

Heterogeneity in Price Setting and the Real Effects of Monetary Shocks

Carlos Carvalho*

Department of Economics
Princeton University

This version: September 2005

First version: February 2004

Abstract

This paper analyzes the implications of heterogeneity in price setting for the real effects of monetary shocks. Starting from otherwise standard sticky price and sticky information models, I introduce ex-ante heterogeneity in terms of price setting frictions, and compare the resulting dynamics with those of identical firms economies under alternative calibrations. Both the qualitative and the quantitative results show that heterogeneity leads monetary shocks to have substantially larger and more persistent real effects. In particular, reproducing the dynamics of a truly heterogeneous economy with a model based on identical firms requires unrealistically large degrees of price setting frictions.

*Previously entitled “Heterogeneity in Price Setting and its Effects on Price and Inflation Inertia.” I would like to thank David Romer, Ricardo Reis, Michael Woodford, Vasco Cúrdia, participants from ESWC 2005, LAMES 2004, EEA Meeting 2004 and from the Student Macro/International Workshop at Princeton, and two anonymous referees for helpful comments. All errors are my own. Financial support from Princeton University, under the Harold Willis Dodds Merit Fellowship in Economics, is gratefully acknowledged. Address for correspondence: Department of Economics, Princeton University, Fisher Hall, Princeton, NJ 08544-1021, USA. E-mail: cvianac@princeton.edu.

1 Introduction

Standard models of sticky prices and sticky information usually do not involve any explicit attempt to model heterogeneity in firms' price setting behavior. They are usually assumed to be ex-ante identical, except for differences in the timing of price adjustments or information updating. However, at least for the sticky price models, which can be confronted with micro datasets on price setting behavior, there is ample evidence that firms do in fact differ substantially in terms of the frequency of price adjustments (see Blinder et al., 1998 and Bils and Klenow, 2004 for the US economy; Dhyne et al., 2004, and references cited therein for the Euro area). So, apart from analytical convenience, the only reason not to take heterogeneity explicitly into account would be if it did not matter qualitatively in aggregate terms, or at least not quantitatively.

In this paper I argue that this is not the case. Starting from otherwise standard sticky price and sticky information models, I introduce ex-ante heterogeneity in terms of price setting frictions, and compare the resulting dynamics with those of identical firms economies under alternative calibrations. Both the qualitative and the quantitative results show that heterogeneity leads monetary shocks to have substantially larger and more persistent real effects.

Recently, the basic versions of models of price setting have been transformed or extended in several directions, in search of mechanisms which can generate more persistent dynamics in response to monetary shocks.¹ The role of heterogeneity in these models, however, has not yet been properly explored. To what extent can heterogeneity be a source of persistence by itself? Are there important differences between ex-ante and ex-post heterogeneity? Once there is heterogeneity, how should we match these models to the data? Are the implications different for different types of monetary shocks?

Some recent papers which involve heterogeneity in terms of price setting behavior are Ohanian et al. (1995), Bils and Klenow (2004), Bils et al. (2003) and Barsky et al. (2003). Taylor (1993), in particular, extended his original model (1979, 1980) to account for contract lengths of different durations. However, none of these papers focuses on isolating the role of heterogeneity. This requires comparing models with heterogeneous firms with otherwise equivalent models in which all firms are identical. Exceptions are Aoki (2001) and Benigno (2004), who explore this comparison to show that ex-ante heterogeneity in the context of Calvo pricing does affect optimal monetary policy, and Dixon and Kara (2005a), who study heterogeneity in the context of Taylor staggered wage setting. On the empirical front, Jadresic (1999) presents econometric evidence that heterogeneity improves the performance of sticky price models when applied to U.S. data, and Coenen and Levin (2004) document promising performance of DSGE models with heterogeneity in price rigidity, using data for Germany. In a different framework in which firms follow state- rather than time-dependent pricing

¹A common departure from the basic settings has been to change Calvo's pricing to account for some sort of indexation (Yun (1996), Gali and Gertler (1999), Woodford (2003)). Sticky information models, starting with Mankiw and Reis (2002), were themselves developed as part of this research effort. Other examples are Calvo, Celasun and Kumhof (2003), who assume that firms choose (linear) price paths rather than levels, and the related paper by Devereux and Yetman (2003), who compare the dynamic implications of fixed-prices versus predetermined price paths in a Calvo-style framework.

rules, Caballero and Engel (1991, 1993) show that ex-ante (“structural”) heterogeneity affects the dynamic response of the economy to shocks in an important way. In particular, they show that under some circumstances it may slow down the economy’s adjustment process. Heterogeneity also plays a role in terms of guaranteeing existence and uniqueness of equilibrium.²

In this paper I use simple continuous-time versions of four price setting models to address the questions raised above. The first two (sticky price) models build on the seminal contributions of Calvo (1983) and Taylor (1979, 1980). The other two are sticky information models, based on Mankiw and Reis (2002) and Dupor and Tsuruga (2005). I compare the dynamic response of the various heterogeneous economies to those of identical firms economies under alternative calibrations, for two classes of AR(1) type shocks: level and growth rate shocks to nominal aggregate demand.

In general, the dynamics of the heterogeneous economies depend on the underlying distribution of frictions. For the case of permanent level and growth rate shocks, however, I am able to derive the implications of heterogeneity for the cumulative output effects of monetary shocks for arbitrary distributions. Albeit particular, these results turn out to illustrate qualitatively the role of heterogeneity in the more general cases. For the other results, I use the statistics reported recently by Bils and Klenow (2004) for the US economy to calibrate the relevant distribution for each model.

The first result is that the usual calibration of identical firms models, which is based on average frequencies of price adjustment (or of price plan revisions for the sticky information models), always understates the real effects of monetary shocks relative to the underlying heterogeneous economy. The reason is that such effects are more directly related to the relevant **average durations** rather than frequencies, and, because of Jensen’s inequality, average durations exceed the inverse of the corresponding average frequencies.

Accounting for this bias, however, does not suffice in general. For growth rate shocks, heterogeneity has a direct effect on the real effects of monetary shocks in addition to the bias engendered by Jensen’s inequality. Moreover, in the presence of strategic complementarities in price setting (or real rigidities), the interaction of firms with higher and lower frictions amplifies the role of heterogeneity in generating persistence, for all types of shocks.

How large are these effects likely to be in quantitative terms? What fraction of persistence not accounted for by models with identical firms can be explained with realistic degrees of heterogeneity? To address these questions I perform the following experiment: given the empirical distribution of price rigidities reported by Bils and Klenow (2004), I find the average duration and degree of strategic complementarities in the identical firms economy that minimize the distance of its impulse response functions to those of the heterogeneous economy. Despite the fact that the average duration of price rigidity reported by Bils and Klenow (2004) is approximately 2.2 quarters, the “best fitting average duration” for the identical firms models ranges from 5 to 11 quarters, depending on the model, the type of shock, and the degree of strategic complementarity in the original heterogeneous economy.

A straightforward, but important implication of these results is that we must be careful when

²Konieczny and Skrzypacz (2004) also present a model with state-dependent pricing, in which heterogeneity in the frequency of price adjustments arises from the fact that firms in different markets face consumers with different search intensities.

relating parameters of models based on identical firms to micro evidence on price setting behavior, and when interpreting estimates of parameters based on identical firms models. While they may be able to provide a reasonable description of a more complex, heterogeneous reality, this is likely to require parameter values which will seem unrealistic if interpreted literally. The fact is that, given the empirical evidence documenting a high degree of heterogeneity and the fact that it does matter for the dynamic response of monetary economies to shocks, the parameters of (mispecified) identical firms models cannot be seen as “structural,” and should be treated accordingly.

The rest of the paper is organized as follows. Section 2 presents the basic setup and introduces the four price setting models in separate subsections. Section 3 presents the steady states, introduces monetary shocks, and derives the equations which characterize the dynamic response of the economy to such shocks for each model. In the following section I provide qualitative and analytical results which illustrate the role of heterogeneity. Section 5 presents quantitative results on the importance of the effects analyzed in the previous sections. It shows that heterogeneity interacts with real rigidities to make the process of adjustment to shocks substantially more sluggish, and that the parameters of identical firms models are “distorted” when trying to mimic the dynamics of truly heterogeneous economies. The last section concludes. All proofs are in the appendix.

2 Models

This section presents the basic setup, with the assumptions which are common to the models of price setting that will be introduced in specific subsections. The modeling strategy is to introduce ex-ante heterogeneity in terms of price setting frictions into otherwise standard sticky price and sticky information models. For brevity and to highlight the central results of the paper, I model the demand side of the economy in the simplest possible way and use reduced form equations which can be derived from first principles as in Blanchard and Kiyotaki (1987), and Ball and Romer (1989), for instance.³ The models are set in continuous time.

2.1 Basic setup

In the economy there is a continuum of imperfectly competitive firms divided into groups, which differ with respect to the friction they face to set prices. More precisely, in the two models with sticky prices (which build on Calvo, 1983, and Taylor, 1979, 1980), each group has a different frequency of price adjustment, while in the versions of the model with sticky information (building on Mankiw and Reis, 2002, and Dupor and Tsuruga, 2005), each group updates information with a different frequency.

Firms will be indexed by their group, n , and by $i \in [0, 1]$. Each group is indexed by $n \in [0, n^*]$, which also determines the intensity of the friction that they face. In the models with sticky prices, n measures the time interval during which a new price is expected to remain fixed, and ranges, correspondingly, from “continuous adjustment” to “adjustment at intervals of (expected) length n^* .”

³See also Woodford (2003).

In the cases of sticky information, n is the expected time interval between dates of information updating, ranging from “continuous information updating” to “updating at intervals of (expected) length n^* .” The distribution of firms across groups is summarized by a density function $f(\cdot)$ on $[0, n^*]$, with cdf $F(\cdot)$.⁴ The degree of heterogeneity will be measured by the dispersion of such distribution.

In the absence of frictions to price adjustment and information collection, it is assumed that the optimal level of the individual relative price, which is the same for all firms, is given by:⁵

$$p^*(t) - p(t) = \theta y(t), \quad (1)$$

where p^* is the individual frictionless optimal price, p is the aggregate price level and y is the output gap (which equals aggregate demand, because the natural rate is normalized to be identically equal to zero). For simplicity, $p(t)$ is evaluated according to:

$$p(t) = \int_0^{n^*} f(n) \int_0^1 x_{n,i}(t) di dn,$$

where $x_{n,i}(t)$ is the price charged by firm i from group n at time t .

To focus on the supply side of the model, I assume that nominal aggregate demand, $m(t) = y(t) + p(t)$, follows an exogenous process.⁶

Combining nominal aggregate demand and equation (1) yields:

$$p^*(t) = \theta m(t) + (1 - \theta)p(t). \quad (2)$$

In a frictionless world, each firm would choose $x_{n,i}(t) = p^*(t)$ and the resulting aggregate price level would be $p(t) = m(t)$. Thus, aggregate output and individual prices would be given by $y(t) = 0$ and $x_{n,i}(t) = m(t)$, respectively.

2.2 Calvo pricing

In this subsection I introduce frictions through price setting as in Calvo (1983). For each firm, the opportunity to change prices arrives according to a Poisson process, with rate given by the inverse of the expected duration of price rigidity for the firms’ group ($\frac{1}{n}$).

For simplicity, based on a second order approximation to the loss incurred from not charging the optimal price, firms are assumed to set prices to minimize expected squared deviations from the optimal price:

$$\begin{aligned} p_{n,i}(t) &= \arg \min_x \int_0^\infty e^{-\frac{1}{n}s} E_t [x - p^*(t+s)]^2 ds \\ &= \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}s} E_t p^*(t+s) ds. \end{aligned}$$

⁴ All results hold in the case of discrete or mixed distributions.

⁵ All variables should be interpreted as log-deviations from the deterministic, zero inflation steady state.

⁶ This simplification is also used by Mankiw and Reis (2002), among others. See also Woodford (2003).

The aggregate price level is then given by:

$$p(t) = \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} p_{n,i}(s) ds dn.$$

2.3 Taylor staggered price setting

Building on Taylor (1979, 1980), in this case firms are assumed to set prices for a fixed period of time. Firms from group n set prices for a period of length n . Adjustments are uniformly staggered across time in terms of both firms and groups.

Assuming the same second order approximation to the loss incurred from not charging the optimal price, firms set prices according to:

$$\begin{aligned} p_{n,i}(t) &= \arg \min_x \int_0^n E_t [x - p^*(t+s)]^2 ds \\ &= \frac{1}{n} \int_0^n E_t p^*(t+s) ds. \end{aligned}$$

The aggregate price level is then given by:

$$p(t) = \int_0^{n^*} f(n) \frac{1}{n} \int_0^n p_{n,i}(t-s) ds dn.$$

2.4 Sticky information (Mankiw and Reis, 2002)

Following the sticky information model proposed by Mankiw and Reis (2002), firms can only update their information sets when they receive a ‘‘Poisson signal.’’ The hazard rate for group n equals the inverse of the expected time interval between two subsequent updates ($\frac{1}{n}$).

There are no impediments to price adjustment, so that firms set prices at each instant to minimize the expected squared deviation from the frictionless optimal price, conditional on the information available when they last had a chance to update:

$$\begin{aligned} p_{n,i}(t) &= \arg \min_x E_{s_{n,i}} [x - p^*(t)]^2 \\ &= E_{s_{n,i}} p^*(t), \end{aligned}$$

where $s_{n,i} \leq t$ indicates the time when firm i from group n last updated its information set.

The aggregate price level is then given by:

$$p(t) = \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} E_s p^*(t) ds dn.$$

2.5 ‘‘Staggered sticky information’’ (Dupor and Tsuruga, 2005)

Based on the staggered sticky information model of Dupor and Tsuruga (2005), firms from group n update their information sets at intervals of length n . Information updating dates are uniformly

staggered across time in terms of both firms and groups.

Again, firms are not prevented from changing prices, and so set them exactly as in the previous subsection. However, due to the difference in the distribution of times of information updating, the aggregate price level is now given by:

$$p(t) = \int_0^{n^*} f(n) \frac{1}{n} \int_0^n E_{t-s} p^*(t) ds dn.$$

2.6 Economy-wide expected durations

This subsection derives the economy-wide, or cross-sectional expected duration of price rigidity and of price plans for the sticky price and sticky information models, respectively. In the case of both models with Poisson signals it is trivially equal to the corresponding duration for any individual firm. In the models with uniform staggering, however, these measures differ in a way that will be useful to understand some of the results presented later in the paper.

Starting with the Calvo model, at any point in time every firm from group n expects its price to remain fixed for an interval of length n . The expected duration of price rigidity in the economy as a whole is, therefore, trivially equal to $\bar{n} \equiv \int_0^{n^*} f(n) n dn$. Analogous reasoning leads to the result that the expected duration of a price plan in the standard sticky information economy is also equal to \bar{n} .

Consider now a Taylor type model in which all firms set prices for periods of length n . While it is true that upon setting a new price at t it will remain fixed until $t + n$, at any point in time the duration of price rigidity for a randomly selected firm will be less than n . In fact, given the assumption of uniform staggering of adjustment dates across time, it will be equal to $\frac{n}{2}$. Extending the logic to the case of an economy with heterogeneous firms, the average duration of price rigidity in the economy as a whole will be given by $\frac{\bar{n}}{2}$. Analogously, the average duration of a price plan in an economy with staggered sticky information is given by $\frac{\bar{n}}{2}$.

As highlighted by Dupor and Tsuruga (2005), and Dixon and Kara (2005b), the standard calibration in identical firms models is based on the expected duration of a new price, not the economy-wide duration. As a consequence, it implies twice as much duration of price rigidity in Calvo's model relative to Taylor's (and analogously for the standard sticky information model relative to the staggered sticky information case). This is an important source of differences in quantitative results generated by these models under the standard calibration, and, to a large extent, explain the results found by Kiley (2002).

3 Steady state and monetary shocks

This section describes the steady state and the types of shocks that will be analyzed. I assume that the economy is initially in an inflationary steady state that is expected to last forever, in which nominal aggregate demand grows at a constant rate $\pi \geq 0$. This implies that, after a normalization, $m(t) = \pi t$. In the appendix I derive the equations characterizing individual firm behavior in each of the models, and aggregate them to obtain a full description of the initial steady state in each case.

In each experiment a particular shock to nominal aggregate demand hits the economy at time $t = 0$. Then, for each price setting model I derive the equations which characterize the transition of the economy into the new steady state. I focus on AR(1) type shocks to the level and growth rate of nominal aggregate demand. The particular (but interesting) cases of permanent shocks obtain when the decay parameters are set equal to zero.

3.1 Level shocks

I assume, without loss of generality, that $\pi = 0$,⁷ and that at $t = 0$ nominal aggregate demand is hit by a shock of size \bar{m} , which then decays exponentially at rate $\rho \geq 0$. The particular case of a permanent level shock obtains when $\rho = 0$. For $t \geq 0$, the path for nominal aggregate demand is therefore given by $m(t) = \bar{m}e^{-\rho t}$.

3.1.1 Calvo pricing

After learning of the shock, whenever firm i from group n gets a chance to adjust its price it sets:

$$\begin{aligned} p_{n,i}(t) &= \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}s} E_t p^*(t+s) ds \\ &= \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}s} \left(\theta \bar{m} e^{-\rho(t+s)} + (1-\theta) p(t+s) \right) ds. \end{aligned}$$

The corresponding path for the aggregate price level is defined implicitly by:

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} p_{n,i}(s) ds dn \\ &= \int_0^{n^*} f(n) \int_0^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}r} \left(\theta \bar{m} e^{-\rho(t+r)} + (1-\theta) p(t+r) \right) dr ds dn, \quad (3) \end{aligned}$$

where the second integral in the last expression ranges from 0 (and not from $-\infty$) to t because $p(0) = 0$.

3.1.2 Taylor staggered price setting

Whenever it is time for firms to adjust after $t = 0$ they set:

$$\begin{aligned} p_{n,i}(t) &= \frac{1}{n} \int_0^n E_t p^*(t+s) ds \\ &= \frac{1}{n} \int_0^n \theta \bar{m} e^{-\rho(t+s)} + (1-\theta) p(t+s) ds. \end{aligned}$$

The aggregate price level is, again, defined implicitly. For $0 \leq t \leq n^*$ there are firms with prices

⁷This is just a translation of axis.

set before the shock, and so:

$$\begin{aligned}
p(t) &= \int_0^t f(n) \frac{1}{n} \int_0^n p_{n,i}(t-s) ds dn + \int_t^{n^*} f(n) \frac{1}{n} \int_0^t p_{n,i}(t-s) ds dn \\
&= \int_0^t \frac{f(n)}{n^2} \int_0^n \int_0^n \theta \bar{m} e^{-\rho(t-s+r)} + (1-\theta) p(t-s+r) dr ds dn \\
&\quad + \int_t^{n^*} \frac{f(n)}{n^2} \int_0^t \int_0^n \theta \bar{m} e^{-\rho(t-s+r)} + (1-\theta) p(t-s+r) dr ds dn.
\end{aligned} \tag{4}$$

For $t \geq n^*$ all firms set prices with knowledge of the shock, and $p(t)$ is given by:

$$p(t) = \int_0^{n^*} f(n) \frac{1}{n} \int_0^n \frac{1}{n} \int_0^n \theta \bar{m} e^{-\rho(t-s+r)} + (1-\theta) p(t-s+r) dr ds dn. \tag{5}$$

3.1.3 Sticky information

In this case the relevant distinction is between firms that had the opportunity to update their pricing plans after the shock, and firms that are still setting prices based on their previous information.

Firms which get to update their plans set:

$$\begin{aligned}
p_{n,i}(t) &= E_{s \geq 0} p^*(t) \\
&= \theta \bar{m} e^{-\rho t} + (1-\theta) p(t),
\end{aligned}$$

where $E_{s \geq 0}$ indicates expectation conditional on information sets which include knowledge of the shock.⁸

Therefore, the aggregate price level satisfies:

$$\begin{aligned}
p(t) &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} E_s p^*(t) ds dn \\
&= \int_0^{n^*} f(n) \int_0^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} (\theta \bar{m} e^{-\rho t} + (1-\theta) p(t)) ds dn,
\end{aligned}$$

where again the second integral in the last expression ranges from 0 (and not from $-\infty$) to t because $p(0) = 0$. Solving explicitly for $p(t)$ yields:

$$p(t) = \frac{\theta \bar{m} e^{-\rho t} \int_0^{n^*} f(n) \left(1 - e^{-\frac{1}{n}t}\right) dn}{1 - (1-\theta) \int_0^{n^*} f(n) \left(1 - e^{-\frac{1}{n}t}\right) dn}. \tag{6}$$

3.1.4 Staggered sticky information

The behavior of firms is exactly as in the previous case: whenever it is time for them to update their plans after the shock they set a price path corresponding to the (expected) frictionless optimal price.

⁸ Given that I am analyzing one-time surprise shocks in an otherwise perfect foresight model, all information sets updated after $t = 0$ are equal.

To derive the aggregate price level, note that for $0 \leq t \leq n^*$ some firms are still setting prices based on their pre-shock plans, and so:

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \frac{1}{n} \int_0^n E_{t-s} p^*(t) ds dn \\ &= \int_0^t \frac{f(n)}{n} \int_0^n \theta \bar{m} e^{-\rho t} + (1-\theta) p(t) ds dn + \int_t^{n^*} \frac{f(n)}{n} \int_0^t \theta \bar{m} e^{-\rho t} + (1-\theta) p(t) ds dn. \end{aligned}$$

Solving explicitly for $p(t)$ leads to:

$$p(t) = \frac{\theta \bar{m} e^{-\rho t} \left(F(t) + \int_t^{n^*} f(n) \frac{t}{n} dn \right)}{1 - (1-\theta) \left(F(t) + \int_t^{n^*} f(n) \frac{t}{n} dn \right)}. \quad (7)$$

After $t = n^*$ all firms have updated their plans, and therefore:

$$p(t) = m(t) = \bar{m} e^{-\rho t}. \quad (8)$$

3.2 Growth rate shocks

In the case of a growth rate shock, $\dot{m}(t) \equiv \frac{\partial m(t)}{\partial t}$ jumps at $t = 0$ from π to $\pi + \Delta\pi$, where $\Delta\pi$ is the size of the shock. Thereafter the shock decays exponentially at rate $\lambda \geq 0$, so that $\dot{m}(t) = \pi + \Delta\pi e^{-\lambda t}$, and $m(t) = \int_0^t \dot{m}(s) ds = \pi t + \Delta\pi \frac{1 - e^{-\lambda t}}{\lambda}$. Once more, the particular case of a permanent growth rate shock obtains when $\lambda = 0$.

3.2.1 Calvo pricing

After $t = 0$, whenever firm i from group n gets a chance to adjust its price it sets:

$$\begin{aligned} p_{n,i}(t) &= \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}s} E_t p^*(t+s) ds \\ &= \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}s} \left(\theta \left(\pi t + \pi s + \Delta\pi \frac{1 - e^{-\lambda(t+s)}}{\lambda} \right) + (1-\theta) p(t+s) \right) ds. \end{aligned}$$

The corresponding path for the aggregate price level is defined implicitly by:

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} p_{n,i}(s) ds dn \\ &= \int_0^{n^*} f(n) \int_{-\infty}^0 \frac{1}{n} e^{-\frac{1}{n}(t-s)} (\pi s + \pi n) ds dn \\ &\quad + \int_0^{n^*} \int_0^t \int_0^\infty \frac{f(n)}{n^2} e^{-\frac{1}{n}(t+r-s)} \left(\theta \left(\pi s + \pi r + \Delta\pi \frac{1 - e^{-\lambda(s+r)}}{\lambda} \right) + (1-\theta) p(s+r) \right) dr ds dn. \end{aligned} \quad (9)$$

3.2.2 Taylor staggered price setting

Whenever it is time for firms to adjust after $t = 0$ they set:

$$\begin{aligned} p_{n,i}(t) &= \frac{1}{n} \int_0^n E_t p^*(t+s) ds \\ &= \frac{1}{n} \int_0^n \theta \left(\pi t + \pi s + \Delta\pi \frac{1 - e^{-\lambda(t+s)}}{\lambda} \right) + (1 - \theta) p(t+s) ds. \end{aligned}$$

The aggregate price level is, again, defined implicitly. For $0 \leq t \leq n^*$ there are firms with prices set before the shock, and so:

$$\begin{aligned} p(t) &= \int_0^t \frac{f(n)}{n} \int_0^n p_{n,i}(t-s) ds dn + \int_t^{n^*} \frac{f(n)}{n} \int_0^t p_{n,i}(t-s) ds dn \\ &= \int_0^t \frac{f(n)}{n^2} \int_0^n \int_0^n \theta \left(\pi t - \pi s + \pi r + \Delta\pi \frac{1 - e^{-\lambda(t-s+r)}}{\lambda} \right) + (1 - \theta) p(t-s+r) dr ds dn \quad (10) \\ &\quad + \int_t^{n^*} \frac{f(n)}{n^2} \int_0^t \int_0^n \theta \left(\pi t - \pi s + \pi r + \Delta\pi \frac{1 - e^{-\lambda(t-s+r)}}{\lambda} \right) + (1 - \theta) p(t-s+r) dr ds dn. \end{aligned}$$

For $t \geq n^*$ all firms set prices with knowledge of the shock, and $p(t)$ is given by:

$$p(t) = \int_0^{n^*} \frac{f(n)}{n^2} \int_0^n \int_0^n \theta \left(\pi t - \pi s + \pi r + \Delta\pi \frac{1 - e^{-\lambda(t-s+r)}}{\lambda} \right) + (1 - \theta) p(t-s+r) dr ds dn. \quad (11)$$

3.2.3 Sticky information

Firms which get to update their plans after $t = 0$ set:

$$\begin{aligned} p_{n,i}(t) &= E_{s \geq 0} p^*(t) \\ &= \theta \left(\pi t + \Delta\pi \frac{1 - e^{-\lambda t}}{\lambda} \right) + (1 - \theta) p(t). \end{aligned}$$

The aggregate price level satisfies:

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} E_s p^*(t) ds dn \\ &= \int_0^{n^*} \frac{f(n)}{n} \int_{-\infty}^0 e^{-\frac{1}{n}(t-s)} E_s p^*(t) ds + \int_0^t e^{-\frac{1}{n}(t-s)} E_s p^*(t) ds dn \\ &= \int_0^{n^*} \frac{f(n)}{n} \int_{-\infty}^0 e^{-\frac{1}{n}(t-s)} \pi t ds + \int_0^t e^{-\frac{1}{n}(t-s)} \left(\theta \left(\pi t + \Delta\pi \frac{1 - e^{-\lambda t}}{\lambda} \right) + (1 - \theta) p(t) \right) ds dn. \end{aligned}$$

Solving explicitly for $p(t)$ yields:

$$p(t) = \frac{\int_0^{n^*} f(n) \left(\theta \left(\pi t + \Delta \pi \frac{1-e^{-\lambda t}}{\lambda} \right) \left(1 - e^{-\frac{1}{n}t} \right) + \pi t e^{-\frac{1}{n}t} \right) dn}{1 - (1 - \theta) \int_0^{n^*} f(n) \left(1 - e^{-\frac{1}{n}t} \right) dn}. \quad (12)$$

3.2.4 Staggered sticky information

The aggregate price level for $0 \leq t \leq n^*$ is now given by:

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \frac{1}{n} \int_0^n E_{t-s} p^*(t) ds dn \\ &= \int_0^t \frac{f(n)}{n} \int_0^n \theta \left(\pi t + \Delta \pi \frac{1-e^{-\lambda t}}{\lambda} \right) + (1 - \theta) p(t) ds dn \\ &\quad + \int_t^{n^*} \frac{f(n)}{n} \int_0^t \theta \left(\pi t + \Delta \pi \frac{1-e^{-\lambda t}}{\lambda} \right) + (1 - \theta) p(t) ds dn. \end{aligned}$$

Solving explicitly for $p(t)$ leads to:

$$p(t) = \frac{\theta \left(\pi t + \Delta \pi \frac{1-e^{-\lambda t}}{\lambda} \right) F(t) + \int_t^{n^*} \frac{f(n)}{n} \left(t \theta \left(\pi t + \Delta \pi \frac{1-e^{-\lambda t}}{\lambda} \right) + (n-t) \pi t \right) dn}{1 - (1 - \theta) \left(F(t) + \int_t^{n^*} f(n) \frac{t}{n} dn \right)}. \quad (13)$$

After $t = n^*$ all firms have updated their plans, and therefore:

$$p(t) = \pi t + \Delta \pi \frac{1 - e^{-\lambda t}}{\lambda}. \quad (14)$$

4 Qualitative features and some analytic results

This section presents qualitative and analytic results which illustrate the role of heterogeneity in the models under consideration. In order to isolate its effects, I need to construct a benchmark economy with identical firms, retaining the same degree of frictions, in some sense. But what does that mean exactly? In the sticky price models, does that mean matching the average duration of price rigidity, or the average frequency of price adjustments? Likewise, in the context of sticky information models, should we match the average life of a price plan, or the average frequency with which firms update their plans? While with identical firms the degree of frictions can be equivalently summarized by the relevant average durations or frequencies, with heterogeneous firms this is no longer the case.

To illustrate how important these differences can be, I chose to simulate the dynamic response of three economies with Calvo pricing to a permanent level shock to aggregate demand: one heterogeneous economy and two identical firms economies - one with the same average frequency of price adjustments and the other with the same average duration of price rigidity as the heterogeneous economy.⁹ Since heterogeneity and strategic complementarities in price setting $(1 - \theta)$ interact in

⁹The features highlighted by this example are common to virtually all other cases (combinations of shocks and

quantitatively important ways to generate persistence, to better isolate the role of heterogeneity throughout this section I rule out strategic complementarities, and set $\theta = 1$. The next section shows that real rigidities strengthen the role of heterogeneity in generating persistence.

As can be seen from the equations presented in the previous section, in general the dynamics of the heterogeneous economies will depend on the whole distribution of relevant frictions. To handle this issue I chose to use the statistics on price setting behavior in the US economy reported by Bils and Klenow (2004) to obtain the distribution used in the simulations. For simplicity, the same data will be used to calibrate the relevant distribution for each of the four price setting models, when needed. I take the distribution of the average duration between price changes (converted to years) reported in their appendix to be the relevant distribution for n in each of the models. This is arguably a sensible interpretation of their data in the context of the sticky price models analyzed in this paper. The sticky information models are, however, impossible to reconcile with the micro evidence on nominal price rigidity, since they imply continuous price adjustment. The choice of this distribution in these cases is therefore more arbitrary, and I only use it for convenience. The reader should be aware of the different applicability of this empirical distribution to each of the models, specially since it will also be the basis for the quantitative analysis presented in the next section.

Figure 1 presents the results of the above mentioned simulation. Keep in mind that the qualitative features of this example are common to the other models analyzed in this paper. Relative to the heterogeneous economy, the adjustment process in the identical firms economy with the same average frequency of price changes is clearly too fast.¹⁰ Surprisingly, this is the standard usually adopted to calibrate identical firms models based on micro evidence. The identical firms economy with the average duration of price rigidity found in the data seems to give a better representation of the heterogeneous economy.¹¹ A qualitative difference between the latter two economies is that, initially, adjustment is faster in the heterogeneous economy, because a relatively larger measure of firms with higher frequency of adjustment gets to change prices earlier. As time passes, the distribution of (expected) duration of price rigidity among firms which have not yet adjusted becomes more and more dominated by firms with relatively longer (expected) contract lengths. So, the speed of adjustment slows down through time, and eventually the process becomes more sluggish in the heterogeneous economy.

A natural question to ask is whether there is a general rule to determine which parameterization for an identical firms economy will best mimic the dynamics of a given heterogeneous economy, in terms of its impulse response functions, say. Given the dependence of the latter on the underlying distribution of frictions, the answer is unfortunately, but not surprisingly, negative. It is possible, however, to get some guidance from analytic results, which are presented next.

models), which are not presented for brevity. The only exception is the case of a permanent growth rate shock in a Calvo economy with no strategic complementarities ($\theta = 1$), in which the shock has no real effects in any case.

¹⁰Another possibility is to use the median duration of price rigidity, as advocated by Bils and Klenow (2004). Although not shown here, it also performs poorly in tracking the behavior of the heterogeneous economy.

¹¹Baharad and Eden (2004) also argue in favor of the average duration of price rigidity instead of the average frequency of price adjustments when there is heterogeneity, in the context of a Taylor type model.

4.1 Analytic results: cumulative output effects

As seen previously, in general the dynamics of heterogeneous economies of the type analyzed here depend on the underlying distribution of frictions. However, in the case of permanent shocks there is a sensible measure of the overall effects of monetary disturbances which only depends on a few moments of such distribution: the (normalized) cumulative output effect, which takes into account both the intensity and the persistence of the real effects of monetary shocks.¹² Albeit particular, permanent shocks allow sharp analytic results, which serve as guidelines for understanding the effects of more general AR(1) type shocks.

4.1.1 Permanent shocks

In the case of a permanent level shock, the following two results shed additional light on the comparison between a model with arbitrary heterogeneity, and identical firms models with calibrations based on average durations and average frequencies, as previously illustrated in Figure 1. Note that the results apply to all four price setting models.

Proposition 1 *In the context of permanent level shocks ($\rho = 0$) and no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{\bar{m}} \int_0^\infty \bar{m} - p(t) dt$ is equal to:*

- i) the average duration of price rigidity in the economy, in the case of sticky price models;*
- ii) the average duration of price plans in the economy, in the case of sticky information models.*

This result shows why, without strategic complementarities, the impulse response functions for the identical firms economies with the same average duration of the relevant frictions will, in some sense, track their heterogeneous firms counterparts: for permanent level shocks the cumulative real effects that they imply are the same.

The identical firms economy with the same average frequency of price adjustment or average frequency of price plan revisions, on the other hand, will systematically understate the real effects of monetary shocks in the heterogeneous economy, as shown below:

Proposition 2 *In the context of permanent level shocks ($\rho = 0$) and no strategic complementarities ($\theta = 1$), an arbitrary heterogeneous economy will always display larger (normalized) cumulative output effects (as measured by $\frac{1}{\bar{m}} \int_0^\infty \bar{m} - p(t) dt$) than an identical firms economy with the same average frequency of price adjustments (in the case of sticky price models), or average frequency of price plan revisions (in the case of sticky information models).*

¹²This measure is also discussed, for example, in Christiano et al. (2005). It is widely used in the context of disinflations, being referred to as the Sacrifice Ratio.

This is a direct result of Jensen's inequality. The intuition as to why the average frequency of price adjustments or of price plan revisions can be misleading as an indicator of overall degree of frictions can be developed from the following limiting case: imagine a heterogeneous economy with a non-negligible fraction of firms which adjust prices (or revise price plans) continuously. Then, irrespective of how low the frequencies of adjustment/revision of the remaining firms are, the average frequency in the economy will be infinite. Nevertheless, monetary shocks will clearly have real effects due to the firms with finite adjustment/revision frequencies. The intuition of this extreme example carries through to more realistic distributions: in heterogeneous economies, high average frequencies of price adjustment or price plan revisions need not imply low degrees of friction (note that the implication does hold in identical firms economies). One might conjecture that getting rid of the extremes in the distribution of adjustment/revision frequencies by using the median rather than the average frequency would solve this problem, but this is not what is to be taken from the above results, since **Proposition 1** clearly states that **average durations**, not median frequencies nor median durations, are directly related to the real effects of monetary shocks.

In the case of growth rate shocks, however, taking Jensen's inequality into account and using the average duration instead of the average frequency to calibrate de identical firms economy does not suffice. The reason is that heterogeneity as measured by $\sigma_n^2 \equiv \int_0^{n^*} f(n)(n - \bar{n})^2 dn$ has an additional, direct impact on total output effects, as shown below:

Proposition 3 *In the context of permanent growth rate shocks ($\lambda = 0$) and no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt$ equals:*

i) For the Calvo model:

zero;

ii) For the Taylor model:

$$\frac{1}{12} (\bar{n}^2 + \sigma_n^2);$$

iii) For the sticky information model:

$$(\bar{n}^2 + \sigma_n^2);$$

iv) For the staggered sticky information model:

$$\frac{1}{6} (\bar{n}^2 + \sigma_n^2).$$

The intuition for why heterogeneity has a direct effect on total output effects in the case of growth rate shocks, but not in the case of level shocks, can be developed from the identical firms case. With level shocks, a change in (expected) duration affects the speed of the adjustment process, but not the magnitude of real output effects. In the case of growth rate shocks, however, a higher

(expected) duration increases the magnitude of output effects, and reduces the speed at which they fade away. Jointly, these two “channels” lead total output effects to depend on the square of the relevant (expected) duration. With heterogeneity, the mechanism at place is qualitatively the same, and the overall effect is the weighted average of the effects for each group of firms, thus being proportional to the second moment of the relevant distribution of (expected) durations.

Both sticky price models exhibit a discontinuity in the relation between the cumulative output effect and the persistence of the growth rate shock at the point corresponding to permanent shocks ($\lambda = 0$). This is most clearly seen in the case of the Calvo model, with a shock such that $\Delta\pi = -\pi$. In the case of a permanent shock, firms that get to adjust after $t = 0$ set $p_{n,i}(t) = 0$ forever, because nominal aggregate demand is forecast to remain unchanged. Since at $t = 0$ prices are, on average, equal to zero, and signals are independent of when each firm last adjusted, output is identically zero after the shock. With a temporary shock, however, nominal aggregate demand growth will eventually resume to its previous level π , and firms take this into account when setting prices. Due to nominal rigidities they set higher prices, and so the shock does have real effects.¹³ A less extreme, but conceptually analogous discontinuity occurs in the case of Taylor’s model.

Thus, in the case of temporary shocks, the equivalent of results *i*) (specially) and *ii*) from **Proposition 3** will be qualitatively different. Nevertheless, the result that total output effects are proportional to the second moment of the relevant distribution of durations will hold (approximately) for all models in the case of very persistent (but temporary) shocks.

4.1.2 Temporary shocks

As highlighted earlier, in the case of growth rate shocks sticky price models exhibit a discontinuity in the relation between the cumulative output effect and the persistence of the shock at $\lambda = 0$. It arises from the differences in price setting behavior in permanent versus temporary growth rate shocks.

However, despite these discontinuities and the fact that the impulse response functions implied by the four models are generally different, the following result, surprisingly, holds:

Proposition 4 *In the context of temporary level shocks ($\rho > 0$) with no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{m} \int_0^\infty m(t) - p(t) dt$ is equal to:*
*i) For **both** the Calvo pricing and the sticky information models:*

$$\int_0^{n^*} f(n) \frac{n}{1 + \rho n} dn;$$

*ii) For **both** the Taylor pricing and the staggered sticky information models:*

$$\frac{1}{\rho} \left(1 - \int_0^{n^*} f(n) \frac{1 - e^{-\rho n}}{\rho n} dn \right).$$

¹³There are no such discontinuities in the case of level shocks.

In the context of temporary growth rate shocks ($\lambda > 0$) with no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt$ equals:

iii) For **both** the Calvo pricing and the sticky information models:

$$\int_0^{n^*} f(n) \frac{n^2}{1 + \lambda n} dn;$$

iv) For **both** the Taylor pricing and the staggered sticky information models:

$$\frac{1}{2} \frac{\bar{n}}{\lambda} + \frac{1}{\lambda^2} \left(\int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) dn - 1 \right).$$

Figures 2a,b,d,e show the normalized cumulative output effects of both level and growth rate shocks as a function of the corresponding decay parameter, using the empirical distribution of price rigidity in the US, for all models. The identical firms economies were constructed with the same relevant average **duration**. From Figures 2a,b,c, in the context of level shocks it is apparent that correcting for Jensen's inequality makes the behavior of the heterogeneous and the identical firms economies similar. In the case of growth rate shocks (Figures 2d,e,f) this correction does not suffice, and, except for very short-lived shocks, the real effects of monetary shocks are significantly larger in the heterogeneous economies.

5 Quantitative analysis

Recall that all analytic and qualitative results derived so far rested on the simplifying assumption of no strategic complementarities/real rigidities ($\theta = 1$). Even then, the implications of accounting for heterogeneity are not trivial in quantitative terms. For example, for level shocks, taking the case of $\rho = 0$ as a benchmark, the increase in total output effects when heterogeneity is accounted for exceeds 70% in the case of the US economy: the inverse of the average frequency of price changes reported by Bils and Klenow (2004), which is the usual measure, is 3.8 months, while the average duration of price rigidity is 6.6 months.¹⁴ For growth rate shocks, again focusing on permanent (or very persistent) shocks as a benchmark, the results are much more pronounced: based on the statistics reported by Bils and Klenow (2004), $\sigma_n = 7.1$ months, which means that the ratio of total output effects computed correctly to the usual measure is $\frac{(6.6)^2 + (7.1)^2}{(3.8)^2} = 6.5!$ Even correcting for Jensen's inequality and using the average duration produces cumulative effects which are less than half of the those in the heterogeneous economy.

Introducing strategic complementarities exacerbates the role of heterogeneity even further. To illustrate this result I start by displaying impulse response functions (IRFs) for several combinations

¹⁴For the Euro area, the results reported by Dhyne et al. (2004) are even more pronounced: the usual measure is 6.6 months, while the average duration of price rigidity ranges from 13 to 15.1 months, depending on how the individual country data is aggregated.

of shocks and models. For growth rate shocks I display IRFs of output and inflation, while for level shocks I depict IRFs of output and of the aggregate price level. Note that all the comparisons involve identical firms economies with the same relevant average **duration** as their heterogeneous firms counterparts. So, I am already correcting the usual measure in accounting for Jensen’s inequality. The comparison against identical firms economies with the same average frequencies would imply an even larger role for heterogeneity.

Figures 3a-h, 4a-h, and 5a-h display the results for the cases of Taylor pricing, sticky information, and staggered sticky information, respectively.¹⁵ All cases include IRFs both with and without strategic complementarities. For comparison purposes, I set $\theta = 0.1$ when there are strategic complementarities, since this is a standard calibration in this literature (e.g. Mankiw and Reis, 2002, and Dupor and Tsuruga, 2005).

From these results it is clear that strategic complementarities interact with heterogeneity to generate more persistent real effects of monetary shocks. Real rigidities do make the adjustment process more sluggish, even in the identical firms case. With heterogeneity, however, this is even more so, according to several metrics: the recession troughs are delayed, output is lower than in the identical firms economy essentially during the whole process, and takes much longer to return to the steady state; inflation is, accordingly, also more persistent. The cumulative effects on output, obtained through numerical integration, confirm the interaction of heterogeneity and strategic complementarities. Figures 6a-f present results analogous to figures 2a-f, with $\theta = 0.1$. Now, even for level shocks (Figures 6a-c) it is the case that total output effects are larger in the heterogeneous economy. For growth rate shocks (Figures 6d-f) the differences are amplified even further. As an example, in the case of permanent growth rate shocks, the ratio of cumulative output effects in the heterogeneous economy to those in the identical firms economy goes from around 2.15 for all models when $\theta = 1$ to 3.5 – 6 (depending on the model), when $\theta = 0.1$.

The intuition for these results can be understood in the context of the framework developed by Haltiwanger and Waldman (1991). With no strategic complementarities, relative to an identical firms economy with the same average duration of price rigidity, heterogeneous economies initially display faster adjustment owing to a relatively higher measure of firms with shorter (expected) durations, which get to adjust earlier. As time passes, the distribution of durations among firms which have not yet adjusted prices or updated price plans becomes more and more dominated by firms with relatively longer (expected) durations, slowing down the adjustment process. The weight of each group of firms in this process corresponds directly to the distribution $f(\cdot)$. On the other hand, when there are strategic complementarities, the decisions of firms with shorter durations are influenced by the existence of firms with longer durations, since the former do not want to set prices that will deviate “too much” from the aggregate price in the future. Therefore firms with longer “contract lengths” end up having a disproportionate effect on the price level. That is, strategic complementarities interact with heterogeneity to make the adjustment process even more sluggish.

¹⁵The Calvo model was omitted, because solving the model with strategic complementarities is extremely demanding in computational terms.

5.1 Fitting IRFs with an identical firms model

After developing a better understanding of how heterogeneity in price setting introduces persistence in monetary economies, in this subsection I finally revisit a question posed earlier in the paper: which parameterization for an identical firms economy will best mimic the dynamics of a given heterogeneous economy in terms of its impulse response functions?

For that purpose I conduct the following experiment:¹⁶ given the empirical distribution of (expected) durations used in all previous simulations, and a value for θ in the heterogeneous economy, what are the average (expected) duration (n_{id}) and degree of strategic complementarities ($1 - \theta_{id}$) in the identical firms economy that minimize the integral of squared deviations of its IRFs from the heterogeneous economy’s IRFs.¹⁷ I do this calculation for several values of θ in the heterogeneous economy, for several shocks, and also use absolute deviations instead of squared deviations.

The results are presented in Tables 1a-d (sticky information) and 2a-d (staggered sticky information). Given the analyses of the previous sections, one might have guessed that fitting the IRFs of a truly heterogeneous economy with an identical firms model would require higher (average) durations and perhaps more strategic complementarities than in the heterogeneous economy. This, however, is not exactly the case. A higher average duration is indeed required, and the more so the higher the degree of real rigidities in the heterogeneous economy ($1 - \theta$). Despite the fact that the average duration of price rigidity reported by [Bils and Klenow \(2004\)](#) is approximately 2.2 quarters, the “best fitting average duration” for the identical firms models ranges from 5 to 11 quarters, depending on the model, the type of shock,¹⁸ and the degree of strategic complementarity in the original heterogeneous economy. The degree of strategic complementarity in the identical firms economy, however, is systematically lower than in the heterogeneous economy, but they do vary in the same direction. These results are robust across all combinations of shocks and (sticky information) models.

My understanding of why the average duration, but not the degree of strategic complementarity, turns out to be higher in the identical firms economies is admittedly imperfect. My intuition is as follows. Frictions to price setting, in the form of non-zero durations, are necessary (and sufficient) for monetary shocks to have real effects in these models. Strategic complementarities, on the other hand, are neither necessary nor sufficient: they simply strengthen the role of price setting frictions, but have no effect by themselves (i.e. in the absence of price setting frictions). Therefore, the main channel through which persistence is increased in the identical firms models to approximate the heterogeneous economy must be the average duration. The best fitting level of average durations, however, generates “too much” persistence, given the original degree of strategic complementarities, because of the interaction between firms with lower and higher durations (akin to “responders” and “non-responders” in the terminology of [Haltiwanger and Waldman, 1991](#)). Therefore, θ_{id} increases

¹⁶Due to the computational demands for solving the sticky price models with strategic complementarities, I only perform the experiments for both sticky information models, for which an explicit analytical solution is available. Given the mechanisms underlying the results, however, I conjecture that the same regularities are to be found in the sticky price models.

¹⁷I use IRFs for output.

¹⁸In the simulations I considered permanent shocks, and shocks with decay parameter of 0.5 (half life of 1.4 years).

relative to θ (often to the extent of implying strategic substitutability - $\theta_{id} > 1$ - rather than complementarity) to provide a better fit.

These results show that we must be cautious when interpreting estimates of parameters of price setting frictions and real rigidities based on identical firms models, or calibrating them in such models based on micro evidence. The point is not that identical firms models are unable to provide a reasonable description of a more complex, heterogeneous reality, but rather that this is likely to require parameter values which will seem unrealistic if interpreted literally. Given the empirical evidence documenting a high degree of heterogeneity and the fact that it does matter, the parameters of (mispespecified) identical firms models cannot be seen as “structural,” and should be treated accordingly.¹⁹

6 Concluding Remarks

In this paper, I argued that heterogeneity in price setting frictions should have a larger role in models which attempt to analyze the real effects of monetary shocks. Standard models of nominal price rigidity usually assume that all firms are identical in terms of price setting behavior. This would be a good approximation either if empirically the degree of heterogeneity were small or if, despite significant in the real world, heterogeneity turned out not to matter.

Available empirical evidence points to the existence of a high degree of heterogeneity in price setting frictions. This paper provided analytical results which show that heterogeneity affects the dynamic response of economies to monetary shocks. Relative to an identical-firms economy with the same average degree of frictions, heterogeneous economies initially display faster adjustment owing to a relatively higher measure of firms with shorter “contract lengths,” which get to adjust earlier. As time passes, the distribution of contract lengths among firms which have not yet adjusted becomes more and more dominated by firms with relatively longer contracts, slowing down the adjustment process. When there are strategic complementarities in price setting, the decision to adjust by firms which face lower price setting frictions is influenced by the existence of firms facing higher frictions, which end up having a disproportionate effect on the price level. Strategic complementarities interact with heterogeneity to make the adjustment process even more sluggish.

Using the distribution of the frequency of price adjustments reported recently by Bils and Klenow (2004) to calibrate the distribution of frictions in the four models that I considered, I obtained important quantitative differences between models with identical firms and models with heterogeneity. In particular, I showed that reproducing the dynamics of a truly heterogeneous economy with a model based on identical firms requires substantially larger degrees of price setting frictions.

These results might help shed some additional light on the so called *persistence problem* (Chari et al., 2000). Some recent papers which carry out quantitative evaluations of sticky-price DSGE models based on identical firms find that in order to obtain good empirical performance one needs

¹⁹Smets and Wouters (2003) hint at the possibility that heterogeneity could bias their results (footnote n. 3). Christiano et al. (2005) claim that “inference about nominal rigidities is sensitive to getting the real side of the model right,” but ignore the possibility that the same applies to heterogeneity in terms of price (and perhaps wage) setting.

unrealistically low frequencies of price adjustment. A natural step is to fully assess the empirical relevance of the results obtained in this paper for this issue, by introducing heterogeneity in a standard DSGE model, and taking the model to the data. Promising results in this direction appear in Coenen and Levin (2004).

Another research question motivated by these results, which I am currently working to address, is whether heterogeneity in price setting of the sort considered in this paper can account for the so called Purchasing Power Parity (PPP) puzzle. A recent paper by Imbs et al. (2005) shows, using econometric methods, that aggregation of heterogeneous dynamics in terms of deviations from the law of one price can account for the PPP puzzle. They use econometric methods to calculate the half life of deviations from PPP in a way that accounts for the aggregation bias, and therefore makes their measure comparable to theoretical constructs based on models in which the persistence of deviations from the law of one price is the same across different goods. My results suggest addressing the same question in the opposite direction: writing a model with explicit heterogeneous dynamics in deviations from the law of one price, arising from heterogeneity in price setting frictions, and assessing whether it can match the dynamic properties of deviations from PPP found in the data.

Appendix

I start by characterizing the initial inflationary steady state in each model.

Calvo pricing

In the inflationary steady state, firm i from group n sets its price as

$$\begin{aligned} p_{n,i}(t) &= \int_0^\infty \frac{1}{n} e^{-\frac{1}{n}s} (\pi t + \pi s) ds \\ &= \pi t + \pi n. \end{aligned}$$

The aggregate price level is given by

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} p_{n,i}(s) ds dn \\ &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} (\pi s + \pi n) ds dn \\ &= \pi t, \end{aligned}$$

and output is constant at the natural rate $y(t) = 0$.

Taylor staggered price setting

In Taylor's model, prior to $t = 0$ firm i from group n sets its price as

$$\begin{aligned} p_{n,i}(t) &= \frac{1}{n} \int_0^n (\pi t + \pi s) ds \\ &= \pi t + \frac{\pi n}{2}, \end{aligned}$$

and the aggregate price level is therefore given by

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \left(\frac{1}{n} \int_0^n p_{n,i}(t-s) ds \right) dn \\ &= \int_0^{n^*} f(n) \left(\frac{1}{n} \int_0^n \pi t - \pi s + \frac{\pi n}{2} ds \right) dn \\ &= \pi t. \end{aligned}$$

Output is, therefore, constant at $y(t) = 0$.

Sticky information models

In both sticky information models firms trivially set $p_{n,i}(t) = p(t) = m(t)$, so that $y(t) = 0$.

Proofs of propositions

Proposition 1 *In the context of permanent level shocks ($\rho = 0$) and no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{\bar{m}} \int_0^\infty \bar{m} - p(t) dt$ is equal to:*

- i) the average duration of price rigidity in the economy, in the case of both sticky price models;*
- ii) the average duration of price plans in the economy, in the case of both sticky information models.*

Proof. The first step is to show that:

- a) the effects in the sticky information model are identical to the ones under Calvo pricing, and
- b) that the same is true for staggered sticky information relative to the Taylor model.

This is straightforward. For a) just compare (3) and (6); and for b) compare (4) and (7), and (5) and (8). In all cases set $\rho = 0$ and $\theta = 1$.

Now, derive the expression for the (normalized) cumulative output effect for each pair of models.

-Calvo pricing and sticky information:

$$\begin{aligned}
 \frac{1}{\bar{m}} \int_0^\infty \bar{m} - p(t) dt &= \int_0^\infty 1 - \left(1 - \int_0^{n^*} f(n) e^{-\frac{1}{n}t} dn \right) dt \\
 &= \int_0^\infty \int_0^{n^*} f(n) e^{-\frac{1}{n}t} dn dt \\
 &= \int_0^{n^*} \int_0^\infty f(n) e^{-\frac{1}{n}t} dt dn \\
 &= \int_0^{n^*} f(n) n dn \\
 &= \bar{n}.
 \end{aligned}$$

-Taylor pricing and staggered sticky information:²⁰

$$\begin{aligned}
\frac{1}{\bar{m}} \int_0^\infty \bar{m} - p(t) dt &= \int_0^{n^*} 1 - \left(1 - \int_t^{n^*} f(n) \left(1 - \frac{t}{n} \right) dn \right) dt \\
&= \int_0^{n^*} \int_t^{n^*} f(n) \left(1 - \frac{t}{n} \right) dndt \\
&= \int_0^{n^*} 1 - F(t) dt - \int_0^{n^*} \left(\int_t^{n^*} f(n) \frac{1}{n} dn \right) t dt \\
&= \int_0^{n^*} 1 - F(n) dn - \frac{1}{2} \int_0^{n^*} n f(n) dn \\
&= \frac{1}{2} \int_0^{n^*} f(n) n dn \\
&= \frac{\bar{n}}{2}.
\end{aligned}$$

■

Proposition 2 *In the context of permanent level shocks ($\rho = 0$) and no strategic complementarities ($\theta = 1$), an arbitrary heterogeneous economy will always display larger (normalized) cumulative output effects (as measured by $\frac{1}{\bar{m}} \int_0^\infty \bar{m} - p(t) dt$) than an identical firms economy with the same average frequency of price adjustments (in the case of sticky price models), or average frequency of price plan revisions (in the case of sticky information models).*

Proof. The results follow directly from Jensen’s inequality, combined with *Proposition 1*. For the sticky price models, pick the average frequency of price adjustments in the identical firms economy to match that of the corresponding heterogeneous economy ($\int_0^{n^*} f(n) \frac{1}{n} dn$ for Calvo pricing, and $\int_0^{n^*} f(n) \frac{2}{n} dn$ for the Taylor’s model). Then, the average duration of price rigidity in the heterogeneous economy equals:

i) For Calvo pricing: $\int_0^{n^*} f(n) n dn > \left(\int_0^{n^*} f(n) \frac{1}{n} dn \right)^{-1}$ = average duration of price rigidity in the identical firms economy.

ii) For Taylor pricing: $\int_0^{n^*} f(n) \frac{n}{2} dn > \left(\int_0^{n^*} f(n) \frac{2}{n} dn \right)^{-1}$ = average duration of price rigidity in the identical firms economy.

For the sticky information models do the analogous steps with the average frequency of price plan revisions.

Finally, since the relevant average durations in the heterogeneous economies exceed the durations in the identical firms economies, applying *Proposition 1* completes the proof. ■

Proposition 3 *In the context of permanent growth rate shocks ($\lambda = 0$) and no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt$ equals:*

²⁰To go from the third to the fourth line below, integrate $\int_0^{n^*} \left(\int_t^{n^*} f(n) \frac{1}{n} dn \right) t dt$ by parts, by differentiating $\left(\int_t^{n^*} f(n) \frac{1}{n} dn \right)$ and integrating $t dt$. The same “trick” should be applied to all similar integrals found subsequently.

i) For the Calvo model:

zero;

ii) For the Taylor model:

$$\frac{1}{12} (\bar{n}^2 + \sigma_n^2);$$

iii) For the sticky information model:

$$(\bar{n}^2 + \sigma_n^2);$$

iv) For the staggered sticky information model:

$$\frac{1}{6} (\bar{n}^2 + \sigma_n^2).$$

Proof. i) Calvo pricing:

Set $\theta = 1$ in (9). Then, for $t \geq 0$, $p(t)$ is given by

$$\begin{aligned} p(t) &= \int_0^{n^*} f(n) \int_{-\infty}^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} p_{n,i}(s) ds dn \\ &= \int_0^{n^*} f(n) \left(\int_{-\infty}^0 \frac{1}{n} e^{-\frac{1}{n}(t-s)} (\pi s + \pi n) ds + \int_0^t \frac{1}{n} e^{-\frac{1}{n}(t-s)} ((\pi + \Delta\pi) s + (\pi + \Delta\pi) n) ds \right) dn \\ &= (\pi + \Delta\pi) t \\ &= m(t), \end{aligned}$$

so that output is identically zero.

ii) Taylor model:

Set $\theta = 1$ in (10) and (11). Then:

$$\begin{aligned} \frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\Delta\pi} \int_0^{n^*} (\pi + \Delta\pi) t - (\pi + \Delta\pi) t + \frac{1}{2} \Delta\pi \int_t^{n^*} f(n) \left(t - \frac{t^2}{n} \right) dn dt \\ &= \frac{1}{2} \int_0^{n^*} \int_t^{n^*} f(n) \left(t - \frac{t^2}{n} \right) dn dt \\ &= \frac{1}{2} \left(\int_0^{n^*} t (1 - F(t)) dt - \int_0^{n^*} \left(t^2 \int_t^{n^*} f(n) \frac{1}{n} dn \right) dt \right) \\ &= \frac{1}{2} \left(\frac{1}{2} \int_0^{n^*} f(t) t^2 dt - \int_0^{n^*} \left(\int_t^{n^*} f(n) \frac{1}{n} dn \right) t^2 dt \right) \\ &= \frac{1}{2} \left(\frac{1}{2} \int_0^{n^*} f(n) n^2 dn - \frac{1}{3} \int_0^{n^*} f(n) n^2 dn \right) \\ &= \frac{1}{12} \int_0^{n^*} f(n) n^2 dn \\ &= \frac{1}{12} (\bar{n}^2 + \sigma_n^2). \end{aligned}$$

iii) Sticky information:

Set $\theta = 1$ in (12) and solve for $p(t)$. Then:

$$\begin{aligned}
\frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\Delta\pi} \int_0^\infty \pi t + \Delta\pi t - \left(\pi t + \Delta\pi \int_0^{n^*} f(n) t \left(1 - e^{-\frac{1}{n}t}\right) dn \right) dt \\
&= \int_0^\infty t \left(1 - \int_0^{n^*} f(n) \left(1 - e^{-\frac{1}{n}t}\right) dn \right) dt \\
&= \int_0^\infty \int_0^{n^*} f(n) t e^{-\frac{1}{n}t} dn dt \\
&= \int_0^{n^*} f(n) \int_0^\infty t e^{-\frac{1}{n}t} dt dn \\
&= \int_0^{n^*} f(n) n^2 dn \\
&= (\bar{n}^2 + \sigma_n^2).
\end{aligned}$$

iv) Staggered sticky information:

Set $\theta = 1$ in (13) and (14), and solve for $p(t)$. Then:

$$\begin{aligned}
\frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\Delta\pi} \int_0^{n^*} \pi t + \Delta\pi t - \left(\pi t + \Delta\pi \left(F(t) t + t \int_t^{n^*} f(n) \frac{t}{n} dn \right) \right) dt \\
&= \int_0^{n^*} t \left(1 - F(t) - \int_t^{n^*} f(n) \frac{t}{n} dn \right) dt \\
&= \int_0^{n^*} t (1 - F(t)) dt - \int_0^{n^*} \left(\int_t^{n^*} f(n) \frac{t^2}{n} dn \right) dt \\
&= \int_0^{n^*} f(t) \left(\frac{t^2}{2} - \frac{t^2}{3} \right) dt \\
&= \frac{1}{6} \int_0^{n^*} f(n) n^2 dn \\
&= \frac{1}{6} (\bar{n}^2 + \sigma_n^2).
\end{aligned}$$

■

Proposition 4 *In the context of temporary level shocks ($\rho > 0$) with no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{m} \int_0^\infty m(t) - p(t) dt$ is equal to:*

i) For **both** the Calvo pricing and the sticky information models:

$$\int_0^{n^*} f(n) \frac{n}{1 + \rho n} dn;$$

ii) For **both** the Taylor pricing and the staggered sticky information models:

$$\frac{1}{\rho} \left(1 - \int_0^{n^*} f(n) \frac{1 - e^{-\rho n}}{\rho n} dn \right).$$

In the context of temporary growth rate shocks ($\lambda > 0$) with no strategic complementarities ($\theta = 1$), the (normalized) cumulative output effect as measured by $\frac{1}{\Delta\pi} \int_0^\infty m(t) - p(t) dt$ equals:

iii) For **both** the Calvo pricing and the sticky information models:

$$\int_0^{n^*} f(n) \frac{n^2}{1 + \lambda n} dn;$$

iv) For **both** the Taylor pricing and the staggered sticky information models:

$$\frac{1}{2\lambda} + \frac{1}{\lambda^2} \left(\int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) dn - 1 \right).$$

Proof. i)

a) Calvo pricing:

Set $\theta = 1$ in (3). Then:

$$\begin{aligned} \frac{1}{\bar{m}} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\bar{m}} \int_0^\infty \bar{m} e^{-\rho t} - \bar{m} \int_0^{n^*} f(n) \frac{1}{n^2 \rho^2 - 1} \left(e^{-\frac{1}{n}t} - e^{-\rho t} \right) dn dt \\ &= \int_0^\infty \int_0^{n^*} f(n) \left[e^{-\rho t} - \frac{1}{n^2 \rho^2 - 1} \left(e^{-\frac{1}{n}t} - e^{-\rho t} \right) \right] dn dt \\ &= \int_0^{n^*} f(n) \int_0^\infty \left[e^{-\rho t} - \frac{1}{n^2 \rho^2 - 1} \left(e^{-\frac{1}{n}t} - e^{-\rho t} \right) \right] dt dn \\ &= \int_0^{n^*} f(n) \frac{n}{1 + \rho n} dn. \end{aligned}$$

b) Sticky information:

Set $\theta = 1$ in (6) and solve for $p(t)$. Then:

$$\begin{aligned} \frac{1}{\bar{m}} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\bar{m}} \int_0^\infty \bar{m} e^{-\rho t} - \bar{m} e^{-\rho t} \int_0^{n^*} f(n) \left(1 - e^{-\frac{1}{n}t} \right) dn dt \\ &= \int_0^\infty \int_0^{n^*} f(n) \left[e^{-\rho t} - e^{-\rho t} \left(1 - e^{-\frac{1}{n}t} \right) \right] dn dt \\ &= \int_0^{n^*} f(n) \int_0^\infty e^{-\rho t} e^{-\frac{1}{n}t} dt dn \\ &= \int_0^{n^*} f(n) \frac{n}{1 + \rho n} dn. \end{aligned}$$

ii)

a) Taylor model:

Set $\theta = 1$ in (4) and (5). Then:²¹

$$\begin{aligned}
\frac{1}{\bar{m}} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\bar{m}} \left(\int_0^\infty m(t) dt - \int_0^{\mathbf{n}^*} p(t) dt - \int_{\mathbf{n}^*}^\infty p(t) dt \right) \\
&= \int_0^\infty e^{-\rho t} dt - \int_0^{\mathbf{n}^*} \int_0^t \frac{f(n)}{n^2 \rho^2} (e^{n\rho} - 2 + e^{-n\rho}) dn e^{-\rho t} dt \\
&\quad + \int_0^{\mathbf{n}^*} \int_t^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} (e^{\rho t} + e^{-\rho n} - e^{-\rho(n-t)} - 1) dn e^{-\rho t} dt \\
&\quad - \int_{\mathbf{n}^*}^\infty e^{-\rho t} \int_0^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} (e^{n\rho} - 2 + e^{-n\rho}) dn dt \\
&= \frac{1}{\rho} - \int_0^{\mathbf{n}^*} \int_0^t \frac{f(n)}{n^2 \rho^2} (e^{n\rho} - 2 + e^{-n\rho}) dn e^{-\rho t} dt \\
&\quad - \int_0^{\mathbf{n}^*} \int_t^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} (e^{-n\rho} - 1) dn e^{-\rho t} dt - \int_0^{\mathbf{n}^*} \int_t^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} (1 - e^{-n\rho}) dn dt \\
&\quad - \int_{\mathbf{n}^*}^\infty \int_0^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} e^{-n\rho} (e^{2n\rho} - 2e^{n\rho} + 1) dn e^{-\rho t} dt \\
&= \frac{1}{\rho} + \left(\left[\int_0^t \frac{f(n)}{n^2 \rho^2} (e^{n\rho} - 2 + e^{-n\rho}) dn \frac{e^{-\rho t}}{\rho} \right]_{t=0}^{\mathbf{n}^*} - \int_0^{\mathbf{n}^*} \frac{f(t)}{t^2 \rho^2} \frac{e^{-\rho t}}{\rho} (e^{\rho t} - 2 + e^{-\rho t}) dt \right) \\
&\quad - \left(\left[- \int_t^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} (e^{-n\rho} - 1) dn \frac{e^{-\rho t}}{\rho} \right]_{t=0}^{\mathbf{n}^*} - \int_0^{\mathbf{n}^*} \frac{e^{-\rho t}}{\rho} \frac{f(t)}{t^2 \rho^2} (e^{-\rho t} - 1) dt \right) \\
&\quad - \left(\left[\int_t^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} (1 - e^{-n\rho}) dn \right]_{t=0}^{\mathbf{n}^*} - \int_0^{\mathbf{n}^*} \frac{f(t)}{t \rho^2} (e^{-\rho t} - 1) dt \right) \\
&\quad - \frac{e^{-\rho \mathbf{n}^*}}{\rho} \int_0^{\mathbf{n}^*} \frac{f(n)}{n^2 \rho^2} e^{-n\rho} (e^{2n\rho} - 2e^{n\rho} + 1) dn \\
&= \frac{1}{\rho} \left(1 - \int_0^{\mathbf{n}^*} f(n) \frac{1 - e^{-\rho n}}{\rho n} dn \right)
\end{aligned}$$

b) Staggered sticky information

²¹ Again, the only non-trivial step is to integrate by parts expressions such as $\int_0^{\mathbf{n}^*} \int_0^t f(n) \frac{e^{-\rho t}}{n^2 \rho^2} e^{-n\rho} (e^{2n\rho} - 2e^{n\rho} + 1) dn dt$ or $\int_0^{\mathbf{n}^*} \int_t^{\mathbf{n}^*} f(n) \frac{e^{-\rho t}}{n^2 \rho^2} (e^{t\rho} + e^{-n\rho} - e^{t\rho} e^{-n\rho} - 1) dn dt$. The trick is to rearrange such expressions as, for example,

$\int_0^{\mathbf{n}^*} \left(\int_0^t h_1(n, \rho) dn \right) h_2(t, \rho) dt$, and integrate by parts by differentiating $\left(\int_0^t h_1(n, \rho) dn \right)$ and integrating $h_2(t, \rho) dt$.

Set $\theta = 1$ in (7) and (8) and solve for $p(t)$. Then:

$$\begin{aligned}
\frac{1}{\bar{m}} \int_0^\infty m(t) - p(t) dt &= \frac{1}{\bar{m}} \int_0^{n^*} \bar{m}e^{-\rho t} - \bar{m}e^{-\rho t} \left(F(t) + \int_t^{n^*} f(n) \frac{t}{n} dn \right) dt \\
&= \int_0^{n^*} e^{-\rho t} - \left(e^{-\rho t} F(t) + e^{-\rho t} \int_t^{n^*} f(n) \frac{t}{n} dn \right) dt \\
&= \int_0^{n^*} e^{-\rho t} (1 - F(t)) dt - \int_0^{n^*} e^{-\rho t} t \int_t^{n^*} f(n) \frac{1}{n} dn dt \\
&= \left[-(1 - F(t)) \frac{e^{-\rho t}}{\rho} \right]_{t=0}^{n^*} - \int_0^{n^*} \frac{e^{-\rho t}}{\rho} f(t) dt - \\
&\quad \left(\left[\int_t^{n^*} f(n) \frac{1}{n} dn \left(-\frac{1}{\rho^2} - \frac{t}{\rho} \right) e^{-\rho t} \right]_{t=0}^{n^*} - \int_0^{n^*} \left(\frac{1}{\rho^2} + \frac{t}{\rho} \right) e^{-\rho t} f(t) \frac{1}{t} dt \right) \\
&= \frac{1}{\rho} - \int_0^{n^*} \frac{e^{-\rho t}}{\rho} f(t) dt - \frac{1}{\rho^2} \int_0^{n^*} f(t) \frac{1}{t} dt + \int_0^{n^*} \left(\frac{1}{\rho^2} \frac{1}{t} + \frac{1}{\rho} \right) e^{-\rho t} f(t) dt \\
&= \frac{1}{\rho} \left(1 - \int_0^{n^*} f(n) \frac{1 - e^{-\rho n}}{\rho n} dn \right).
\end{aligned}$$

iii)

a) Calvo pricing:

Set $\theta = 1$ in (9). Then:

$$\begin{aligned}
\int_0^\infty \frac{m(t) - p(t)}{\Delta\pi} dt &= \frac{1}{\Delta\pi} \int_0^\infty \pi t + \Delta\pi \frac{1 - e^{-\lambda t}}{\lambda} - \pi t - \int_0^{n^*} f(n) \frac{e^{-\lambda t} - 1 + \left(1 - e^{-\frac{1}{n}t}\right) n^2 \lambda^2}{\lambda (n^2 \lambda^2 - 1)} \Delta\pi dn dt \\
&= \int_0^\infty \int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda t}}{\lambda} - \frac{e^{-\lambda t} - 1 + \left(1 - e^{-\frac{1}{n}t}\right) n^2 \lambda^2}{\lambda (n^2 \lambda^2 - 1)} \right) dn dt \\
&= \int_0^{n^*} f(n) \frac{n^2}{1 + n\lambda} dn.
\end{aligned}$$

b) Sticky information:

Set $\theta = 1$ in (12) and solve for $p(t)$. Then:

$$\begin{aligned}
\int_0^\infty \frac{m(t) - p(t)}{\Delta\pi} dt &= \frac{1}{\Delta\pi} \int_0^\infty \pi t + \Delta\pi \frac{1 - e^{-\lambda t}}{\lambda} - \left(\pi t + \Delta\pi \int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda t}}{\lambda} \right) \left(1 - e^{-\frac{1}{n}t} \right) dn \right) dt \\
&= \int_0^\infty \frac{1 - e^{-\lambda t}}{\lambda} - \int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda t}}{\lambda} \right) \left(1 - e^{-\frac{1}{n}t} \right) dndt \\
&= \int_0^\infty \int_0^{n^*} f(n) \frac{1 - e^{-\lambda t}}{\lambda} e^{-\frac{1}{n}t} dndt \\
&= \int_0^{n^*} f(n) \int_0^\infty \frac{1 - e^{-\lambda t}}{\lambda} e^{-\frac{1}{n}t} dt dn \\
&= \int_0^{n^*} f(n) \frac{n^2}{1 + \lambda n} dn.
\end{aligned}$$

iv)

a) Taylor model:

Set $\theta = 1$ in (10) and (11). Then:

$$\begin{aligned}
\int_0^\infty \frac{m(t) - p(t)}{\Delta\pi} dt &= \int_0^{n^*} \frac{\pi t}{\Delta\pi} + \frac{(1 - e^{-\lambda t})}{\lambda} - \frac{\pi t}{\Delta\pi} - \int_0^t \frac{f(n)}{n^2 \lambda^2} \left(n^2 \lambda + (e^{-\lambda(t+n)} - e^{-\lambda t}) \frac{e^{\lambda n} - 1}{\lambda} \right) dndt \\
&\quad - \int_0^{n^*} \int_t^{n^*} \frac{f(n)}{n^2 \lambda^2} \left(n \lambda t + (e^{-\lambda(t+n)} - e^{-\lambda t}) \frac{e^{\lambda t} - 1}{\lambda} \right) dndt + \int_{n^*}^\infty \frac{\pi t}{\Delta\pi} + \frac{1 - e^{-\lambda t}}{\lambda} dt \\
&\quad - \int_{n^*}^\infty \frac{\pi t}{\Delta\pi} + \int_0^{n^*} \frac{f(n)}{n^2 \lambda^2} \left(n^2 \lambda + (e^{-\lambda(t+n)} - e^{-\lambda t}) \frac{e^{\lambda n} - 1}{\lambda} \right) dndt \\
&= -\frac{1}{\lambda^2} + \frac{e^{-\lambda n^*} + \lambda n^*}{\lambda^2} + -\frac{1}{\lambda} (n^* - \bar{n}) - \frac{e^{-\lambda n^*}}{\lambda^2} \int_0^{n^*} f(n) \frac{1 - e^{-\lambda n}}{\lambda n} \frac{e^{\lambda n} - 1}{\lambda n} dn \\
&\quad + \int_0^{n^*} \frac{f(n)}{\lambda^2} \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right)^2 dn - \frac{1}{\lambda} \int_0^{n^*} \frac{n}{2} f(n) dn + \frac{1}{\lambda^2} \int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) dn \\
&\quad + \frac{1}{\lambda^2} \int_0^{n^*} \frac{f(n)}{\lambda n} \frac{e^{-\lambda n} - 1}{\lambda n} dn + \int_0^{n^*} \frac{f(n)}{\lambda n} \frac{e^{-\lambda n}}{\lambda^2} \frac{1 - e^{-\lambda n}}{\lambda n} dn - \frac{e^{-\lambda n^*}}{\lambda^2} \\
&\quad - \frac{e^{-\lambda n^*}}{\lambda^2} \int_0^{n^*} f(n) \left(\frac{e^{-\lambda n} - 1}{\lambda n} \frac{e^{\lambda n} - 1}{\lambda n} \right) dn \\
&= \frac{1}{2\lambda} + \frac{1}{\lambda^2} \left(\int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) dn - 1 \right).
\end{aligned}$$

b) Staggered sticky information:

Set $\theta = 1$ in (13) and (14), and solve for $p(t)$. Then:

$$\begin{aligned}
\int_0^\infty \frac{m(t) - p(t)}{\Delta\pi} dt &= \frac{1}{\Delta\pi} \int_0^{n^*} \pi t + \Delta\pi \frac{1 - e^{-\lambda t}}{\lambda} - \pi t - \Delta\pi \left(F(t) \frac{1 - e^{-\lambda t}}{\lambda} + \frac{1 - e^{-\lambda t}}{\lambda} \int_t^{n^*} f(n) \frac{t}{n} dn \right) dt \\
&= \int_0^{n^*} \frac{1 - e^{-\lambda t}}{\lambda} - \left(F(t) \frac{1 - e^{-\lambda t}}{\lambda} + \frac{1 - e^{-\lambda t}}{\lambda} \int_t^{n^*} f(n) \frac{t}{n} dn \right) dt \\
&= \int_0^{n^*} \frac{1 - e^{-\lambda t}}{\lambda} - \frac{1 - e^{-\lambda t}}{\lambda} \left(F(t) + \int_t^{n^*} f(n) \frac{t}{n} dn \right) dt \\
&= \int_0^{n^*} \frac{1 - e^{-\lambda t}}{\lambda} (1 - F(t)) dt - \int_0^{n^*} \frac{1 - e^{-\lambda t}}{\lambda} t \left(\int_t^{n^*} f(n) \frac{1}{n} dn \right) dt \\
&= -\frac{1}{\lambda^2} + \int_0^{n^*} f(t) \left(\frac{t}{\lambda} + \frac{e^{-t\lambda}}{\lambda^2} \right) dt + \frac{1}{\lambda^3} \int_0^{n^*} f(t) \frac{1}{t} dt \\
&\quad - \int_0^{n^*} f(t) \frac{1}{t} \frac{1}{\lambda^3} \left(t\lambda e^{-t\lambda} + \frac{1}{2} t^2 \lambda^2 + e^{-t\lambda} \right) dt \\
&= \frac{1}{2\lambda} + \frac{1}{\lambda^2} \left(\int_0^{n^*} f(n) \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) dn - 1 \right).
\end{aligned}$$

■

References

- [1] Aoki, K. (2001), "Optimal Monetary Policy Responses to Relative-Price Changes," *Journal of Monetary Economics* 48: 55-80.
- [2] Baharad, E. and E. Benjamin (2004), "Price Rigidity and Price Dispersion: Evidence from Micro Data," *Review of Economic Dynamics* 7: 613-641.
- [3] Ball, L. and D. Romer (1989), "Equilibrium and Optimal Timing of Price Changes," *Review of Economic Studies* 56: 179-198.
- [4] Barsky, R., C. House and M. Kimball (2003), "Do Flexible Durable Goods Prices Undermine Sticky Price Models?," NBER Working Paper #9832.
- [5] Benigno, P. (2004), "Optimal Monetary Policy in a Currency Area," *Journal of International Economics* 63: 293-320.
- [6] Bils, M. and P. Klenow (2004), "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy* 112: 947-985 (also NBER Working Paper #9069).
- [7] Bils, M., P. Klenow and O. Kryvstov (2003), "Sticky Prices and Monetary Policy Shocks," *Federal Reserve Bank of Minneapolis Quarterly Review* 27: 2-9.
- [8] Blanchard, O. and N. Kiyotaki (1987), "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review* 77: 647-666.
- [9] Blinder, A., E. Canetti, D. Lebow and J. Rudd. (1998), *Asking about Prices: A New Approach to Understanding Price Stickiness*, Russel Sage Foundation.
- [10] Caballero, R. and E. Engel (1991), "Dynamic (S, s) Economies," *Econometrica* 59: 1659-1686.
- [11] _____ (1993), "Heterogeneity and Output Fluctuations in a Dynamic Menu-Cost Economy," *Review of Economic Studies* 60: 95-119.
- [12] Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12: 383-98.
- [13] Calvo, G., O. Celasun, and M. Kumhof (2003), "A Theory of Rational Inflationary Inertia," in P. Aghion, R. Frydman, M. Woodford and J. Stiglitz, eds., *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press.
- [14] Chari, V., P. Kehoe and E. McGrattan (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?," *Econometrica* 68: 1151-1179.
- [15] Christiano, L., M. Eichenbaum and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 115: 1-45.

- [16] Coenen, G. and A. Levin (2004), “Identifying the Influences of Nominal and Real Rigidities in Aggregate Price-Setting Behavior,” ECB Working Paper Series no. 418.
- [17] Devereux, M. and J. Yetman (2003), “Predetermined Prices and the Persistent Effects of Money on Output,” *Journal of Money, Credit and Banking* 35: 729-741.
- [18] Dhyne, E., L. Álvarez, H. Le Bihan, G. Veronese, D. Dias, J. Hoffman, N. Jonker, P. Lünnemann, F. Rumler and J. Vilmunen (2004), “Price Setting in the Euro Area: Some Stylised Facts from Individual Consumer Price Data,” mimeo.
- [19] Dixon, H. and E. Kara (2005a), “Persistence and Nominal Inertia in a Generalized Taylor Economy: How Longer Contracts Dominate Shorter Contracts,” ECB Working Paper Series no. 489.
- [20] _____ (2005b), “How to Compare Taylor and Calvo Contracts: A Comment on Michael Kiley,” forthcoming in the *Journal of Money, Credit and Banking*.
- [21] Dupor, B. and T. Tsuruga (2005), “Sticky Information: The Impact of Different Information Updating Assumptions,” forthcoming in the *Journal of Money, Credit and Banking*.
- [22] Gali, J. and M. Gertler (1999), “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics* 44: 195-222.
- [23] Haltiwanger, J. and M. Waldman (1991), “Responders Versus Non-Responders: A New Perspective on Heterogeneity,” *Economic Journal* 101: 1085-1102.
- [24] Imbs, J., H. Mumtaz, M. Ravn and H. Rey (2005), “PPP Strikes Back: Aggregation and the Real Exchange Rate,” *Quarterly Journal of Economics* CXX: 1-43.
- [25] Jadresic, E. (1999), “Sticky Prices: An Empirical Assessment of Alternative Models,” IMF Working Paper # 99/72.
- [26] Kiley, M. (2002), “Partial Adjustment and Staggered Price Setting,” *Journal of Money, Credit and Banking* 34: 283-298.
- [27] Konieczny, J. and A. Skrzypacz (2004), “Search, Costly Price Adjustment and the Frequency of Price Changes – Theory and Evidence,” mimeo.
- [28] Mankiw, G., and R. Reis (2002), “Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve,” *Quarterly Journal of Economics* 117: 1295-1328.
- [29] Ohanian, L. A. Stockman and L. Kilian (1995), “The Effects of Real and Monetary Shocks in a Business Cycle Model with Some Sticky Prices,” *Journal of Money, Credit and Banking* 27: 1209-1234.
- [30] Taylor, J. (1979), “Staggered Wage Setting in a Macro Model,” *American Economic Review* 69: 108-113.

- [31] _____ (1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88: 1-23.
- [32] _____ (1993), *Macroeconomic Policy in a World Economy: From Econometric Design to Practical Operation*, W. W. Norton.
- [33] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.
- [34] Yun, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37: 345-370.

Calvo Pricing

Permanent Level Shock: $\bar{m} = -0.1, \rho = 0, \theta = 1$

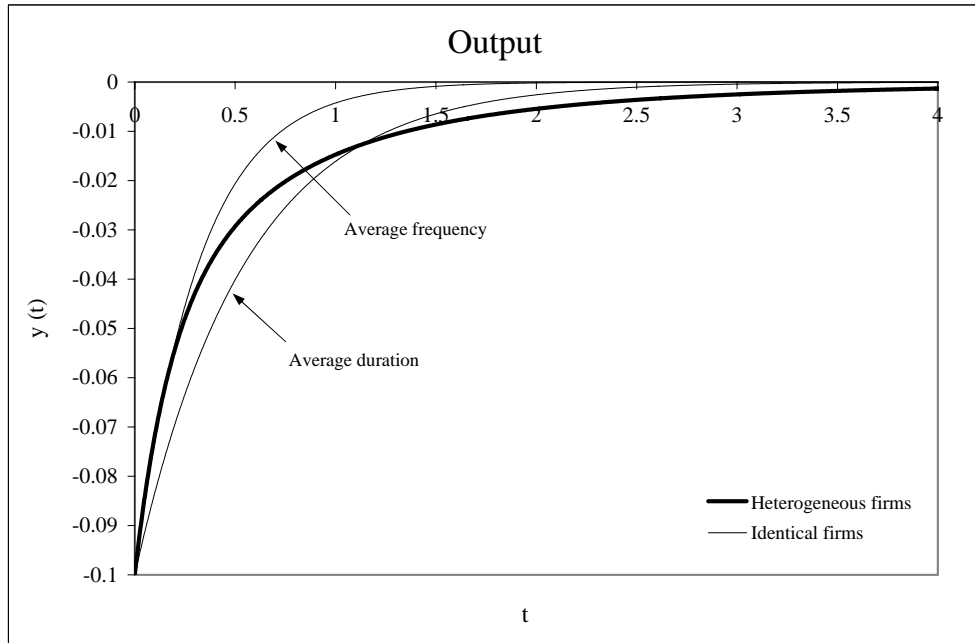


Figure 1

Normalized Cumulative Output Effects (NCOE), without strategic complementarities ($\theta = 1$)

Level Shocks

Growth Rate Shocks

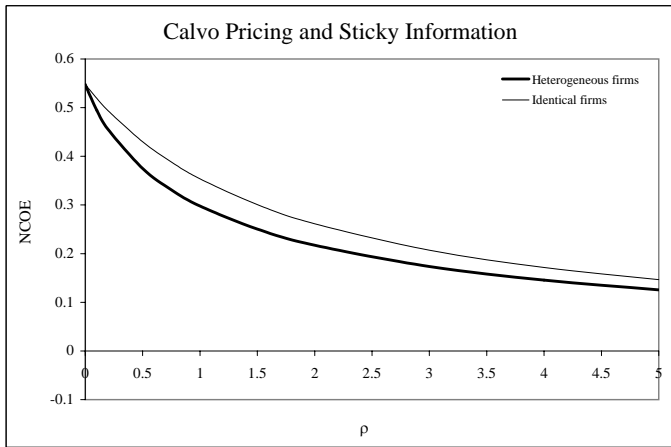


Figure 2a

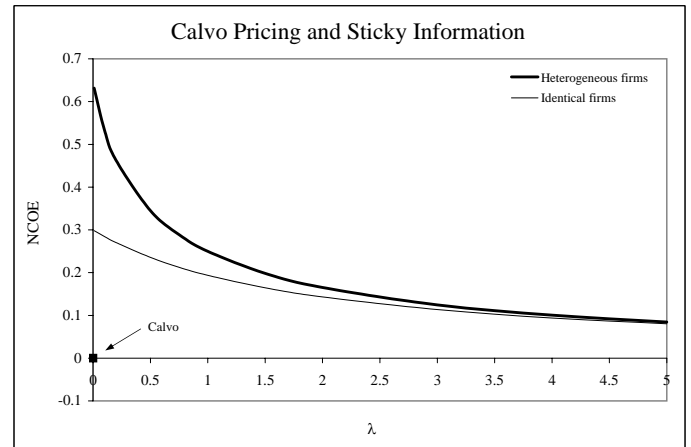


Figure 2d

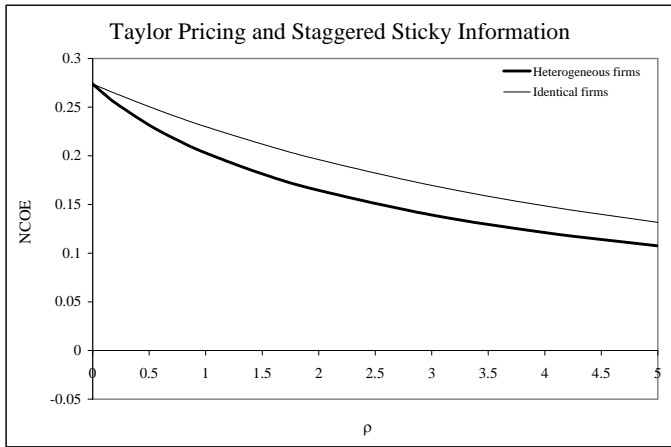


Figure 2b

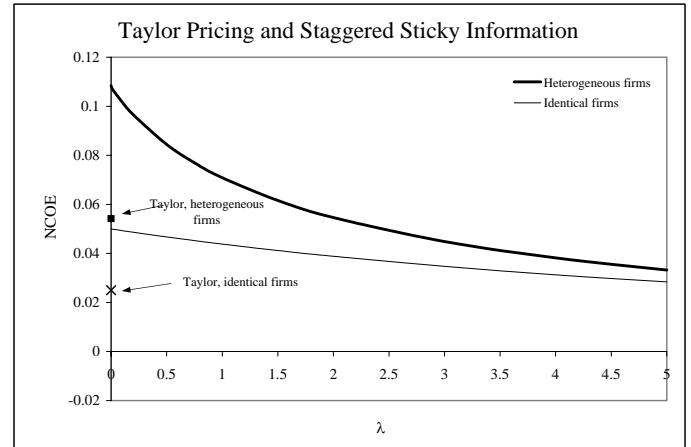


Figure 2e

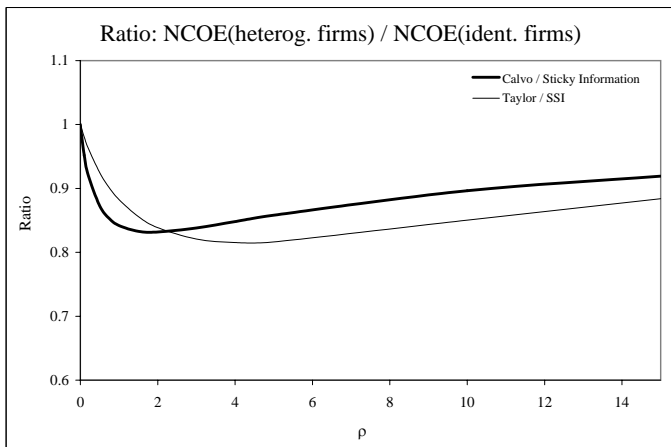


Figure 2c

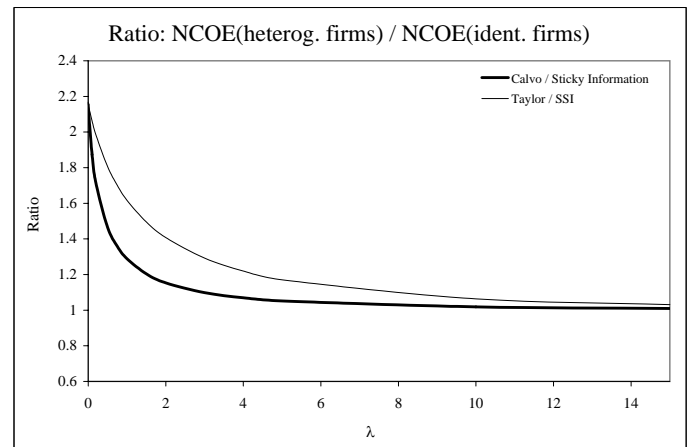


Figure 2f

Taylor Pricing

Permanent Growth Rate Shock: $\pi = 0.1, \Delta\pi = -0.1, \lambda = 0$

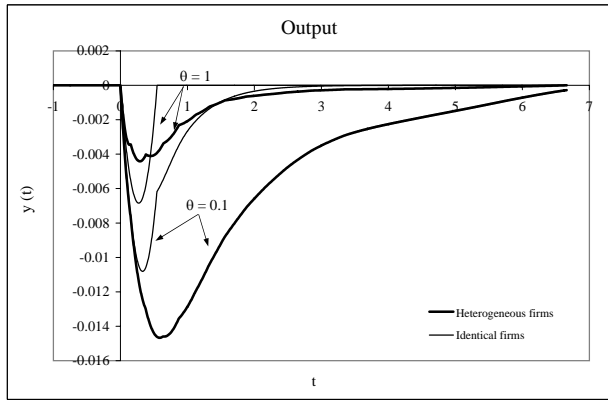


Figure 3a

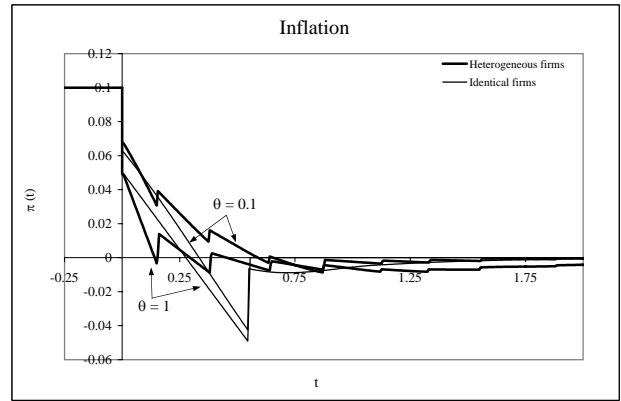


Figure 3b

Temporary Growth Rate Shock: $\pi = 0.1, \Delta\pi = -0.1, \lambda = 0.5$

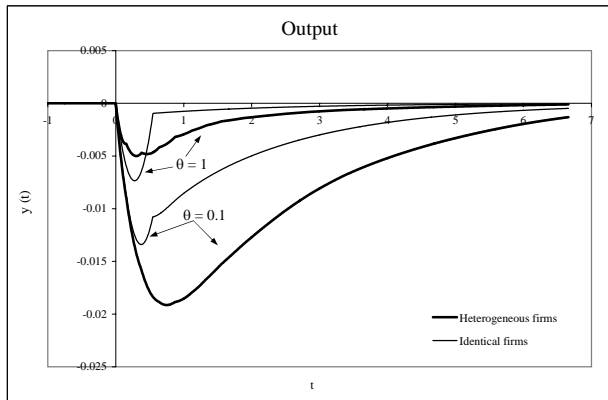


Figure 3c

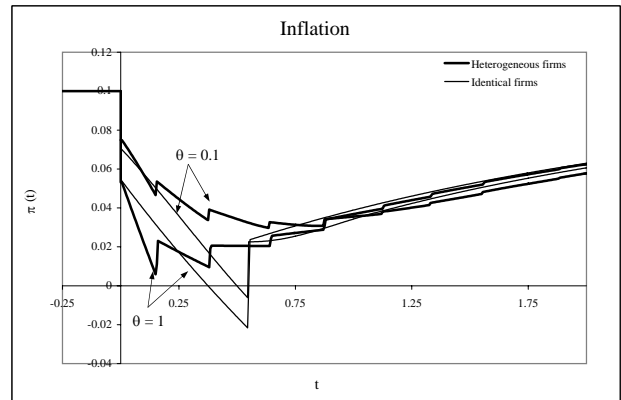


Figure 3d

Permanent Level Shock: $\bar{m} = -0.1, \rho = 0$

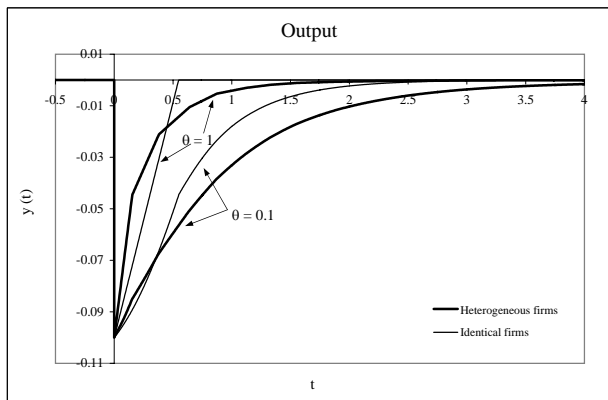


Figure 3e

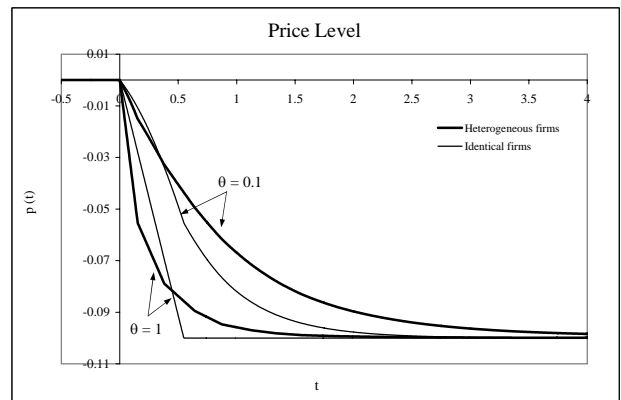


Figure 3f

Temporary Level Shock: $\bar{m} = -0.1, \rho = 0.5$

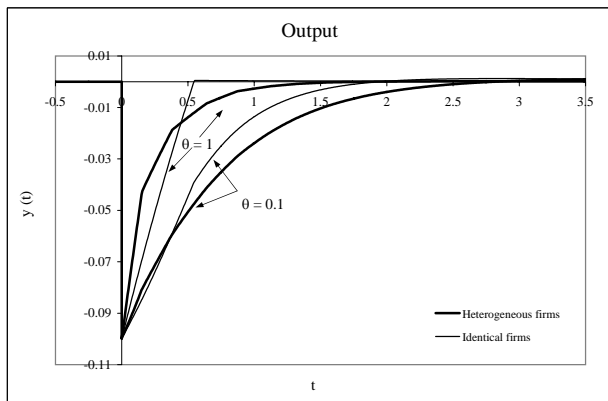


Figure 3g

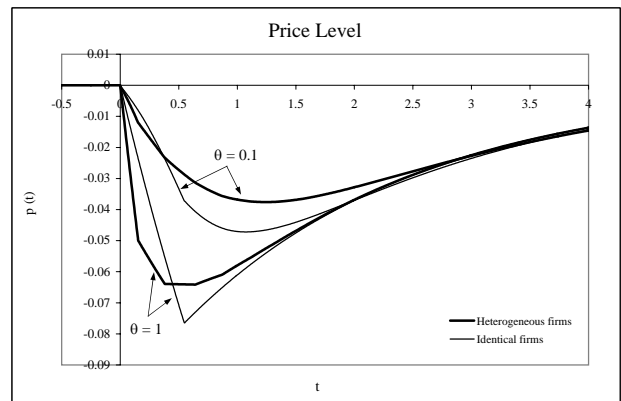


Figure 3h

Sticky Information (Mankiw and Reis, 2002)

Permanent Growth Rate Shock: $\pi = 0.1, \Delta\pi = -0.1, \lambda = 0$

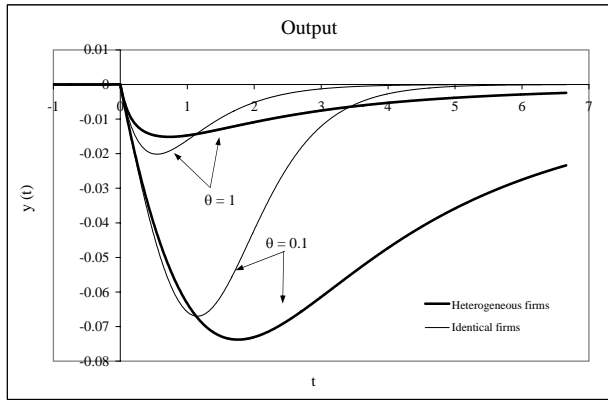


Figure 4a

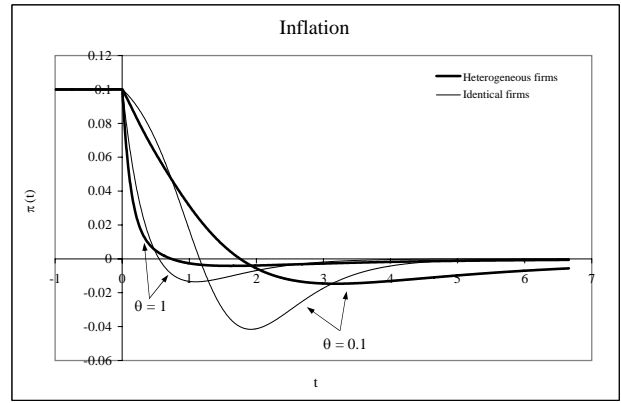


Figure 4b

Temporary Growth Rate Shock: $\pi = 0.1, \Delta\pi = -0.1, \lambda = 0.5$

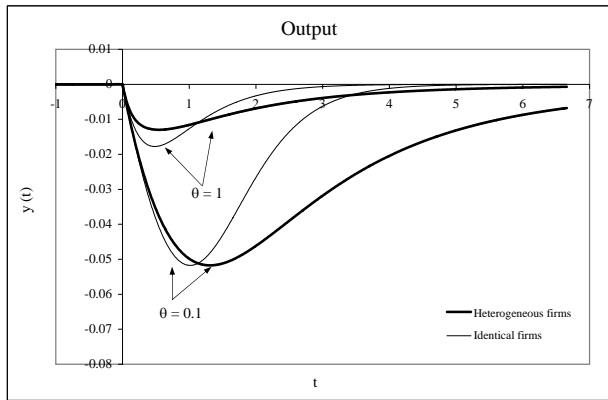


Figure 4c

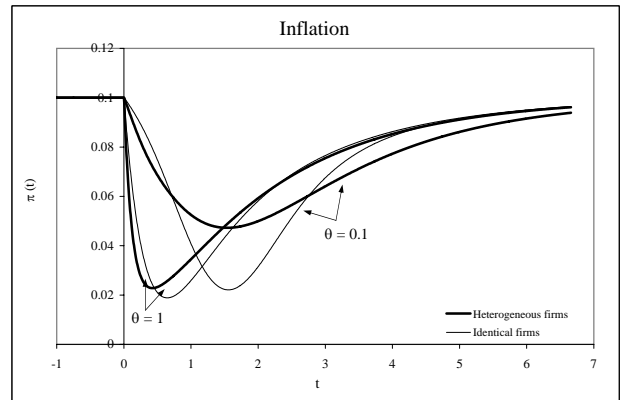


Figure 4d

Permanent Level Shock: $\bar{m} = -0.1, \rho = 0$

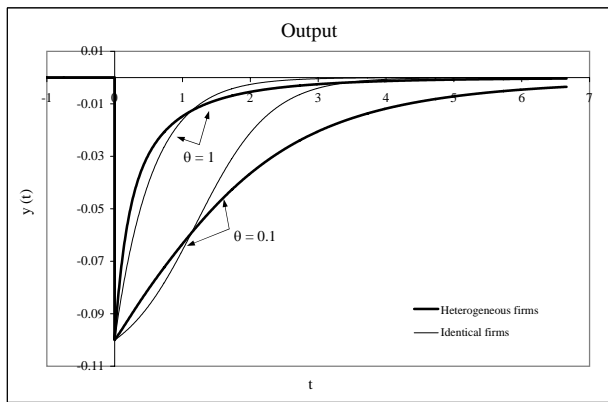


Figure 4e

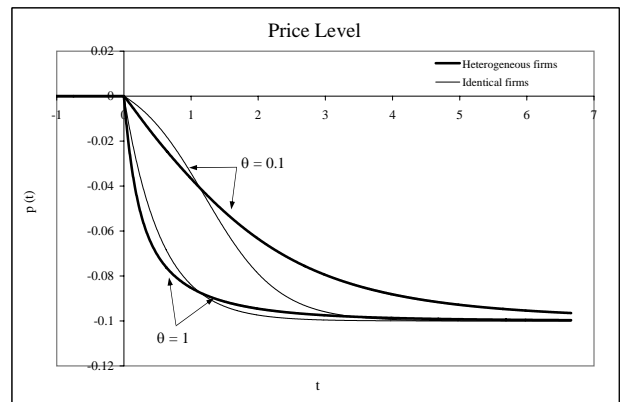


Figure 4f

Temporary Level Shock: $\bar{m} = -0.1, \rho = 0.5$

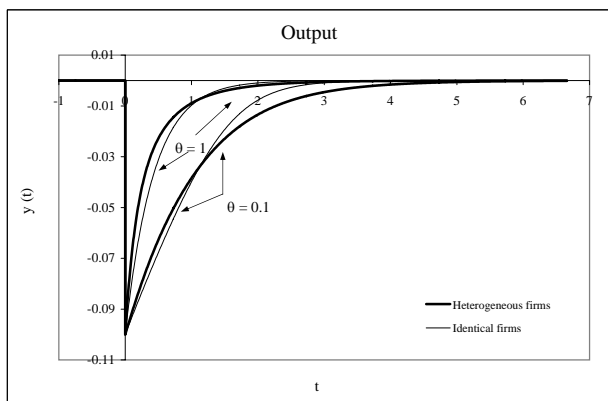


Figure 4g

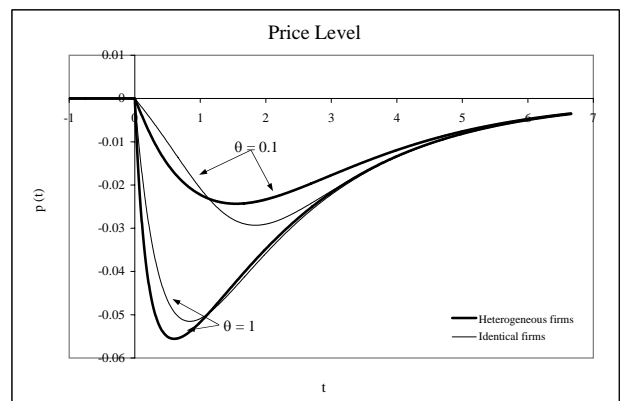


Figure 4h

Staggered Sticky Information (Dupor and Tsuruga, 2005)

Permanent Growth Rate Shock: $\pi = 0.1, \Delta\pi = -0.1, \lambda = 0$

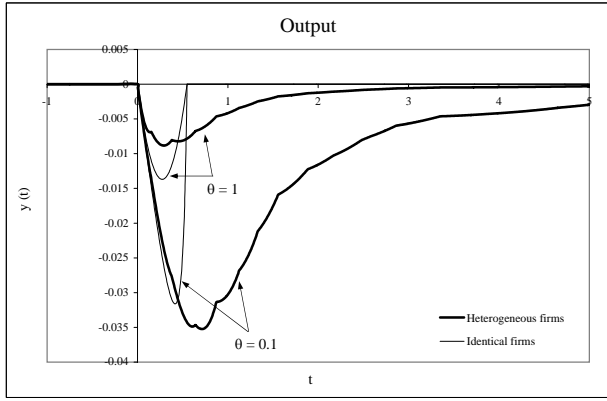


Figure 5a

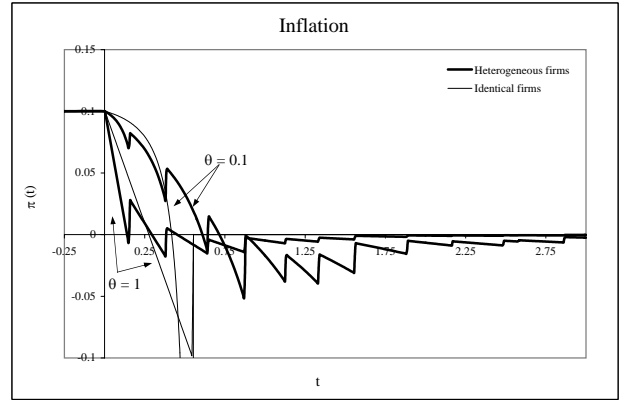


Figure 5b

Temporary Growth Rate Shock: $\pi = 0.1, \Delta\pi = -0.1, \lambda = 0.5$

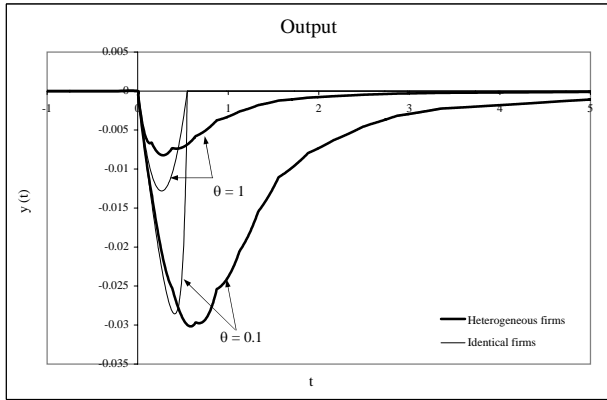


Figure 5c

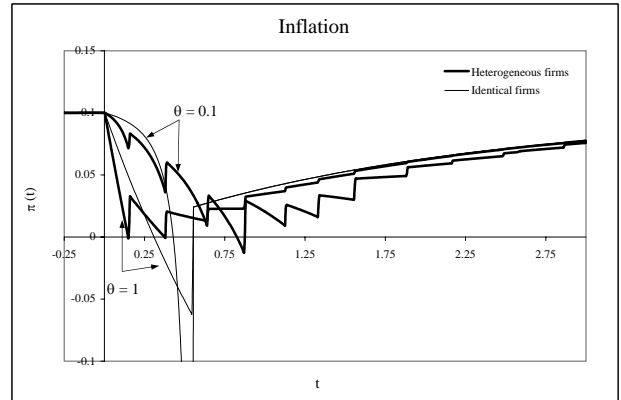


Figure 5d

Permanent Level Shock: $\bar{m} = -0.1, \rho = 0$

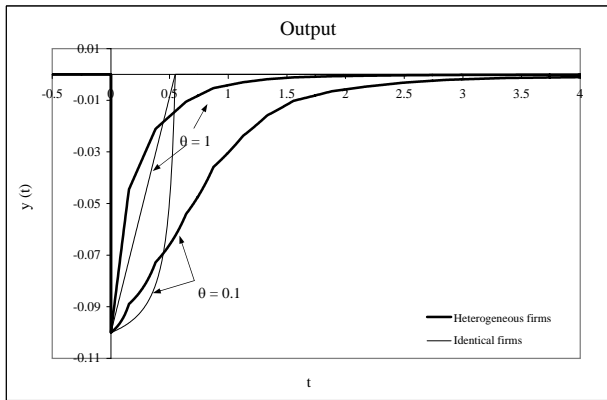


Figure 5e

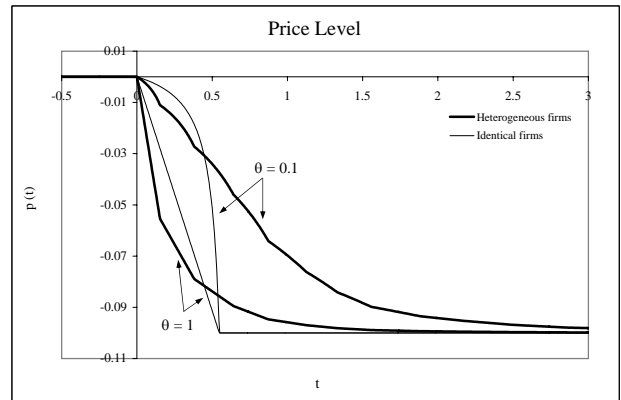


Figure 5f

Temporary Level Shock: $\bar{m} = -0.1, \rho = 0.5$

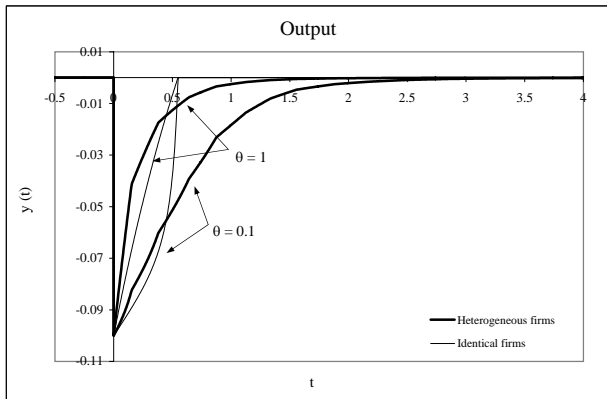


Figure 5g

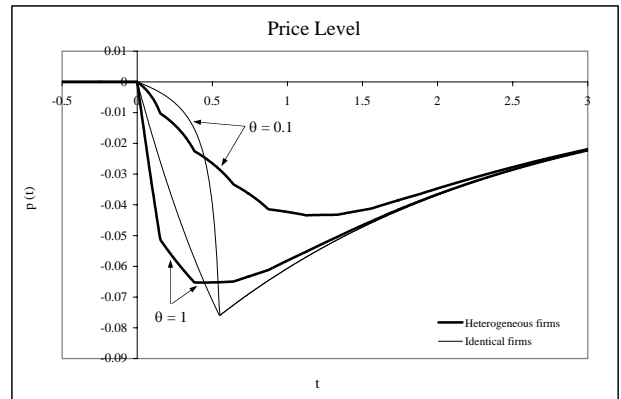


Figure 5h

Normalized Cumulative Output Effects (NCOE), with strategic complementarities ($\theta = 0.1$)

Level Shocks

Growth Rate Shocks

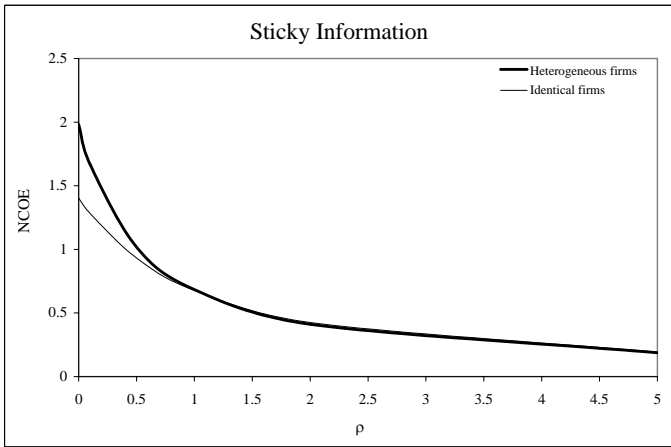


Figure 6a

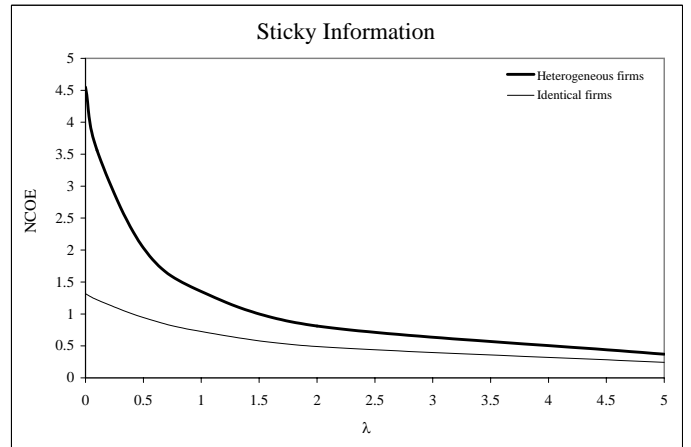


Figure 6d

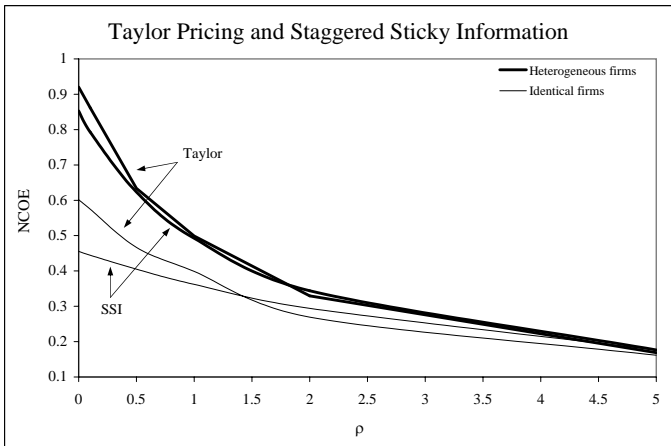


Figure 6b

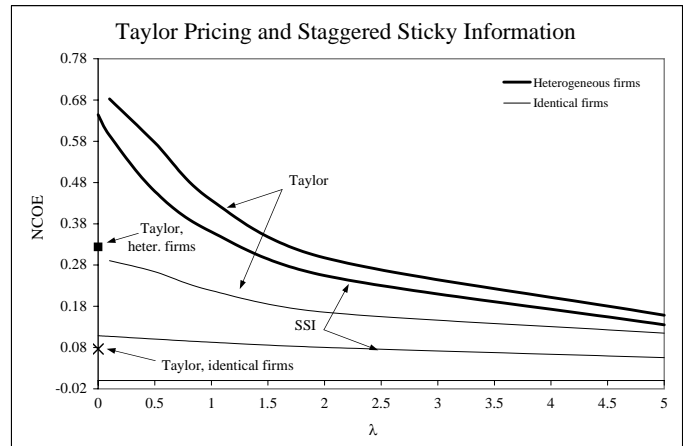


Figure 6e

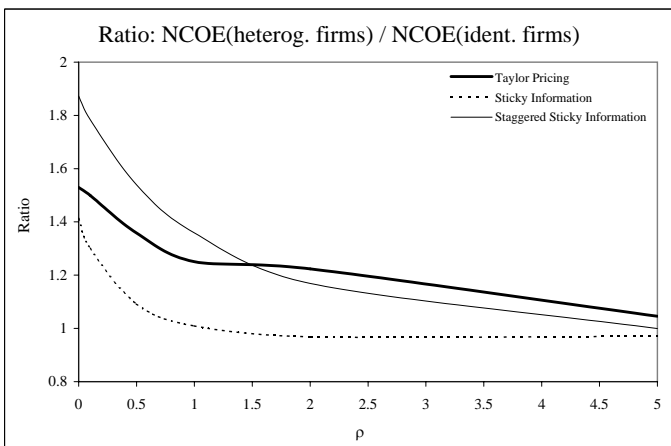


Figure 6c

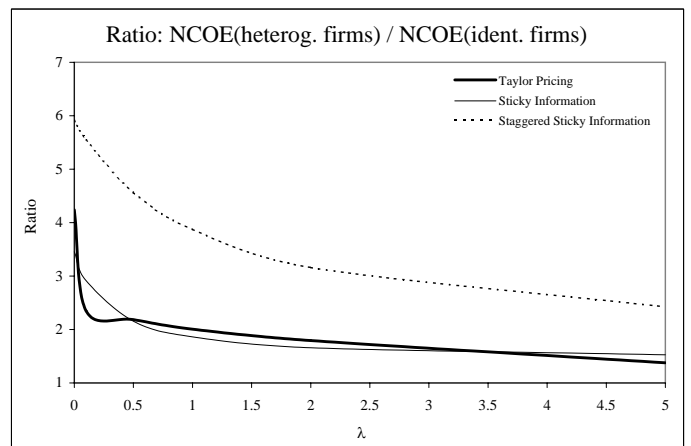


Figure 6f

Sticky Information (Mankiw and Reis, 2002)

Permanent Growth Rate Shock ($\lambda = 0$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	2.36	1.56	2.21	1.32
0.15	2.23	2.03	2.10	1.74
0.20	2.15	2.47	2.03	2.15
0.50	1.96	4.97	1.88	4.51
1.00	1.87	8.95	1.81	8.29

Table 1a

Temporary Growth Rate Shock ($\lambda = 0.5$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	1.90	0.96	1.84	0.90
0.15	1.82	1.37	1.77	1.24
0.20	1.77	1.65	1.73	1.57
0.50	1.66	3.62	1.64	3.54
1.00	1.61	6.83	1.61	6.80

Table 1b

Permanent Level Shock ($\rho = 0$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	1.72	0.78	1.70	0.75
0.15	1.65	1.07	1.63	1.05
0.20	1.61	1.36	1.60	1.35
0.50	1.52	3.11	1.51	3.08
1.00	1.50	6.07	1.46	5.80

Table 1c

Temporary Level Shock ($\rho = 0.5$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	1.49	0.60	1.49	0.60
0.15	1.47	0.89	1.47	0.89
0.20	1.46	1.18	1.45	1.17
0.50	1.45	2.92	1.40	2.78
1.00	1.43	5.76	1.33	5.20

Table 1d

Obs: durations are reported in years.

Staggered Sticky Information (Dupor and Tsuruga, 2005)

Permanent Growth Rate Shock ($\lambda = 0$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	2.74	4.74	2.60	4.05
0.15	2.24	4.28	2.37	4.54
0.20	2.18	5.36	2.17	5.00
0.50	1.75	7.40	1.86	8.35
1.00	1.63	12.40	1.73	13.94

Table 2a

Temporary Growth Rate Shock ($\lambda = 0.5$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	2.24	2.88	2.32	3.03
0.15	2.14	3.89	2.11	3.52
0.20	1.86	3.57	1.99	4.05
0.50	1.67	6.83	1.76	7.32
1.00	1.60	11.84	1.55	11.00

Table 2b

Permanent Level Shock ($\rho = 0$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	1.78	1.52	1.86	1.69
0.15	1.65	1.95	1.73	2.10
0.20	1.60	2.38	1.65	2.52
0.50	1.48	5.09	1.46	4.88
1.00	1.40	9.30	1.32	8.10

Table 2c

Temporary Level Shock ($\rho = 0.5$)

θ	Absolute deviations		Squared deviations	
	n_{id}	θ_{id}	n_{id}	θ_{id}
0.10	1.64	1.28	1.70	1.36
0.15	1.57	1.72	1.60	1.78
0.20	1.52	2.16	1.54	2.19
0.50	1.42	4.75	1.37	4.39
1.00	1.31	8.30	1.23	7.25

Table 2d

Obs: durations are reported in years.