The Dynamic Beveridge Curve

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August 2005

Abstract

In aggregate U.S. data, exogenous shocks to labor productivity induce highly persistent and hump-shaped responses to both the vacancy-unemployment ratio and employment. We show that the standard version of the Mortensen-Pissarides matching model fails to replicate this dynamic pattern due to the rapid responses of vacancies. We extend the model by introducing a sunk cost for creating new job positions, motivated by the well-known fact that worker turnover exceeds job turnover. In the matching model with sunk costs, vacancies react sluggishly to shocks, leading to highly realistic dynamics.

JEL codes: E32, J63, J64

Keywords: Unemployment, Vacancies, Labor Adjustment, Matching

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1 Introduction

The Mortensen-Pissarides job matching model has become the standard framework for analyzing macroeconomic labor adjustment. Recent research has focused on the ability of this model to explain various empirical facts about unemployment and job vacancies.[1] A largely overlooked question concerns whether the model can account for the *dynamic* characteristics of labor market adjustment over the business cycle. For example, aggregate employment is widely viewed as a lagging indicator of the business cycle by policy makers and business analysts. Business cycles are also linked to a characteristic pattern of unemployment and vacancy movements. As shown in Figure 1, this pattern involves long counterclockwise loops in the space of vacancies and unemployment, with the vacancy-unemployment ratio rising in booms and falling in recessions.[2]

It is reasonable to imagine that the Mortensen-Pissarides model captures these features of aggregate labor market adjustment, since it incorporates a sluggish labor reallocation process that generates qualitatively plausible movements in unemployment and vacancies.[3] This paper shows, however, that the model in its standard form cannot account for the cyclical pattern of unemployment and vacancies when calibrated to U.S. data, nor can it capture the associated dynamics of employment adjustment. Modifying the model by introducing a sunk cost for the creation of new job positions,

[1]For example, recent papers by Costain and Reiter (2005), De Bock (2005), Hagedorn and Manovskii (2005), Krause and Lubik (2004), Hall (2005), Rudanko (2005), Shimer (2005) and Silva and Toldeo (2005) evaluate the model's ability to account for the volatility of unemployment and vacancies given plausible shocks to labor productivity, while Bastos (2004), Fujita (2003), Fujita (2004), Menzio (2004) and Yashiv (2005) consider other aspects of the model. See also Cole and Rogerson (1999) and Mortensen (1994).

[2]The negative empirical relationship between unemployment and vacancies is known as the Beveridge curve in recognition of the contribution of W.H. Beveridge. For a background discussion on the Beveridge curve, see Abraham (1987), Blanchard and Diamond (1989), Jackman, Layard, and Pissarides (1989) and the references therein. Bleakley and Fuhrer (1997) provide an updated empirical evaluation of the Beveridge curve. In this paper we focus on the dynamic characteristics of vacancy-unemployment relationship at business cycle frequencies.

[3]Pissarides (2000) (pp. 26-33) provides a qualitative analysis of the the dynamic properties of the model.

however, dramatically improves the performance of the model in matching the empirical dynamics.

We begin our analysis by estimating a three-variable VAR at quarterly frequency for the U.S. consisting of aggregate labor productivity, the vacancy-unemployment ratio and employment. In the context of the matching model, the vacancy-unemployment ratio reflects the intensity of search activities in the matching market, and thus it influences employment through match formation. Procyclical movements in the vacancy-unemployment ratio capture the adjustment pattern depicted in Figure 1. This specification allows us to trace the dynamic effects of labor productivity shocks on matching intensity, as measured by the vacancy-unemployment ratio, and employment. Note that in the context of the matching model, our identified cyclical productivity shock can be broadly interpreted as resulting from impulses to demand or technology.[4]

The estimates reveal that the productivity driving process identified in our VAR has a high positive correlation with matching intensity and employment at lags of 0-2 quarters and 1-3 quarters, respectively. Correspondingly, the impulse responses for the two variables exhibit hump shapes, with matching intensity and employment peaking after four and five quarters, respectively. In the period of the shock, matching intensity jumps by about a third of the way toward its peak response, while employment does not jump.

We next consider the ability of the Mortensen-Pissarides model to account for these patterns. A discrete-time version of the model is calibrated to U.S. data, and simulated data are obtained using a productivity driving process that approximates the estimated process. The resulting dynamic correlations are sharply peaked at zero lag, in stark contrast to the empirical correlations. The impulse responses derived from the simulated data exhibit excessively large jumps in the period of the shock, and the subsequent responses fail to capture the empirical hump shapes. Thus, the standard matching model fails to capture the dynamic characteristics of the data.

The poor performance of the standard model can be traced to the rapid adjustment of vacancies following a shock. Firms in the model incur vacancy posting costs only on a period-by-period basis, and there are no other costs associated with creating job

[4]Shimer (2005) offers a similar interpretation concerning shocks to labor productivity.

positions. Thus, they can respond freely to changes in productivity, causing vacancies to closely track the driving process.

Evidence suggests, however, that job positions behave differently from vacancies. Davis et al. (1996), for example, emphasize that jobs themselves are created and destroyed at a rate substantially lower than the rate at which firms open up and fill vacancies. This makes sense if firms bear additional costs for creating job positions, over and above the cost of posting vacancies. Such costs could tend to slow the adjustment of positions in response to shocks, leading to sluggishness in vacancy adjustment.

To assess the importance of this mechanism for the dynamic performance of the matching model, we extend the model in a simple way by introducing sunk job creation costs. We assume that heterogeneous opportunities for creating new positions become available each period. To exploit these opportunities, potential entrant firms must pay a sunk cost before they can post a vacancy. Positions created in this way remain active, whether filled or unfilled, until they are eliminated by obsolescence. In particular, when a worker quits, the firm can post a vacancy to refill the position without incurring the sunk cost. This provides a meaningful distinction between job flows and worker flows. Moreover, potential entrant firms and firms with established positions will have different incentives to post vacancies.

Simulated data from the calibrated sunk cost model reproduce the dynamic correlations for matching intensity and employment with great precision. Impulse responses exhibit realistic hump shapes, with the correct timing of peaks. The initial jump in matching intensity closely matches the empirical estimate, while the counterfactual jump in employment is significantly reduced relative to the standard model.

The superior performance of the sunk cost model is explained by the fact that sunk costs cause vacancies to respond more smoothly following shocks. On impact, the increase in vacancies is dampened due to the limited availability of opportunities whose returns exceed the sunk cost. Since the shock persists, however, newly arising opportunities will have profitability above the steady state level in the periods following the shock. This leads to further increases in vacancies, even as productivity decreases toward the steady state. Similar reasoning applies with respect to negative shocks. These smooth changes in vacancies underlie the realistic persistence exhibited by the simulated data.

Related literature. In their pioneering work, Blanchard and Diamond (1989, 1990) study the dynamic adjustment of vacancies and unemployment using VAR models that incorporate the labor force as a third variable. The impulse responses estimated by these authors capture the hump-shaped responses of vacancies and unemployment, consistent with our empirical analysis using the single matching intensity variable. Fujita (2004) studies vacancy persistence using an identification approach that builds on the method of Blanchard and Diamond (1989, 1990), and he shows that the Mortensen-Pissarides model cannot generate realistic persistence. In the present paper, we utilize a different identification procedure, based on direct use of labor productivity data, and we also consider employment dynamics. Further, we show that the deficiencies of the model can be corrected by introducing a sunk job creation cost.

Fujita (2003), Hornstein, Krusell, and Violante (2004) and Yashiv (2005) consider job matching models in which firms bear costs for adjusting the stock of job positions. In Fujita (2003), sluggish adjustment results from an initial job planning stage, while Yashiv (2005) introduces sluggishness by means of a standard convex adjustment cost. Neither of these papers considers direct costs of job creation, nor do they develop the distinction between job flows and worker flows. Hornstein, Krusell, and Violante (2004) focus on steady-state economic growth rather than cyclical adjustment.

The remainder of the paper proceeds as follows. Section 2 presents the empirical findings from the estimated VAR. The standard matching model and the sunk cost extension are developed in Section 3, and results are presented in Section 4. Section 5 concludes.

2 Cyclical shocks and adjustment

In this section we assess the cyclical characteristics of unemployment, vacancies and employment for the U.S. between 1951 and 2004. The first step is to determine an appropriate measure of cyclical shocks. Shimer (2005) has argued that average labor productivity can be used as a cyclical indicator for purposes of evaluating the matching model. This is compelling in that productivity incorporates a broad range of factors that influence the returns to employment. Productivity can itself be influenced by the labor market, however, so we must account for potential endogeneity in evaluating the effect of productivity on labor market variables.

We next specify an empirical relationship between productivity and the labor market variables. A large body of empirical work has demonstrated that hiring flows are well approximated by a constant returns-to-scale function of unemployment and job vacancies.[5] This implies that hiring may be regarded as a function of the vacancy-unemployment ratio. Thus, it makes sense to summarize the relationship between unemployment and vacancies in terms of the vacancy-unemployment ratio; we refer to the latter as *matching intensity*. This leads to the following reduced-form VAR:

$$A(L)\begin{bmatrix} \ln z_t \\ \ln vu_t \\ \ln emp_t \end{bmatrix} = \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^{vu} \\ \varepsilon_t^e \end{bmatrix}, \tag{1}$$

where $\ln z_t$, $\ln vu_t$ and $\ln emp_t$ denote the logs of labor productivity, the vacancy-unemployment ratio and the employment-population ratio, respectively; ε_t^z, ε_t^{vu} and ε_t^e are the reduced-form residuals of the three equations; and $A(L)$ is a lag polynomial matrix, with $A(0)$ being the identity matrix. Each variable is detrended by regressing on Chebyshev polynomial time trends. The sample period is 1951:Q1 to 2004:Q3.[6]

To identify the exogenous component of productivity, we use the recursive ordering $\ln z_t$, $\ln vu_t$ and $\ln emp_t$, so that innovations to labor productivity are treated as exogenous with respect to matching intensity and employment in the current quarter.[7] The

[5]See Petrongolo and Pissarides (2001) for a detailed survey of research on the estimation of matching functions.

[6]Labor productivity is measured as real GDP divided by civilian employment, 16 years and over. For the vacancy-unemployment ratio, we use the index of newspaper help-wanted advertising divided by the number of unemployed, 16 years and over. The employment-population ratio consists of civilian employment, 16 years and over, divided by civilian noninstitutional population. Data were obtained from the FRED II database maintained by the Federal Reserve Bank of St. Louis. The sample period is restricted by the availability of the help-wanted index. The model was estimated for all combinations of lag lengths (up to four) and polynomial trends (up to fourth order). Based on the AIC, we choose lag lengths of three quarters and third-order time trends in each equation.

[7]Our identification of cyclical shocks can be motivated by the reasonable restriction that innovations to the vacancy-unemployment ratio should not have a positive contemporaneous effect on productivity. Among the six possible orderings of the three variables, this restriction eliminates the three orderings in which $\ln vu_t$ is ordered before $\ln p_t$. The results for the remaining three orderings are nearly

exogenous productivity series, denoted by $\ln \widehat{z}_t$, is determined by

$$\widehat{A}_{11}(L) \ln \widehat{z}_t = \widehat{\varepsilon}_t^z, \tag{2}$$

where $\widehat{A}_{11}(L)$ is the estimated value of the polynomial in the first row and first column of $A(L)$ and $\widehat{\varepsilon}_t^z$ indicates the fitted residuals from (1). Note that (2) extracts the exogenous cyclical component of $\ln z_t$ by suppressing the feedbacks to productivity from matching intensity and employment that are associated with the terms $\widehat{A}_{12}(L)$ and $\widehat{A}_{13}(L)$ in the second and third columns of the first row of the matrix polynomial.

The conditional correlations of $\ln vu_t$ and $\ln emp_t$ with $\ln \widehat{z}_t$ at various leads and lags provide a summary measure of the dynamic relationship between cyclical shocks, matching intensity and employment. These correlations are computed from the following structural system:

$$\begin{bmatrix} \widehat{A}_{11}(L) & 0 & 0 \\ \widehat{A}_{21}(L) & \widehat{A}_{22}(L) & \widehat{A}_{23}(L) \\ \widehat{A}_{31}(L) & \widehat{A}_{32}(L) & \widehat{A}_{33}(L) \end{bmatrix} \begin{bmatrix} \ln \widehat{z}_t \\ \ln vu_t \\ \ln emp_t \end{bmatrix} = \begin{bmatrix} \widehat{\varepsilon}_t^z \\ 0 \\ 0 \end{bmatrix}, \tag{3}$$

where $\widehat{A}_{2j}(L)$ and $\widehat{A}_{3j}(L)$ give the estimated values of the polynomials in the j^{th} column of the second and third rows, respectively.[8] Figure 2 reports the correlations graphically. Observe that matching intensity and employment are highly correlated with lagged values of exogenous productivity, with peak correlations at lags of 0-2 quarters and 1-3 quarters, respectively.

Impulse responses to a one-standard-deviation productivity shock can be computed by calculating the trajectories implied by (3). Results are reported in Figure 3. As seen in the top panel, $\ln \widehat{z}_t$ jumps by about 0.7 percent as a result of the shock, then returns monotonically to its steady state after oscillating slightly for two quarters.[9] The dynamic pattern of unemployment and vacancy adjustment is captured in the

indistinguishable from one another. For concreteness, we focus on one particular choice from among these three.

[8]Since the identified productivity shock $\widehat{\varepsilon}_t^z$ may have contemporaneous effects on the other two variables in the structural form (3), the elements $\widehat{A}_{21}(0)$ and $\widehat{A}_{31}(0)$ may be nonzero, in contrast to the restriction $A_{21}(0) = A_{31}(0) = 0$ in the reduced form (1).

[9]Note that the estimated impulse response for $\ln \widehat{z}_t$ is very close to the one generated by the technology process $\ln z_t = .95 \ln z_{t-1} + \varepsilon_t$, with $\sigma_z = .007$, that is standard in RBC analysis.

middle panel as a hump-shaped response of the variable $\ln vu_t$. Vacancies initially jump relative to unemployment by about 5 percent. The vacancy-unemployment ratio continues to rise rapidly for four quarters, with a peak at roughly 12 percent above the steady state, after which it falls fairly rapidly for eight quarters or so. The variable $\ln emp_t$, in the bottom panel, does not jump in the period of the shock, but otherwise its response closely mimics the response of $\ln vu_t$, with a one-quarter lag and a peak of about 0.35 percent above the steady state. This indicates that the adjustment of employment is closely tied to the dynamics of unemployment and vacancies.

We now link these results back to adjustments of unemployment and vacancies themselves. The matching intensity variable $\ln vu_t$ can be decoupled into separate vacancy and unemployment components by estimating a bivariate VAR in which the fitted exogenous productivity process and its innovations appear as independent variables:

$$B(L)\begin{bmatrix} \ln vac_t \\ \ln unemp_t \end{bmatrix} = C(L)\ln \widehat{z}_t + D\widehat{\varepsilon}_t^z + \begin{bmatrix} \varepsilon_t^v \\ \varepsilon_t^u \end{bmatrix},$$

where $\ln vac_t$ and $\ln unemp_t$ represent the logs of vacancies and unemployment, respectively; ε_t^v and ε_t^u are innovations to these two variables; and $B(L)$, $C(L)$ and D indicate a polynomial matrix, a polynomial vector and a real vector, respectively. Variables have been detrended by regressing on Chebyshev polynomial time trends.[10]

Figure 4 depicts the responses of vacancies and unemployment to a one-standard-deviation shock to productivity. Observe that vacancies and unemployment respond in opposite directions, but by the same magnitude, in the period of the shock, with vacancies rising by 2 percent and unemployment falling by 2 percent. The variables move as rough mirror images of each other over the next 12 quarters, with unemployment decreasing at a slightly faster rate in the first year. The vacancy response peaks at 5.5 percent in the third period following the shock, and the unemployment response troughs at -6.5 percent in the fourth quarter. It follows that just under half of the overall adjustment of matching intensity over the first 12 quarters is accounted for by changes in vacancies. By the 13^{th} quarter, vacancies have substantially returned to their steady state, while unemployment maintains an extended gap of about one

[10] In this estimation we use lag lengths of three quarters and third-order time trends in order to maintain consistency with the previous specification.

percentage point below its steady state.

3 Matching Models

3.1 Standard model

Model description. In this section we present a version of the standard matching model.[11] The model consists of a unit mass of workers and an infinite mass of firms. In any given period, workers may be either matched with a firm or in the unemployment pool searching for a match. Firms are either matched with workers, in the vacancy pool searching for a match, or in a pool of potential entrant firms that are not actively searching. Matched worker-firm pairs engage in production, while workers and firms in the matching pools seek to form new matched pairs. Potential entrant firms may choose to enter the vacancy pool in any period.

While in the unemployment pool, a worker receives a flow benefit of b per period, which may be interpreted as utility from leisure. Each firm in the vacancy pool pays a posting cost of c per period. The net number of new matches created in period t is given by the matching function $m(u_t, v_t)$, where u_t and v_t indicate the sizes of the period t unemployment and vacancy pools, respectively. The function m satisfies the customary properties.[12] A worker and firm that are matched in period t begin a new employment relationship in period $t+1$.

At the start of period t, each worker-firm match undergoes two distinct separation hazards. First, separation occurs with probability λ^d as a consequence of *obsolescence*. In this case, the worker enters the period t unemployment pool and the firm enters the pool of potential entrants. Second, separation may occur for reasons that do not

[11] This is essentially a discrete-time version of the model described in Chapter 1 of Pissarides (2000). In defining the job separation rate, we introduce a distinction between permanent obsolescence and quits, and we also distinguish between quit rates of newly formed and ongoing matches. These distinctions are immaterial for the standard model, but they will play an important role in the sunk cost model introduced in the following section.

[12] That is, m is twice continuously differentiable, strictly increasing and strictly concave in each argument, and homogeneous of degree one. Further, $m(0, v_t) = m(u_t, 0) = 0$ and $m(u_t, v_t) \leq \min\{u_t, v_t\}$ for all u_t and v_t.

destroy the job, which we refer to as a *quit*. For continuing matches, quits occur with probability λ^q, while they occur with the higher probability λ^n for matches that were newly-formed in the preceding period; this reflects the higher observed separation hazards among low-tenure employment relationships. Following a quit, the worker enters the period t unemployment pool and the firm enters the period t vacancy pool. Agents that enter the period t matching pools as a consequence of separation are eligible to be rematched in period t.

If the worker-firm match survives the separation hazards, then the agents negotiate a contract that divides the period t surplus according to the Nash bargaining solution, where π gives the worker's bargaining weight and the threat point is a quit. Given that the worker and firm agree to continue, the match incurs a flow capital cost of κ and engages in production in period t. The output of the match is given by the productivity level z_t, which evolves according to a Markov process.

Equilibrium. Let $\theta_t = v_t/u_t$ denote the vacancy-unemployment ratio. For a worker who begins period t in the unemployment pool, the probability of ending period t in a job match is

$$\frac{m(u_t, v_t)}{u_t} = m(1, \theta_t) = p(\theta_t),$$

where we have made use of the linear homogeneity of the function m. Let S_t indicate the value of surplus for a match that survives the separation hazards in period t. A worker in the unemployment pool receives the benefit b along with a proportion of the surplus from any match made in in period t that survives into period $t + 1$. Thus, the expected present value of current and future receipts for an unemployed worker is given by

$$U_t = b + \beta E_t\Big[p(\theta_t)(1 - \lambda^n)(1 - \lambda^d)\pi S_{t+1} + U_{t+1}\Big], \tag{4}$$

where β indicates the discount factor.

Similarly, for a firm that begins period t in the vacancy pool, the probability of ending period t in a job match is

$$\frac{m(u_t, v_t)}{v_t} = m\Big(\frac{1}{\theta_t}, 1\Big) = q(\theta_t),$$

and the expected present value of the firm's current and future receipts is

$$V_t = -c + \beta E_t \Big[q(\theta_t)(1-\lambda^n)(1-\lambda^d)(1-\pi)S_{t+1} + (1-\lambda^d)V_{t+1} \Big]. \tag{5}$$

Note that the firm receives the outside option value V_{t+1} only if separation occurs as a consequence of a quit, whereas the worker receives U_{t+1} after either obsolescence or a quit. This follows from the fact that the job ceases to exist following obsolescence.

A job match that survives the separation hazards in period t obtains the following expected present value:

$$N_t = z_t - \kappa + \beta E_t \Big[(1-\lambda^q)(1-\lambda^d)S_{t+1} + U_{t+1} + (1-\lambda^d)V_{t+1} \Big]. \tag{6}$$

Equilibrium surplus is thus defined by

$$S_t = N_t - U_t - V_t. \tag{7}$$

Because entry into the vacancy pool entails zero sunk cost, competitive pressure from potential entrant firms will drive the equilibrium value of a vacancy to zero, i.e., $V_t = 0$ must hold for all t. Using (5), this gives rise to the following free entry condition:

$$\frac{c}{q(\theta_t)} = \beta E_t \left[(1-\lambda^n)(1-\lambda^d)(1-\pi)S_{t+1} \right]. \tag{8}$$

Equations (4) and (6)-(8) determine equilibrium paths of U_t, N_t, S_t and θ_t for a given process z_t. The equilibrium solution determines the law of motion for unemployment:

$$u_t = u_{t-1} + [\lambda^d + (1-\lambda^d)\lambda^q](1-u_{t-1}) - p(\theta_{t-1})(1-\lambda^d)(1-\lambda^n)u_{t-1}. \tag{9}$$

Vacancies adjust in each period in order to maintain the relationship $v_t/u_t = \theta_t$.

3.2 Sunk cost model

Motivation. The standard model has the key property that firms incur no costs to create job positions. Thus, unmatched firms are dissuaded from entering the vacancy pool only by the posting cost c. Equivalently, every unmatched firm has an implicit position to fill, and the model makes no distinction between the creation of positions and the filling of vacancies.

Evidence on job and worker flows, however, points to important differences between job positions and vacancies. Davis, Haltiwanger and Schuh (1996), in particular, have stressed that the rate at which workers move across jobs greatly exceeds the rate at which filled jobs are created and destroyed. This difference can be measured by comparing steady-state match separation versus job destruction rates. Drawing on several sources of information, we calculate below that nearly 19 percent of employed workers separate from their jobs over the ensuing quarter, while the rate of job destruction itself is only about 8 percent per quarter. Thus, the "churning" of workers across filled job positions amounts to over 10 percent of employment each quarter.

Such large-scale churning stems from a strong tendency of employers to maintain job positions when workers depart. Indeed, Blanchard and Diamond (1990) argue that roughly 85 percent of quits are replaced, while Holzer (1989) finds that firms with a higher rate of worker turnover tend to post vacancies at a higher rate. Evidently, job positions become more valuable once they are created, indicating the presence of sunk costs for job creation. It is therefore sensible to extend the standard model by introducing sunk costs for creating new job positions. This modification leads to a meaningful distinction between job and worker flows, and also generates sluggishness in the adjustment of positions that carries over to vacancies.

Model description and equilibrium. The standard model is augmented as follows. At the start of each period t, a number of opportunities for creating new job positions is randomly distributed among the potential entrant firms. Each opportunity allows an entrant to create one position by incurring a sunk cost of K. The positions differ in their attractiveness; for simplicity, we assume that heterogeneity of positions is reflected by differences in sunk costs. Let the continuous function $F(K)$ give the total mass of opportunities in a period that have sunk cost no greater than K.

When an entrant receives an opportunity, it observes the aggregate productivity level for period t, and then chooses whether or not to create the position at a cost of K. If it chooses to create the position, then it may enter the vacancy pool for period t; moreover, it does not have to pay the sunk cost for this position in any future period, whether or not it is matched with a worker. However, at the start of each period the position becomes obsolete with probability λ^d, and it ceases to exist thereafter. If the entrant chooses not to create the position, then the opportunity becomes unavailable in future periods.[13]

For the sunk cost model, the free entry condition (8) is replaced by

$$V_t = \widehat{K}_t, \tag{10}$$

where $\widehat{K}_t$ is the *entry margin*. Potential entrants with $K \leq \widehat{K}_t$ at the start of period t will choose to enter the vacancy pool, while those with $K > \widehat{K}_t$ will choose to stay out. *New openings*, denoted by e_t, are determined by

$$e_t = \int_0^{\hat{K}_t} dF(K). \tag{11}$$

The implied law of motion for vacancies is

$$v_t = (1 - \lambda^d)\Big[v_{t-1} + \lambda^q(1 - u_{t-1}) - (1 - \lambda^n)q(\theta_{t-1})v_{t-1}\Big] + e_t. \tag{12}$$

Equilibrium paths of U_t, N_t, S_t, $\widehat{K}_t$, v_t and e_t are determined by equations (4)-(7), (10)-(11), and the identity $v_t/u_t = \theta_t$, given the z_t process and the predetermined variables u_{t-1} and v_{t-1}. Vacancies become a state variable in this model due to the fact that only finitely many jobs can profitably be created in any given period, even though the value of an established vacancy is strictly positive.

3.3 Functional forms

We adopt the following functional forms. For the matching function we use the form introduced by den Haan et al. (2000):

$$m(u_t, v_t) = \frac{u_t v_t}{(u_t^l + v_t^l)^{1/l}}. \tag{13}$$

Since this function always lies below unity, it does not require truncation, in contrast to the standard Cobb-Douglas specification. The stochastic process for productivity

[13] We do not model the determination of $F(K)$, nor do we allow a firm to hold its opportunity for more than one period, for reasons of parsimony. Under these assumptions there is no need to keep track of the distribution of available opportunities across potential entrants. Feedbacks between this distribution and search activities represent a potentially important additional source of propagation effects, however, as discussed in Conclusion.

assumes the standard first-order autoregressive form:

$$\ln z_t = (1-\rho)\ln z + \rho \ln z_{t-1} + \varepsilon_t,$$

where ε_t is normally distributed with mean zero and standard deviation σ_ε. Note that the realization of ε_t is observed by all workers and firms in the economy at the start of each period t.

Finally, for the sunk cost model we take the opportunity function $F(K)$ to be linear in K with slope $1/\overline{K}$, i.e.:

$$F(K) = \frac{K}{\overline{K}}.$$

Note that the standard matching model can be recovered as a limiting case by letting $\overline{K}$ approach zero.[14]

3.4 Measurement and calibration

To evaluate the two versions of the matching model, we generate artificial data that conforms to the U.S. data considered in Section 2. Our identification procedure establishes a direct correspondence between the theoretical driving process $\ln z_t$ and the exogenous productivity component $\ln \widehat{z}_t$. The theoretical matching pools v_t and u_t, however, do not represent represent end-of-period stocks, but rather cumulations of all workers and firms available for matching in period t. Measured end-of-period stocks are obtained from the following formulas:

$$\begin{aligned} v_t^m &= v_t - m(u_t, v_t), \\ u_t^m &= u_t - m(u_t, v_t). \end{aligned}$$

The variable $\ln(v_t^m/u_t^m)$ thus corresponds to the empirical matching intensity variable $\ln vu_t$. Finally, the measured employment level is

$$n_t^m = 1 - u_t^m,$$

[14] This may be seen as follows. For small values of $\overline{K}$, inflows of entrant firms would be very large if V_t were not close to zero. Such large inflows would raise θ_t, however, and thereby drive V_t to zero. Thus, V_t must be close to zero, and the standard free entry condition $V_t = 0$ must hold in the limit.

and $\ln n_t^m$ corresponds to the employment variable $\ln emp_t$.[15]

We next summarize the calibration of the model at quarterly frequency; details of these calculations are given in the Appendix, and the chosen parameter values are presented in Table 1. First, information from Fallick and Fleischman (2004) and Anderson and Meyer (1994) is used to construct estimates of quarterly separation rates for new and continuing job matches. These are given by 42.12 percent and 12.79 percent for the first and subsequent quarters of a match, respectively, giving an overall average separation rate of 18.89 percent per quarter.

To measure quarterly job finding rates we must account for the existence of workers who are out of the labor force but still available for work. As argued in the Appendix, it is reasonable to equate the number of such available workers to the number of officially unemployed workers. Under this assumption, we can construct a monthly transition matrix for workers between the states of employed at a new employer, employed at the same employer, unemployed, and out of the labor force. This matrix yields quarterly rates of transition from unemployment and out of the labor force to employment of 58.34 percent and 65.79 percent, respectively. The implied overall job finding rate is $p(\theta) = .6206$ per quarter.

We next combine this job finding rate with the vacancy rate estimated from the Job Opening and Labor Turnover Survey (JOLTS) to obtain a vacancy filling rate of $q(\theta) = .8541$ per quarter. From these estimates we calculate the value $l = 2.413$ for the matching function parameter. Further, we can extract the obsolescence rate λ^d by combining our estimated separation and vacancy filling rates with the average job destruction rate of roughly 8 percent per quarter, obtained from Faberman (2004). This gives an estimate of $\lambda^d = .063$ per quarter. The associated quit rates for new and continuing matches are $\lambda^n = .382$ and $\lambda^q = .069$ per quarter, respectively.

We adopt standard values for the parameters β, π, z, ρ and σ_z; see Table 1. To select the parameters c, κ and b in the standard model, we impose two restrictions in addition to the free entry condition (8): per-period wages in the steady state must

[15] As a check on our calibration, note that the measured unemployment rate in the steady state is $u^m = .0996$. This is a reasonable value, in view of the fact that under our calibration the worker matching pool includes a group of out of the labor force workers equal in size to the officially unemployed group.

match the measured labor share of 65 percent; and the unemployment benefit must amount to 65 percent of net match output, $z - \kappa$. Note that the latter value lies between the values of 40 percent and 94.3 percent suggested by Shimer (2005) and Hagedorn and Manovskii (2005), respectively, and close to the value of 75 percent proposed by Costain and Reiter (2005). These restrictions yield the values $c = .193$, $\kappa = .293$ and $b = .460$ for the standard model. For the sunk cost model, we use the free entry condition (10) and the other two restrictions to pin down the parameters $\overline{K}$, κ and b for given values of c. In turn, we choose c to match the standard deviation of $\ln(v_t^m/u_t^m)$ generated by simulated data to the standard deviation of the empirical variable $\ln vu_t$. The resulting parameter values are $\overline{K} = .682$, $\kappa = .288$, $b = .463$ and $c = .183$.

4 Results

The standard and sunk cost models are solved using the nonlinear global projection method called the collocation parameterized expectation algorithm (PEA); see Christiano and Fisher (2000) for a general discussion of this method. Details of the procedure are given in the Appendix. The second moment statistics of the model economies are based on 500 simulated samples. Each sample consists of 400 periods, where the first 200 observations are ignored to randomize initial conditions, and the last 200 observations are used to compute the statistics.

4.1 Empirical evaluation

Dynamic correlations. Figure 5 presents the dynamic correlations of productivity with matching intensity and employment, calculated from simulated data, for the standard and sunk cost models, along with the empirical correlations originally reported in Figure 2. The productivity-matching intensity correlations generated by the standard model, shown in the upper panel, exhibit an unrealistically sharp peak at zero lag, and fail to capture the flatness of the empirical correlations between lags of zero and two quarters. Further, as seen in the lower panel, the model makes the counterfactual prediction of almost perfect contemporaneous correlation between productivity and employment, and the correlations at lags of 2 and 3 quarters are too low. These results clearly show that the standard model does not generate realistic dynamics.

The sunk cost model, in contrast, yields correlation profiles that essentially duplicate the patterns seen in the data: the simulated correlations are flat over the correct ranges of lags, with the correct phasing. Notably, the sunk cost model does not produce the sharp peaks at zero lag that are characteristic of the standard model. Quantitatively, the correlations of matching intensity with current and lagged productivity are somewhat too high in the sunk cost model, as may be seen in the upper panel of Figure 5. The model produces a remarkably close match with the productivity-employment correlations, however, as the lower panel shows.

Impulse responses. The deficiencies of the standard model can be further illustrated using impulse responses. Figure 6 overlays the model-based impulse responses for a one standard deviation shock to productivity with the VAR-based impulse responses reported in Figure 3. The top panel shows the dynamics of our calibrated productivity driving process in comparison with the estimated process. The middle panel indicates the response of matching intensity. On impact, the simulated variable $\ln(v_t^m/u_t^m)$ jumps upward by over twice as much as the estimated response. Following this, it returns monotonically to the steady state. Thus, the standard model significantly overstates the effect of the shock on impact, and entirely misses the subsequent hump-shaped response pattern. Reflecting this behavior, the simulated variable $\ln n_t^m$ jumps upward by a large amount in the period of impact, and its subsequent upward movements are slight and short-lived. While the presence of matching frictions introduces some persistence into the employment response, the amount of added persistence is quantitatively tiny in the standard model.

Impulse responses for the sunk cost model, together with the empirical impulse responses, are graphed in Figure 7. As seen in the middle panel, the jump in $\ln(v_t^m/u_t^m)$ matches the empirical value closely, and the subsequent response also displays a realistic hump-shaped pattern: matching intensity rises for four quarters, then returns to the steady state. The third panel shows that the response of $\ln n_t^m$ continues to exhibit the counterfactual upward jump in the period of impact, but the size of the jump is about half that of the standard model.[16] Moreover, the subsequent dynamics closely

[16]Under our timing assumptions, the productivity shock induces a contemporaneous change in entry

match the empirical hump shape. Overall, the sunk cost model does a much better job capturing the dynamic characteristics of the employment adjustment process.

4.2 Analysis

Vacancy adjustment and new openings. The dynamic properties of the standard and sunk cost models are linked to the behavior of vacancies. This is illustrated in Figure 8, which considers the vacancy adjustments associated with the calibrated standard and sunk cost models. The upper panel depicts the impulse response of the measured vacancy stock v_t^m for a one-standard-deviation productivity shock, and the bottom panel shows the corresponding net changes in the vacancy stock. The variables are expressed as level deviations from their steady-state values.[17]

In the standard model, the vacancy stock spikes upward in the period of the shock, and then decreases monotonically in tandem with productivity. This means the initial large inflow of vacancies is immediately reversed by outflows, as the bottom panel shows. Since there is no sunk cost for entering the matching pool, potential entrant firms can freely alter their job creation decisions to compete away rents following a shock. Consequently, the vacancy stock adjusts directly with productivity.

In the sunk cost model, the initial upward jump is much smaller, and vacancies continue to rise for several periods following the shock. The bottom panel of Figure 8 shows how the increases in vacancies gradually diminish, in contrast to the abrupt adjustment exhibited by the standard model. This smooth adjustment of the changes in vacancies underlies the persistent responses of the vacancy stock, matching intensity and employment exhibited by the sunk cost model.

Smooth adjustment arises because of the way sunk job creation costs affect the responses of potential entrant firms. The initial upward jump in vacancies following a

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into the vacancy pool, and the job matches generated by these added vacancies are counted as higher employment at the end of the period. The measured jumps in both matching intensity and employment would be reduced by a measurement convention that incorporated some averaging of beginning- and end-of-period levels.

[17] Other graphs in the paper are expressed as log deviations from steady-state values. In this instance, considering level deviations allows us to decompose net vacancy changes into separate gross inflows and outflows.

positive productivity shock is limited by the availability of profitable opportunities for creating new openings, reflected in the opportunity function. Moreover, persistently higher productivity pushes the equilibrium entry margin $\widehat{K}_t$ above its steady state value. This raises the volume of new openings relative to the steady state, as may be seen in (11), and leads to further increases in vacancies. As the entry margin returns to the steady state along with productivity, the increases in vacancies gradually diminish. Similar reasoning applies with respect to negative productivity shocks: lower productivity drives down the entry margin and new openings, causing vacancies to decrease for several periods following a shock.

The quantitative importance of new openings can be seen by decomposing the net changes in the measured vacancy stock into separate outflow and inflows, using (12):

$$\begin{aligned}\Delta v_t^m &= \underbrace{-\lambda^d v_{t-1}}_{obsolescense} \underbrace{-\left[m_t - (\lambda^d + (1-\lambda^d)\lambda^n)m_{t-1}\right]}_{hires} \\ &\quad + \underbrace{(1-\lambda^d)\lambda^q(1-u_{t-1})}_{repostings} + \underbrace{e_t}_{new\_openings} .\end{aligned}$$

Observe that the net changes comprise gross outflows due to obsolescence and hires, together with gross inflows due to repostings following quits and new openings. Figure 9 plots the impulse responses for these gross flows, which make up the impulse response for the sunk cost model shown in the bottom panel of Figure 8. The graph clearly shows that vacancy adjustment is almost entirely driven by new openings and hires. Further, the inflows from new openings lead the outflows from hires, as a consequence of the matching process. In the four quarters following the shock, new openings inflows exceed hiring outflows, accounting for the vacancy increases observed in Figure 8.

Empirical evidence. Although new openings play a crucial role in shaping vacancy stock behavior in the sunk cost model, the available vacancy data do not permit a direct empirical assessment of this role. The Job Openings and Labor Turnover Survey (JOLTS), however, does provide information about vacancy stocks, quits, layoffs and hires for 2001:Q1 to 2005:Q1, and this allows us to obtain an indirect measure of new openings via the following stock-flow relationship:

$$vac_t = (1-\lambda^d)vac_{t-1} + quits_t - hires_t + e_t. \tag{14}$$

Our analysis of steady state separation rates suggests that $\lambda^d = .063$ provides a reasonable estimate of the vacancy withdrawal rate. We can combine this figure with the JOLTS data to impute an estimate of e_t from (14).

Consider the first quarter of 2001, which we view as the most typical within the limited JOLTS sample. For this quarter, the ratio of cumulative hires to end-of-quarter vacancy stock is about 3.5. Our imputed inflow of new vacancies amounts to 1.5 times the end-of-quarter stock. Thus, new openings amount to nearly half of total hires within the quarter. Figure 10 plots indices of imputed new openings, vacancies, hires and quits, treating 2001:Q1 as the base period. The four series fluctuate by comparable amounts over the sample period, and, in particular, new openings exhibit significant variability. Moreover, new openings move strongly upward in 2003:Q2, leading the upward movement of vacancies by four quarters. The upward movements of hires and quits lag those of new openings and are less steep. Based on this limited evidence, it appears that new openings adjust sooner and by a greater magnitude in comparison with the other components.

4.3 Amplification of shocks

Recent literature has focused on the ability of the matching model to amplify productivity shocks in order to explain the volatility of the vacancy-unemployment ratio. This question may be addressed using our calibrations of the standard and sunk cost models. Table 2 presents the standard deviations of matching intensity, employment, vacancies and unemployment obtained from the detrended data, along with corresponding measurements from simulated data for the standard and sunk cost models. The standard deviation of the productivity process is 0.019 in all three cases.

Under the calibrated parameter values, the volatilities in the standard model exceed the empirical levels for all variables except unemployment. As for the sunk cost model, recall that our calibration procedure matches the standard deviation of $\ln(v_t^m/u_t^m)$ to that of $\ln vu_t$. The standard deviations of the four variables are all lower than those of the standard model, but employment and vacancies remain somewhat more volatile than in the data.

Shimer (2005) shows that the amplification of productivity shocks is sensitive to the value of b, with larger values of b leading to greater amplification. To assess the sensitivity of amplification under our calibration, we reevaluate the standard model using Shimer's suggested value of $b = .40$, with the other parameters adjusted to maintain the calibration requirements.[18] Under the alternative parameter values, the volatility of matching intensity is reduced to 0.199 from 0.363. Broadly speaking, our results thus confirm that the matching model may generate insufficient volatility of the vacancy-unemployment ratio, but the amplification mechanism is not nearly so weak as suggested by Shimer (2005).

5 Conclusion

This paper has evaluated the cyclical dynamics of job matching intensity, as measured by the vacancy-unemployment ratio, and employment, where business cycles are driven by exogenous shocks to labor productivity. The two variables respond to shocks in similar ways, with employment responses lagging matching intensity responses by one quarter. Both variables display the "hump-shaped" dynamics that are commonly observed throughout the empirical business cycle literature.

We show that the standard matching model, as exemplified by Pissarides (2000, ch. 1), cannot account for the observed dynamic patterns. Because potential entrant firms are able to respond easily to shocks, the vacancy pool adjusts in tandem with productivity. As a consequence, most of the adjustment of matching intensity and employment occurs in the period of the shock, leading to counterfactual dynamic correlations and impulse responses. Introducing a sunk cost for creating job positions causes the stock of vacancies to adjust sluggishly, leading to realistic dynamic behavior. Our results suggest that sunk job creation costs may play a central role in explaining cyclical adjustment. Moreover, the modified matching model can generate highly realistic employment dynamics without resort to any kind of consumption-smoothing mechanism.

The paper has relied on a number of simplifying assumptions that we believe are worth evaluating in future research. First, the arrival of new opportunities for creating

[18] In particular, κ and c have been changed to .245 and .352, respectively, while the other parameters remain the same.

job openings is taken to be exogenous and constant across periods. A more complete theoretical model would tie this distribution to underlying investments made by firms (e.g., in physical capital or R&D). This creates further possibilities for longer-run feedbacks from the labor market to productivity that might serve as important additional sources of propagation. Relatedly, the assumed equivalence of newly-created and pre-existing job positions can be modified by incorporating a vintage structure, whereby new jobs enjoy higher productivity. This permits the endogenous obsolescence of jobs and the turnover of workers to be considered as separate flows within a common framework.[19] Finally, we have ignored the effects of cyclical variation in the relative sizes of the pools of unemployed workers and workers out of the labor force but available for work, and in the flows between these pools. Changes in the characteristics of the latter pool may, however, represent another important source of longer-run propagation effects.

6 Appendix

6.1 Calibration

Separation rates. The monthly average separation rate for job matches can be calculated as the average of monthly outflows from job matches divided by total employment. Monthly outflows consist of worker transitions to new job matches, unemployment, and out of the labor force. Fallick and Fleischman (2004) have recently provided measures of aggregate U.S. stocks and flows of workers, derived from the Current Population Survey (CPS) for 1994 and 1996-2003, that include these three transitions. The numbers reported by Fallick and Fleischman imply outflows from unemployment that are inconsistent with steady state, so we adjust them slightly to achieve consistent steady-state flows and stocks. Adjusted flows and stocks are reported in Table 3. The

[19] Aghion and Howitt (1994), Caballero and Hammour (1994) and Mortensen and Pissarides (1998), for example, analyze endogenous obsolescence in models that combine embodied technological progress with search/matching frictions. None of those papers distinguish between worker and job turnover. In recent work, Hornstein, Krusell, and Violante (2004) adopt a specification similar to ours for purposes of analyzing the unemployment experiences of the U.S. and Europe. They focus on comparison of steady states, however, rather than cyclical adjustment.

units represent percentages of noninstitutional civilian population aged 16 and over. Observe that 4.15 new matches are created each month, while total employment is 63.15; thus, the implied steady-state separation rate is 4.15/63.15 = .0674 per month.

This figure is roughly corroborated by Anderson and Meyer (1994), who consider a large panel of workers derived from U.S. state unemployment insurance records for 1978-1984. In their sample, 17.23 percent of job matches experience a permanent separation each quarter, which translates into an average monthly separation rate of .0611.

Anderson and Meyer (1994) also find that 43.42 percent of job matches observed during a quarter have duration of less than one year, which cannot hold unless these newer matches experience separation at rates in excess of .0611. We incorporate this fact in a parsimonious way by specifying a higher separation rate in the first month of a match.[20] We calculate values for monthly separation rates in the first and subsequent months such that, in the steady state, 43.42 percent of matches have duration of less than one year when the flow of new matches is 4.15 per month. This yields monthly separation rates of .3663 and .0446 in the first and subsequent months, respectively, which translate into quarterly separation rates in the first quarter and subsequent quarters as follows:

$$\lambda^d + (1 - \lambda^d)\lambda^n = .4212, \tag{15}$$

$$\lambda^d + (1 - \lambda^d)\lambda^q = .1279. \tag{16}$$

Job finding and vacancy filling rates. As seen in Table 3, a significant number of workers flow directly into employment from out of the labor force each month. Thus, our measurements must account for a pool of workers who are out of the labor force but still available for work. Since a total of 2.45 workers flow into the labor force each month, there must be at least this many available workers at the start of the month. Further, assume that proportion ω of the workers who remain out of the labor force

[20] This calibration procedure matches the distribution of employment durations, but not the duration-specific survival probabilities, since separations are crowded into the first month. Fitting the survival probabilities would require match-specific separation rates, and the number of categories of employed workers would correspondingly expand. In this paper we have opted instead for the simplest specification that allows new matches to separate at a higher rate.

at the end of the month are also available, corresponding to those whom the CPS records as not engaging in search activity but still willing to take a job if contacted by a searching firm. Evidence from the 1994 CPS, discussed in Castillo (1998, p. 36), indicates that this group amounts to anywhere between 22.5 and 77.5 percent of the officially unemployed population, yielding a range of ω between .025 and .085. We settle on the value of ω that equates the number of officially unemployed workers to the number of available workers, i.e., $3.4 = 2.45 + 31\omega$, or $\omega = .0306$.

Monthly steady-state transition rates for newly separated, unemployed and available workers into new employment, unemployment and out of the labor force may be calculated from Table 3. For example, monthly job finding rates for newly separated, unemployed, and available workers are $1.6/4.15 = .386$, $.85/3.4 = .25$ and $1.7/3.4 = .5$, respectively. To calculate quarterly transition rates, we construct a monthly transition matrix for the worker states of employed at new employer, employed at same employer, unemployed, and out of labor force but available. Using this matrix, we calculate that a worker who begins the quarter in the unemployed state will end the quarter in one of the two employed states with probability .5834. The corresponding figure for an available worker is .6579. Averaging over unemployed and available workers gives an overall job finding rate of $p(\theta) = .6206$ per quarter. Using this job finding rate together the total separation rates for new and established matches, i.e., $\lambda^d + (1 - \lambda^d)\lambda^n$ and $\lambda^d + (1 - \lambda^d)\lambda^q$, respectively, in the steady-state flow balance equation for unemployment, we can determine the steady-state unemployment rate in the model u. This allows us to obtain the number of matches from $m = p(\theta)u$ and the measured unemployment rate $u^m = u - m$.

Next we use evidence from JOLTS to determine the vacancy filling rate $q(\theta)$. JOLTS measures the vacancy rate as follows:

$$vac\_rate = \frac{v^m}{1 - u^m + v^m}.$$

According to the JOLTS, the monthly average level of the vacancy rate is 2.5 percent over the period of December 2000 through May 2005. Given the widely-recognized weakness of the U.S. labor market between 2001 and 2003, the average vacancy rate over the longer horizon is likely to have been somewhat greater. We therefore set $vac\_rate = .03$. Using this evidence together with the previously calculated value for

u^m allows us to solve the above equation for v^m, and from this we obtain $q(\theta) = \frac{m}{v} = \frac{m}{v^m+m} = .8541$. Having determined the steady-state values of u, v and m, the efficiency parameter of the matching function l is solved using the matching function, yielding $l = 2.413$. Furthermore, using the numerical values for λ^d, λ^q, u, v and $q(\theta)$ in the steady-state version of the law of motion for vacancies (12), we can calculate the steady-state value for new openings $e = .058$.

Job flows. In the sunk cost model, we need to distinguish the obsolescence rate λ^d from the quit rates λ^n and λ^q.[21] To pin down these three parameters, we combine the worker turnover rates with the job flow rates reported by the Business Employment Dynamics (BED) statistics. According to Faberman (2004), the quarterly job flow rates in the private sector averaged around 8 percent over the period 1990 through 2003. We equate this measured job destruction rate to the corresponding magnitude in the model:

$$\lambda^d + (1-\lambda^d)(1-q(\theta))\left[\frac{m}{1-u+m}\lambda^n + \frac{1-u}{1-u+m}\lambda^q\right] = .08. \tag{17}$$

The first term on the left-hand side measures job obsolescence, while the second term reflects jobs that become vacant as a result of quits, and remain vacant at the end of the period. Since job destruction in the BED is computed from employment changes over a quarter, both sources of employment change must be included. Using the previously determined values of $q(\theta)$, u, and m, equations (15), (16), and (17) may be solved for the values $\lambda^d = .063$, $\lambda^n = .382$ and $\lambda^q = .069$.

Other parameters. We adopt the standard values $\beta = .99$ and $\pi = .50$ for the discount factor and worker bargaining weight. Productivity is normalized by setting $z = 1$, and the parameters ρ and σ_ε are selected to match the dynamics generated by our Section 2 estimates.

For the standard model we make use of the value function equations (4)-(7) in the steady state, together with the free entry condition $V = 0$, to specify the remaining

[21] In view of the free entry condition $V_t = 0$, the equilibrium conditions for the standard model are influenced only by the combined separation rates $\lambda^d + (1-\lambda^d)\lambda^n$ and $\lambda^d + (1-\lambda^d)\lambda^q$. Thus, we do not need to separately measure λ^d for purposes of evaluating the standard model.

parameters. The flow posting cost c is determined by (8). The flow capital cost κ is chosen so that the per-period wage payment to the worker matches the measured labor share of .65:

$$\left[\lambda^d + (1-\lambda^d)\lambda^q\right]\pi S + (1-\beta)U = .65. \tag{18}$$

The unemployment benefit b is selected so that $b = .65(z-\kappa)$, i.e., the unemployment benefit is 65 percent of net match output. This lies between the values of 40 percent and 94.3 percent suggested by Shimer (2005) and Hagedorn and Manovskii (2005), respectively, and close to the value of 75 percent proposed by Costain and Reiter (2005). This procedure yields the values $c = .193$, $\kappa = .293$ and $b = .460$.

For the sunk cost model we must also specify the parameter $\overline{K}$ of the opportunity function. For a given value of c, the parameters κ and b are chosen to satisfy (18) and $b = .65(z-\kappa)$, while $\overline{K}$ is chosen so that the free entry condition $V = \widehat{K}$ is consistent with our previously estimated value of e:

$$.058 = \frac{V}{\overline{K}}.$$

It remains to specify the parameter c. There is little direct evidence as to the level of flow posting costs. The responsiveness of $\widehat{K}_t$ to changes in z_t is highly sensitive to c, however, so we use second moment properties of the model to pin down this parameter. From the VAR model (1), the estimated standard deviation of $\ln vu$, conditional on $\ln \widehat{z}_t$, is .299. Thus, we choose c so that the standard deviation of the vacancy-unemployment ratio, based on end-of-period stocks, in the simulated data lies as close as possible to the empirical value. This yields the values $\kappa = .288$, $b = .463$, $\overline{K} = .682$ and $c = .183$. The implied standard deviation of the vacancy-unemployment ratio is .303. Table 1 summarizes the choices of parameter values for both models.

6.2 Solution Method

Standard Model. For the standard model, the aggregate state of the economy for period t is a set of variables $\{m_{t-1}, u_{t-1}, z_t\}$.[22] We set the aggregate state space to $[m-$

[22]Note that m_{t-1} is in the set of period-t state variables because new matches and preexisting matches are subject to different separation hazard rates. In the sunk cost model below, v_{t-1} is also a state variable, and therefore we do not need to treat m_{t-1} as a separate state variable.

$0.1, m+0.1\big] \times \big[u-0.1, u+0.1\big] \times \big[\exp\big(-4\sqrt{\sigma_z^2/(1-\rho^2)}\big), \exp\big(4\sqrt{\sigma_z^2/(1-\rho^2)}\big)\big]$, where m and u are the steady-state values of the number of new matches and unemployment. We parameterize the right-hand side of equations (4), (6) and (8) as a tensor product of second-order Chebyshev polynomials of the three state variables. Note that each function has 27 ($= 3^3$) unknowns, and thus there is a total of $81 (= 27 \times 3)$ unknowns to be determined.

Consider starting at an arbitrary grid point in the state space. For an initial guess of the unknown parameters of the approximating functions, we use the unemployment law of motion (9) to obtain the next period unemployment u_t. Using the approximating function for the right-hand side of the free entry condition (8) and the initial guess of its unknown parameters, we can obtain v_t. The u_t and v_t allow us to determine the number of matches formed m_t. Once we obtain the next period values of the state variables, we compute the conditional expectations appearing in the value functions from the distribution of the productivity shock ϵ_t. The conditional expectations associated with the productivity shock are computed via the Gauss-Hermite quadrature with 5 nodes.

The conditional expectations for each value function are evaluated at 27 grid points that are chosen by finding zeros of Chebyshev polynomials of each of the three state variables, and taking all possible combinations of the zeros. The new set of coefficients of the approximating functions are obtained by equating the right-hand side of equations (4), (6) and (8) to the values of the approximating functions at the 27 grid points. Since there are 27 coefficients in each approximating function, this uniquely pins down the new set of coefficients. This process is iterated until convergence of the 81 Chebyshev coefficients is achieved. The convergence criterion is set to 10^{-8}. Finally, we simulate a long time series $(200,000$ observations) using the obtained solution functions in order to check that the economy remains within the specified state space.

Sunk Cost Model. We solve the sunk cost model in a similar way. The period-t state variables in this economy consist of $\{u_{t-1}, v_{t-1}, z_t\}$. The state space is defined as $\big[v-0.1, v+0.1\big] \times \big[u-0.1, u+0.1\big] \times \big[\exp\big(-4\sqrt{\sigma_z^2/(1-\rho^2)}\big), \exp\big(4\sqrt{\sigma_z^2/(1-\rho^2)}\big)\big]$, where v and u are the steady-state values of vacancies and unemployment. This time we parameterize the right-hand side of equations (4), (5) and (6), again as a tensor product of second-order Chebyshev polynomials of the state variables. Because there are three state variables and three functions to be parameterized, there is the same number of unknowns (= 81).

Using the initial guess of the set of unknown parameters for the parameterized equation (5), the entry conditions (10) and (11) pin down the entry level e_t at each grid point. Using the law or motion for vacancies (12), we can then obtain the next period value of vacancies v_t corresponding to the initial grid point. The next period unemployment u_t is also obtained from equation (9). Given the distribution of the productivity shock ϵ_t, we can compute the conditional expectation of the right-hand side of equations (4), (5) and (6). Each of the three conditional expectations is evaluated at 27 grid points, and the new set of coefficients is obtained by equating the right-hand side of equations (4), (5) and (6) to the values of the approximating functions at those grid points. The iteration process continues until convergence of the coefficients is achieved.

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Table 1: Parameter Values

symbol	description	calibrated value	
		standard	sunk cost
l	efficiency parameter of the matching function	2.413	same
λ^d	rate of obsolescence	0.063	same
λ^n	separation rate for new matches	0.382	same
λ^q	separation rate for preexisting matches	0.069	same
β	discount factor	0.99	same
π	bargaining weight of workers	0.5	same
z	steady-state value of labor productivity	1.0	same
ρ	autoregressive parameter of $\ln z_t$	0.93	same
σ_z	standard deviation of the productivity shock	0.007	same
c	flow vacancy posting cost	0.193	0.183
κ	flow capital cost	0.293	0.288
b	utility from leisure	0.460	0.463
$\overline{K}$	slope parameter of the opportunity function	n.a.	0.682

Table 2: Volatilities

data	$\ln v_t/u_t$	$\ln emp_t$	$\ln v_t$	$\ln u_t$
	0.299	0.0087	0.131	0.179

model	$\ln v_t^m/u_t^m$	$\ln n_t^m$	$\ln v_t^m$	$\ln u_t^m$
standard	0.363	0.0161	0.225	0.141
sunk cost	0.303	0.0134	0.181	0.119

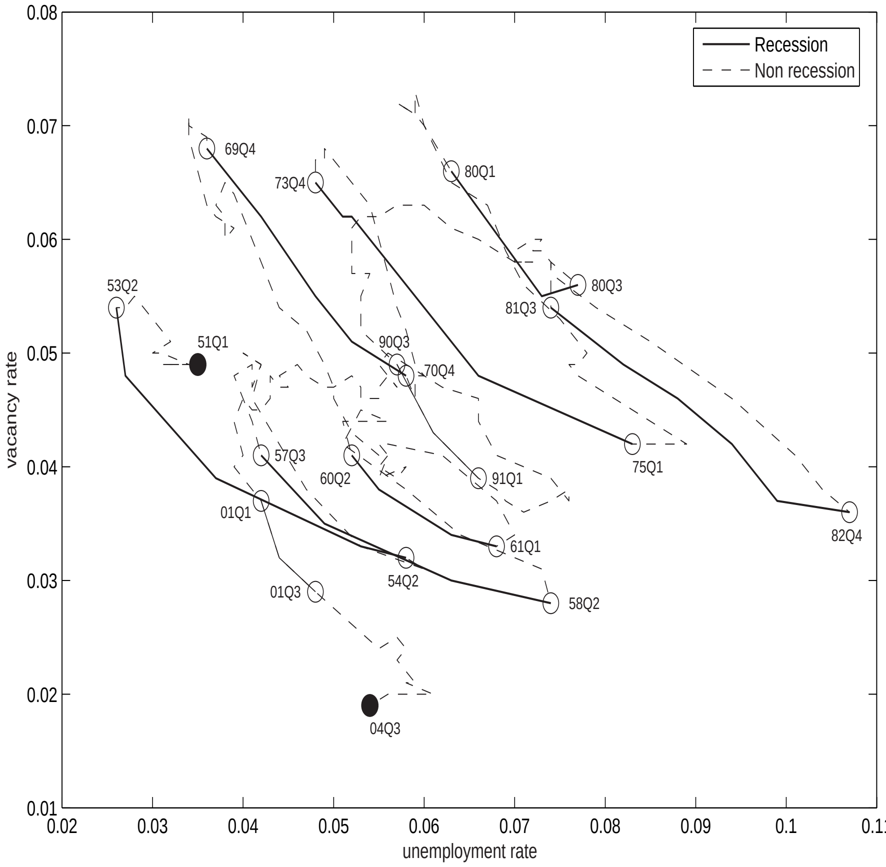
Notes: Volatilities of the empirical data are conditional standard deviations.

Table 3: Labor market transition matrix

		End of Month				
		E^s	E^n	U	N	
Start	$E^s \cup E^n$	59.0	1.60	0.85	1.70	63.15
of	U		0.95	1.70	0.75	3.40
Month	N		1.60	0.85	31.00	33.45
		59.0	4.15	3.40	33.45	

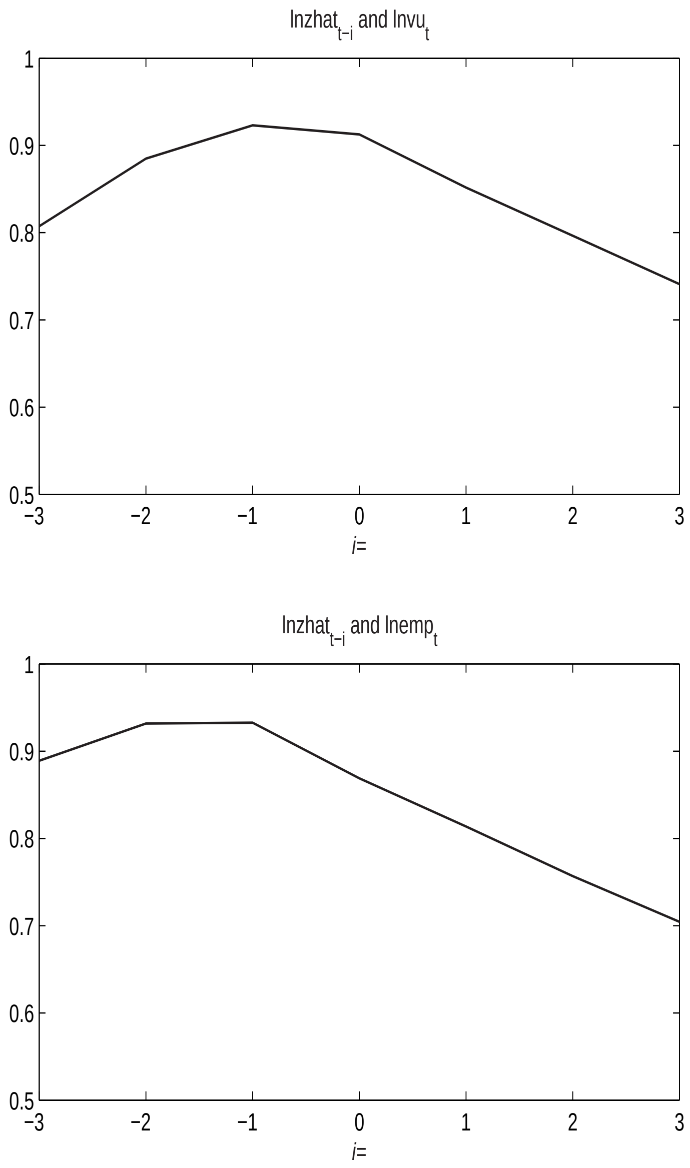
Notes: Computed from Fallick and Fleischman, Table 2 (2001, p11). E^s: same employer, E^n: new employer, U: unemployed, N: out of labor force. Outflows from U in their table are too large to be consistent with steady state. To adjust for this, we subtracted 0.05 from $U \rightarrow E^n$ flows and $U \rightarrow N$ flows, and added 0.05 to $E^n \rightarrow U$ and $N \rightarrow U$ flows. Units are percentages of civilian noninstitutional population aged 16 and over.

Figure 1: Beveridge curve



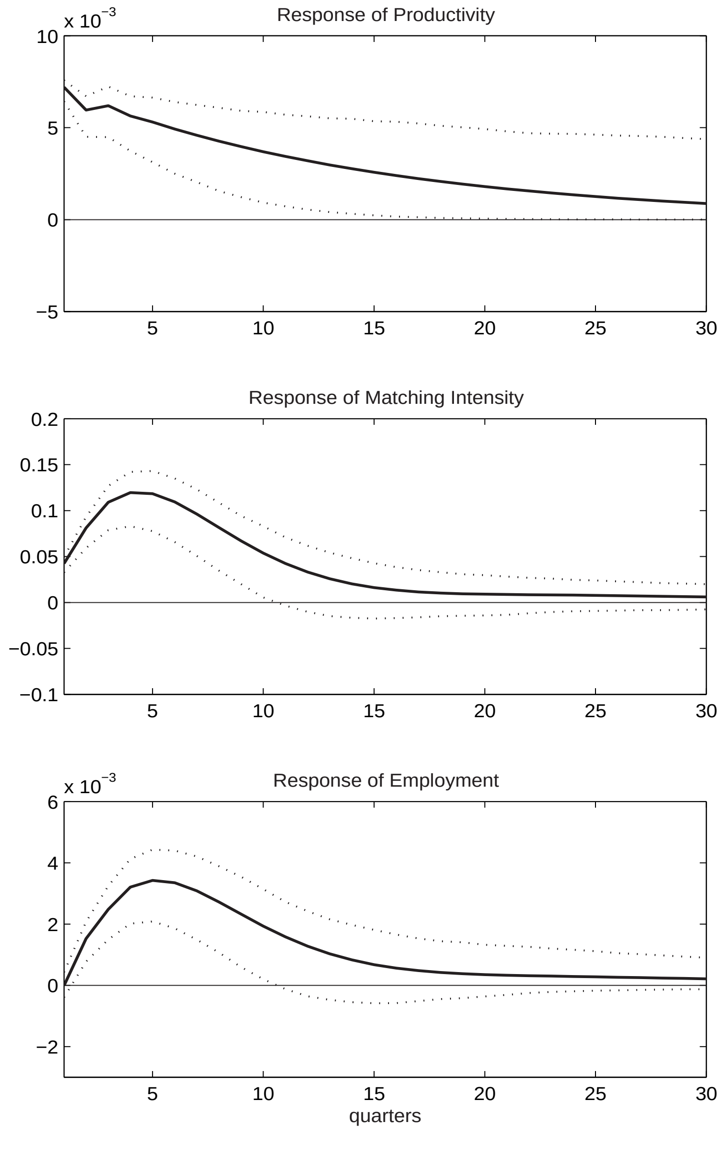
Notes: Quarterly averages of seasonally adjusted monthly data. The vacancy rate is the number of newspaper help-wanted ads divided by the sum of employment and help-wanted ads. Sample period: 1951Q1-2004Q3.

Figure 2: Empirical dynamic correlations



Notes: Plotted are conditional correlations.

Figure 3: VAR-based impulse responses to one-s.d. productivity shock



Notes: Dotted lines are 90% confidence bands computed via Monte-Carlo simulations with $1,000$ replications.

Figure 4: Responses of vacancies and unemployment to one-s.d. productivity shock

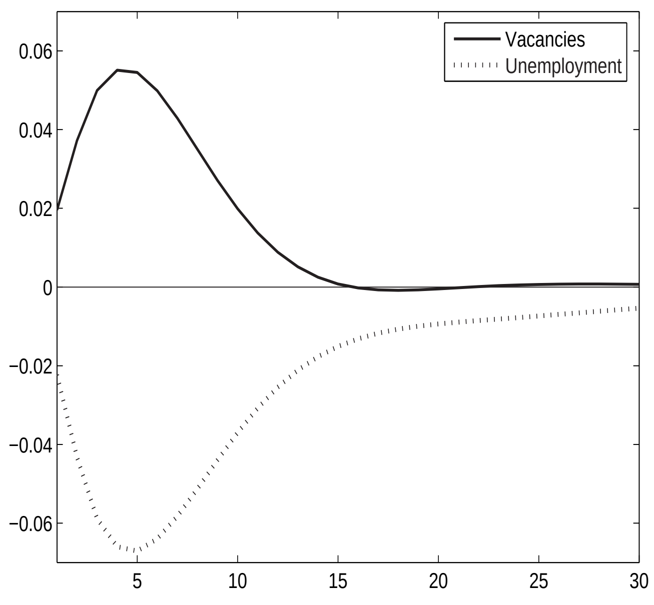


Figure 5: Comparison of dynamic correlations

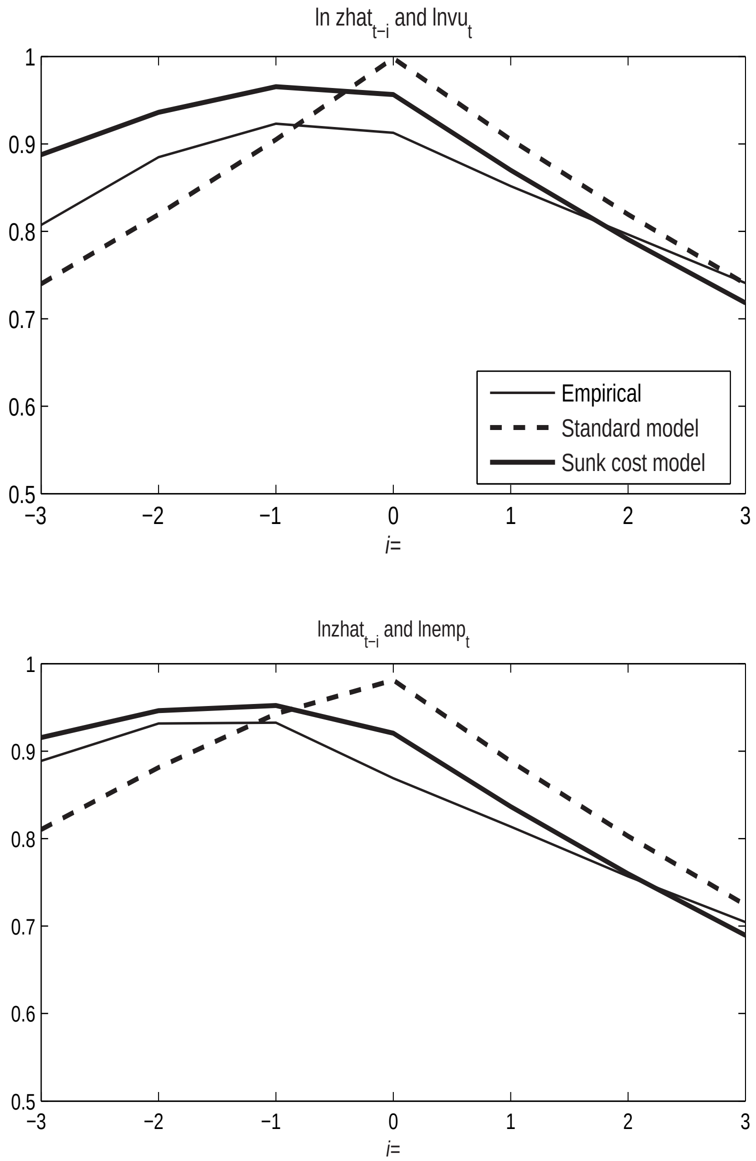


Figure 6: Comparison of the impulse responses: standard model

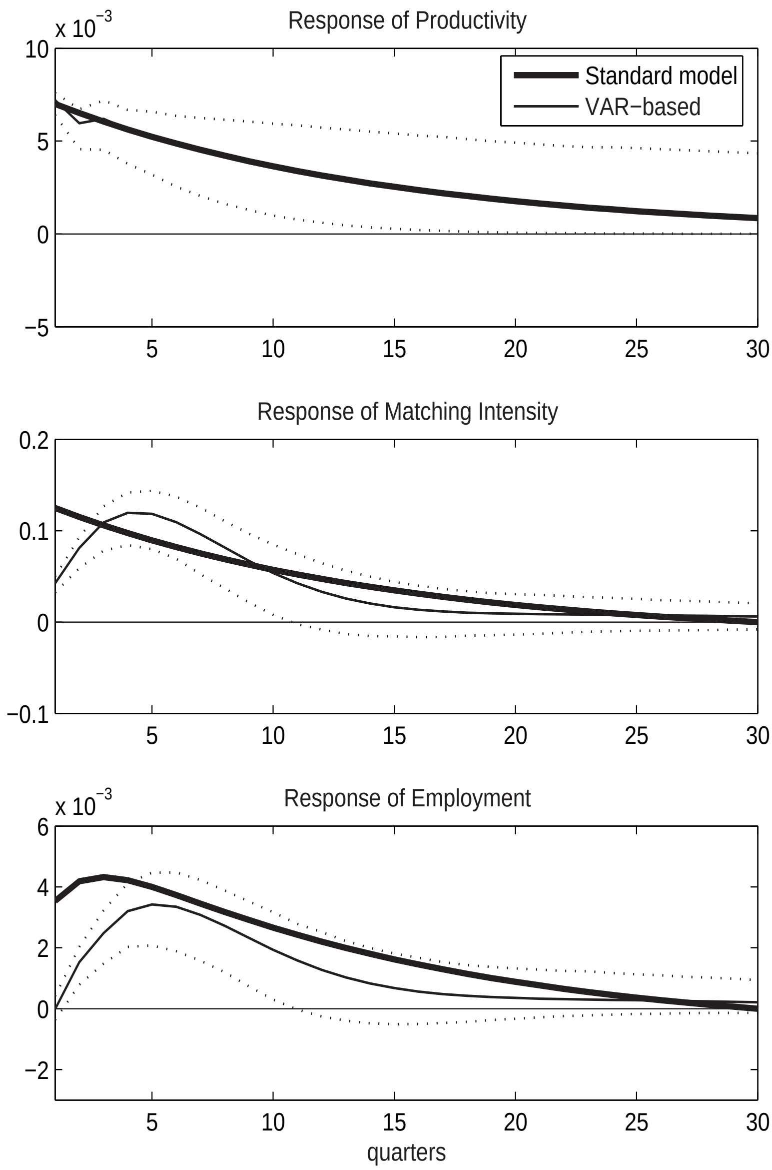


Figure 7: Comparison of the impulse responses: sunk cost model

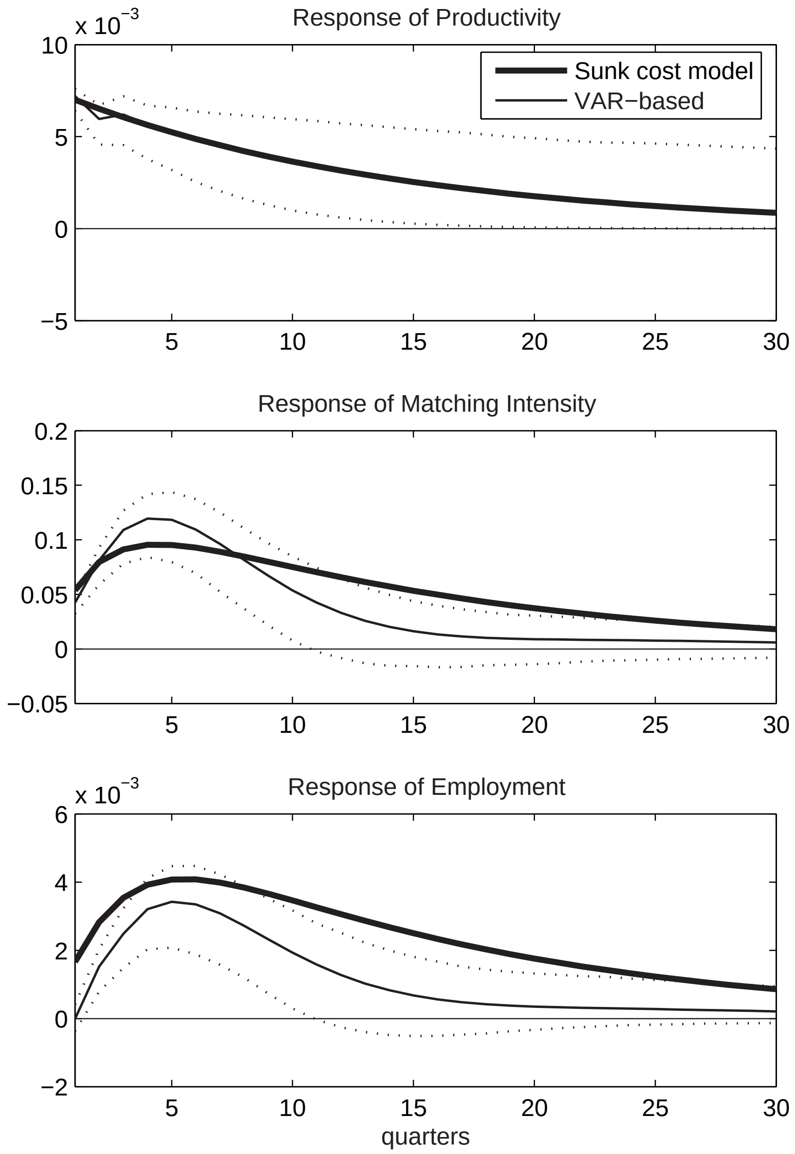
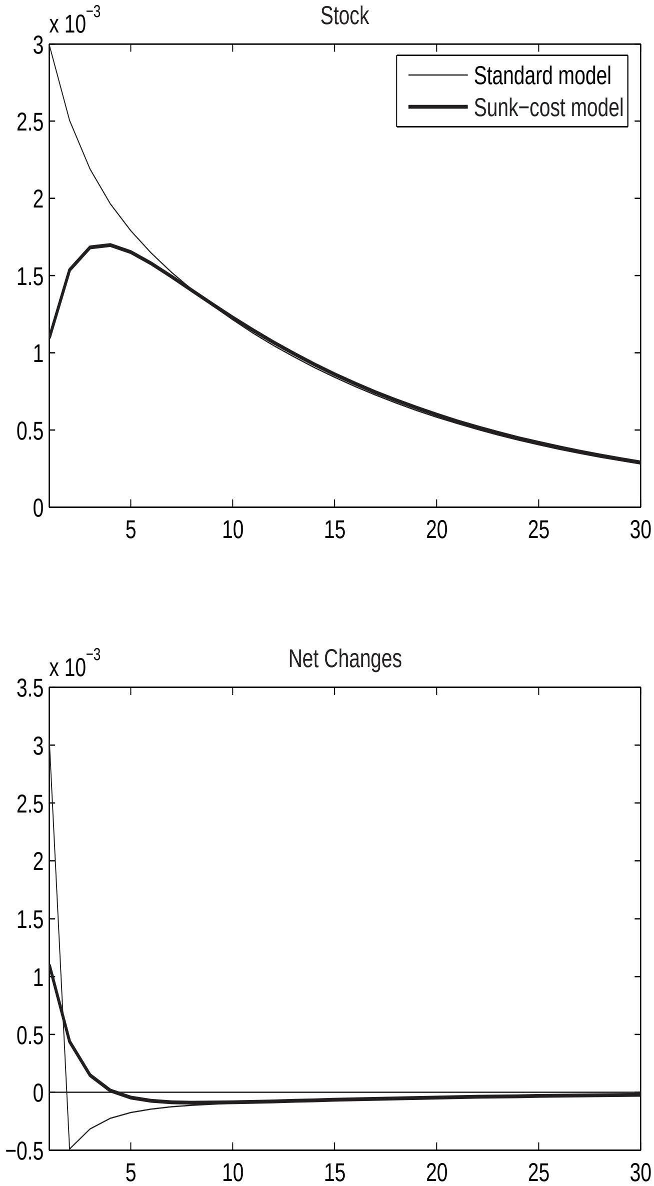
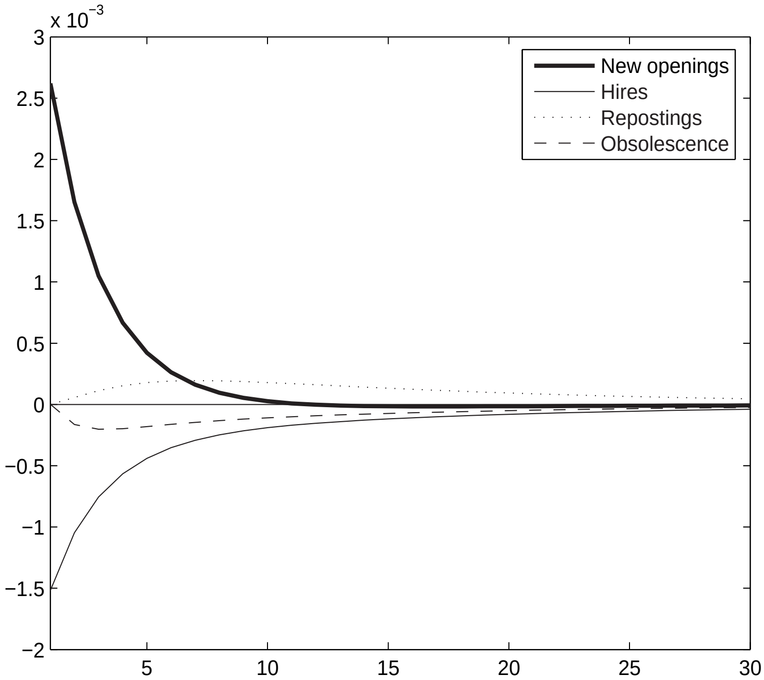


Figure 8: Vacancy responses



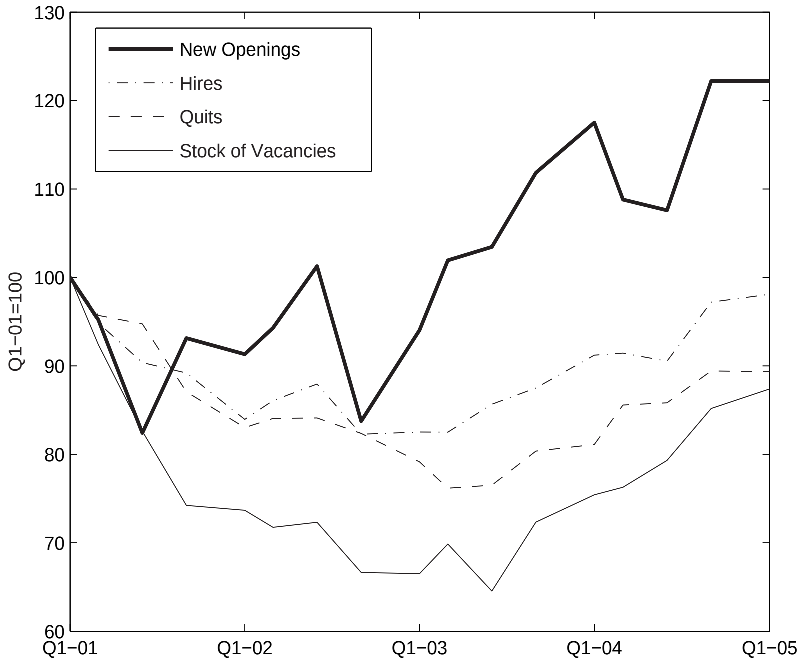
Notes: Level deviations from the steady-state values.

Figure 9: Gross flows of vacancies in the sunk cost model



Notes: Level deviations from the steady-state values. Outflows are plotted as negative values.

Figure 10: JOLTS data



Notes: New postings are imputed from the vacancy stock-flow relationship (14) using the JOLTS data on quits, hires and end-of-the-period stock of vacancies, and the calibrated value of the withdrawal rate λ^d. The above figure plots the indices that treat 2001:Q1 as the base period.