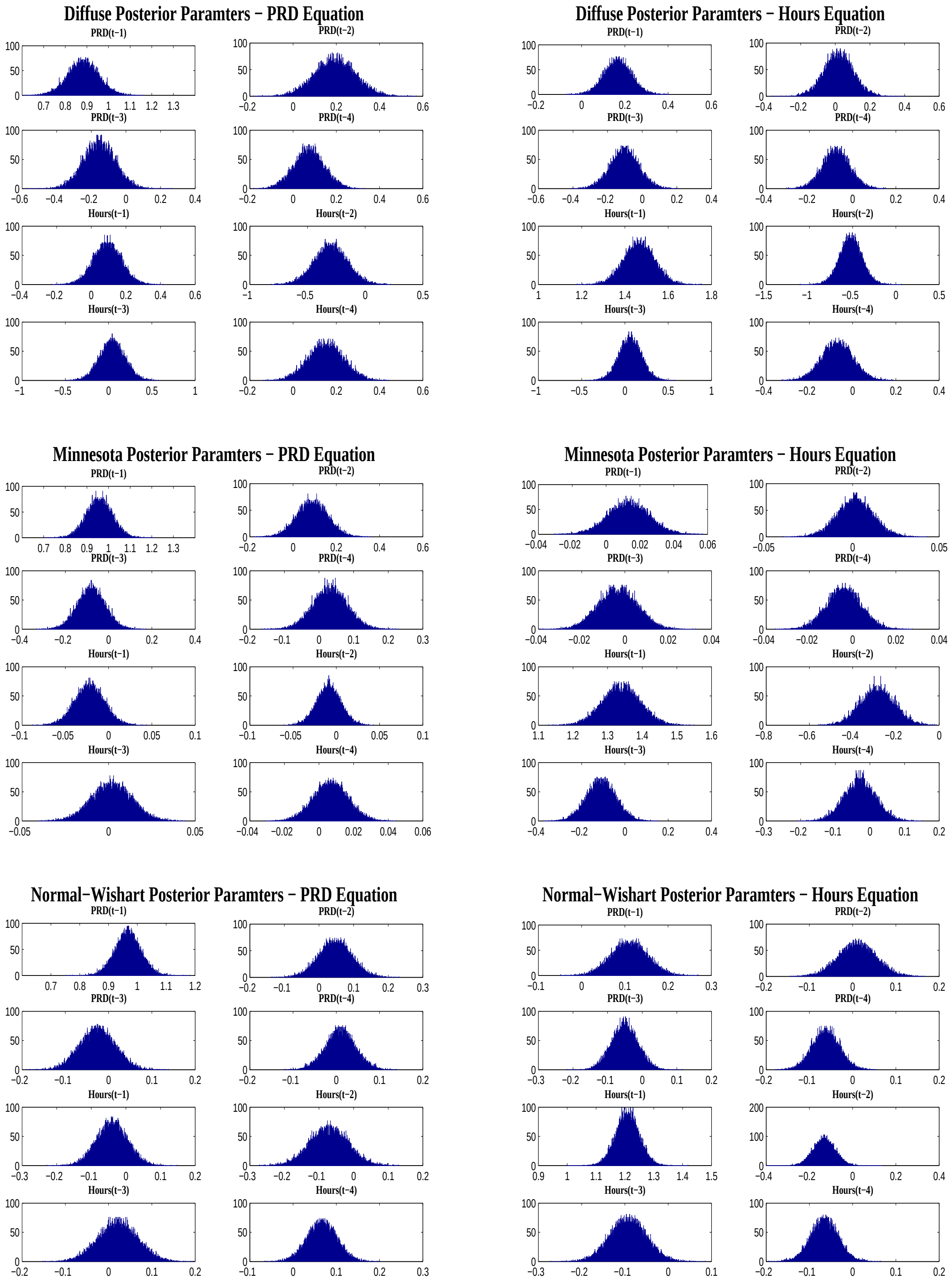

Figure 12: Diffuse, Minnesota and Normal-Wishart

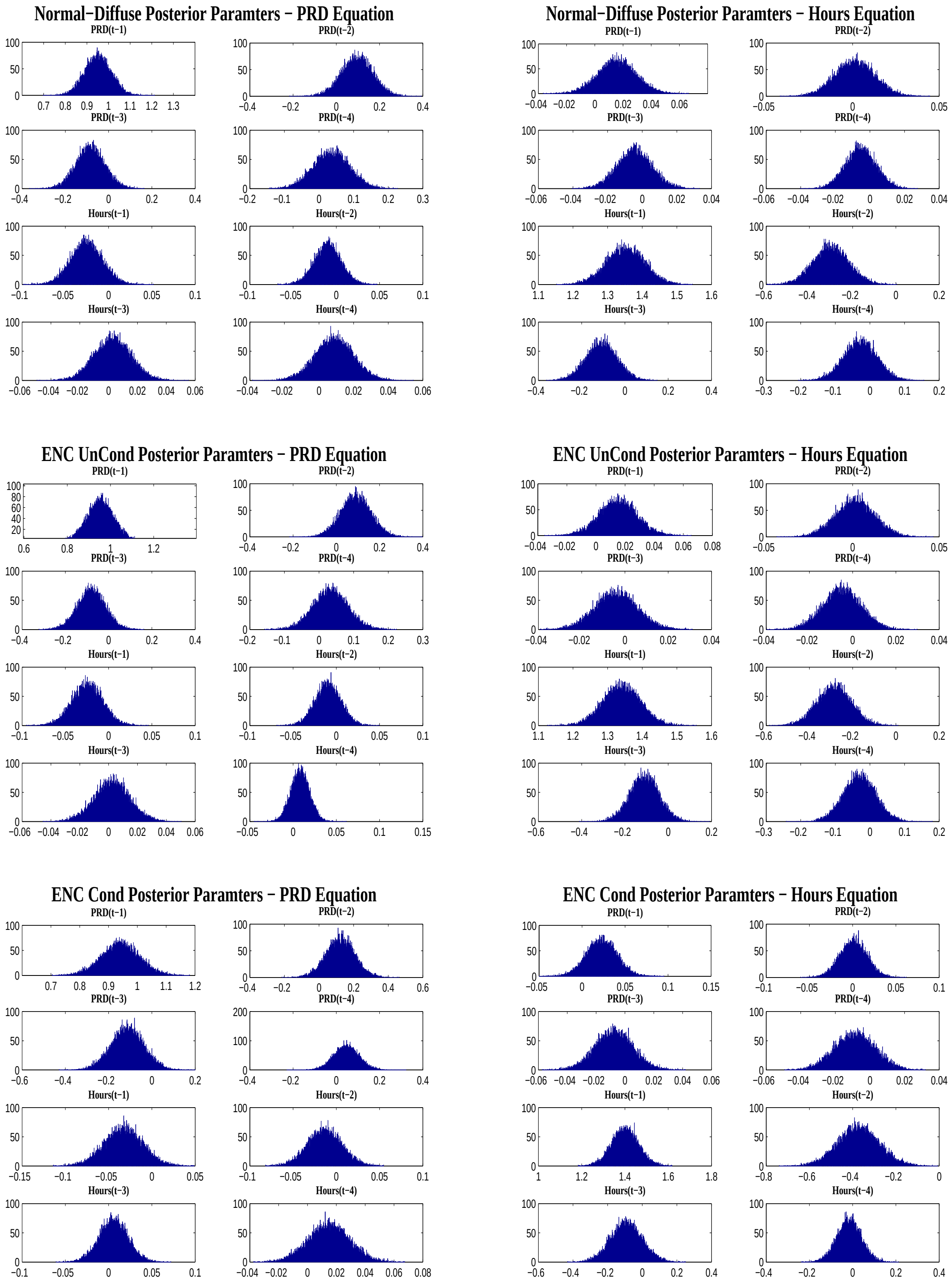

Figure 13: Normal-Diffuse, Extended Natural Conjugate Conditional and Unconditional