Explaining the Negative Coefficient Associated with Human Capital in Augmented Solow Growth Regressions

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Abstract

In this paper we consider different explanations for why the coefficient associated with human capital is often negative in growth regressions once country-specific effects are controlled for, whereas the coefficient in question is strongly positive in cross-sectional or panel results based on the pooling estimator. In turn, we explore: (i) additional sources of unobserved heterogeneity stemming from country-specific rates of labor-augmenting technological change, (ii) measurement error in the human capital series being used, and (iii) the lack of variability in the human capital series once the usual covariance transformations are implemented. Remaining unobserved country-specific heterogeneity and measurement error alone are shown to be inadequate explanations. The lack of variability in the human capital series is tackled using a modified version of the Hausman-Taylor (1981) approach whose identifying assumptions are found to be reasonable in the context of the Solow model.

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1 Introduction

Since the seminal empirical contributions by Mankiw, Romer and Weil (1992, henceforth MRW) and Benhabib and Spiegel (1994), there has been a fundamental tension between cross-sectional and panel data results concerning the impact of education on the process of economic growth. Results based on cross-sectional data over 25 year time spans (or longer), such as those presented by MRW, indicate a strong positive effect of various measures of human capital on economic growth. In contrast, once country-specific fixed effects are controlled for, as in Benhabib and Spiegel (1994) or Islam (1995), the coefficient associated with human capital becomes either statistically indistinguishable from zero or negative and statistically significant at the usual levels of confidence.[1] Given the high proportion of government expenditures devoted to education, the question that immediately arises, as it was cogently put by Pritchett (2001) is: Where has all the education gone?[2]

The reason for including human capital in an empirical implementation of the Solow growth model –the point of departure for the contribution of MRW– was to reduce the point estimate of the coefficient associated with physical capital, held to be much too high in light of the mean value of labor's share in GDP across countries and across time periods.[3] In a restricted Solow growth regression estimated over the period 1960-1985, the point estimate of α, the share of capital in GDP, was found by MRW to be equal to 0.6.[4] Including human capital in the specification brought it down to the much more acceptable level of 0.31, with education's share coming in at 0.28.[5] As such, the augmented Solow specification on cross-sectional data can be said to have accomplished its mission.

With the increasing availability of internationally comparable panel data, however, it became difficult to justify estimating growth regressions on cross sections, given that the data, as well as the appropriate econometric techniques, allowed one to control for country-specific unobserved heterogeneity. As is well-known, failure to control for individual effects tends to bias point estimates upwards, when the individual effects in question are positively correlated with the variable whose marginal impact one is trying to estimate. As such, panel estimation through some sort of covariance transformation (such as fixed effects) provides one with an additional tool that can, *a priori*, bring down the point estimate of the coefficient associated with physical capital, and provide more robust estimates of the marginal impact of human capital on growth (presumably reducing, though not, hopefully, eliminating it).

The puzzle being tackled in this paper stems from the fact that, once country-specific fixed effects are controlled for, the baby has been thrown out along with the bath-

[1] As Benhabib and Spiegel (1994, p. 154) put it, "the coefficient for human capital is insignificant and enters with the wrong sign....whether we use the Kyriacou, Barro-Lee, or literacy data sets as proxies for the stock of human capital," while Islam (1995, p. 1153) states that "the coefficient on the human capital variable now appears....with the wrong sign ....Whenever researchers have attempted to incorporate the temporal dimension of human capital variables into growth regressions, outcomes of either statistical insignificance or negative sign have surfaced."

[2] Pritchett uses one human capital stock and instruments using another in his specification. This method, known as the "indicator variable" approach, is well described in Wooldridge (2002).

[3] The issue of the "appropriate" value of capital's share has been considered by a number of authors. Hamilton and Monteagudo (1998) consider a vintage capital model that explains the lack of correspondence between the coefficient on capital in the estimation and the share of capital in GDP (see their references on p. 506). Gollin (2001) revises the estimates of labor's share of income (usually based on employee compensation) using data on self-employment and small enterprises, and shows that conventional estimates are likely to be severely biased for poor countries.

[4] MRW (1992, Table I, p. 414); Islam (1995, Table 1, p. 1141) obtains $\widehat{\alpha}$ =0.83.

[5] MRW, 1992, Table II, p. 420.

water: the marginal impact of human capital on growth, within the admittedly limiting confines of the augmented Solow growth model, becomes negative.[6] A similar finding by Hamilton and Monteagudo (1998) leads them to the rather unpalatable conclusion that: "The suggestion that countries can significantly improve their growth by further investments in public education does not seem to be supported by the data."[7]

The purpose of this paper is, first, to understand why human capital's role vanishes once country-specific effects are controlled for and, second, to provide an empirical answer that restores human capital to the key positive role that is predicted by almost all growth theories. It is worth stressing that the reasoning, and the empirical results, presented in this paper apply to the augmented Solow model of economic growth. On the one hand, this approach is rather limiting, in that richer empirical specifications are possible if one considers more sophisticated theoretical underpinnings. On the other, the augmented Solow model provides a simple unifying framework within which to analyze the role of human capital: moreover, if human capital is not a significant determinant of growth even within the augmented Solow model, its purported positive role hinges on much more tentative and specific mechanisms (such as the capacity to adopt new technologies). In addition, despite the popularity of endogenous growth theories as theoretical constructs within which the determinants of growth can be understood, it is difficult to test them structurally: the Solow model can certainly not be criticized in this respect.[8]

The structure of this paper is as follows. In part 2, we set out the basic empirical specification of the augmented Solow model. In part 3 we consider the two simple covariance transformations habitually used to control for country-specific heterogeneity (within and first-differences) and discuss the upward biases that arise when these corrections are not implemented. We also consider additional sources of country-specific heterogeneity that are not addressed by these procedures. Given the impact of controlling for country-specific effects on the coefficient associated with human capital, the main conclusion of this section is that some other source of negative bias is exacerbated by the usual covariance transformations.

In part 4, we consider the classic errors in variables problem that may affect the education variable (and which is inevitable, given the method by which the Barro-Lee dataset was constructed), and show how this problem may bias the coefficient associated with human capital downwards. We then move on to instrumental variables estimation

[6] See Islam (1995, Table V, p. 1151) where the coefficient associated with human capital becomes negative and statistically significant for his NONOIL sample; it is statistically indistinguishable from zero in the INTER and OECD samples.

[7] Hamilton and Monteagudo (1998, p. 508).

[8] For a critical review of the contribution of the endogenous growth literature to our understanding of economic growth, see Bardhan (1996). On the other hand, Klenow and Rodriguez-Clare (1997) stress the recent exaggerated use of the Neoclassical model in explaining differences in growth performance. Krueger and Lindahl (2001) provide a good discussion of the different manners in which human capital is entered into growth regressions.

using the Arellano-Bond (1991a, 1991b) GMM estimator. We show that this approach does not solve the human capital puzzle, in that the coefficient associated with human capital remains either negative and statistically significant.

In part 5, our focus is on the low variance of the human capital variable, once the within or the first-difference transformations have been performed. We show that most of the variance in the Barro-Lee education variable stems from the initial level of education, and that the process that generates human capital can be approximated by constant, country-specific rates of growth of human capital. The impact of this dramatic fall in variance is that the effect of human capital on economic growth becomes almost impossible to identify, and that measurement error may become relatively large. This also implies that the Arellano-Bond estimator is biased downward in the same direction as the first-difference estimator.

In part 6, we propose a modified version of the Hausman-Taylor (1981) estimator in which the impact of time-invariant covariates can be identified in panel data while controlling for individual effects through the use of covariance transformations of the variables themselves as instruments, which we combine with the orthogonality conditions of the Arellano-Bond (1991a, 1991b) estimator. We also implement the more classical Blundell-Bond (1998) estimator. We show that these two estimators solve the human capital puzzle, and yield point estimates of the coefficient on human capital that are more consistent with *a priori* expectations than are those provided by other estimation methods. Part 7 concludes.

2 The basic Augmented-Solow empirical specification

Let the production technology for country i at time t be given by the usual Cobb-Douglas functional form with labor-augmenting technological change $Y_{it} = K_{it}^{\alpha} H_{it}^{\varphi} (L_{it} A_{it})^{1-\alpha-\varphi}$, where Y_{it} is GDP, K_{it} is the stock of physical capital, H_{it} is the stock of human capital, L_{it} is population, and A_{it} represents the level of technology (here, the productivity of labor). As is usual, we assume constant population growth $n = \dot{L}_{it}/L_{it}$, a constant depreciation rate δ, and an exogenous rate of labor augmenting technological progress $g = \dot{A}_{it}/A_{it}$ (MRW, 1992, and Islam, 1995). Assuming neoclassical savings behavior (in both physical and human capital) yields the pair of dynamic factor accumulation equations

$$\dot{\hat{k}}_{it} = s_K \hat{k}_{it}^{\alpha} \hat{h}_{it}^{\varphi} - (n+g+\delta)\hat{k}_{it}, \quad \dot{\hat{h}}_{it} = s_H \hat{k}_{it}^{\alpha} \hat{h}_{it}^{\varphi} - (n+g+\delta)\hat{h}_{it}, \tag{1}$$

where $\widehat{x}_{it} \equiv x_{it}/A_{it}L_{it}$, represents variables expressed in terms of *efficiency units* of labor, and s_K and s_H are the investment rates in physical and human capital, respectively.

Since s_H observable in the data, the usual practice in the empirical growth literature is to assume that one has an acceptable proxy for the steady-state level of human capital, and to work solely with the $\dot{\hat{k}}_{it}$ equation. Imposing the condition $\dot{\hat{k}}_{it} = 0$ yields the steady-state level of physical capital per efficiency unit of labor as $\hat{k}^*_{it} = \left(\frac{s_K}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \hat{h}^{*\,\frac{\varphi}{1-\alpha}}_{it}$, and therefore steady-state GDP *per capita* as

$$y^*_{it} = \left(\frac{s_K}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} \hat{h}^{*\,\frac{\varphi}{1-\alpha}}_{it} A_{it}, \tag{2}$$

where $\hat{h}^*_{it}$ represents the steady-state level of human capital per efficiency unit of labor. By a first-order Taylor expansion around the steady-state in terms of convergence from time $t-1$ to time t, by letting the investment ratio and the rate of population growth be functions of i and t, and by appending a disturbance term, one obtains the usual estimating equation:

$$\begin{aligned} \Delta \ln y_{it} &= -\gamma_0 \ln y_{it-1} + \gamma_1 \left[\ln s_{Kit} - \ln(n_{it} + g + \delta)\right] + \gamma_2 \ln \hat{h}^*_{it} \\ &\quad + g + \gamma_0 g(t-1) + \gamma_0 \ln A_{i1} + \mu_i + \eta_t + \varepsilon_{it}, \end{aligned} \tag{3}$$

where λ is the rate of convergence towards the steady-state, $\mu_i + \eta_{it} + \varepsilon_{it}$ is the composite disturbance term, and where $\gamma_0 \equiv 1 - \exp\{-\lambda\}$, $\gamma_1 \equiv (1 - \exp\{-\lambda\}) \frac{\alpha}{1-\alpha}$, $\gamma_2 \equiv (1 - \exp\{-\lambda\}) \frac{\varphi}{1-\alpha}$. For notational simplicity in what follows, we shall also define $\xi_{it} = \gamma_0 \ln A_{i1} + \mu_i + \varepsilon_{it}$, since η_{it} will be accounted for by times dummies.

This paper will focus on the sign of γ_2, the coefficient associated with human capital in the augmented Solow model, as well as with the point estimate of φ. The usual practice in the empirical growth literature is to replace $\hat{h}^*_{it}$ by h_{it}, the average number of years of schooling in the population above 15 years of age at the *end* of the period considered. In what follows, we approximate this by the Barro-Lee (1993, 1996, 2001, henceforth BL) measure of human capital. The growth rate of GDP per capita (in constant domestic currency) comes from the World Bank, the initial level of GDP per capita comes from the Heston-Summers (1988) dataset, the source for the annual population growth rate and the investment rate in physical capital is the GDN.[9] Equation (3) constitutes the basic empirical specification that underlies all econometric studies of the augmented-Solow model, including the remainder of this paper.[10]

In order to estimate equation (3) using cross-sectional data as in MRW (1992), a strong identifying restriction needs to be imposed. Indeed, the only identifying restriction possible here is to assume that $\gamma_0 \ln A_{i1} + \mu_i$ is identical across countries. Panel data allows one to relax this restriction, as noted by Islam (1995). This, and other identifying restrictions are the subject of the next section.

[9] Our dataset is available upon request.

[10] Our choice of dependent and explanatory variables (particularly in terms of the price indices used to evaluate the variables in question) is based on the motivations set out very clearly in Nuxoll (1994).

3 Unobserved, country-specific heterogeneity

3.1 Country-specific initial levels of technology

The principal contribution of Islam (1995) was to estimate equation (3) using country-specific effects thereby controlling for differences stemming from heterogeneity across countries in the initial value of $\ln A_{1i}$. This is because the within transformation sweeps out the term $\gamma_0 \ln A_{1i} + \mu_i$, which would otherwise be included in the disturbance term, leading to biased estimates of the coefficients because of the correlation thereby induced between the explanatory variables and the error term.

In the absence of the within transformation, the bias in least squares estimate of the coefficient associated with human capital (γ_2) in the basic growth regression is given by $p\lim \hat{\gamma}_{2OLS} = \gamma_2 + \frac{cov[\gamma_0 \ln A_{1i} + \mu_i, \hat{e}^h_{it}]}{\sigma^2_{e_h}}$, where $\sigma^2_{e_h}$ is the variance of the residual from the auxiliary regression of human capital on the other included regressors which shall denote by X_{it}.[11]

Since it is likely that the initial level of technology and the level of human capital are positively correlated (after purging the effect of the other covariates), it follows that $cov[\gamma_0 \ln A_{1i} + \mu_i, \hat{e}^h_{it}] > 0$ and estimation of the growth regression by OLS should lead to an *upward* bias in the estimate of γ_2. The within and first-difference procedures are the two main covariance transformations generally used to account for this bias, although both suffer from their respective limitations.

The main weakness of the within transformation, as first noted by Anderson and Hsiao (1981), is that the resulting estimator will be inconsistent if some variables at time t are correlated with random shocks in any period $s \leq t$. We shall return to this problem later in the context of the issue of GMM estimation and autocorrelation. Nota also that the first-difference transformation results *by construction* in correlation between $\ln y_{it-1} - \ln y_{it-2}$(the differenced lagged-dependent variable) and $\varepsilon_{it} - \varepsilon_{it-1}$ (the differenced error term), an issue that will be explicitly addressed below in the context of GMM estimation. For the moment, these sources of bias will be ignored.[12]

Estimation results, over eight five-year periods (1960-2000), corresponding to pooling (estimation by OLS in levels), the within procedure, and first-differencing are presented in columns 1, 2 and 3 of Table 1, and largely reproduce those obtained by other authors. In particular, we obtain a negative and statistically significant coefficient associated with human capital using the within procedure and a negative and statistically insignificant

[11] That is $\sigma^2_{e_h} = var[\hat{e}^h_{it}] = var[h_{it} - X_{it}\hat{\omega}_{OLS}]$, where $\hat{\omega}_{OLS}$ is the coefficient vector from the auxiliary regression. See Griliches and Hausman (1986, p.97) and Hsiao (1986, p. 64, equation 3.9.3).

[12] This approach is similar to that used by Hamilton and Monteagudo (1998, p. 500-502), who estimate over two ten-year periods (1960-70, 1975-85) using the MRW data, while allowing parameter estimates to vary by decade. They then impose an increasingly stringent set of restrictions, ending up with a first-differenced form that imposes the theoretical constraints suggested by the augmented Solow model.

Table 1: Restricted Estimation of the Augmented Solow Models: 1960-2000, Eight five-year periods. Simple Covariance Transformations

Estimation method	Pooling	Within	First diff.	Sec. diff.
α	0.8447	0.4974	0.2250	0.1075
	(0.000)	(0.000)	(0.000)	(0.000)
φ	0.1459	−0.1922	−0.0839	−0.0189
	(0.003)	(0.001)	(0.101)	(0.612)
λ	0.0001	0.0063	0.0218	0.0392
	(0.000)	(0.000)	(0.000)	(0.000)
H_0: $\alpha + \varphi - 1 = 0$	0.009	−0.6951	−0.858	−0.9114
	(0.795)	(0.061)	(0.000)	(0.000)
$\overline{R}^2$	0.476	0.265	0.345	0.520
VIF (collinearity diagnostic)	3.008	3.008	1.068	1.112
No of observations	737	737	635	535

P-values in parentheses below coefficients

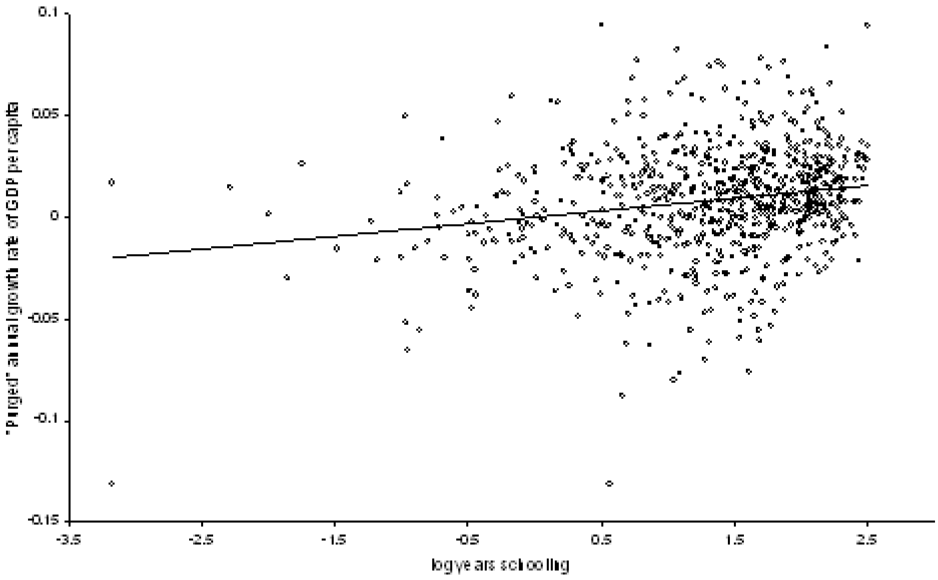
coefficient in first-differences. In Figures 1 through 3, we present graphs of the type popularized by Robert Barro, in which the growth rate of GDP per capita, purged of the effects of all explanatory variables except the variable of interest (education), is plotted on the vertical axis, with education being plotted on the horizontal axis. The regression line also appears in the figure, and passes through the origin by construction: its slope is equal to the value of $\widehat{\gamma}_2$ (the coefficient associated with human capital) estimated by each procedure.

Note that, despite what appear to be outliers (in Figures 1 and 3), the unbounded nature of the influence function associated with the within and first-difference estimators does not lie behind the negative γ_2 coefficient. For example, when one re-estimates the equation in first-differences by least absolute deviations (LAD), rather than by least squares, a method that is robust to leptokurtic (i.e., "fat tailed") disturbance terms, the estimated value of γ_2 is equal to $\hat{\gamma}_{2dLAD} = -0.0188$ with an associated t-statistic of -3.415 (the same result obtains, qualitatively, when one estimates by LAD after the within transformation). Controlling for influential observations therefore simply reinforces the puzzling negative coefficient associated with human capital.[13]

Note that the specification that we are using is questionable on a number of other counts. First, the Cobb-Douglas functional form assumes decreasing marginal returns to

[13] Temple (1999b) is able to obtain a positive coefficient on human capital on the Benhabib and Spiegel (1994) dataset, using OLS on first-differenced data, following use of least trimmed squares which allows him to eliminate 14 outliers. This specification does not, however, correspond to the augmented Solow model and involves only 64 observations (our first-differenced results involve 635 observations).

Figure 1: Annual growh rate of GDP per capita and human capital : Pooling ($\widehat{\gamma}_{2OLS}$= 0.0062)



the stock of education and a log-log specification, whereas the standard Mincerian micro specification yields a log-linear form.[14] According to Topel (1999, p. 2972), this type of specification error explains the negative $\widehat{\varphi}$ coefficient obtained by Benhabib and Spiegel (1994).[15] In order to see whether this is the case here, we re-estimated in pooling, within and first-differences using a log-linear specification. As shown in Table 4 in the Appendix, the coefficient associated with human capital, though positive, remains statistically indistinguishable from zero once time-invariant country-specific heterogeneity is taken into account.[16,17]

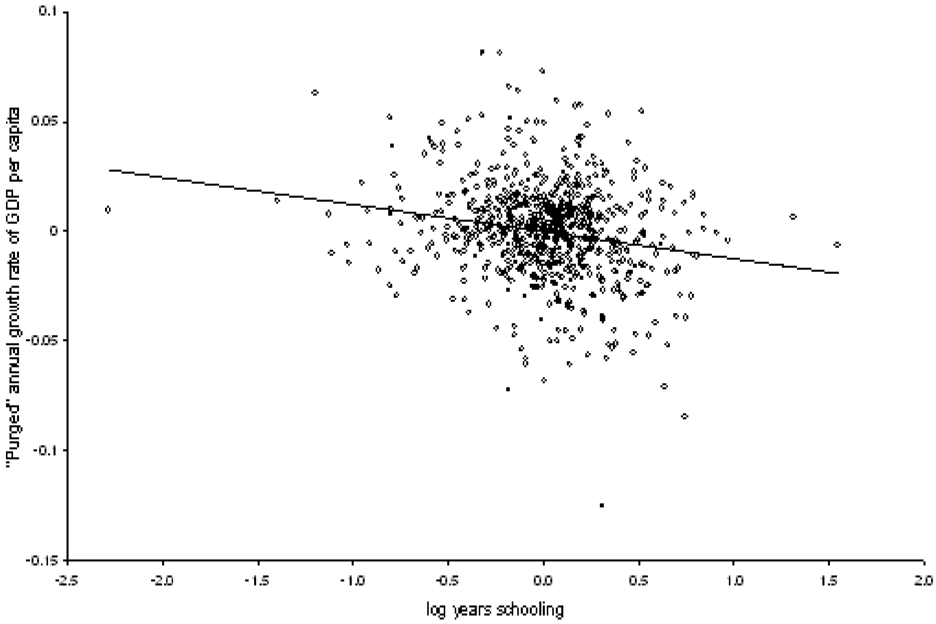
[14] Our specification also assumes perfect substitutability among workers with the same level of schooling, and the predominance of the quantitative aspects of education over its qualitative side (see Pritchett (2001) concerning this last point).

[15] Bils and Klenow (2000), Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) explicitly use a Mincerian specification linking the stock of human capital to the level of education in their macro-level specifications. They do so by constructing a stock of human capital in value terms, where the "price" is given by the returns to education found in micro studies. Barro and Lee (2001) have critized this approach whereas Topel (1999), as well as Krueger and Lindhal (2001) estimate growth models where the stock of human capital enters in logarithmic form.

[16] Topel (1999, p. 2969), using a "log-linear" specification, obtains a positive and statistically significant coefficient on human capital in first-differences. Note however that his specification either omits the investment rate (in physical capital) or restricts it to being equal to a given coefficient determined *ex ante*. Gemmell (1996) disaggregates the effects of primary, secondary and higher education, and finds positive effects on growth of a single level of educational attainment (in increasing order) for LDCs, middle income and OECD countries, respectively.

[17] Note also that (i) if human capital affects technological progresss and (ii) if the latter is constant

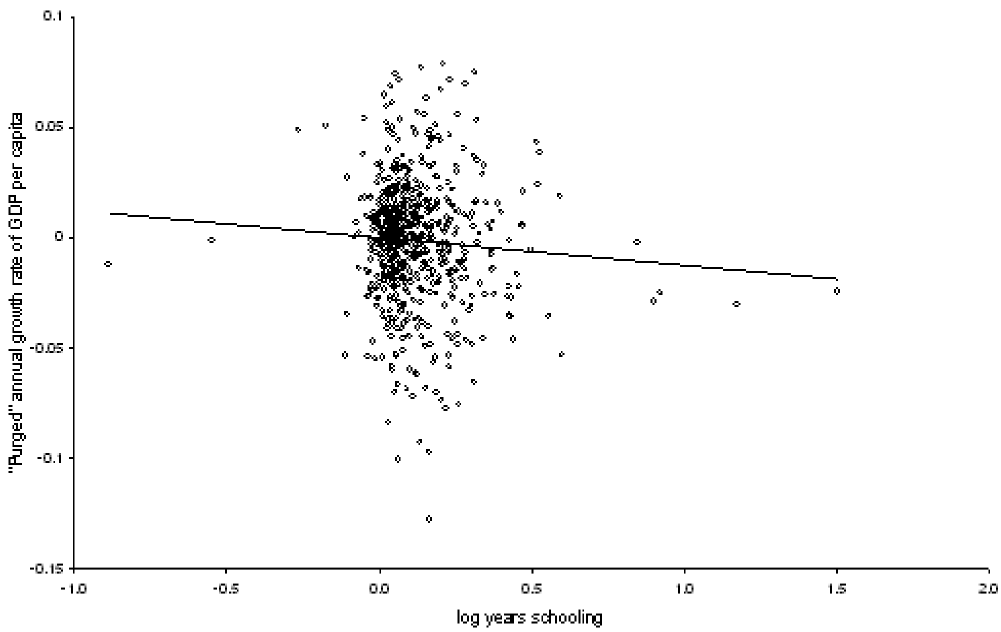
Figure 2: Annual growh rate of GDP per capita and human capital:
Within ($\widehat{\gamma}_{2W}= -0.0122$)



Second, it could be that our results are specific to the BL human capital data. There are several alternative sources of proxies for human capital, though many are based solely on flow data and use the perpetual inventory method to arrive at a stock measure. This is true of the schooling data of Lau, Jamison and Louat (1991), Lau, Bhalla and Louat (1991), or those of Nehru, Swanson and Dubey, (1995). The Kyriacou (1991) data are constructed by estimating a relationship between lagged enrollment rates and the subsequent average level of schooling of the population. In contrast, the series provided by BL (1993, 1996, 2001), de la Fuente and Doménech (2000), and Cohen and Soto (2001) are, at least in part, based on census data or national educational attainment surveys. Their greater accuracy therefore renders them preferable to the other sources mentioned above. Given that the de la Fuente and Doménech (2000) data are confined to OECD countries, Cohen and Soto (2001) remains the only viable alternative. In the Appendix, we therefore present results corresponding to reestimating our basic specifications using

over the five-year periods, then a portion of the impact of education cannot be identified. See Sianesi and Van Reenen (2002, p. 64) or Topel (2001, p. 94) on this topic. Krueger and Lindhal (2001, p. 1124-1127) and Cohen and Soto (2001) omit the physical capital variable. Their justification for doing so is that the high degree of complementarity between human and physical capital makes it unlikely that their separate effects can be identified. We include the variance inflation factor (VIF) for the human capital variable for all estimations presented in Table 1. If we consider the pooling and the within results, the VIFs are almost identical. It is therefore not an increase in collinearity stemming from the within transformation that is driving the human capital puzzle.

Figure 3: Annual growh rate of GDP per capita and human capital: First-differences ($\widehat{\gamma}_{2d} = -0.0125$)



the Cohen and Soto (2001) human capital data (in contrast to BL, the Cohen-Soto data are decade averages). As should be obvious from the results presented in Table 4, the coefficient associated with human capital remains statistically indistinguishable from zero once the within or first-differencing transformations are performed.[18]

3.2 Country-specific rates of labor-augmenting technological change

A potential source of bias not accounted for by Islam (1995) is constituted by country-specific *rates* of technological progress.[19] If we replace g with g_i, and consider a first-order Taylor expansion of $\ln(n_{it} + g_i + \delta)$ around $\ln(n_{it} + \delta)$, the basic growth regression can be rewritten as:

$$\begin{aligned} \Delta \ln y_{it} &= -\gamma_0 \ln y_{it-1} + \gamma_1 \left[\ln s_{Kit} - \ln(n_{it} + \delta)\right] + \gamma_2 \ln h_{it} \\ &\quad -\gamma_1 (n_{it} + \delta)^{-1} g_i + g_i + \gamma_0 g_i (t-1) + \gamma_0 \ln A_{i1} + \mu_i + \eta_t + \varepsilon_{it}. \end{aligned} \tag{4}$$

[18] Cohen and Soto (2001), in contrast, obtain a statistically significant positive coefficient. Note however that (i) they do not estimate a conditional convergence equation as in (3) and (ii) that their sample is poorly documented. It would appear for example that all sub-Saharan African countries are omitted.

[19] This issue is considered explicitly by Lee, Pesaran and Smith (1998) who consider a stochastic version of the Solow growth model. It is also worth emphasizing that the assumption of country-specific rates of technological change is linked to the debate concerning σ-convergence.

Neither the within procedure nor first-differencing eliminates this source of bias. Since it is likely that the level of human capital is positively correlated with the country-specific rate of technological progress, failure to account for this problem is likely to bias estimates of the impact of human capital on growth *upwards*.[20]

The solution to this problem is to move to *second-difference*s, which will eliminate $\gamma_0 g_i$ from equation (4), and to assume multiplicative country-specific fixed effects to account for the remaining source of heterogeneity $\gamma_1 g_i \Delta^2(n_{it}+\delta)^{-1}$ since the equation to be estimated by least squares is now given by:

$$\begin{aligned}\Delta^3 \ln y_{it} &= -\gamma_0 \Delta^2 \ln y_{it-1} + \gamma_1 \left[\Delta^2 \ln s_{Kit} - \Delta^2 \ln(n_{it}+\delta)\right] \\ &\quad +\gamma_2 \Delta^2 \ln h_{it} - \gamma_1 g_i \Delta^2 (n_{it}+\delta)^{-1} + \Delta^2 \eta_t + \Delta^2 \varepsilon_{it}.\end{aligned} \tag{5}$$

Estimates of the parameters of the growth regression in second-differences with multiplicative country-specific effects are presented in columns 4 of Table 1. Again, we obtain a negative and statistically insignificant coefficient associated with human capital

Note that there is some evidence that the specification in terms of labor-augmenting technological change employed in the basic MRW specification is itself misplaced. Boskin and Lau (2000) find, for the G7 countries, that "technical progress is simultaneously purely tangible capital and human capital augmenting, that is, generalized Solow-neutral .... Technical progress has been capital, not labor, saving." On the other hand, this should not present particular problems in the context of empirical implementations of the augmented Solow model since different forms of technological progress cannot be identified.[21]

It is also worth noting that other sources of unobserved heterogeneity can readily be found in the augmented Solow model. The most obvious stems from the linearization around the steady-state used to move from equation (2) (the steady-state level of GDP per capita) to equation (3) (the basic growth regression). This is because, while it is customary to write the annual rate of convergence towards the steady as a constant $\lambda = (n+g+\delta)(1-\alpha-\varphi)$, one should really be writing $\lambda_{it} = (n_{it}+g+\delta)(1-\alpha-\varphi)$ or $\lambda_{it} = (n_{it}+g_{it}+\delta)(1-\alpha-\varphi)$.

The speed of convergence should therefore vary over time. It should also vary across countries. The first problem is considered implicitly by Hamilton and Monteagudo

[20] More precisely, and as with the bias stemming from uncontrolled for differences in the initial level of technology, we assume that the residual from the auxiliary regression is, like human capital itself, positively correlated here with the country-specific rate of technological change.

[21] In the basic augmented Solow specification, if we change the production function so that it is specified in terms of Solow-neutral technological change, $Y_{it} = A_{it} K_{it}^{\alpha} H_{it}^{\varphi} (L_{it})^{1-\alpha-\varphi}$, with all other assumptions remaining the same, the country-specific term in the growth regression becomes $[(1+\varphi)/(1-\alpha)] \ln A_{i1} + \mu_i$. The within procedure or first-differencing will therefore eliminate this source of bias. The same discussion goes for country-specific rates of technological change in the second-differencing procedure.

(1998), who allow for coefficients that vary over the two time periods of their estimations.[22] It is also dealt with partially by Rappaport (2000), who explicitly considers variations over time in the speed of convergence, although his empirical specification is chosen (rightly, in his case) for its tractability rather than its faithfulness to the theoretical construct of the Solow model. The second problem (country-specific rates of convergence) is implicitly tackled in Durlauf, Kourtellos and Minkin (2001) in that their non-parametric approach allows all coefficients to vary over countries, as a function of the initial level of GDP per capita. However, as they do not seek to impose the restrictions implied by the Cobb-Douglas functional form, they do not furnish one with estimates of country-specific heterogeneity in the rate of convergence. It is interesting to note, in terms of the human capital puzzle, that their estimate of γ_2 is positive for values of log GDP per capita lying roughly between 6.3 ($544) and 7.5 ($1,808), and is negative otherwise.[23]

3.3 Simple covariance transformations: a first assessment

These results highlight the main issue tackled by this paper, namely the instability of the sign of the coefficient associated with human capital, which ranges from being positive and statistically significant (pooling results), to being negative and statistically significant (within). The upshot of these three simple covariance transformations is that there must be other sources of bias, not controlled for by the within, first-differencing, or second-differencing procedures, which bias estimates of φ *downwards.* Moreover, these potential sources of bias may be exacerbated by the procedures in question. The natural candidate is of course measurement error in the human capital variable.

4 Measurement error

4.1 Explicit hypotheses

At this point, it is worthwhile explicitly stating those hypotheses under which the within and first-differenced results will be unbiased, as well as alternative, weaker, hypotheses that will be considered at greater length in what follows.[24]

[22] Hamilton and Monteagudo (1998, equation 9, p. 498-500).

[23] Durlauf, Kourtellos and Minkin (2001, Figure 1, p. 934). An additional source of bias in the standard tests of the augmented Solow model involves the imposed functional form. Duffy and Papageorgiou (2000) show that a CES functional form is preferred over the usual Cobb-Douglas specification, although they use a human capital-adjusted measure of the labor input (i.e. education does not enter as a separate input or, more precisely, its coefficient is restricted to being the same as that associated with labor) and do not consider the augmented Solow model *per se* since their focus is on an aggregate production function.

[24] We also assume that $(\gamma_0 \ln A_{i1} + \mu_i)$ and ε_{it} have the familiar components structure: $E[\varepsilon_{it}] = 0$, $E[\gamma_0 \ln A_{i1} + \mu_i] = 0$, $E[(\gamma_0 \ln A_{i1} + \mu_i)', \varepsilon_{it}] = 0$.

ASSUMPTION 1 (exogeneity):

$$E[\ln h'_{it}\varepsilon_{is}] = E[\ln(n_{it}+g+\delta)'\varepsilon_{is}] = E[\ln s'_{Kit}\varepsilon_{is}] = 0, \quad \forall \; s,t$$

ASSUMPTION 2 (weak exogeneity):

$$E[\ln h'_{it}\varepsilon_{is}] = E[\ln(n_{it}+g+\delta)'\varepsilon_{is}] = E[\ln s'_{Kit}\varepsilon_{is}] = 0, \quad \forall \; s > t.$$

ASSUMPTION 3 (correlated effects):

$$E[\ln h'_{it}(\gamma_0 \ln A_{i1}+\mu_i)] \neq 0$$
$$E[\ln(n_{it}+g+\delta)'(\gamma_0 \ln A_{i1}+\mu_i)] \neq 0$$
$$E[\ln s'_{Kit}(\gamma_0 \ln A_{i1}+\mu_i) \neq 0.$$

Both the within and first-differencing procedures are explicitly designed to deal with ASSUMPTION 3 (correlated effects), and the within procedure will yield unbiased estimates when ASSUMPTION 1 (exogeneity) holds. On the other hand, the within procedure will be biased when ASSUMPTION 1 (exogeneity) is not satisfied, while first-differencing induces correlation between the differenced lagged dependent variable and the differenced error term, as previously noted, even when ASSUMPTION 1 is satisfied. ASSUMPTION 2 (weak exogeneity) is crucial in allowing one to overcome this particular hurdle using instrumental variable or GMM estimation. This issue will be addressed in section 5.

4.2 Problems with the Barro-Lee data

As mentioned earlier, the BL variable was only partly generated using census information on school attainment, and missing observations were inferred from enrollment ratios (as well as from adult illiteracy rates which allow one to construct a good proxy of the no-schooling category). The BL variable is therefore necessarily affected by a measurement error problem because (i) the enrollment rates furnished by UNESCO and used to fill in missing observations are acknowledged to be of poor quality (Krueger and Lindahl (2001, p. 1114), (ii) the survival rates used to compound the flow data are assumed to be the same at each level of schooling (BL, 2001, p. 545), (iii) immigrants are assumed to have the same average level of schooling as the mean level of the local population (Cohen and Soto (2001, p. 12) and (iv) schooling levels are divided into only seven categories. An illustration of the consequences of these problems is provided by Cohen and Soto (2001, Tables A.3 and A.4, p. 31) who show that the correlation between changes in the level of schooling, between their data and BL, is less than 10%, for the OECD countries.

4.3 Consequences in terms of attenuation bias

Assume that one observes an error-ridden measure of human capital h'_{it} given by the true value of human capital h_{it} plus an error term:[25]ASSUMPTION 4 (classical measurement error):

$$\ln h'_{it} = \ln h_{it} + u_{it},$$

where u_{it} is distributed i.i.d. with mean zero and variance σ_u^2.

ASSUMPTION 4 implies that the using within or first-differences estimators leads to a *downward* bias in the estimate of $\widehat{\gamma}_2$. Nelson (1995) shows that the *vector* of OLS parameters is also asymptotically biased towards zero when several variables are affected.[26] While this does explain why the coefficient associated with human capital might be biased downwards, it implies that, far from being overestimated, the coefficient associated with physical capital may be *underestimated* (the opposite of what is usually believed). Note that it may be the case that the two sources of bias (upward from the failure to control for unobserved country-specific heterogeneity, downward for measurement error) cancel each other out in the pooling results.[27]

4.4 Arellano-Bond estimation

The traditional cure for an errors in variables problem is, of course, estimation by instrumental variables.[28] Recall that first-differencing induces correlation between $\Delta \ln y_{it-1} = \ln y_{it-1} - \ln y_{it-2}$ and $\Delta\varepsilon_{it} = \varepsilon_{it} - \varepsilon_{it-1}$, since $\ln y_{it-1}$ is correlated with ε_{it-1}. We now make the following identifying assumption:

ASSUMPTION 5 (no autocorrelation in the error term):

$$E[\varepsilon_{it}\varepsilon_{is}] = 0, \; s \neq t.$$

In the absence of serial correlation in ε_{it}, and under ASSUMPTION 2 a valid instrument for $\Delta \ln y_{it-1}$ is given by $\ln y_{it-2}$. This is because $\ln y_{it-2}$ is orthogonal to $\Delta\varepsilon_{it} = \varepsilon_{it} - \varepsilon_{it-1}$. Moreover, given ASSUMPTION 5, $\ln y_{it-3}$ is also a valid instrument for $\Delta \ln y_{it-1}$, as is any $\ln y_{it-n}$, $n \geq 2$. This is expressed by the following orthogonality condition (recall that $\xi_{it} = \gamma_0 \ln A_{i1} + \mu_i + \varepsilon_{it}$):

[25] Temple (1999b) considers the robustness of the MRW cross-sectional results to classical measurement error, using the Klepper and Leamer (1984) reverse regression technique as well as classical method of moments estimators (Carroll, Ruppert and Stefanski, 1995). He does not, however, consider the robustness of the panel data literature. He finds that estimates of $\widehat{\varphi}$ lie between 0.15 and 0.38 (p. 371).

[26] See Dagenais (1994) for a discussion concerning the combination of serially correlated errors in the equation's disturbance term and measurement error in one of the variables.

[27] A similar argument could be made for the remaining unobserved heterogeneity stemming from g_i and measurement error on human capital in the within and first-difference estimations.

[28] As is well-known (Griliches and Hausman, 1986), different covariance transformations that control for the country-specific fixed effect can be combined in order to obtain consistent estimators for γ_2 and σ_u^2. Unfortunately, this approach did not yield a positive value of $\hat{\gamma}_2$.

ORTHOGONALITY CONDITION 1 :

$$E[\ln y'_{it-n}\Delta\xi_{it}] = 0,\ n \geq 2.$$

In terms of the other explanatory variables, we pose the following additional orthogonality conditions, which simply formalize ASSUMPTION 2 (weak endogeneity) in GMM terminology:

ORTHOGONALITY CONDITION2 :

$$E[\ln h'_{it-n}\Delta\xi_{it}] = E[\ln (n_{it-n}+g+\delta)'\Delta\xi_{it}] = E[\ln s'_{Kit-n}\Delta\xi_{it}] = 0,\ n \geq 2.$$

Note that using the human capital variable lagged two periods and more as instruments will be valid only when ASSUMPTION 4 and ASSUMPTION 5 *both* hold, that is when there is no autocorrelation in the disturbance term in the growth regression as well as no autocorrelation in the measurement error affecting human capital.

The Arellano-Bond (1991a, 1991b, henceforth AB) estimator combines the instruments defined by ORTHOGONALITY CONDITIONS 1 and 2 in an optimal manner through the use of the generalized method of moments (GMM) estimator.[29] It consists in using suitably lagged levels of the variables as instruments, after equation (3) has been first-differenced.

In the first column of Table 2 (AB_1), we present results corresponding to the AB estimator.[30] We obtain a negative and statistically significant coefficient associated with the human capital variable ($\hat{\varphi} \approx -0.231$). The absence of first-order serial correlation in the disturbance term of the growth equation in levels is implied, in the growth equation expressed in first-differences, by: (i) the *presence* of negative first-order serial correlation and (ii) the *absence* of second-order serial correlation.[31] In addition, the overidentifying restrictions are not rejected by the Hansen test. We then re-estimate our specification while (i) dropping the most recent instruments (AB_2) and (ii) using only variables lagged two and three periods (AB_3). Results, presented in columns 2 and 3 of Table 2, yield a negative and statistically significant estimate of $\hat{\varphi}$.

[29] Note that some authors reject the use of lagged right-hand-side variables altogether as instruments, even in the absence of serial correlation concerns. For example, Rappaport (2000) notes that "the potential for a reverse causal link from the current income level to any of the "stock" conditioning variables (i.e., right-hand-side variables constrained to a finite time derivative) should be of great concern in any instrumental variable procedure based on the Arellano-Bond approach". As he puts it: "To the extent that an included right-hand-side stock variable is a normal good, its level will increase with income; *education* and public capital seem obvious examples. The persistence of stock variables along with optimization by forward-looking agents rule out using lagged values as instruments" (Rappaport, 2000, p. 13).

[30] We use the two-step estimators that are robust to an unknown form of heteroskedasticity, and apply the Windmeijer (2000) correction in order to reduce the bias that may affect the second-step standard errors.

[31] AB (1991a, pp. 281-2).

Table 2: Restricted Estimation of the Augmented Solow Model: 1960-2000. Arellano-Bond (1991) estimator

Estimation method	DIFF-GMM		
	AB_1	AB_2	AB_3
α	0.1617 (0.211)	0.1123 (0.504)	0.2619 (0.096)
φ	−0.2318 (0.058)	−0.3644 (0.076)	−0.3315 (0.039)
λ	0.0134 (0.001)	0.0183 (0.001)	0.0115 (0.010)
H_0: $\alpha + \varphi - 1 = 0$	−1.070 (0.000)	−1.252 (0.000)	−1.069 (0.000)
No of observations	557	473	557
Hansen (p-value)	0.299	0.129	0.198
m_1 ($\sim N(0,1)$)	−3.80	−2.59	−3.79
m_2 ($\sim N(0,1)$)	0.37	0.63	0.14

P-values in parentheses below coefficients, Windmeijer (2000) correction

AB_1 : Instruments = variables lagged two periods and more

AB_2 : Instruments = variables lagged three periods and more

AB_3 : Instruments = variables lagged two and three periods.

A well-known example of the application of this estimator to the augmented Solow model is the paper by Caselli, Esquivel and Lefort (1996, henceforth CEL), who also find, estimating over the 1960-1985 time period a negative and statistically significant coefficient associated with the human capital variable ($\hat{\varphi} \approx -0.25$).[32]

Manifestly, the AB estimator does *not* provide one with a solution to the human capital puzzle. As we shall see below, part of the problem lies in the low variability of the human capital variable once first-differencing.

Given Montecarlo evidence, our results (and those of CEL) using AB are perhaps not surprizing. This is because Blundell and Bond (1998) have shown that diff-GMM is likely, in finite samples, to be biased towards the within estimator (that is, downwards).[33] This will be the case if (i) N is finite and T is small, (ii) the number of moment conditions is

[32] CEL (1996, Table 3, p. 376).

[33] For a discussion of the properties of IV estimators in finite samples, see Nelson and Startz (1990a,b), Buse (1992) or Staiger and Stock (1997). Intuitively, finite sample bias stems from the fact that the reduced form coefficients are estimated and not fixed parameters.

large with respect to the time dimension and, (iii) the instruments are weak. Since first-differencing, in near unit root processes, leads to instruments that are weakly correlated with the regressors,[34] it is likely that the AB estimator implemented here is biased towards least squares applied to the model expressed in first-differences.[35]

In the following section, we consider the properties of the growth process followed by our human capital variable. Table 5 in the Appendix provides a more detailed description of the time series properties of the BL education variable.

5 Low Variability in the Human Capital Variable

The within transformation results in a substantial reduction in the variance of the human capital variable, which goes from $\sigma_h^2 = 0.692$ in levels to $\sigma_{\tilde{h}}^2 = 0.121$ in deviations with respect to country-specific means. Basing identification on within-country variation results in a substantial loss in variance that could be the cause of insignificant coefficients associated with the human capital variable. The situation is even worse when one carries out first-differencing, with $\sigma_{\Delta h}^2$ =0.0230. This dramatic fall in variance is illustrated in Figure 4, where we plot different kernel density estimates of the human capital variable following various transformations (all variables have had their unconditional mean subtracted, which explains why all the kernels are centered on zero): it is obvious that the within and first-difference transformations correspond to substantial mean-preserving decreases in the "spread" (in the sense of Rothschild and Stiglitz) of the distribution of $\ln h_{it}$, with respect to the situation in levels (graphically, the estimated distributions become much more concentrated around zero). As one would expect from the respective variances reported above, this decrease in the spread is much more noticeable for the first-difference transformation than for the within transformation

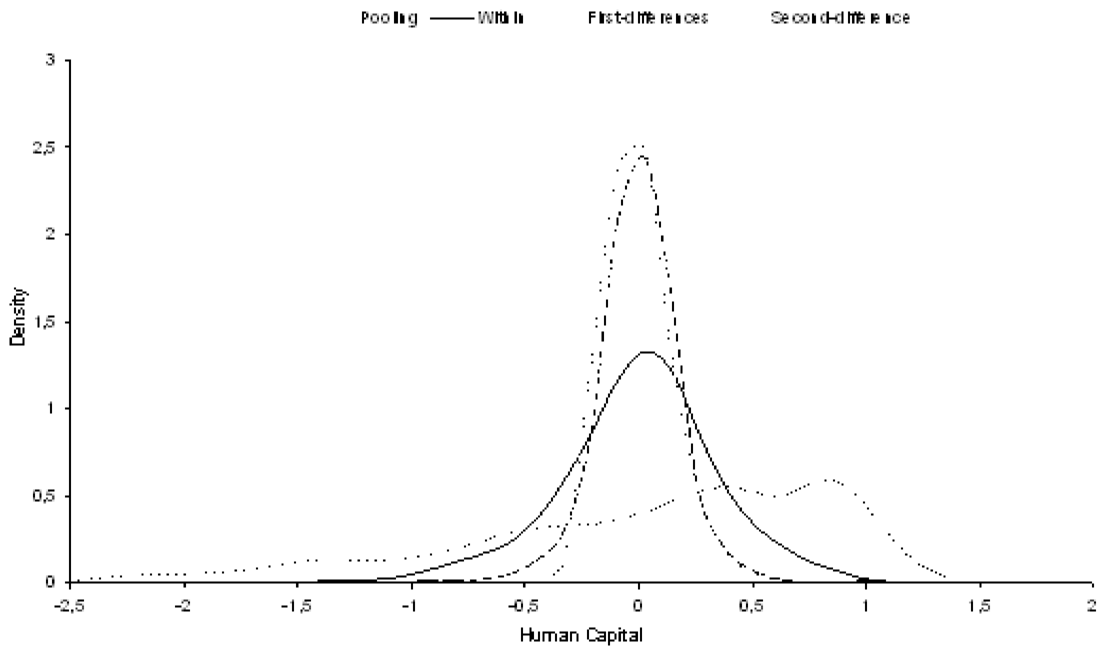
5.1 The growth process of human capital

The preceding findings in terms of the variances associated with various covariance transformations of the human capital variable naturally leads one to investigate its statistical properties more closely. Recall that the within transformation purges the human capital variable of its country-specific mean over the period. All that remains are within-country changes in human capital, and if that rate of growth is roughly constant (the variable is in logs), the within transformation will have removed inter-country differences due to differences in the initial level of education, leaving only relatively small differences in the between-period growth rate of human capital. The same is true of the first-difference transformation. In order to illustrate this point formally, consider

[34] For an excellent survey of the difficulties that arise with weak instruments and ways to deal with them, see Stock *et al* (2002) and Hahn and Hausman (2003).

[35] This is analogous to the bias, towards the OLS estimator, of a standard IV procedure (see Blundell *et al*, 2000, pp.10-11).

Figure 4: The changing distribution of $\ln h_{it}$. Kernel density estimates of log education: Pooling, within, first-differences, second differences



the following exponential growth process for human capital: $h_{it} = h_{i1} \exp\{a_i t\}$, where h_{i1} is the (country-specific) initial level of human capital, and a_i is its (country-specific) growth rate. If we consider the first-difference transformation, our estimating equation may be re-expressed as :

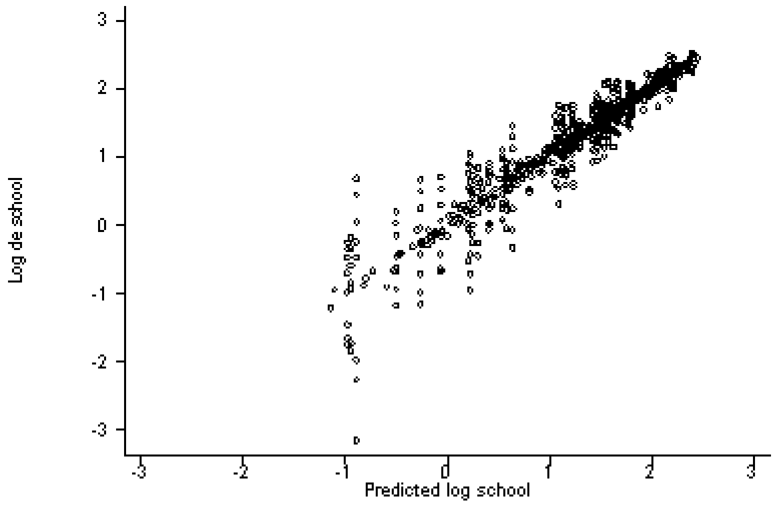
$$\begin{aligned} \Delta^2 \ln y_{it} &= -\gamma_0 \Delta \ln y_{it-1} + \gamma_1 \left[\Delta \ln s_{Kit} - \Delta \ln(n_{it} + g + \delta)\right] \\ &\quad + \gamma_0 g + \gamma_2 a_i + \Delta\eta_t + \Delta\varepsilon_{it}. \end{aligned} \tag{6}$$

What is clear in equation (6) is that *the entire effect of human capital in the regression will be identified through the variations in* a_i.[36] How great can one expect the fall in variance of the human capital variable to be when one moves from estimation in levels to estimation in first-differences, when human capital follows the process defined above? Let $var[a_i] = \sigma_a^2$, and $var[\ln h_{i1}] = \sigma_{h_1}^2$. Then it can be shown that the variance of the logarithm of human capital in a pooling regression over T periods is given by

[36] Note that, if this were indeed the true process generating human capital, the effect of the latter would not be identifiable at all in the equation estimated in second-differences. This is indeed what happens, in the sense that the standard error associated with human capital becomes extremely large when one moves to second-differences; see the results presented in Table 1, column 4.

$T\sigma^2_{h_1} + \sigma^2_a \sum_{n=1}^{n=T} n^2$.[37] Now consider a regression in first-differences. The variance of the first-difference of the logarithm of human capital, where the equation is estimated over $T-1$ periods, is given by $\sigma^2_{\Delta h_t} = (T-1)\sigma^2_a$. The ratio of the variances of log human capital in levels and log human capital in first-differences is therefore given by $\frac{\sigma^2_{h_t}}{\sigma^2_{\Delta h_t}} = \frac{T}{T-1}\frac{\sigma^2_{h_1}}{\sigma^2_a} + \frac{1}{T-1}\sum_{n=1}^{n=T} n^2$.[38] Here $T = 8$ which implies that $\sigma^2_{h_t}/\sigma^2_{\Delta h_t} = 29 + \frac{8}{7}(\sigma^2_{h_1}/\sigma^2_a)$. Thus, if human capital follows the exponential process we are assuming, one expects the variance that is performing the function of identification to fall by a factor of *at least* 29. This is indeed what happens when one performs the first-difference transformation: the resulting ratio of variances is approximately equal to 30 (here $\sigma^2_{h_1}$ =1.013).

Figure 5: The growth of human capital as a country-specific, exponential process $h_{it} = h_{i0}\exp\{(a_i + \theta_{it})t\}$; actual *versus* predicted value of $\ln h_{it}$



How good an approximation of the behavior of the human capital variable is the process that we are assuming? In order to assess its empirical validity, we simply estimated country-specific, time-invariant growth rates of human capital by OLS. The results are represented graphically in Figure 8, where we plot the actual value of $\ln h_{it}$ against its predicted: as should be obvious from the Figure 8, the fit is good.

5.2 Measurement error in the growth rate of human capital

[37] See Appendix 2 for the derivation.

[38] It is interesting to note that this expression provides part of the explanation for why the coefficients (and especially their standard errors) vary as the time frame changes (2 twenty-year periods, 4 ten-year periods, etc.) over which growth regressions in first differences are estimated.

Note that the preceding argument is a powerful explanation for the *imprecision* of the estimates of the coefficient associated with human capital, after the first-difference transformation. It does not, however, explain a negative and statistically significant coefficient.[39] In order to do so, measurement error must again be invoked. If the measurement error takes a form such that its magnitude is relatively important, relative to that of the transformed human capital variable, then (i) the process generating the human capital variable, (ii) measurement error and (iii) the first-difference transformation which results in a dramatic fall in the variance of the human capital variable may explain the negative coefficient associated with human capital.

Suppose that there is measurement error in the country-specific *growth rate* of human capital. We pose this as $\ln h'_{it} = \ln h_{i0} + (a_i + \theta_{it})t$, where θ_{it} is i.i.d.$\sim N(0, \sigma^2_\theta)$. Under this assumption, the equation in first-differences is given by

$$\begin{aligned}\Delta^2 \ln y_{it} = -\gamma_0 \Delta \ln y_{it-1} + \gamma_1 \left[\Delta \ln s_{Kit} - \Delta \ln(n_{it} + g + \delta)\right] \\ +\gamma_0 g + \gamma_2 a_i + \gamma_2 \theta_{it} + \Delta\eta_t + \Delta\varepsilon_{it}.\end{aligned} \tag{7}$$

Ignoring problems stemming from uncontrolled-for heterogeneity in the growth rate of labor productivity (g_i), the bias resulting from the measurement error on the growth rate of human capital is then given by $p\lim \hat{\gamma}_{2d} = \gamma_2 - \left[2\gamma_2\sigma^2_\theta/(\sigma^2_{e_{\Delta h}} + \sigma^2_\theta)\right]$, where $\sigma^2_{e_{\Delta h}}$ is the variance of the residuals from the auxiliary regression of $\Delta \ln h_{it}$ on the other explanatory variables, expressed in first-differences). The key issue is that σ^2_θ may be of the same order of magnitude as $\sigma^2_{\Delta h_t}$ (or more precisely, $\sigma^2_{e_{\Delta h}}$): it will nevertheless be extremely small (by a factor of 30, as shown above) with respect to $\sigma^2_{h_t}$.

The point being made here is that the instrumental variables method that one is looking for must simultaneously deal with the measurement error problem (and, therefore, orthogonalize, $\Delta \ln h'$ with respect to the error term), and inject enough "between" variance (i.e., cross-country variance) for the impact of human capital to be precisely identified after the first-difference transformation, which deals with Assumption 3 (correlated effects) but leaves very little variance in the transformed variable. Given that external instruments are unavailable, the next logical step is to consider instrumental variable estimators that use covariance transformations themselves as instruments, first proposed by Hausman and Taylor (1981) although this approach will have to be modified in order to take the orthogonality conditions given by Conditions 1 and 2 into account. Before, we present the Blundell-Bond (998) estimator.

6 Alternative estimators

6.1 Blundell and Bond (1998)

[39] In order to counter this problem, Mairesse (1990, p. 92, 1993, p. 435) suggests carrying out a "between" estimation after performing the first-difference transformation. Results corresponding to the between regression on first-differences are presented in column 5 of Table 4.

By adding a quasi-stationarity assumption, Arellano and Bover (1995) and Blundell and Bond (1998, henceforth BB) show that it is legitimate to write the model and levels and use lagged first-differenced variables as instruments. ASSUMPTIONS 2 and 3 (correlated effects and weak exogeneity), plus the assumption that country-specific effects are uncorrelated with the explanatory variables in first differences, allows one to add the following two orthogonality conditions:

ORTHOGONALITY CONDITION 3:

$$E[\Delta \ln h'_{it-1}(\xi_{it})] = E[\Delta \ln(n_{it-1} + g + \delta)'(\xi_{it})] = E[\Delta \ln s'_{Kit-1}(\xi_{it})] = 0.$$

ORTHOGONALITY CONDITION 4:

$$E[\Delta \ln y'_{it-1}(\xi_{it})] = 0.$$

CONDITIONS 3 and 4 imply that the explanatory variables (apart from the lagged dependent variable), in first differences and lagged one period, are orthogonal to the individual effects.[40] A sufficient condition is the mean-stationarity of the variables in question.[41] Since the mean stationarity of y_{it-n} is doubtful in the present context, we prefer the weaker necessary condition that $E[\Delta \ln y'_{i1}(\xi_{it})] = 0$ which, combined with ORTHOGONALITY CONDITION 3, ensures the validity of CONDITION 4. Combining CONDITIONS 1 TO 4 in an optimal manner yields the BB estimator (or System-GMM estimator).[42,43]

6.2 Hausman-Taylor meets Arellano-Bond, again

To the best of our knowledge, no use of the Hausman-Taylor (1981, henceforth HT) estimator has been made in the empirical growth literature, and this is surprising. Although Judson (1995) does mention their paper, she confines her estimations to the within, variance components (random effects), and GLS estimators. HT provide consistent and efficient estimators for the coefficients associated with time-invariant variables when these variables are correlated with unobserved heterogeneity, when we have

[40] Note that we only use the first differences lagged one period. The additional instruments that could be constructed using additional lags are redundant because of the first two orthogonality conditions. See Blundell and Bond (1998).

[41] Introducing time dummies weakens the assumptions needed concerning stationarity. In this case, it is sufficient that the mean of each variable grow in the same manner over time.

[42] The Montecarlo evidence presented by BB and Blundell *et al* (2000) show that if the data are near unit root processes or the variance of the individual effects is large relative to that of the disturbance term, the system-GMM estimator performs significantly better than diff-GMM. This is particularly true of the results presented in Blundell *et al*, which show the gains in precision that are obtained through use of the system-GMM estimator in the presence of weakly exogenous regressors that are correlated with the individual effects.

[43] Ahn and Schmidt (1995) show that it is possible to construct additional (linear and non-linear) orthogonality conditions based on the assumptions of no serial correlation and homoskedasticity, further improving the precision of the AB estimator. However, it is redundant to add their linear orthogonality conditions based on the absence of serial correlation to the conditions already embeded in the system-GMM estimator.

no external exogenous instruments, and when some explanatory variables satisfy ASSUMPTION 1 (exogeneity with repect to the disturbance term). The principle of this procedure consists in using the transformations of exogenous time-varying explanatory variables in terms of deviations with respect to their country-specific means and their country-specific means as instruments. In addition, only deviations with respect to their country-specific means of variables correlated with country-specific effect are admissible instruments. In other words, this approach is an ingenious manner of multiplying the number of available instruments.[44] However, it assumes that some of the explanatory variables are EXOGENOUS with respect to the disturbance term.

In the case under consideration, the point is not to identify the effect of time invariant variables while controling for individuals effects, rather we wish to use the decomposition suggested by HT as "normal" instrumental variables through which to identify the impact of a variable which exibits a low degree of variability.

Assume that the right-hand-side variables satisfy ASSUMPTION 2 (weak exogeneity) and ASSUMPTION 3 (correlated effects). Consider the following projection matrix P_{T-j}, which transforms time-varying variables into their individual *conditional* means from time 1 to time $t-j$; that is :

$$P'_{T-j}X_{it} = (t-j)^{-1}\sum\nolimits_{\tau=1}^{\tau=t-j} X_{i\tau} \equiv X_{i\bullet(t-j)}. \tag{8}$$

One can think of P_{T-j} as being the product of two matrices,$P_{T-j} = P_A S_{T-j}$, where P_A is a conventional idempotent matrix (of dimension $[(T-j)N] \times [(T-j)N]$) that transforms a $(T-j)N \times 1$ vector of variables into its individual means, and S_{T-j} is a $[(T-j)N] \times TN$ matrix that deletes the most recent j observations from each individual. If we premultiply a time-invariant variable by P_{T-j}, we simply obtain a $(T-j)N \times 1$ vector of the $T-j$ earliest elements of the variable itself, where the most recent j observations for each individual will have been deleted. In what follows, most of the discussion will be phrased in terms of the case where $j = 1$.

Now consider o the “within” transformation :

$$\tilde{X}_{it-1(t-1)} \equiv X_{it-1} - X_{i\bullet(t-1)}. \tag{9}$$

One can think of this as premultiplying the variables by the “anihilator matrix“ $Q_{T-1} = (\mathbf{I}_{(T-1)N} - P_{T-1})S_{T-1}$, where $\mathbf{I}_{(T-1)N}$ is the identity matrix of dimension $(T-1)N \times (T-1)N$. Q_{T-1} transforms a variable, after deleting the observation at time t for each individual, into deviations of the variable lagged one period, with respect to its individual mean, measured from $t-1$ backwards. Of course, if we premultiply a time-invariant Z_i vector by Q_{T-1}, we simply get a $T-1$ dimensional vector of zeroes. These two transformations of variables yield the following orthogonality condition:

[44] These results have been extended by Amemiya and McCurdy (1986) and Breusch, Mizon and Schmidt (1989).

Orthogonality condition 5

$$E[\tilde{X}'_{it-1(t-1)}(\xi_{it})] = 0.$$

Note that if some variables, noted X_{1it}, uncorrelated with the country-specific effect were available (which is not the case here), we could add the following orthogonality condition:

Orthogonality condition 6

$$E[X'_{1i\bullet(t-j)}(\xi_{it})] = 0.$$

Combining Conditions 1, 2 and 5 in an optimal manner yields the AH estimator.[45]

6.3 Results

In Table 3, we present results using the BB estimator (columns 1 to 3), followed by the modified version of HT (columns 4 and 5)[46]. The usual diagnostic statistics are reported in the lower part of the table, and do not reject in all five cases presented.

In column 1, the BB estimator uses first differences lagged one period as instruments for the levels equations, whereas the first-differenced equations use variables lagged two periods and more. In order to account for potential serial correlation in the measurement error affecting the human capital variable, column 2 presents results when the more recent lags are dropped.[47]

Given that Sevestre (2002) or Bowsher (2002) have shown that (i) a proliferation of instruments and/or (ii) weak instruments can lead to the systematic under-rejection of the null of their orthogonality with respect to the error term, the results presented in column 3 use the investment rate and the population growth rate lagged *only* two and three periods, while the human capital variable is confined to three and four period lags for the first-differenced equations.[48] The investment rate and the population growth rate (the human capital variable) in first differences lagged one period (two periods) still constitute the instruments for the level equations As with the first two columns, the Hansen test does not reject the null, though it does begin to approach a $p-$value of 10%. The coefficient associated with the human capital variable ($\widehat{\varphi}$) is positive in

[45] Note that several authors have suggested modified versions of the HT matrix of instruments. See in particular Arellano and Bover (1995).

[46] We also use the two-step estimators that are robust to an unknown form of heteroskedasticity, and apply the Windmeijer (2000) correction in order to reduce the bias that may affect the second-step standard errors.

[47] It would have been possible to use a common factor representation of equation (3) by assuming an autoregressive structure for the disturbance term. It is however impossible, even when the common factor restrictions are not rejected (which they are not), to deduce unique values of our three structural coefficients α, φ and λ. See Blundell *et al* (2000).

[48] On the other hand, Hahn and Hausman (2003) show that finite sample bias leads to an upward bias in the Sargan test statistic. This should lead one to *over*-reject.

Table 3: Human Capital augmented Solow model: 1960-2000. Blundell and Bond (1998) and a modified Hausman and Taylor (1981) estimator

	System-GMM			AH-GMM	
	BB_1	BB_2	BB_3	AH_1	AH_2
α	0.8504 (0.000)	0.8396 (0.000)	0.8561 (0.000)	0.8291 (0.069)	0.7907 (0.000)
φ	0.1693 (0.134)	0.2456 (0.104)	0.1898 (0.208)	0.2012 (0.052)	0.2787 (0.023)
λ	0.0013 (0.115)	0.0014 (0.175)	0.0014 (0.249)	0.0004 (0.001)	0.0019 (0.076)
H_0: $\alpha + \varphi - 1 = 0$	0.0198 (0.798)	0.0853 (0.406)	0.0459 (0.645)	0.0018 (0.043)	0.0023 (0.035)
Hansen (p-value)	0.512	0.413	0.109	0.2805	0.2139
m_1 ($\sim N(0,1)$)	-4.99	-4.98	-4.91	-4.73	-4.22
m_2 ($\sim N(0,1)$)	-0.05	-0.05	0.00	0.24	0.29

P-values in parentheses below coefficients

INSTRUMENTS

BB_1 : s_{kit}, y_{it-1} and h_{it} in levels lagged two periods and more,
first differences lagged 1 period

BB_2 : s_{kit} and y_{it-1} in levels lagged two periods and more,
first differences lagged 1 period,
h_{it}, in levels lagged 3 periods and more,
first differences lagged 2 periods

BB_3 : s_{kit} and y_{it-1}, in levels lagged 2 and 3 periods,
first differences lagged 1 period
h_{it}, in levels lagged 3 and 4 periods,
first differences lagged 2 periods

AH_1 : s_{kit}, y_{it-1} and h_{it} : $X_{it-1} - X_{i\bullet(t-1)}$
s_{kit}, y_{it-1} and h_{it}, in levels lagged 2 and 3 periods

AH_2 : s_{kit} and y_{it-1} : $X_{it-1} - X_{i\bullet(t-1)}$
h_{it} : $X_{it-2} - X_{i\bullet(t-2)}$
s_{kit} and $y_{it-\tau}$ in levels lagged 2 and 3 periods,
h_{it}, in levels lagged 3 and 4 periods

all three cases, though it is not estimated with great precision. The point estimate is, however, relatively stable, and varies between 0.16 and 0.25. The coefficient associated with physical capital ($\widehat{\alpha}$) is slightly greater than 0.80, convergence obtains at an annual rate of 0.13%, while the null of constant returns is not rejected. Generally speaking, the results are similar to those obtained busing the pooling estimator (column 1 of Table 1).

Columns 4 and 5 of Table 3 present the results using the AH estimator. In column 4, our instruments are variables lagged one period, expressed as deviations with respect to the country-specific means over the previous $T-1$ periods for the levels equations whereas the first-differenced equations use variables laggued two and three periods. In column 5, we drop the most recent lag of the human capital variable for the levels equations as well as for the first-differenced equations. Irrespective of the matrix of instruments that is used, the coefficient associated with the human capital variable is positive and statistically significant at the usual levels of confidence (in the results presented in the Table, it varies between 0.2012 and 0.2787). The diagnostic statistics do not reject the validity of our instruments, the implied capital share is close to that obtained using the BB estimator, the null of constant returns is not rejected, and the estimated annual rate of convergence (in column 5) is equal to 1.9%.

The upshot of these results is that the coefficient associated with the human capital variable is relatively stable, whether we apply BB or our modified version of HT. On the other hand, the latter estimation procedure results in standard errors that are much smaller.[49]

Based on the empirical results presented in this paper, our informed conjecture concerning the source of the negative coefficients associated with human capital once country-specific effects are controlled for is the following:

1. the results using the pooling, within and first-difference estimators are downward biased because of their failure to account for measurement error (section 4);

2. the pooling results are, however, also affected by an upward bias stemming from the failure to account for unobserved country-specific heterogeneity (section 3);

3. the AB results are downward-biased (towards the within results) because the instruments being used are weakly correlated with the explanatory variables expressed in first-differences; Blundell and Bond (1999), note that this will be the case when the variables are near unit root processes and the variance of the individual effects is large relative to the variance of the remaining disturbance term;

[49] See Hahn *et al* (2001) for an AB type estimator that uses "long" differences. Given that the finite sample bias of 2SLS is a function, *ceteris paribus*, of the correlation of the instruments with the jointly endogenous RHS variable (the partial R^2 of the reduced form) and the correlation between the reduced form and structural equation disturbance terms, using a system of equations in long differences with the appropriate choice of instruments (lagged residuals from the structural equation and other lagged variables) reduces this bias.

as shown in the appendix (Table 6), the human capital series is indeed close to displaying a unit root;[50]

4. the annual rate of convergence, as well as the capital share, are relatively stable over the various IV procedures; note however that IV estimation results in a point estimate of $\widehat{\varphi}$ that is greater than what is obtained under pooling; *a priori*, we would have expected the lower and upper bounds on $\widehat{\varphi}$ to have been given by the within and pooling results, respectively;

5. if our results are to be believed, it is then necessarily the case that measurement error biases the pooling results *downwards* more than correlated effects bias them *upwards*;

6. the conclusions reached by Islam (1995) and CEL (1996) may therefore be largely due to the statistical problems highlighted here that are not taken into account by their procedures.

7 Concluding Remarks

In this paper, we have attempted to explain why, once conventional panel estimation techniques such as the within procedure or first-differencing are performed, the coefficient associated with human capital, which is positive and statistically significant in cross-sectional or pooling regressions, becomes either statistically indistinguishable from zero or negative and statistically significant. After reviewing the forms of bias that are likely to arise in the augmented Solow model, we showed that the crucial issue revolves around the lack of variability in the education variable once country-specific heterogeneity is accounted for, and how standard covariance transformations result in the measurement error that affects human capital becoming the dominant source of identifying variance. We have proposed a modified version of the HT approach. An application of this estimator as well as the one suggested by Blundell and Bond (1998) allows one to solve the negative human capital coefficient puzzle, although further testing would be desirable.

The contributions of this paper to the literature on economic growth are, we believe, two-fold. First, we have shown, sometimes (and unfortunanely) rather laboriously, that a clear understanding of the underlying data-generating process is essential for one to be able to choose the right empirical instrument. The Barro-Lee human capital variable is an extremely useful creation which does, however, bring with it important problems, that have led to an econometric puzzle that has baffled growth-specialists in recent years.

Second, from the methodological perspective, we have shown that the Hausman-Taylor approach can be fruitfully applied to the empirics of economic growth. It is a

[50] See Hall (2001) for surveys of panel unit root tests.

viable alternative, when modified to take into account the milder identifying assumptions. Further investigations will involve exploring the broader instrument sets made possible by our approach, as well as developing a battery of hypothesis tests that will provide further checks on the validity of the identifying assumptions.

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Appendix 1: Robustness

Table 4: Robustness

Estimation method	"log-linear" specification of education		Cohen and Soto data		Mairesse's suggestion
	Pooling	First differences	Pooling	First differences	Between
α	0.4962 (0.000)	0.2213 (0.000)	0.5987 (0.000)	0.4473 (0.000)	0.5301 (0.000)
φ	-0.227 (0.429)	0.0197 (0.305)	-0.232 (0.001)	-0.2114 (0.087)	-0.0730 (0.749)
λ	0.0059 (0.000)	0.0217 (0.000)	0.0049 (0.000)	0.0091 (0.000)	0.0250 (0.019)
$\mathrm{H}_0 = \alpha + \varphi - 1 = 0$	-0.4810 (0.000)	-0.7589 (0.000)	-0.6457 (0.000)	-0.7640 (0.000)	-0.5428 (0.053)
$\overline{R}^2$	0.2559	0.354	0.4080	0.4375	0.0845
No of observations	737	635	326	236	100

P-values in parentheses below coefficients

1^{st} and 2^{nd} column : "log-linear" specification of the stock of human capital, Barro-Lee data.

3^{st} and 4^{nd} column : Cohen and Soto data. Four ten-year periods, 1960-2000.

5^{nd} column : Mairesse'ssuggestion, Barro-Lee data.

Appendix 2: the fall in variance associated with first-differences

The variance of human capital in a pooling regression over T periods when $h_{it} = h_{i0}\exp\{a_i t\}$ is given by :

$$\begin{aligned}
\sigma^2_{h_t} &= \textstyle\sum_{t=1}^{t=T} var[\ln h_{it}] \\
&= \textstyle\sum_{t=1}^{t=T} var[\ln h_{i1} + a_i t] \\
&= T var[\ln h_{i0}] + \textstyle\sum_{t=1}^{t=T} var[a_i t] \\
&= T\sigma^2_{h_1} + var[a_i] + var[a_i 2] \\
&\quad +... + var[a_i(T-1)] + var[a_i T] \\
&= T\sigma^2_{h_1} + \sigma^2_a + 4\sigma^2_a + ... + (T-1)^2\sigma^2_a + T^2\sigma^2_a \\
&= T\sigma^2_{h_1} + \sigma^2_a \textstyle\sum_{n=1}^{n=T} n^2.
\end{aligned}$$

In first-differences over $T-1$ periods, the corresponding expression is obtained as :

$$\begin{aligned}
\sigma^2_{\Delta h_t} &= \sum_{t=1}^{t=T-1} var[\ln h_{it} - \ln h_{it-1}] \\
&= \sum_{t=1}^{t=T-1} var[a_i] \\
&= (T-1) var[a_i] \\
&= (T-1)\sigma^2_a.
\end{aligned}$$

Appendix 3: Time series properties of the BL variable

Table 5: AR(1) specifications for the BL variable

	Pooling	Within	AB	BB
h_{it}				
$h_{it}(-1)$	0.9163	0.7039	0.7914	0.8683
	(0.006)	(0.0251)	(0.0548)	(0.0192)
$m_1 \, (\sim N(0,1))$	-4.20	-4.04	-3.60	-4.04
$m_2 \, (\sim N(0,1))$	-0.56	-0.39	-0.52	-0.54
Hansen $(p - value)$			0.697	0.761
Diff-Sargan $(p - value)$				0.556

P-values in parentheses

AB: h_{it} in levels lagged three and four periods

BB: h_{it} in levels lagged three and four periods

first differences lagged two periods