Investment Ratio and Growth

B. Bhaskara Rao

rao_b@usp.ac.fj

University of the South Pacific, Suva (Fiji)

Abstract

In growth and development policy investment ratio is an important policy instrument. However, there is no well defined framework to determine what should be the investment ratio for a given growth target. This paper explains the potential of Solow (1956) and Solow (1957) to explain the relationship between the target growth rate and investment ratio. Hypothetical data are used for illustration.

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1. INTRODUCTION

A legacy of the Harrod-Domar growth models for growth and development policy is that to increase the growth rate, the ratio of investment to output (investment ratio) should be increased. In contrast, the neo classical growth model of Solow (1956) and his growth accounting framework (GAF), based on Solow (1957), imply that the key to permanently raise the growth rate is to raise total factor productivity (TFP). An increase in the investment ratio only increases the growth rate during the transition of the economy from one steady to another steady state. However, it is well known that the East Asian countries, especially Singapore, have achieved high growth rates for several years in spite of modest increases in TFP and mainly through high investment ratios. Therefore, it would be interesting to understand the relationship growth and investment ratio. We first use the simpler GAF and then the more informative growth model framework (GMF) of Solow (1956). We show that the simple GAF gives good approximate empirical results for the medium term of 5 to 10 periods/years.

2. THE TWO FRAMEWORKS

GAF is essentially used as an identity to estimate TFP and TFP based on GAF is known as the Solow residual. Therefore, GAF can be used to estimate the required rate of growth of capital, as a residual, provided reliable estimates are available for other parameters and variables in this identity.

We assume a Cobb-Douglas production function with constant returns and Hicks neutral technical progress:

$$Y_t = A_t\ K^{\alpha}\ L^{(1-\alpha)} \tag{1}$$

where Y is output, A is stock of knowledge, K is capital and L is labour. It implies that

$$\dot{Y}_t = g + \alpha \dot{K}_t + (1-\alpha)\dot{L}_t \tag{2}$$

where g the rate of growth of TFP and dot on the other variables are proportionate changes. Equation (2) can be solved for $\dot{K}_t$ to get:

$$\begin{aligned} \dot{K}_t &= \frac{\dot{Y}_t - (1-\alpha)\dot{L}_t - g}{\alpha} \\ &= \frac{\theta - (1-\alpha)n - g}{\alpha} \end{aligned} \tag{3}$$

where θ is the rate of growth of output (our target), n is rate of growth of employment and g is growth of TFP. Thus, the required rate of growth of (net) capital, for a given target growth rate (θ) can be computed as a residual if the values of all other parameters and variables are known.

Using the definition of gross investment, the investment ratio can be expressed as:

$$\begin{aligned}\frac{I_t}{Y_t} &= \frac{I_t^{net}}{Y_t} + \frac{\delta K_t}{Y_t} \\ &= \frac{I_t^{net}}{Y_t} + \delta \Pi_t \end{aligned} \tag{4}$$

where δ is depreciation rate and Π is the capital-output ratio. Substituting for I^{net}, from the discrete version of (3) into (4), gives:

$$\begin{aligned}\frac{I_t}{Y_t} = s &= \frac{[\theta - (1-\alpha)n - g]}{\alpha} \frac{K_{t-1}}{Y_t} + \delta \Pi_t \\ &= \left[\frac{(\theta - (1-\alpha)n - g)}{\alpha\ (1+\theta)} + \delta(1+\gamma) \right] \Pi_{t-1} \end{aligned} \tag{5}$$

where s is investment ratio and γ is the rate of change in the capital output ratio and equals zero if this ratio is constant. That capital output ratio is fairly constant is a stylized fact.

Given the target rate of growth and the values for other variables, equation (5) can determine the required investment rate. A main limitation of GAF is that, since it is an identity, it ignores the dynamic adjustments in the growth model. Therefore, it implies that the target rate of growth can be maintained indefinitely. However, if in reality such dynamic adjustments are small and time consuming, GAF is a good predictor of the growth rate in the medium term of 5 to 10 periods. Given its simplicity, it is an attractive alternative approach to determine the investment rate for a given growth target.

The main driving force in the economy, moving it towards its steady state, is the fall in the marginal productivity of capital as capital stock increases. Such effects are taken into account in the neo classical growth model of Solow (1956). However, it is not obvious, from its familiar textbook versions, how long the relationship between growth rate and investment ratio will be significantly above the new steady state growth implied by TFP or TFP and factor accumulation when the behavior of the level of output is of interest.

This relationship can be derived, for example, by asking what should be the investment ratio to double output, say over 14 periods, implying that the average growth rate during these 14 periods is 5%. For this purpose, we use the following closed form solution for output for the neo classical growth model:[1]

$$Y_t = A_0 e^{gt}\ L_0 e^{nt}\left[\frac{s}{n+g+\sigma}\Big(1-e^{(1-\lambda t)}\Big)+\Big(\frac{Y_0}{A_0}\Big)^{[(1-\alpha)/\alpha]}e^{-\lambda t}\right]^{\frac{\alpha}{1-\alpha}} \tag{6}$$

where s is investment ratio and $\lambda = (1-\alpha)(n+g+\delta)$, $t = 0\cdots t$ is time. To simplify our observation that GAF is satisfactory for medium term policy to determine the investment ratio s to achieve a given target rate of growth (on the average) we shall use a simple simulation exercise with plausible values for the parameters and variables.

3. SIMULATION RESULTS

We start with closed form solution in equation (6), say for a developing country; and assume that growth of employment $n = 0.01$, TFP $g = 0.0125$, share of profits $\alpha = 0.425$ and depreciation rate $\sigma = 0.095$. Needless to say these values can be changed with more reliable estimates based on actual data. If the investment ratio $s = 0.15$, this economy takes about 24.25 periods to double its income with an implied growth rate $\theta = 0.0286$. On the other hand if the investment rate $s = 0.25$, it takes about 14 periods to double income with an implied average growth rate $\theta = 0.05$ for the 14 periods. In other words a 0.01 increase in the investment ratio, on the average, increases the growth rate by 0.275 and the implied elasticity of growth with respect to investment ratio is 3.1. Such increases in the growth rate, however, are not perpetual and converge, in the limit, to the rate of 0.0225, implied by the assumed TFP and factor accumulation.

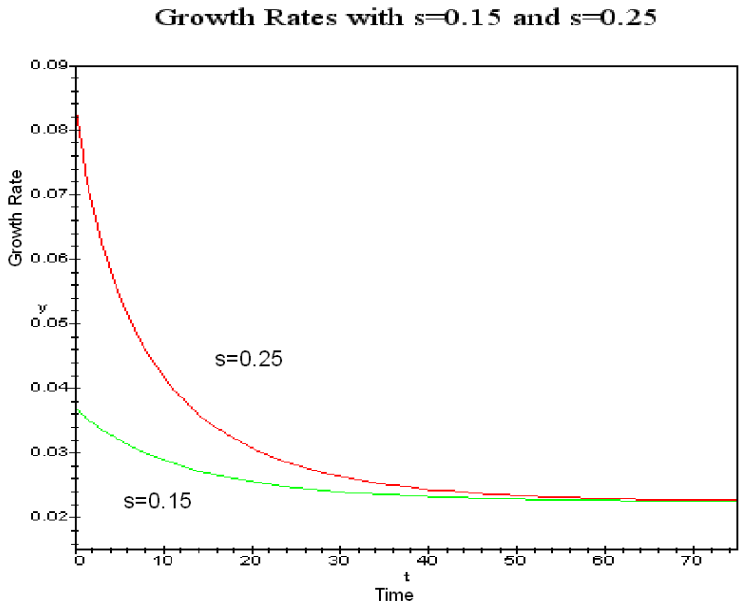
It is important to note from the simulation the average growth rate during the first 14 periods. With $s = 0.25$ the average growth rate is 0.050 and with $s = 0.15$ 0.026. The growth profiles of the economy with $s = 0.15$ and $s = 0.25$ are given below in Figure 1. It is assumed that initially the economy is in the steady state, with a growth rate of 0.0225.

A noteworthy feature is that when s increases by a substantial amount, high growth rates continue for several periods and the economy takes several decades to converge to its steady state. When $s = 0.25$ growth after 20 periods is still above 3%. In about 50 periods it reaches 0.95% of its steady state value. Consequently, it may be said that the need to increase the investment ratio to raise the growth rate, during the intermediate periods of up to 10 to 15

[1] This solution is given in Jones (2002). Jones acknowledges that originally this was due to Sato (1963).

periods, is also an important policy option in addition to the current emphasis on policies to increase TFP. This is of importance because considerable time and political will are necessary to raise TFP and therefore increasing the investment ratio is a pragmatic option for the short and intermediate policies.

FIGURE 1
Time in horizontal axis & Growth on vertical axis



Suppose, we use GAF to capture the effects of a rise in the investment ratio. How good are the predictions of the growth rates, say over 5 to 10 years? To examine this, it is reasonable to emphasize on the predictions of the average growth rates from these GAF and GMF because GAF predicts a constant growth rate for all the periods.

To generate predictions of growth rates with GAF, we assume constant capital output ratio i.e., $\gamma = 0$. Therefore, we select a value for the capital output ratio (Π) that satisfies the assumption that the growth rates of output and net capital are the same. This yielded $\Pi = 1.7529$ and we shall use the approximate value of $\Pi = 0.75$. Furthermore, the rate of growth of TFP used earlier ($g = 0.0125$) needs adjustment because it is equal to $(1 - \alpha) * g`$ where $g`$ is the measure based on Hicks neutral TFP. Therefore, $g` = [g/(1 - \alpha)] = 0.0217$ in GAF. With these adjustments GAF predicts that 5% growth rate can be achieved with $s = 0.255$ and this is close to $s = 0.25$ from GMF.

The only weakness of GAF is that it implies that this growth rate of 5% is constant, since it does not incorporate the dynamic adjustments that drive the economy towards a new steady state. Therefore, if such adjustments are fast, GAF is unlikely to accurately predict the dynamics of the growth rate. The average growth rate in GMF for the first 5, 10 and 14 periods, respectively, are 0.064, 0.055 and 0.050. Therefore, while GAF somewhat under predicted the average growth rate for the first 5 periods, its prediction for the first 10 and 14 periods is very good. Furthermore, the possibility of the economy converging to the new steady state during even 10 periods is remote. Therefore, it may be said that GAF is a simple and useful framework in the medium term for determining investment ratios for given growth targets. The differences between the predictions from these two frameworks will be further reduced if in fact the implied rate of decline in the marginal productivity of capital is not rapid enough to take the economy to its new steady state in 10 or 15 periods. In fact such slow adjustments might have enabled the East Asian economies to achieve high growth rates for substantial periods with high rates of factor accumulation and low rates of TFP.

4. CONCLUSIONS

In this paper we developed two frameworks, using Solow (1956) and Solow (1957), to examine the relationship between the growth rate and investment ratio. This helps to determine investment ratios required to achieve given growth rate targets. Investment ratio seems to be still important in policies for growth and development. We found that both frameworks imply that substantial improvement in the actual growth rates can be achieved for several periods by increasing the investment ratio. Although the framework based on Solow (1956) is generally preferred, GAF, based on Solow (1957), is also satisfactory for 5 to 14 year period predictions. The widely accepted view that only improvements to TFP can permanently raise growth rates is valid for the long run of 50 periods or more. Although such long run implications should not be ignored, it is also important to note that improvements to the investment ratio are also important for the medium term policies.

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