Imperfectly Credible Disinflation under Endogenous Time-Dependent Pricing*

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Abstract

The real effects of an imperfectly credible disinflation depend critically on the extent of price rigidity. Therefore, the study of how policymakers' credibility affects the outcome of an announced disinflation should not be dissociated from the analysis of the determinants of the frequency of price adjustments. In this paper we examine how credibility affects the outcome of a disinflation in a model with endogenous time-dependent pricing rules. Both the initial degree of price ridigity, calculated optimally, and, more notably, the changes in the duration of price spells during disinflation play an important role in the explanation of the effects of imperfect credibility. We initially consider the costs of disinflation when the degree of credibility is fixed, and then allow agents to use Bayes rule to update beliefs about the "type" of monetary authority that they face. In both cases, the interaction between the endogeneity of time-dependent rules and imperfect credibility increases the output costs of disinflation. The pattern of the output response is more realistic in the case with learning.

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1 Introduction

Lack of credibility has, for a long time, been pointed out as an important ingredient in explaining real effects of disinflation (e.g. Sargent, 1983). It arises when a monetary authority that is serious about disinflating faces distrust from the private sector. Yet, price rigidity is necessary for an imperfectly credible disinflation to have meaningful real effects. If prices are fully flexible, monetary policy essentially has no real effects, and the lack of credibility does not matter.[1]

Additionally, the *extent* of price rigidity matters for the effect of imperfect credibility. Consider an economy during an imperfectly credible disinflation in which individual prices are fixed for extremely short periods of time. Then, the price optimally set by each firm tends to be very similar to the price that would be set under full credibility, since there is relatively little uncertainty about the monetary policy regime in the very short run. The real effects of imperfect credibility in this case are not very important. However, the same is not true of an economy where prices are fixed for long periods of time. Since policy uncertainty tends to build up with time, in that case there is a much higher probability of a policy reversal between price adjustments. This uncertainty affects pricing decisions, leading to substantial differences between the individual prices set during an imperfectly credible and a perfectly credible disinflation.[2]

Because the role of credibility depends on the frequency of price changes, conclusions about the effect of imperfect credibility that are based on models where this frequency is chosen arbitrarily will reflect this arbitrary choice. In addition, since a disinflation typically involves a policy regime change, analyses based on such models are inherently subject to the Lucas critique. Not only should the degree of price rigidity respond to the change in regime, it should also depend on its credibility. For those reasons, the study of the role of credibility in disinflation episodes should not be dissociated from the analysis of the determinants of the frequency of price changes.

In this paper we analyze how a policymaker's credibility affects the outcome of a disinflation in a model in which the extent of price rigidity is endogenous. In our model firms face frictions that make it optimal to choose ex-ante the time of the next price change. As a result, the time period between price adjustments - the duration of the spells of price rigidity

[1] In standard models with a unique equilibrium. With multiple equilibria, credibility, as sunspots, may move the economy from one equilibrium to another.

[2] This applies to models where not all firms have the *option* to react instantaneously and with full information to an eventual policy reversal. It applies both to time-dependent models with nominal rigidity, as in Taylor (1979, 1980) and Calvo (1983), and sticky information models, as in Mankiw and Reis (2002), and Reis (2006). It does not apply to state-dependent pricing models, as in Caplin and Spulber (1987), where information is continuously available.

- responds to changes in the economic environment.

Credibility affects the costs of disinflation through a direct and an indirect effect on prices. The *direct* effect is through the expectation of the path of marginal costs until the time of the next price change, given the frequency of price changes. It appears in models based on exogenous time-dependent pricing rules (e.g. Ball 1995, and Erceg and Levin 2003). As we argued above, the magnitude of this effect hinges on the duration of price spells. Our framework naturally brings discipline to the analysis, since such spells are determined endogenously.[3] The *indirect* effect arises in our model with endogenous pricing rules because changes in the frequency of price changes during the disinflation also affect the individual prices chosen. With policy regime shifts, as it happens with a new disinflationary policy, this effect becomes important.

In Section 2 we derive the optimal pricing rule under the assumption that firms cannot obtain, process and react to new information *nor* adjust prices based on their old information unless they incur a real lump sum cost, as in Bonomo and Carvalho (2004). We provide more explicit foundations to our earlier approach, and extend it to derive the optimal pricing rule during an imperfectly credible disinflation. The resulting pricing strategy is an endogenous time-dependent rule, where each time a firm incurs the information/adjustment cost, it sets a price and chooses ex-ante when next to gather and process information to decide on a new price. We refer to such chosen times as *pricing dates*.[4]

We view the assumption of a single information/adjustment cost as a tractable way to incorporate information frictions and adjustment costs that appear to be present in price setting decisions, as documented by Zbaracki et al. (2004). The resulting pricing rule displays time-dependency that resembles the "pricing seasons" described by those authors, and nominal rigidities that are consistent with microeconomic evidence on individual prices (e.g. Bils and Klenow 2004 for recent evidence for the U.S. economy).

Our endogenous time-dependent pricing rule has important implications for the literature that aims to discriminate empirically between alternative models of price setting (e.g.

[3] One could argue that the arbitrariness in specifying (exogenously) the duration of price spells could be avoided by calibrating the frequency of price changes to the microeconomic evidence. However, this would restrict the scope of analysis to economic environments similar to the ones that produced the evidence used in the calibration. In contrast, our approach allows us to calibrate the primitive parameters of the model to the available evidence, and compute the frequency of price changes for different economic environments.

[4] A pure adjustment cost ("menu cost") would give rise to state-dependent pricing (e.g. Barro 1972, Sheshinski and Weiss 1977), whereas a pure information cost would lead to the choice of price paths in between optimally chosen information-gathering dates, as in Caballero (1989) and Reis (2006). Ball, Mankiw, and Romer (1988) analyze a model with endogenous contract lengths in inflationary steady states as a tractable approximation to state-dependent pricing. In a related paper, Romer (1990) proposes an optimally chosen frequency of price adjustment in a Calvo-type model as a tractable (albeit suboptimal) alternative to state-dependent policies. For a recent application of Romer's model, see Levin and Yun (2007).

Klenow and Kryvtsov 2008). In our model, the frequency of price changes responds to the economic environment in ways that resemble a state-dependent pricing model. In particular, it increases with inflation, in line with both time-series and cross-country evidence.[5] Thus, such empirical evidence cannot be used to distinguish between state- and time-dependent pricing behavior. This necessarily requires exploring alternative implications of these models.

Our main interest is to analyze the mechanism through which an imperfectly credible disinflation affects output in a setting in which price setting decisions are optimal. We take credibility as exogenous, and model imperfect credibility as a discrepancy between private agents' beliefs about the likelihood that the monetary authority abandons the disinflation, and the objective likelihood. Such beliefs affect aggregate outcomes through their effect on the choice of the time interval between pricing dates, and prices set by firms.[6] For tractability, we model disinflation as a policy shift that changes the growth rate of nominal aggregate demand instantaneously, without making explicit the details of the transmission mechanism.[7]

In Section 3 we examine the case where the degree of credibility is fixed, so that price setters' beliefs do not change, despite the fact that the disinflation policy is never abandoned. For a given frequency of price changes, imperfect credibility increases the costs of disinflation because agents believe that there is some probability that the stabilization will be abandoned before their next pricing date, and therefore set prices higher than in the case of full credibility. To properly measure this *direct* effect, we set the exogenous frequency of price changes equal to the one that would be optimal for the inflationary environment that prevailed prior to the disinflation.

We assess the *indirect* effect of credibility by examining the case in which pricing rules are endogenous. We find the costs of disinflation to be higher in this case. With endogenous rules, when faced with lower expected inflation after the disinflation is launched, firms optimally choose to change prices less frequently. This raises the probability of a policy reversal occurring between pricing dates, amplifying the difference between individual prices set under perfect and imperfect credibility.

In Section 4 we introduce learning. The assumption that agents do not update their

[5]See, for instance, Dhyne et al. (2006), Gagnon (2007), Konieczny and Skrzypacz (2005), Lach and Tsiddon (1992), and Nakamura and Steinsson (2008).

[6]There is another line of investigation about the effects of imperfect credibility on disinflation, which focuses on explaining credibility. Those models usually have a simple aggregate supply structure, and rely on the discretionary nature of monetary policy. Recent examples are Siu (2008) and Westelius (2005). Backus and Driffill (1985a,b) provided earlier contributions.

[7]For other purposes it might be worthwhile to embed our endogenous time-dependent pricing rule in a model with an explicit transmission mechanism, and study the effects of disinflation when it is implemented in alternative ways (e.g. lowering an inflation target, adopting a currency peg etc).

beliefs, despite useful for gaining insight, is not realistic. It generates the unappealing result that after disinflation output remains permanently below potential. One should expect the monetary authority to gain credibility through time, as agents observe that disinflation continues and update their beliefs about its resolve to deliver on the promise to disinflate. We model the evolution of agents' beliefs through Bayesian learning. The result is a more realistic output path in which the monetary authority gains credibility, and the recession is gradually eliminated. Moreover, the main result of the paper, that endogeneity of pricing rules and lack of credibility interact to generate higher disinflation costs, continues to hold.

The literature that links imperfect credibility and price rigidity explicitly starts with Ball (1995), who argues that both ingredients are necessary to explain the costs of disinflation. He focuses on average effects of disinflation when agents' beliefs are in fact correct (i.e. they know the distribution of abandonment times). Erceg and Levin (2003) explain the output costs during the Volcker disinflation with a model where agents have to learn about a structural change in the interest rate rule. Both papers use exogenous pricing rules. Nicolae and Nolan (2006) model a credibility problem similar to ours, but assume simple learning schemes instead of Bayesian updating. Moreover, they limit the choice of pricing rules: prices are adjusted either every period or every other period. Finally, Almeida and Bonomo (2002) analyze the output costs of disinflation under imperfect credibility and state-dependent pricing. In that model, price setters observe monetary policy and reconsider their pricing decisions continuously, under full information. As a result, imperfect credibility has only a small effect through its impact on the optimal pricing rule.

2 The model

We start from a model with a representative consumer who derives utility from a Dixit-Stiglitz composite of different varieties of a consumption good. She incurs disutility from supplying labor in a competitive market to a continuum of monopolistically competitive firms. Each firm hires labor to produce its variety of the consumption good using a technology that is subject to productivity shocks. Firms face frictions that make it optimal to undertake pricing decisions infrequently, as we discuss extensively in the next subsection. As is now common in the literature, we assume a cashless economy (e.g. Woodford 2003).

In Appendix A we develop the model from fundamentals, and derive the following loglinear expression for the frictionless optimal price that a firm i would charge if it did not face pricing frictions, p^*_{it}:[8]

$$p^*_{it} = \mathcal{Y}_t - y^n_t + e_{it}, \tag{1}$$

where $\mathcal{Y}_t$ is nominal aggregate demand, y^n_t is the natural output rate, and the e_{it}'s are mutually independent, zero mean firm-specific shocks.

In a flexible price equilibrium, i.e. if none of the firms faced pricing frictions, the price charged by firm i at time t, p^f_{it}, would in effect evolve according to (1). Then, the aggregate price in such equilibrium, p^f_t, would be given by:[9]

$$p^f_t = \int_0^1 p^f_{it} di = \mathcal{Y}_t - y^n_t.$$

In such equilibrium, aggregate output and individual prices would be given by y^n_t and $p^f_{it} = p^f_t + e_{it}$, respectively. In contrast, in our economy output will deviate from y^n_t due to the frictions that make infrequent pricing decisions optimal. Letting p_t denote the price level that results from aggregation of the actual prices charged by individual firms, output (y_t) will be given by:

$$y_t = \mathcal{Y}_t - p_t = y^n_t + \left(p^f_t - p_t\right).$$

For simplicity, in the subsequent sections we abstract from aggregate shocks that would affect the natural rate of output. Thus, any variation in the level of output in our economy is a result of the pricing frictions to which we turn in the next subsection.

2.1 Optimal time-dependent pricing rule

The microeconomic evidence on nominal price rigidity has usually been rationalized by the existence of menu costs of changing prices. As it is well known, this leads to pricing decisions that are state-dependent. However, available evidence based on interview studies (Blinder et al. 1998) and direct measurement through field work (Zbaracki et al. 2004) shows the importance of other types of costs associated with price setting decisions, such as information gathering, decision making, and internal communication costs. Those costs prevent the continuous information gathering and processing that are necessary for the implementation of purely state-dependent pricing strategies.

Non-convex information and decision making costs lead to infrequent pricing decisions, and time-dependency (Reis 2006). However, in the absence of adjustment costs, the optimal

[8] Throughout the paper, lowercase variables denote log-deviations of the respective quantity from the deterministic steady-state, as detailed in Appendix A. For expositional simplicity, we omit the expression "log-deviation from the steady state", refering directly to the names of the corresponding variables.

[9] This follows from a loglinear approximation to the aggregate price index (see Appendix A).

pricing rule calls for the choice of a price path at each decision date. This implication is at odds with the microeconomic evidence on nominal price rigidity.

A model endowed with both information and adjustment costs should capture the time-dependency uncovered in recent work (e.g. the pricing seasons documented in Zbaracki et al. 2004) and at the same time generate nominal price rigidity.[10] A tractable model with these features is analyzed by Bonomo and Carvalho (2004), who assume that firms cannot obtain, process and react to new information *nor* adjust prices based on their old information unless they incur a lump sum cost. Here we provide better foundations for our earlier approach, and extend it to obtain the optimal pricing rule under an imperfectly credible disinflation.

Every time a firm decides to gather and process information *and/or* adjust its price it incurs a real fixed cost, which we refer to as the *pricing cost*. Therefore, information collection and processing, and price adjustments are undertaken infrequently, and the optimal pricing policy amounts to choosing a sequence of pricing dates. At each such date the firm decides on the next pricing date and sets a price that will be fixed until then.[11] The choice of the optimal time interval between pricing dates weights the benefits of updating information and changing prices frequently against the pricing cost.

In Appendix B we formulate this problem from first principles and show that under certain conditions it can be approximated by the following dynamic programing problem:

$$V\left(s_t^i\right) = \min_{z_i,\tau_i} E_t\left[\int_0^{\tau_i} e^{-\rho r}\left[z_{it} - \left(p_{it+r}^* - p_{it}^*\right)\right]^2 dr + e^{-\rho\tau_i}\left(F + V\left(s_{t+\tau_i}^i\right)\right)\right], \qquad (2)$$

where V is the present value of profit losses due to existence of pricing costs, F is the (normalized) pricing cost as a share of steady state profits, τ_i denotes the time until the next pricing date, and $z_{it} \equiv x_{it} - p_{it}^*$ denotes the discrepancy between the price set at t, x_{it}, and its frictionless optimal level p_{it}^*. The term $\left(z_{it} - \left(p_{it+r}^* - p_{it}^*\right)\right)^2$ is proportional - to a second order approximation - to the flow of profits foregone due to price rigidity. The relevant state of the economy is denoted by s_t^i, with jth component s_{jt}^i, and its law of motion is described by $s_{t+\Delta t}^i = \Omega\left(s_t^i, \eta_{t,t+\Delta t}^i\right)$, where $\eta_{t,t+\Delta t}^i$ is the set of innovations that hit firm i and the economy between t and $t + \Delta t$. The state of the economy matters for flow values through its effect on the distribution of future frictionless optimal prices before the next

[10] Models that combine information and adjustment costs tend to be complex. Woodford (2008) analyzes a model with menu costs and informational frictions in the form of costly information processing, in the spirit of Sims (2003).

[11] This behavior is consistent with the evidence in Zbaracki et al. (2004). One should note that in this setting any new information that becomes available to the firm is not taken into account until the next pricing date. This is also a feature of the inattention model of Reis (2006). In contrast, when inattention arises due to information processing constraints in the spirit of Sims (2003), as in Woodford's (2008) model, firms continuously incur costs to receive partial information.

pricing date (p^*_{it+r}, for $0 < r < \tau$) conditional on the information available at time t.

The first order conditions for problem (2) are:

$$z^*\left(s_t^i\right) = \frac{\rho}{1 - e^{-\rho\tau^*\left(s_t^i\right)}} \int_0^{\tau^*\left(s_t^i\right)} e^{-\rho r} E_t\left(p^*_{it+r} - p^*_{it}\right) dr, \tag{3}$$

and

$$E_t\left[\left(z^*\left(s_t^i\right) - \left(p^*_{it+\tau^*\left(s_t^i\right)} - p^*_{it}\right)\right)^2\right] = \rho F + \rho E_t V\left(s^i_{t+\tau^*\left(s_t^i\right)}\right) - \frac{\partial}{\partial \tau} E_t V\left(s^i_{t+\tau^*\left(s_t^i\right)}\right), \tag{4}$$

and the envelope conditions with respect to the components of s_t^i are:

$$\frac{\partial V\left(s_t^i\right)}{\partial s^i_{jt}} = \left[\int_0^{\tau^*\left(s_t^i\right)} \frac{\partial E_t\left[z^*_{it} - \left(p^*_{it+r} - p^*_{it}\right)\right]^2}{\partial s^i_{jt}} e^{-\rho r} dr\right] + e^{-\rho\tau^*\left(s_t^i\right)} \frac{\partial}{\partial s^i_{jt}} E_t V\left(s^i_{t+\tau^*\left(s_t^i\right)}\right).$$

Equations (2), (3), and (4) together with the envelope conditions fully characterize the optimal pricing rule, as long as the second order conditions are satisfied. Equation (3) gives the optimal discrepancy. It should be set equal to a weighted average of expected increments in the frictionless optimal price until the next pricing date. Equation (4) characterizes the optimal time interval until the next pricing date. It states that the expected marginal profit loss from postponing the next pricing decision (left hand side) should be equal to the expected marginal benefit of doing so (right hand side).

2.2 Inflationary steady state

In analyzing disinflation we start from an inflationary steady state characterized by a constant rate of inflation. Let π be the constant growth rate of nominal aggregate demand. If we differentiate equation (1), and use the assumption of constant natural output rate, and the fact that in steady state the price level grows at the same rate as nominal income, we obtain:[12]

$$dp^*_{it} = \pi dt + de_{it}. \tag{5}$$

Realistically we think of the idiosyncratic shocks e_i as following persistent, but stationary processes. However, modeling them as mean-reverting processes would add a state variable to the firm's problem without changing the main insights regarding aggregate dynamics. Thus, we adopt the Brownian motion as a convenient approximation of short run dynamics,

[12]Note that, in contrast with the *deterministic* steady state, the inflationary steady state features firm-specific shocks.

that is:

$$de_{it} = \sigma d\widetilde{W}_{it},$$

where $\widetilde{W}_{it}$'s are mutually independent, standard Brownian motions.

With this assumption, the conditional distribution of $z - \left(p^*_{it+r} - p^*_{it}\right)$ given information at t is normal with mean $z - \pi r$ and variance $\sigma^2 r$. It depends only on the time elapsed since time t, and is the same for all firms. As a result, the dynamic problem (2) in the inflationary steady state can be parameterized by π and written as:

$$V_\pi = \min_{z,\tau} E_t \left[\int_0^\tau \left(z - \left(p^*_{it+r} - p^*_{it}\right)\right)^2 e^{-\rho r} dr + e^{-\rho \tau} \left(F + V_\pi\right) \right], \tag{6}$$

where V_π represents the (constant) value function for the steady state problem with nominal aggregate demand growth rate equal to π. In the inflationary steady state, the value function and the optimal z and τ are the same for all firms, because they depend on the parameters of the stochastic process for p^*_i and not on its realizations.

The first order conditions are:

$$z^* = \frac{\rho}{1 - e^{-\rho\tau^*}} \int_0^{\tau^*} E_t \left(p^*_{it+r} - p^*_{it}\right) e^{-\rho r} dr, \tag{7}$$

$$E_t \left[z^* - \left(p^*_{it+\tau^*} - p^*_{it}\right)\right]^2 - \rho \left(V_\pi + F\right) = 0. \tag{8}$$

Manipulating (5), (6), (7), and (8), we arrive at the following equations, which define τ^* implicitly, and z^*:

$$\rho \frac{\int_0^{\tau^*} \left\{ \pi^2 \left[\left(\frac{1}{\rho} - \frac{e^{-\rho\tau^*}}{1-e^{-\rho\tau^*}} \tau^* \right) - r \right]^2 + \sigma^2 r \right\} e^{-\rho r} dr + F}{1 - e^{-\rho\tau^*}} = \pi^2 \left(\frac{1}{\rho} - \frac{e^{-\rho\tau^*}}{1 - e^{-\rho\tau^*}} \tau^* - \tau^* \right)^2 + \sigma^2 \tau^*,$$

$$z^* = \pi \left(\frac{1}{\rho} - \frac{e^{-\rho\tau^*}}{1 - e^{-\rho\tau^*}} \tau^* \right).$$

Based on the above pair of equations, one can show that the optimal time interval between pricing dates is decreasing in $|\pi|$ and σ, and increasing in F. In addition, higher idiosyncratic uncertainty makes such time interval less sensitive to inflation (Bonomo and Carvalho 2004).

In our simulations, we set $\sigma = 3\%$ and calibrate F so that with $\pi = 3\%$, $\sigma = 3\%$ and $\rho = 2.5\%$ a year, firms make pricing decisions once a year. As a result we set $F = 0.000595$. This frequency of price changes seems to be a reasonable characterization of price setting behavior in low inflation environments. It is consistent with the findings of Dhyne et al. (2006) for the Euro area, and with earlier evidence for the U.S. economy (e.g. Carlton, 1986 and Blinder et al., 1998), although it is lower than the frequency of price changes reported by Bils and Klenow (2004) for the U.S. economy.

In order to check the robustness of our calibration, we also compute the optimal time between pricing dates for high and very high inflation rates. The model performs well when confronted with the Israeli experience reported by Lach and Tsiddon (1992), and it also fits the Brazilian hyperinflation experience of the 80's (Ferreira, 1994). With inflation rates of 77% per year the model predicts spells of price rigidity of 2.6 months, against 2.2 months reported by Lach and Tsiddon (1992). With annual inflation of 210% the spells implied by the model go down to 1.68 months, against 1.38 months reported by Ferreira (1994). Thus, in accounting for the effects of inflation on the frequency of price changes the performance of our endogenous time-dependent model is comparable to that of the menu cost model analyzed by Golosov and Lucas (2007).

Endogenous time-dependent pricing rules have important implications for the literature that aims to discriminate between alternative models of price setting based on micro data (e.g. Klenow and Kryvtsov 2008). In particular, the empirical finding that the frequency of price changes responds to the economic environment cannot be taken as evidence in favor of purely state-dependent pricing behavior.[13] Thus, making progress in this area will require exploring alternative implications of these models based on which they can be distinguished.

2.3 Optimal pricing rule under imperfectly credible disinflation

In this subsection we derive the optimal pricing rule during disinflation. The dynamic program formulated in (2) encompasses imperfect credibility in general, which enters the problem through the expectations operator. It is more realistic to assume that agents believe that the new disinflation policy will be abandoned with some (non-zero) probability, than to assume full credibility. We model imperfect credibility by positing that in each finite time interval agents attribute a constant probability of a policy reversal. Thus, from the agents' perspective, the growth rate of nominal aggregate demand after the new policy is implemented changes with the first arrival of a Poisson process with constant rate h. In case the disinflation is abandoned, agents believe that the old policy is resumed and maintained forever. In this section we consider agents' beliefs to be fixed, in the sense that the perceived probability of abandonment over an interval of a given length is always the same. We relax this assumption in Section 4.

[13] In the simple version of the model that we use in this paper to study an imperfectly credible disinflation, the optimal time between pricing dates in the inflationary steady state is constant. More generally, however, the optimal time interval until the next pricing date depends on the *state* of the economy on the current pricing date (see Appendix B).

Despite agents' beliefs, the monetary authority never reneges on the promise to disinflate. Therefore, after the stabilization policy is launched at $t = 0$, the actual process for nominal aggregate demand, $\mathcal{Y}_t$, is given by:

$$\begin{aligned} d\mathcal{Y}_t &= \pi' dt, \\ \mathcal{Y}_0 &= 0, \end{aligned}$$

where π' is the targeted growth rate for nominal income, and where we introduce the normalization $\mathcal{Y}_0 = 0$. We refer to the case of $\pi' = 0$ as "full disinflation," while $0 < \pi' < \pi$ corresponds to a "partial disinflation." We abstract from the details of the transmission mechanism of monetary policy, and implicitly assume that the monetary authority sets its policy instrument so as to generate the postulated disinflation path for nominal aggregate demand.

In contrast, nominal aggregate demand according to agents' beliefs, $\mathcal{Y}_t^b$, evolves as:

$$\begin{aligned} d\mathcal{Y}_t^b &= (\pi' + (\pi - \pi') \, \mathbb{1}_{\{N_t \geqslant 1\}}) dt, \\ \mathcal{Y}_0^b &= 0, \end{aligned}$$

where N_t is a Poisson counting process with constant arrival rate h, and $\mathbb{1}_{\{.\}}$ is the indicator function. With this notation, $N_t = 0$ if the disinflation has been maintained up to time t, and $N_t \geq 1$ otherwise.

With this formulation we can interpret h as a measure of credibility, with *high* values representing *low* credibility. The subjective probability that stabilization will last until time t is given by e^{-ht}. Thus, for example, if $h = 0.5$ (at an annual rate), the subjective probability that the stabilization will last more than one year is 61%. The polar cases of perfect and no credibility correspond to $h = 0$ and $h = \infty$, respectively.

In general, solving for the optimal pricing rule requires solving an optimization and an aggregation problem simultaneously: the optimal pricing rule depends on the expected path for the aggregate price level and other aggregate variables, which in turn result from the aggregation of agents' behavior in equilibrium. However, if the optimal pricing problem can be expressed solely as a function of exogenous variables, the optimization and aggregation problems can be solved sequentially, in that order.

Our model economy satisfies that condition, due to the absence of strategic complementarity or substitutability in price setting, and of any other dependence of the optimal pricing problem on endogenous variables.[14] When making pricing decisions firms only care about the

[14] This follows from our assumptions on preferences and technology, which are spelled out in the Appendix.

evolution of nominal aggregate demand, and therefore we can solve for the optimal pricing rule independently of equilibrium considerations.[15] Moreover, the fact that we model the frictionless optimal price as a random walk, combined with the assumption that (eventual) policy shifts involve instantaneous jumps between regimes, and that policy reversals arrive according to a constant hazard process, simplify the pricing problem substantially.

The relevant state of the economy after the disinflation is launched can be summarized by the Poisson counting process N_t, which indicates whether disinflation has been abandoned up to time t ($N_t \geq 1$) or not ($N_t = 0$). If a policy reversal has occurred before time t, the pricing problem becomes identical to that of the original inflationary steady state. Otherwise, the problem of a firm on a pricing date incorporates the possibility of the disinflation being abandoned sometime in the future:

$$V_h(N_t) = \begin{cases} \min_{z_h, \tau_h} \left[G_h(z_h, \tau_h) + e^{-\rho \tau_h} \left(F + e^{-h\tau_h} V_h(0) + \left(1 - e^{-h\tau_h}\right) V_\pi \right) \right], \text{ if } N_t = 0 \\ \\ V_\pi, \text{ if } N_t \geq 1, \end{cases} \tag{9}$$

where

$$\begin{aligned} G_h(z, \tau) \equiv\ & e^{-h\tau} \left[\int_0^\tau \left((z - \pi' r)^2 + \sigma^2 r \right) e^{-\rho r} dr \right] \\ & + \int_0^\tau \left[\begin{array}{c} \int_0^r \left((z - \pi' r)^2 + \sigma^2 r \right) e^{-\rho r} dr + \\ \int_r^\tau \left((z - \pi(r - r) - \pi' r)^2 + \sigma^2 r \right) e^{-\rho r} dr \end{array} \right] h e^{-hr} dr. \end{aligned} \tag{10}$$

In (9), $G_h(z, \tau)$ is the expected cost due to deviations from the frictionless optimal price during the next interval of length τ, starting with the discrepancy z. If a policy reversal occurs in the near future (i.e. before $t + \tau$), agents will account for it on their *next* pricing date.[16] Then, the new pricing decision will be made under conditions identical to the original inflationary steady state. This results in the value function V_π. In (10), the first line of the expression refers to the subjective probability that the stabilization will be maintained during the next interval of length τ multiplied by the cost in this case. The second line gives the cost if abandonment occurs before the next pricing date. It considers each possible abandonment time $t + r$, and adds the resulting costs weighted by the (subjective) likelihood of each event.

[15] The absence of interactions in pricing decisions is common in state-dependent pricing models, where aggregation can be cumbersome (e.g. Caplin and Leahy 1991, Almeida and Bonomo 2002, and Golosov and Lucas 2007). Caplin and Leahy (1997), and Gertler and Leahy (2006) are two noticeable exceptions.

[16] In Bonomo and Carvalho (2004), firms are allowed to reevaluate their pricing policies when the disinflation is announced. This leads to important changes in the results for high - but not for low - inflation environments. We conjecture that a similar conclusion obtains with respect to abandonment under imperfect credibility, for the same reasons outlined in that paper.

The first order conditions are derived in a straightforward way:

$$z_h^* = \frac{\rho}{1 - e^{-\rho \tau_h^*}} \int_0^{\tau_h^*} \left[\pi' r + (\pi - \pi') \left(r - \frac{1 - e^{-hr}}{h} \right) \right] e^{-\rho r} dr, \tag{11}$$

$$\begin{aligned} &(z_h^* - \pi' \tau_h^*)^2 + \sigma^2 \tau_h^* + h e^{-h \tau_h^*} (V_\pi - V_h(0)) - \rho F - \rho \left(e^{-h\tau_h^*} V_h(0) + \left(1 - e^{-h\tau_h^*}\right) V_\pi \right) \\ &+ \int_0^{\tau_h^*} ((\pi' - \pi)(\tau_h^* - r))^2 h e^{-hr} dr + 2(\pi' - \pi)(z_h^* - \pi' \tau_h^*) \int_0^{\tau_h^*} (\tau_h^* - r) h e^{-hr} dr = 0. \end{aligned} \tag{12}$$

From (6), (9), (10), (11) and (12) we obtain a nonlinear equation in τ_h^*, which can be solved numerically. Then, with τ_h^* we can compute z_h^* using (11).

Figure 1 shows the optimal time interval between pricing dates as a function of the level of credibility in a full disinflation, for two levels of initial inflation ($\pi = 0.1$ and $\pi = 0.2$). It shows that the lower the credibility is (the higher h), the shorter the duration of price spells is. A lower level of credibility implies higher expected inflation, increasing the expected profit loss from having a fixed price for a spell of a given duration. Thus, if the duration is unchanged the expected marginal loss at the next pricing date will exceed the expected marginal benefit of postponing the pricing date. This leads firms to reduce the time interval between pricing dates in order to restore the balance between the marginal benefit and cost of postponing a price change.

3 Aggregate results

3.1 Aggregation methodology[17]

We assume that, prior to disinflation, pricing dates are distributed uniformly over time. Having solved for the optimal pricing rule before and after the disinflation is announced, we can compute the sequence of pricing dates chosen by firms that change prices at any given time. Thus, to obtain the aggregate price level at any point in time after $t = 0$, we can trace back the last pricing date of all firms, and aggregate the corresponding prices.[18]

More formally, let $g(\cdot)$ be the function which gives the next pricing date: $g(t) = t + \tau_t^*$, where τ_t^* denotes the optimal spell chosen at time t.[19] In order to calculate the aggregate

[17] This subsection builds on Bonomo and Carvalho (2004).

[18] Firms are subject to idiosyncratic shocks, and so even firms that share a pricing date will set different prices. However, because these differences cancel out, when aggregating we only have to account for the component that is common to all firms that share a given pricing date.

[19] In the disinflation of the previous section, with fixed beliefs, τ_t^* is constant and equal to τ_h^*. However, this will no longer be the case under learning, in Section (4). Therefore we explain the aggregation method

price level at an arbitrary time after the disinflation announcement, we use the function g to relate the measure of firms which set their prices on a specific pricing date u to the measure of firms at times before u that would have chosen u as their next pricing date (those times are $g^{-1}(u)$). For that purpose, let $\Gamma(t)$ be the correspondence that assigns to t the set of pricing dates when the current prices were chosen:

$$\Gamma(t) = \{t' : t' \leq t \text{ and } g(t') > t\}. \tag{13}$$

Let $g^{-1}(S)$ be the inverse image of the set S under g. Then, $g^{-1}(\Gamma(t))$ is the set of pricing dates for which the next pricing date would be in $\Gamma(t)$. To evaluate the average price at t we need to know the probability measure v of the firms which last adjusted at subsets of $\Gamma(t)$. We can easily relate this measure to the measure φ of subsets of $g^{-1}(\Gamma(t))$, since v is the image measure of φ under g. Then, we have:

$$p_t = \int_{\Gamma(t)} x_r v(dr) = \int_{g^{-1}(\Gamma(t))} x_{g(r)} \varphi(dr), \tag{14}$$

where x_r is the average price of firms which set prices at time r. We apply (14) recursively by relating distributions and pricing dates during disinflation to preceding times. We proceed in this way until we arrive at a set $\Gamma^{-n}(t) \equiv g^{-n}(\Gamma(t))$ such that the measure of firms adjusting at the subset of pricing dates of $\Gamma^{-n}(t)$ corresponds to the uniform distribution of the initial inflationary steady state.

We implement the aggregation algorithm just described computationally, as follows. We discretize time so that one year has 1000 possible pricing dates. The optimal interval between pricing dates obtained in the previous section is rounded accordingly, so that both the domain and image of g coincide with the time grid. We set a final date far enough for the transition to the new steady state to be completed, and, given g, move forward in time to find the subset of dates in which some firms actually make pricing decisions.[20] For each such pricing date, we construct the set defined in (13), and aggregate firms' prices according to (14). In between pricing dates, the aggregate price level remains constant.

using a notation that can be applied to both cases.

[20]If the time interval between pricing dates remained constant despite the disinflation, any future time would be a pricing date for some firms, given that we start from a uniformly staggered distribution of pricing dates. This would simplify aggregation tremendously. However, because the optimal spell of price rigidity changes after the disinflation, this is not the case in our model, and we need to keep track of when firms choose to adjust.

3.2 Results

We start by illustrating why taking into account the optimality of pricing rules might be relevant for assessing the direct effect of imperfect credibility appropriately. In Figure 2, we compare the output effects of perfectly and imperfectly credible disinflations, fixing the same arbitrary duration of price spells for two different initial inflation rates.[21] It is apparent that the direct effect is more important for higher inflation rates. The reason is that, given the same time interval between pricing dates, agents set higher prices because of the risk of facing higher inflation in case the stabilization is abandoned before their next pricing date. In Figure 3, on the other hand, the duration of price spells is fixed at the optimal level for each initial inflation rate. The relation between inflation and the direct effect of imperfect credibility is now unclear. The reason is that the spells of price rigidity are shorter for higher initial inflation rates and so, despite the fact that inflation would be higher in the case of a policy reversal, the probability that this event happens before the next pricing date is now smaller.

These results illustrate the importance of the extent of price rigidity to the assessment of the direct effect of imperfect credibility. Therefore, in all of our subsequent experiments, we fix the duration of price spells under exogenous rules at the optimal level implied by our model for the initial inflationary steady state. This is a suitable assumption for the experiments we analyze, which are unexpected disinflations. We start from an inflationary steady state which is expected to last, and so it makes sense to use spells of price rigidity which are compatible with that steady state. This allows us to properly assess the indirect effect of imperfect credibility, by appropriately taking the direct effect into account.

Figure 4 depicts the output effects of a full disinflation with our baseline calibration for two levels of credibility ($h = 0.5$, and $h = 2$), with both endogenous and exogenous pricing rules. The case of perfect credibility ($h = 0$) is presented for comparison purposes. As expected, with imperfect credibility the recession generated is larger. It is clear that endogeneity of pricing rules reinforces this result. This happens because the time interval between pricing dates increases after the disinflation begins, as firms optimally respond to lower expected inflation. With perfect credibility, as shown in Bonomo and Carvalho (2004), in the case of full disinflation and no strategic complementarity in price setting, the output costs of disinflation are the same with endogenous or exogenous pricing rules. The reason is that every firm that adjusts after the disinflation is announced knows that the aggregate component of their frictionless optimal price will remain constant. Then, individual prices are set taking into account only the idiosyncratic component of such optimal price, and the time

[21]Contract lengths are fixed at the level corresponding to the optimum for $\pi = 3\%$.

interval between pricing dates has no aggregate impact. With imperfect credibility this result ceases to be true, since agents attribute some probability that the monetary authority will abandon the stabilization before their next pricing date, in which case inflation will resume. With endogenous pricing rules, prices are optimally set for a longer interval when compared with exogenous rules, which implies a higher (subjective) probability of abandonment before the next pricing date. Therefore, prices are set at higher levels and the recession is larger. This is a result of the interaction between imperfect credibility and the optimally chosen frequency of repricing.

If credibility is lower, the duration of price spells increases less after the disinflation is announced, and so the differences between endogenous and exogenous pricing rules are attenuated. On the other hand, the differences relative to the case of perfect credibility are amplified due to the direct effect of imperfect credibility, as can be noted in Figure 4.

In Figure 5 we explore the role of idiosyncratic uncertainty. In the case of a perfectly credible full disinflation, idiosyncratic shocks are required for the time interval between pricing dates to be finite after the policy change. Otherwise, with zero inflation and no uncertainty, there would be no reason to incur the cost to make pricing decisions. With imperfect credibility, however, this is no longer the case, since the possibility of a policy reversal leads firms to revisit their pricing decisions irrespective of idiosyncratic uncertainty. The lower σ is, the more the frequency of price changes responds to inflation. So, when $\sigma = 0$ the differences between endogenous and exogenous pricing rules are amplified. This comparison is illustrated in Figure 5, against our benchmark value $\sigma = 3\%$.

These results on the effects of different levels of credibility and idiosyncratic uncertainty illustrate important general features of the interaction between imperfect credibility and the optimal pricing rule, which also apply to the other results that we present. To avoid having too many simulations, however, we illustrate them only through the previous experiments.

A partial disinflation presents some qualitative differences when compared to a full disinflation. The reason is that, with nominal rigidity in individual prices, the expected discrepancy while there is no individual price adjustment only remains constant when the inflation drift is zero. So, in contrast with the full disinflation case, in a partial disinflation a longer time interval between pricing dates will induce firms to set higher prices even with full credibility. With partial disinflation and imperfect credibility, continuing inflation and the probability of a policy reversal interact with the time interval between pricing dates, and affect pricing decisions. Given the optimally chosen longer spell of price rigidity, firms incorporate both the (higher) probability of abandonment and ongoing inflation when setting their prices. As a consequence, the recession tends to be larger.

Figure 6 shows the result of a partial disinflation under imperfect credibility for both exogenous and endogenous pricing rules. As expected, the latter generate a larger recession, but also output cycles. These cycles result from gaps in the new distribution of pricing dates, which are generated by the sudden increase in the optimal time interval between pricing decisions.[22]

4 Disinflation with learning

The results analyzed so far correspond to a situation in which the monetary authority never reneges on the announced disinflation, but nevertheless agents continue to believe that there is always the same probability of a policy reversal. Thus, the recession continues indefinitely, which is clearly unrealistic.

This result arises from the conjunction of two assumptions: initial beliefs that do not correspond to the true type of the monetary authority,[23] and lack of updating of such beliefs as disinflation evolves.

Discrepancies between agents' beliefs and the actual type of the monetary authority capture the essence of the problem faced by a monetary authority that is really serious about disinflating, but has low credibility. Lack of updating of beliefs, on the other hand, is clearly an extreme and unrealistic assumption, which we drop in this section.

We analyze how credibility evolves during disinflation, and how this interacts with optimal price setting to determine the output costs of disinflation. Initially, all agents hold the same beliefs about the type of the monetary authority that they face. After the disinflation is launched, on every pricing date firms update their beliefs, taking into account whether or not disinflation has been abandoned. Updating is done according to Bayes' rule.

In the next subsection we present the framework with learning, and derive the optimal pricing rule. We then specialize to the case of a monetary authority who is fully committed to disinflate, but initially lacks credibility. We compare the costs of disinflation under endogenous and exogenous pricing rules.

4.1 Optimal pricing rule

We assume that there are two possible types for the monetary authority, characterized by the constant hazard rate for the Poisson process according to which it reneges on the promise to disinflate: $\overline{h} > \underline{h} \geqslant 0$. We assume that when the disinflation policy is launched at $t = 0$,

[22] Note that those gaps also occur in the case of full disinflation. However, they cause no output oscillation since on average firms keep their prices constant.

[23] In this section we interpret h as indexing the possible behavioral types that the monetary authority can assume. For instance, a monetary authority that never reneges is of type $h = 0$.

agents have the same belief about the type of monetary authority they face. We denote by μ the prior probability of the monetary authority being of type $\underline{h}$.

At any time $t > 0$, whenever firms incur the pricing cost to gather and process information and make pricing decisions, they observe whether disinflation has been abandoned and, conditional on no abandonment, form the posterior μ_t, according to Bayes' rule:[24]

$$\begin{aligned}
\mu_t &\equiv \Pr\{h = \underline{h} | N_t = 0\} \\
&= \frac{\Pr\{h = \underline{h}, N_t = 0\}}{\Pr\{h = \underline{h}, N_t = 0\} + \Pr\{h = \overline{h}, N_t = 0\}} \\
&= \frac{\mu e^{-\underline{h}t}}{\mu e^{-\underline{h}t} + (1-\mu) e^{-\overline{h}t}}. \qquad (15)
\end{aligned}$$

Now the set of state variables for the pricing problem is augmented by the posterior belief μ_t, given by (15). Given the parameters $\underline{h}$, $\overline{h}$ and the initial belief μ, the posterior is a function only of the time elapsed since disinflation was launched. If a policy reversal has occurred before time t, the pricing problem becomes identical to that of the original inflationary steady state. Otherwise, the problem of a firm on a pricing date incorporates the possibility of the disinflation being abandoned sometime in the future, according to its beliefs:

$$V_\mu(N_t, t) = \begin{cases} \min\limits_{z_t, \tau_t} \left[\begin{array}{c} \mu_t G_{\underline{h}}(z_t, \tau_t) + (1-\mu_t) G_{\overline{h}}(z_t, \tau_t) \\ +e^{-\rho\tau_t} \left(\begin{array}{c} F + \left(\mu_t e^{-\underline{h}\tau_t} + (1-\mu_t) e^{-\overline{h}\tau_t}\right) V_\mu(N_t, t+\tau_t) \\ + \left(1 - \left(\mu_t e^{-\underline{h}\tau_t} + (1-\mu_t) e^{-\overline{h}\tau_t}\right)\right) V_\pi \end{array} \right) \end{array} \right], & \text{if } N_t = 0 \\ V_\pi, \ \text{if } N_t \geq 1. \end{cases} \qquad (16)$$

We solve the above problem numerically, as described in Appendix C.

4.2 Results

We focus on the case of a monetary authority that is fully committed to disinflate (i.e., of type $\underline{h} = 0$) but faces a credibility problem at the time of the policy change ($\overline{h} > 0$, $\mu < 1$).

Figure 7 presents the path for the optimal time interval between pricing dates during a full disinflation. When the disinflation begins at $t = 0$, firms who are on a pricing date choose to fix prices for longer periods when compared to the inflationary steady state. The

[24] Agents also update their beliefs when they learn that the disinflation has been abandoned. However, since we assume that in that case the previous inflationary steady state resumes irrespective of the true type of the monetary authority, such beliefs become irrelevant.

initial jump is a reaction to the announcement of the new policy, which lowers expected inflation. As the disinflation evolves, the monetary authority gains credibility and firms who make pricing decisions subsequently choose progressively longer spells of price rigidity. In the limit, as $t \to \infty$, agents end up believing that the monetary authority is actually not going to renege, and so the optimal frequency of price changes approaches the new steady state.

The paths for output under both endogenous and exogenous pricing rules are presented in Figure 8. They share the general features of the full disinflation case without learning (Figure 4), with one noticeable exception: now, as credibility builds up, output reverts towards the steady state level. Once more, the recession is larger under endogenous pricing rules.

The differences between those results and the ones for a full disinflation without learning hinge on the process of updating of beliefs. According with our assumptions, on pricing dates firms update their beliefs about the type of the monetary authority they face. Because firms have different pricing dates, at each point in time there is a distribution of beliefs among price-setters, which can be represented by $\{\mu_{it}\}_{i\in[0,1]}$, where $\mu_{it} \equiv \Pr\{h = \underline{h}|N_{t_i} = 0\}$, and $t_i \leq t$ represents firm i's last pricing date.

We summarize the evolution of this distribution of beliefs during disinflation by its mean ($\overline{\mu}_t \equiv \int_{0t}^{1} \mu_{it} di$) and standard deviation ($\sigma_t^{\mu} \equiv \sqrt{\int_0^1 (\mu_{it} - \overline{\mu}_t)^2 di}$), which we present in Figure 9. When the disinflation is launched all agents hold the same belief, given by the common prior μ. As disinflation evolves, price-setters who undertake price revisions update their beliefs μ_{it} upwards, and therefore the average belief $\overline{\mu}_t$ increases, at the same time as σ_t^{μ} starts to indicate dispersion in the corresponding distribution. This process continues for a while, with beliefs becoming more dispersed as firms choose to reprice less often and make decisions on different pricing dates, until a point where the tendency reverts and beliefs start to converge, albeit non-monotonically. Meanwhile, the average belief $\overline{\mu}_t$ increases steadily towards unity.

5 Conclusion

The role of credibility in monetary disinflations depends critically on the extent of price rigidity. This paper evaluates the effect of imperfect credibility of a disinflation policy in a model in which the time period between individual price adjustments is chosen optimally ex-ante. As a result we are able to evaluate both the direct effect of credibility, for a given frequency of price adjustments, and the indirect effect, which is engendered by the optimality of the pricing rule. The latter is important, as the effects of imperfect credibility and endogeneity of pricing rules interact to generate larger costs of disinflation. When the model is augmented with learning, it generates a realistic output pattern for the disinflation process.

Those results are encouraging enough to justify further research, both theoretical and empirical, based on endogenous time-dependent pricing rules. In empirical terms, an important challenge is to find ways to distinguish between different price setting specifications. Our results show that state- and endogenous time-dependent pricing models share many similarities in terms of the behavior predicted at the microeconomic level. Yet, their aggregate implications can differ dramatically. In theoretical terms, from a normative perspective our results point to the importance of analyzing the welfare implications of alternative disinflation strategies under optimal pricing rules.

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Appendix A

Here we derive the frictionless optimal price in a general equilibrium framework with firm-specific shocks.

A representative consumer maximizes the following utility function:

$$E_{t_0} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left[\log(C_t) - H_t\right] dt,$$

subject to the budget constraints:

$$B_t = B_0 + \int_0^t W_r H_r dr - \int_0^t \left(\int_0^1 P_{ir} C_{ir} di \right) dr + \int_0^t T_r dr + \int_0^t \Lambda_r dQ_r + \int_0^t \Lambda_r dD_r, \text{ for } t \geq 0,$$

where the composite consumption good over which utility is defined is given by:

$$C_t \equiv \left[\int_0^1 C_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

with $\theta > 1$, and where C_{it} is the consumption of variety i, P_{it} is its price, H_t is the supply of labor, which is remunerated at wage W_t, B_t is total financial wealth, T_t denotes total net transfers, including any lump-sum flow transfer from the government, and profits received from the firms, which are owned by the representative consumer. Q_r is the vector of prices of traded assets, D_r is the corresponding vector of cumulative dividend processes, and Λ_r is the trading strategy, which we assume satisfies conditions that preclude Ponzi schemes. The price index associated with the composite consumption good, P_t, is given by:

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{17}$$

In this setting, the demand for an individual product has the following familiar relation with aggregate demand:

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} C_t. \tag{18}$$

Each firm hires labor to produce its variety of the consumption good according to the following production function:

$$Y_{it} = A_{it} H_{it},$$

where A_{it} is firm i's productivity process. It is decomposed as:

$$A_{it} = \exp\left\{\varepsilon_{it}\right\} = \exp\left\{\varepsilon_t + \xi_{it}\right\},$$

where ε_t is the aggregate productivity component given by $\varepsilon_t \equiv \int_0^1 \varepsilon_{it} di$, and ξ_{it} is the firm-specific component.[25] We assume that firm-specific components have the same law of motion, and are mutually independent.

If producer i could adjust prices continuously, she would choose a price P_{it}^* to maximize profits according to the usual markup rule:

$$\frac{P_{it}^*}{P_t} = \frac{\theta}{\theta - 1} \frac{Y_t}{A_{it}}, \tag{19}$$

where Y_t / A_{it} is the real marginal cost of producing Y_{it}. We refer to P_{it}^* as firm i's *frictionless optimal price*. Substituting the demand function (18) into (19) leads to:

$$\left(\frac{Y_t}{Y_{it}} \right)^{\frac{1}{\theta}} = \frac{\theta}{\theta - 1} \frac{Y_t}{A_{it}}, \tag{20}$$

where we made use of the fact that if firm i had flexible prices, the charged price P_{it} would equal the frictionless optimal price. We have also used the equilibrium conditions $Y_{it} = C_{it}$, so that:

$$Y_t \equiv \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = C_t. \tag{21}$$

In this economy, a *flexible price equilibrium* is the one that obtains when all prices are flexible, so that (20) holds for each i, with aggregate output Y_t given by (21). The corresponding level of aggregate output is what we refer to as the *natural* level of output, Y_t^n. This is similar to the standard concept in the literature (e.g. Woodford, 2003, chapter 3). However, notice that here each individual output level in general differs from Y_t^n due to the existence of firm-specific shocks.[26]

Proceeding analogously as in Woodford (2003), we define the *deterministic* steady state level of production, $\overline{Y}$, as the output level in the symmetric flexible price equilibrium when $\varepsilon_{it} = 0$ for all i. So, it satisfies:

$$\overline{Y} = \frac{\theta - 1}{\theta}.$$

In order to obtain a more explicit characterization of the flexible price equilibrium we loglinearize both sides of equation (20) around the deterministic steady state levels, and

[25] Note that our decomposition and the definition of ε_t imply that ξ_{jt}'s have zero mean in the cross-section of firms.

[26] Equations (20) and (21) define implicitly a function $\Theta : \left(\{Y_{kt}\}_{k \neq j}, A_{jt} \right) \rightarrow Y_{jt}$. For a given realization A_{jt}, define $\Theta_{jt} \left(\{Y_{kt}\}_{k \neq j} \right) \equiv \Theta \left(\{Y_{kt}\}_{k \neq j}, A_{jt} \right)$. For given realizations A_{jt} for all j, an equilibrium is a fixed point of $\{\Theta_{jt}\}_{j \in [0,1]}$.

rearrange to get:

$$y_{it} = (1-\theta)\, y_t + \theta \varepsilon_{it}, \tag{22}$$

where $y_{it} \equiv \log\left(\frac{Y_{it}}{\overline{Y}}\right)$, $y_t \equiv \log\left(\frac{Y_t}{\overline{Y}}\right)$.[27]

Loglinearizing (21), and using (22), we obtain a relation between natural output and the aggregate level of productivity:

$$y_t^n = \varepsilon_t. \tag{23}$$

To derive a relation between the frictionless optimal price for firm i and the output gap, $y_t - y_t^n$, note that:

$$\begin{aligned} p_{it}^* - p_t &= \log \frac{\theta}{\theta - 1} + \log Y_t - \log A_{it} \\ &= -\log \overline{Y} + \log Y_t - \varepsilon_{it} \\ &= y_t - \varepsilon_{it}. \end{aligned}$$

Finally, decompose ε_{it} into its aggregate and firm-specific components and use (23) to replace ε_t, obtaining:

$$p_{it}^* - p_t = y_t - y_t^n + e_{it}, \tag{24}$$

where $e_{it} \equiv -\xi_{it}$.

Since our focus is on the supply side of the model, we take (log) nominal aggregate demand $\mathcal{Y}_t \equiv p_t + y_t$ to be an exogenous process. Substituting $y_t = \mathcal{Y}_t - p_t$ into (24) we arrive at the expression for the frictionless optimal price that we use in the main text:

$$p_{it}^* = \mathcal{Y}_t - y_t^n + e_{it}. \tag{25}$$

Appendix B

Formally, the pricing problem of a firm may be written as:[28]

$$\widetilde{V}(s_{t_0}) = \max_{\left\{\left(t_j, X_{t_j}\right)\right\}_{j=1}^{\infty}} E_{t_0} \sum_{j=0}^{\infty} e^{-\rho(t_j - t_0)} \left\{ E_{t_j} \left[\int_{t_j}^{t_{j+1}} e^{-\rho r} \Pi \left(\frac{X_{t_j}}{P_{t_j+r}}, Y_{t_j+r}, A_{t_j+r} \right) dr \right] - e^{-\rho(t_{j+1}-t_j)} \widehat{F} \right\},$$

so that $\widetilde{V}(s_{t_0})$ denotes the attained present value of real profits Π, net of pricing costs $\widehat{F}$, when the state of the economy is s_{t_0} and $\left\{\left(t_j, X_{t_j}\right)\right\}_{j=1}^{\infty}$ denotes the sequence of pricing dates and nominal prices set at each of those dates.

[27] In what follows, lowercase variables denote log-deviations from the deterministic steady state.
[28] Initially we drop the i subscripts in order to simplify the notation.

Let $V^*(s_{t_0})$ denote the attained present value of profits of an hypothetical identical firm in the same economy that does not face any pricing cost. Then,

$$V^*(s_{t_0}) = E_{t_0}\left[\int_{t_0}^{\infty} e^{-\rho r}\Pi\left(\frac{P^*_{t+r}}{P_{t+r}}, Y_{t+r}, A_{t+r}\right) dr\right],$$

where P^*_{t+r} is the individual price that maximizes real profits at time $t+r$, i.e. the frictionless optimal price of the firm. With this auxiliary value function, $\widehat{V}(s_{t_0}) \equiv V^*(s_{t_0}) - \widetilde{V}(s_{t_0})$ is the minimized present value of the real profit losses due to the existence of pricing costs, and our problem can be stated equivalently as one of minimizing the present value of such losses:

$$\widehat{V}(s_{t_0}) = \min_{\left\{\left(t_j, X_{t_j}\right)\right\}_{j=0}^{\infty}} E_{t_0}\sum_{j=0}^{\infty} e^{-\rho(t_j-t_0)}\left\{E_{t_j}\int_{t_j}^{t_{j+1}} e^{-\rho r}\left[\begin{array}{c}\Pi\left(\frac{P^*_{t_j+r}}{P_{t_j+r}}, Y_{t_j+r}, A_{t_j+r}\right)\\ -\Pi\left(\frac{X_{t_j}}{P_{t_j+r}}, Y_{t_j+r}, A_{t_j+r}\right)\end{array}\right] dr + e^{-\rho(t_{j+1}-t_j)}\widehat{F}\right\}.$$

Defining $\widehat{L}\left(\frac{P^*}{P}, \frac{P_i}{P}, Y, A\right) \equiv \Pi\left(\frac{P^*}{P}, Y, A\right) - \Pi\left(\frac{P_i}{P}, Y, A\right)$ to be the instantaneous real profit loss due to a "suboptimal" price P_i, we can rewrite $\widehat{V}$ as:

$$\widehat{V}(s_{t_0}) = \min_{\left\{\left(t_j, X_{t_j}\right)\right\}_{j=0}^{\infty}} E_{t_0}\sum_{j=0}^{\infty} e^{-\rho(t_j-t_0)}\left\{E_{t_j}\int_{t_j}^{t_{j+1}} e^{-\rho r}\widehat{L}\left(\frac{P^*_{t_j+r}}{P_{t_j+r}}, \frac{X_{t_j}}{P_{t_j+r}}, Y_{t_j+r}, A_{t_j+r}\right) dr + e^{-\rho(t_{j+1}-t_j)}\widehat{F}\right\}.$$

A recursive formulation to this minimization problem is given by the following Bellman equation:

$$\widehat{V}(s_t) = \min_{X,\tau} E_t\left[\int_0^{\tau} e^{-\rho r}\widehat{L}\left(\frac{P^*_{t+r}}{P_{t+r}}, \frac{X}{P_{t+r}}, Y_{t+r}, A_{t+r}\right) dr + e^{-\rho\tau}\left(\widehat{F} + \widehat{V}(s_{t+\tau})\right)\right].$$

Let $\overline{\Pi}$ be the steady state level of real profits in a frictionless economy:

$$\overline{\Pi} \equiv \Pi\left(\frac{P^*_{t_j+r}}{P_{t_j+r}}, \overline{Y}, 1\right) = \Pi\left(1, \overline{Y}, 1\right).$$

We can renormalize the pricing problem by $\overline{\Pi}$ and rewrite it as:

$$\overline{V}(s_t) = \min_{X,\tau} E_t\left[\int_0^{\tau} e^{-\rho r}\overline{L}\left(\frac{P^*_{t+r}}{P_{t+r}}, \frac{X}{P_{t+r}}, Y_{t+r}, A_{t+r}\right) dr + e^{-\rho\tau}\left(\overline{F} + \overline{V}(s_{t+\tau})\right)\right],$$

where $\overline{V}(s_t) \equiv \frac{\widehat{V}(s_t)}{\overline{\Pi}}$, $\overline{L}\left(\frac{P^*}{P}, \frac{X}{P}, Y, A\right) \equiv \frac{\widehat{L}\left(\frac{P^*}{P}, \frac{X}{P}, Y, A\right)}{\overline{\Pi}}$, $\overline{F} \equiv \frac{\widehat{F}}{\overline{\Pi}}$.

Given the primitives for preferences and technology, the expression for flow real profits can be written as:[29]

$$\left(\frac{P_i}{P}\right)^{1-\theta} Y - \frac{W}{P}\frac{Y}{A_i}\left(\frac{P_i}{P}\right)^{-\theta},$$

where P_i is the price *charged* by firm i. We can use the labor supply equation for this economy to express the real wage as a function of aggregate output ($\frac{W}{P} = Y$), and rewrite the expression for flow real profits as:

$$\Pi\left(\frac{P_i}{P}, Y, A_i\right) = \left(\frac{P_i}{P}\right)^{1-\theta} Y - Y\frac{Y}{A_i}\left(\frac{P_i}{P}\right)^{-\theta}.$$

We will later want to approximate the loss function $\overline{L}$. Observe that:

$$\begin{aligned} \overline{L}\left(\frac{P_i^*}{P}, \frac{P_i}{P}, Y, A_i\right) &= \frac{\Pi\left(\frac{P_i^*}{P}, Y, A_i\right) - \Pi\left(\frac{P_i}{P}, Y, A_i\right)}{\overline{\overline{\Pi}}} \\ &= \frac{\Pi\left(\frac{P_i^*}{P}, Y, A_i\right) - \Pi\left(\frac{P_i}{P}, Y, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, Y, A_i\right)} \frac{\Pi\left(\frac{P_i^*}{P}, Y, A_i\right)}{\overline{\overline{\Pi}}}. \end{aligned} \tag{26}$$

The second ratio can be written as

$$\begin{aligned} \frac{\Pi\left(\frac{P_i^*}{P}, Y, A_i\right)}{\overline{\overline{\Pi}}} &= \frac{\left(\frac{P_i^*}{P}\right)^{1-\theta} Y - Y\frac{Y}{A_i}\left(\frac{P_i^*}{P}\right)^{-\theta}}{(1)^{1-\theta}\,\overline{\overline{Y}} - \overline{\overline{Y}}\frac{\overline{\overline{Y}}}{1}(1)^{-\theta}} \\ &= \frac{\left(\frac{P_i^*}{P}\right)^{1-\theta} Y - \frac{\theta-1}{\theta} Y\left(\frac{P_i^*}{P}\right)^{1-\theta}}{\overline{\overline{Y}} - \frac{\theta-1}{\theta}\overline{\overline{Y}}} \\ &= \frac{Y}{\overline{\overline{Y}}}\left(\frac{P_i^*}{P}\right)^{1-\theta} \\ &= \left(\frac{Y}{\overline{\overline{Y}}}\right)^{2-\theta} A_i^{\theta-1}, \end{aligned} \tag{27}$$

where in the second equality we use the facts that:

$$\begin{aligned} \frac{P_i^*}{P} &= \frac{\theta}{\theta-1}\frac{Y}{A_i}, \\ \overline{\overline{Y}} &= \frac{\theta-1}{\theta}. \end{aligned}$$

The first ratio in (26) is the proportional profit loss (relative to the level of profits that

[29] Now we reintroduce the i subscript.

would obtain if the firm had flexible prices) due to the "suboptimal" price. It is convenient to rewrite it as:

$$\frac{\Pi\left(\frac{P_i^*}{P},Y,A_i\right)-\Pi\left(\frac{P_i}{P},Y,A_i\right)}{\Pi\left(\frac{P_i^*}{P},Y,A_i\right)}=1-\frac{\Pi\left(\frac{P_i}{P},Y,A_i\right)}{\Pi\left(\frac{P_i^*}{P},Y,A_i\right)}.$$

The profit ratio in the above expression can be written as:

$$\begin{aligned}\frac{\Pi\left(\frac{P_i}{P},Y,A_i\right)}{\Pi\left(\frac{P_i^*}{P},Y,A_i\right)} &= \frac{\left(\frac{P_i}{P}\right)^{1-\theta}Y-Y\frac{Y}{A_i}\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P_i^*}{P}\right)^{1-\theta}Y-Y\frac{Y}{A_i}\left(\frac{P_i^*}{P}\right)^{-\theta}}\\
&= \frac{\left(\frac{P_i}{P}\right)^{1-\theta}-\frac{\theta-1}{\theta}\frac{P_i^*}{P}\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P_i^*}{P}\right)^{1-\theta}-\frac{\theta-1}{\theta}\frac{P_i^*}{P}\left(\frac{P_i^*}{P}\right)^{-\theta}}\\
&= \theta\frac{\left(\frac{P_i}{P}\right)^{1-\theta}-\frac{\theta-1}{\theta}\frac{P_i^*}{P}\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P_i^*}{P}\right)^{1-\theta}}\\
&= \theta\left(\frac{P_i^*}{P_i}\right)^{\theta-1}-(\theta-1)\left(\frac{P_i^*}{P_i}\right)^{\theta},\end{aligned}$$

so that:

$$\frac{\Pi\left(\frac{P_i^*}{P},Y,A_i\right)-\Pi\left(\frac{P_i}{P},Y,A_i\right)}{\Pi\left(\frac{P_i^*}{P},Y,A_i\right)}=1-\theta\left(\frac{P_i^*}{P_i}\right)^{\theta-1}+(\theta-1)\left(\frac{P_i^*}{P_i}\right)^{\theta}. \tag{28}$$

Combining (27) and (28), we obtain:

$$\overline{L}\left(\frac{P_i^*}{P},\frac{P_i}{P},Y,A_i\right)=A_i^{\theta-1}\left(\frac{Y}{\overline{\overline{Y}}}\right)^{2-\theta}\left[1-\theta\left(\frac{P_i^*}{P_i}\right)^{\theta-1}+(\theta-1)\left(\frac{P_i^*}{P_i}\right)^{\theta}\right].$$

We can rewrite the loss function $\overline{L}$ in terms of log-deviations from the deterministic steady state:

$$G\left(p_i^*-p_i,\varepsilon_i,y\right)=e^{(\theta-1)\varepsilon_i+(2-\theta)y}\left[\left(1-\theta e^{(\theta-1)\left(p_i^*-p_i\right)}\right)+(\theta-1)\,e^{\theta\left(p_i^*-p_i\right)}\right].$$

This allows us to rewrite the optimal pricing problem as:

$$\overline{V}\left(s_t^i\right)=\min_{x_i,\tau_i}E_t\left[\begin{array}{c}\int_0^{\tau_i}e^{-\rho r}e^{(\theta-1)\varepsilon_{it+r}+(2-\theta)y_{t+r}}\left[\left(1-\theta e^{(\theta-1)\left(p_{it+r}^*-x_i\right)}\right)+(\theta-1)\,e^{\theta\left(p_{it+r}^*-x_i\right)}\right]dr\\ +e^{-\rho\tau_i}\left(\overline{F}+\overline{V}\left(s_{t+\tau_i}^i\right)\right)\end{array}\right].$$

The presence of aggregate output in the loss function implies that solving for the optimal pricing rule involves a fixed point problem, even in the absence of strategic complementarity or substitutability in price setting. Furthermore, the firm-specific level of productivity should introduce idiosyncratic variation in the optimal time interval between pricing dates.

We make the problem more tractable by abstracting from such idiosyncratic variation (i.e. setting $\varepsilon_{it} = 0$ when deriving the optimal pricing rule), and assuming $\theta = 2$ in order to eliminate the effect of aggregate output. Then, we take a second-order Taylor expansion of flow profit losses around the path for the frictionless optimal price in order to obtain an approximate dynamic pricing problem:

$$\overline{V}_{app}\left(s_t^i\right) = \min_{x_i,\tau_i} E_t\left[\int_0^{\tau_i} 2e^{-\rho r}\left(p_{it+r}^* - x_i\right)^2 dr + e^{-\rho\tau_i}\left(\overline{F} + \overline{V}_{app}\left(s_{t+\tau_i}^i\right)\right)\right].$$

Finally, defining $V\left(s_t^i\right) \equiv \frac{\overline{V}_{app}\left(s_t^i\right)}{2}$, $F \equiv \frac{\overline{F}}{2}$, and using the discrepancy $z_i \equiv x_i - p_i^*$, we arrive at the pricing problem analyzed in the main text:

$$V\left(s_t^i\right) = \min_{z_i,\tau_i} E_t\left[\int_0^{\tau_i} e^{-\rho r}\left[z_{it} - \left(p_{it+r}^* - p_{it}^*\right)\right]^2 dr + e^{-\rho\tau_i}\left(F + V\left(s_{t+\tau_i}^i\right)\right)\right].$$

Appendix C

Here we present the solution method for (16). The corresponding first order conditions are:

$$z_t^* = \frac{\rho}{1-e^{-\rho\tau_t^*}}\int_0^{\tau_t^*}\left[\pi' r + (\pi - \pi')\left(r - \left(\mu_t\frac{1-e^{-\underline{h}r}}{\underline{h}} + (1-\mu_t)\left(\frac{1-e^{-\overline{h}r}}{\overline{h}}\right)\right)\right)\right]e^{-\rho r}dr, \tag{29}$$

$$\begin{aligned}
&\left(z_t^* - \pi'\tau_t^*\right)^2 + \sigma^2\tau_t^* + \left(\mu_t\underline{h}e^{-\underline{h}\tau_t^*} + (1-\mu_t)\,\overline{h}e^{-\overline{h}\tau_t^*}\right)\left(V_\pi - V_\mu\left(0, t+\tau_t^*\right)\right) \\
&\quad + \left(\mu_t e^{-\underline{h}\tau_t^*} + (1-\mu_t)\,e^{-\overline{h}\tau_t^*}\right)\frac{\partial V_\mu\left(0,t+\tau_t^*\right)}{\partial t} \\
&\quad -\rho\left[F + V_\pi + \left(\mu_t e^{-\underline{h}\tau_t^*} + (1-\mu_t)\,e^{-\overline{h}\tau_t^*}\right)\left(V_\mu\left(0,t+\tau_t^*\right) - V_\pi\right)\right] \\
&\quad + \int_0^{\tau_t^*}\left(\left(\pi'-\pi\right)\left(\tau_t^* - r\right)\right)^2\left(\mu_t\underline{h}e^{-\underline{h}r} + (1-\mu_t)\,\overline{h}e^{-\overline{h}r}\right)dr \\
&\quad +2\left(\pi'-\pi\right)\left(z_t^* - \pi'\tau_t^*\right)\int_0^{\tau_t^*}\left(\tau_t^* - r\right)\left(\mu_t\underline{h}e^{-\underline{h}r} + (1-\mu_t)\,\overline{h}e^{-\overline{h}r}\right)dr = 0.
\end{aligned} \tag{30}$$

Equations (29), (30), and (16) characterize z_t^*, τ_t^* and $V_\mu\left(0, t+\tau_t^*\right)$. To solve this set of equations, we first pick $\bar{t}$ large enough, such that, for $t > \bar{t}$, $V_\mu\left(0,t\right)$ can be taken as approximately constant. This is justified: conditional on no abandonment, the probability that the monetary authority is of type $\underline{h}$ keeps increasing, and the problem becomes more similar to the one analyzed in section (2.3), with $h = \underline{h}$. Formally, $\lim_{t\to\infty} \mu_t = 1$,[30] which implies that $\lim_{t\to\infty} V_\mu(0,t) = V_{\underline{h}}$. So, we solve the set of equations moving backwards in time. For each t we find z_t^*, τ_t^* and use them to compute $V_\mu(0,t)$, which is then used to find z^*, τ^* at earlier times. Alternatively, to avoid numerical derivatives, one can use (29), and (16) to find τ_t^* with a grid search, instead of using (30). This is the method we adopt.

[30] Just rewrite μ_t as $\frac{1}{1+\frac{(1-\mu)}{\mu}e^{-(\overline{h}-\underline{h})t}}$, and recall that $\overline{h} - \underline{h} > 0$.

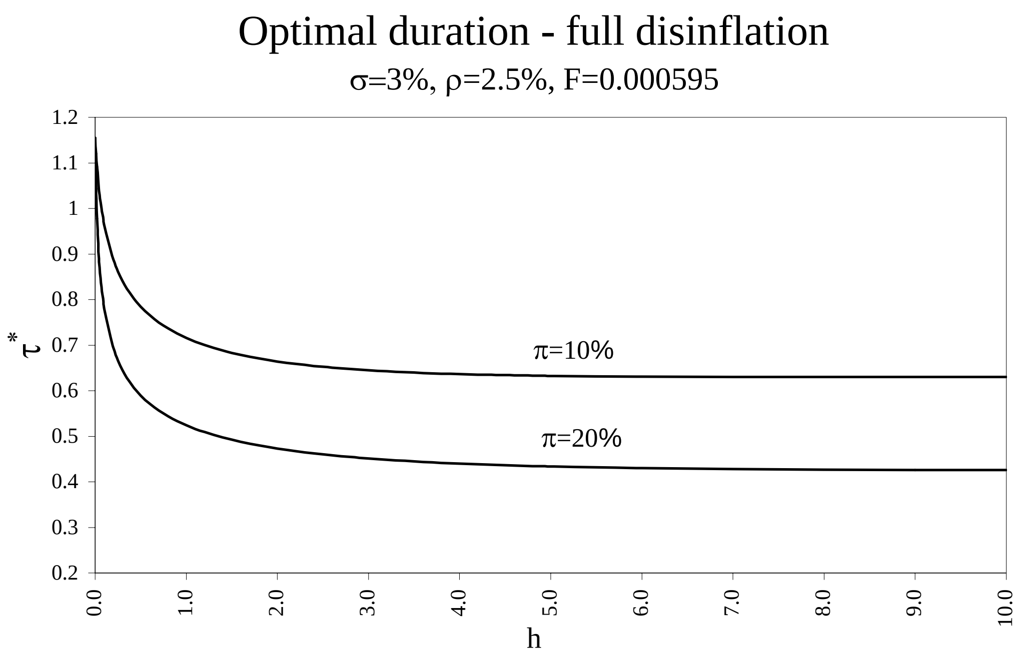

Figure 1

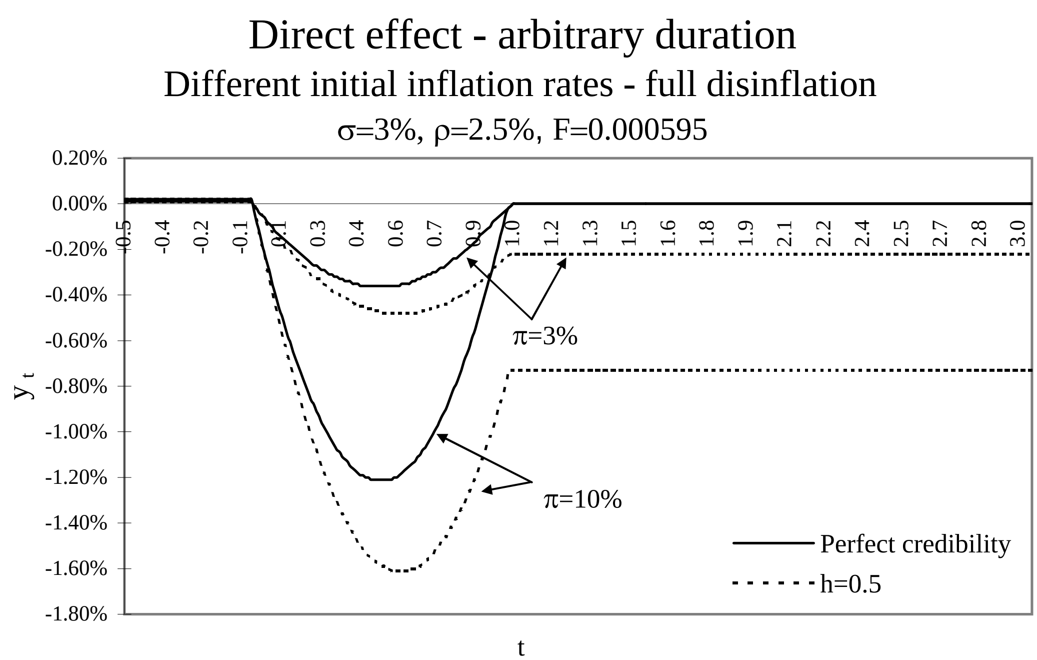

Figure 2

Obs: Duration of price spells are fixed at the model's implied optimal level for π=3%.

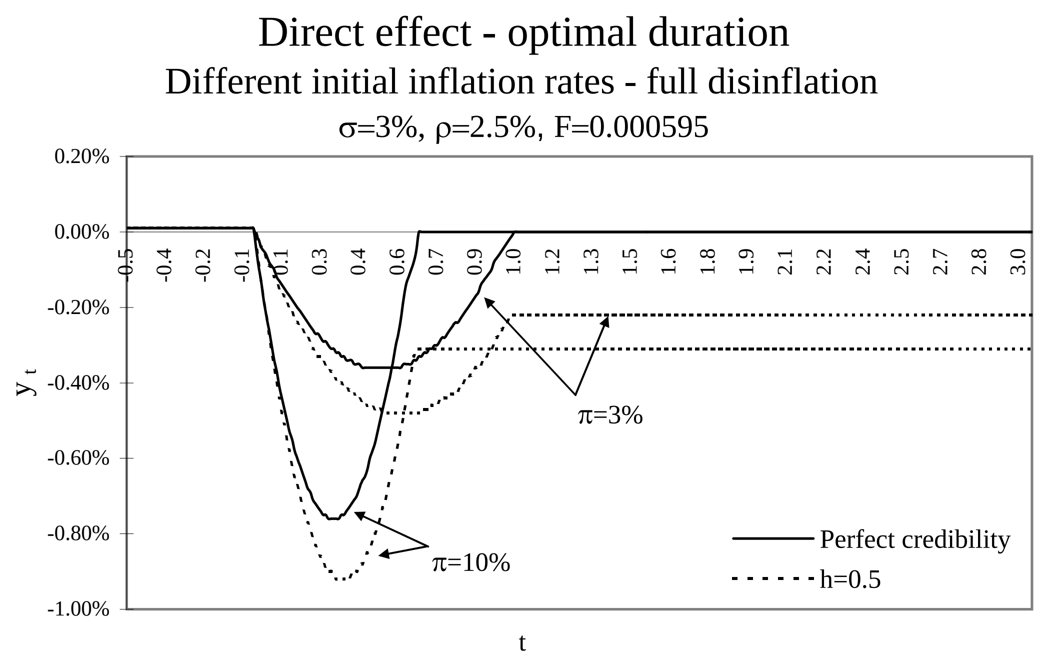

Figure 3

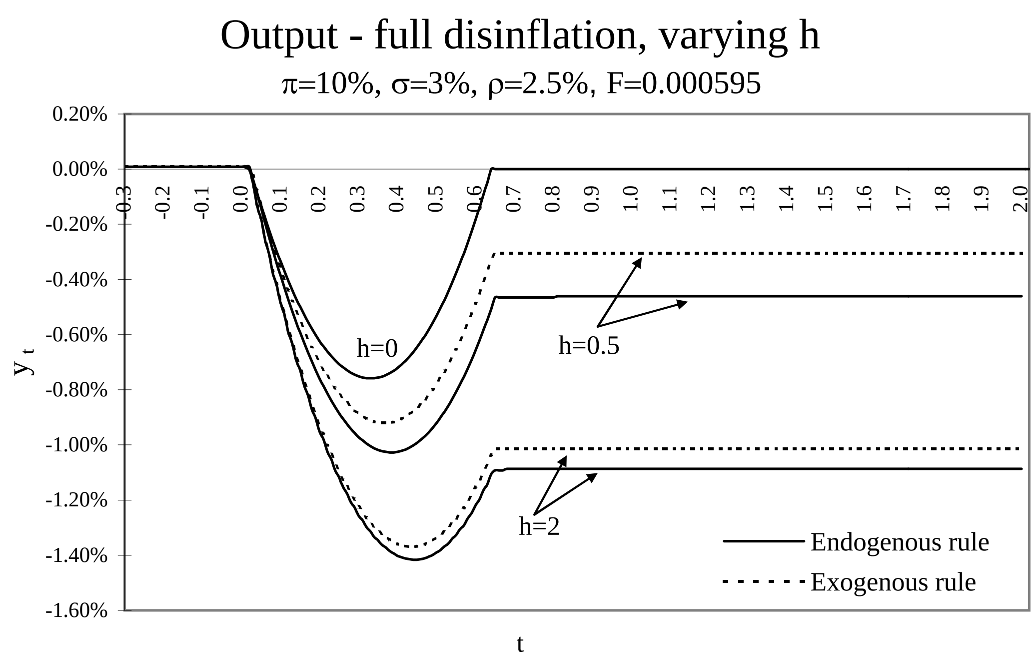

Figure 4

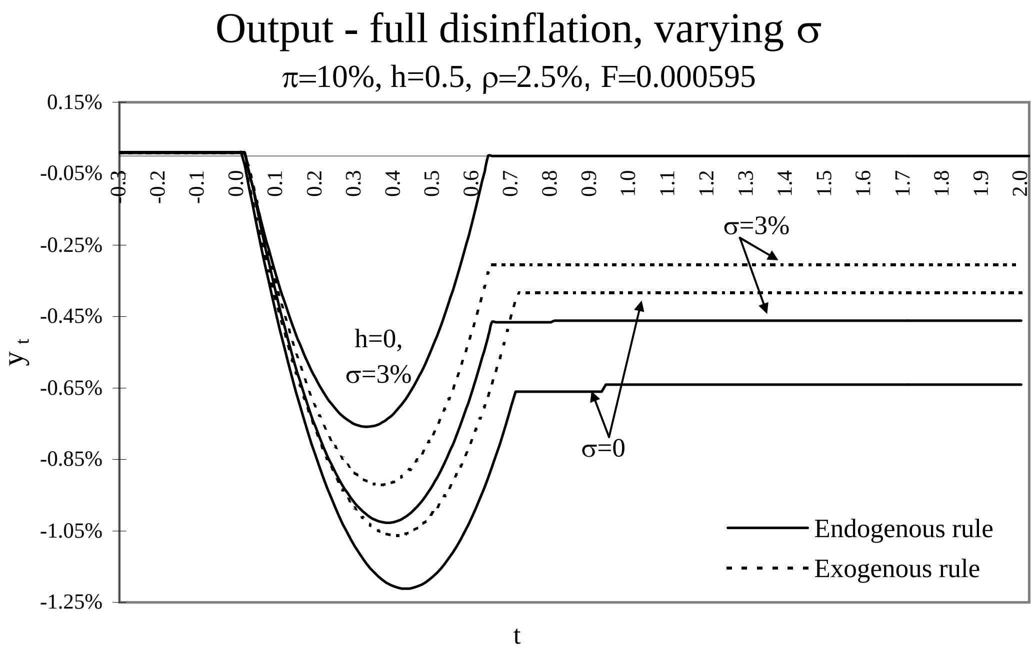

Figure 5

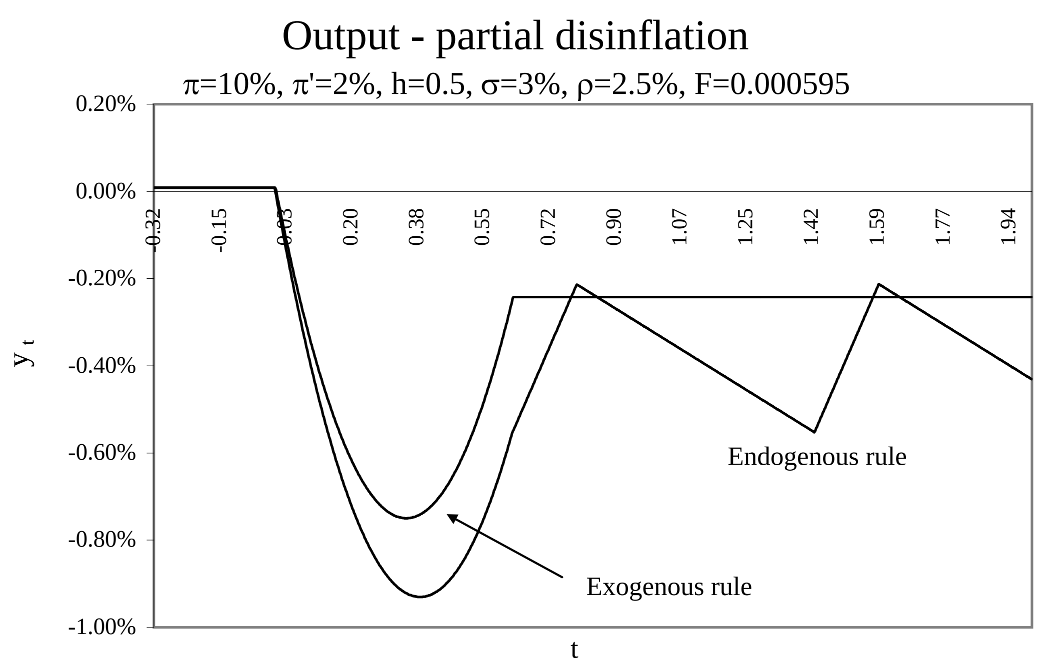

Figure 6

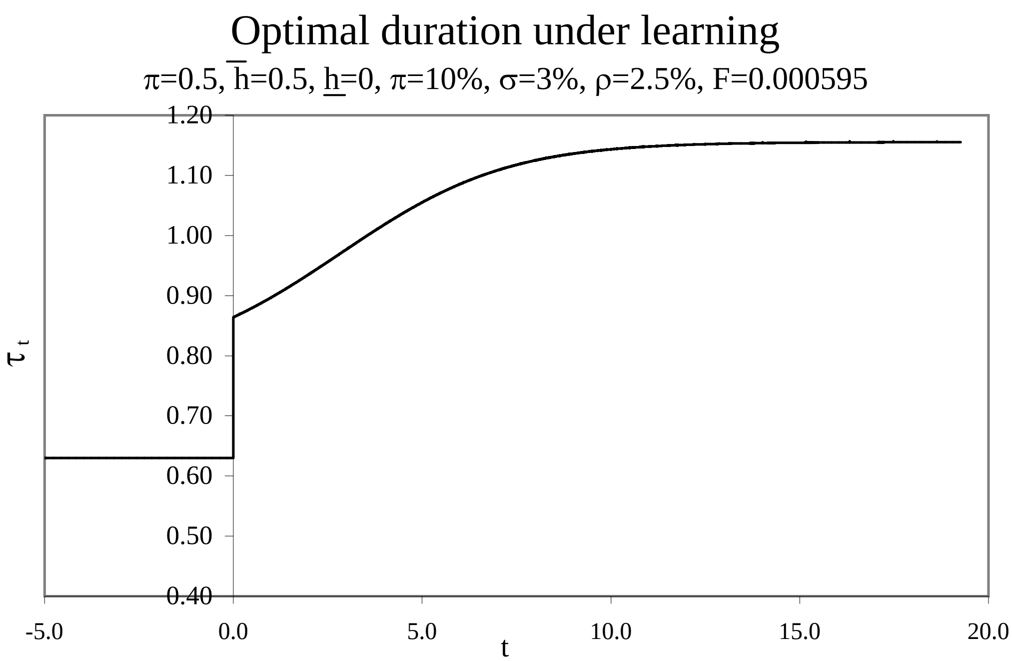

Figure 7

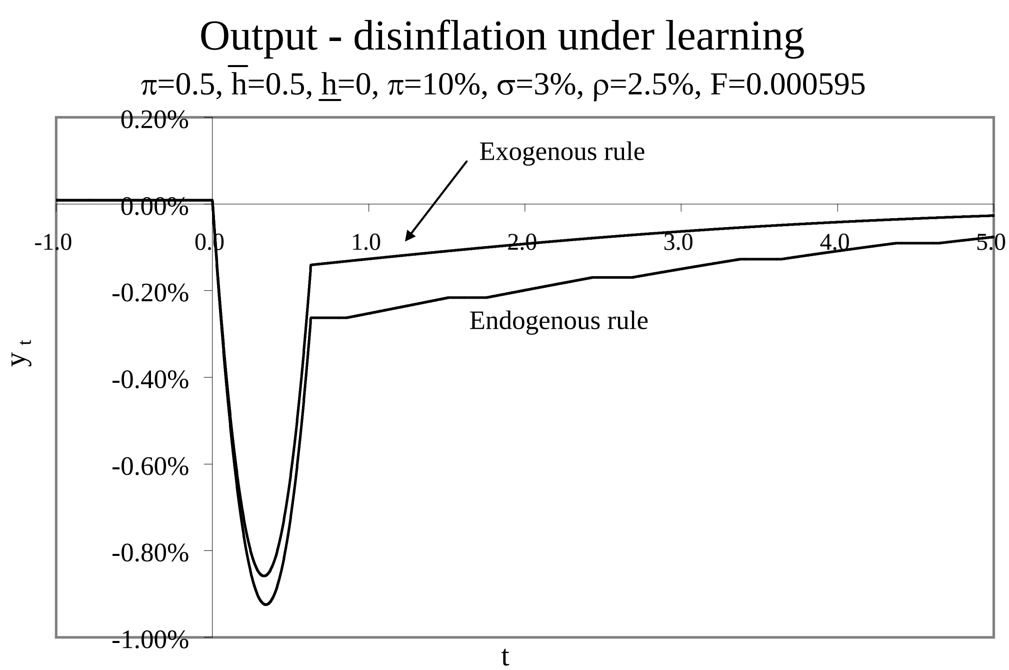

Figure 8

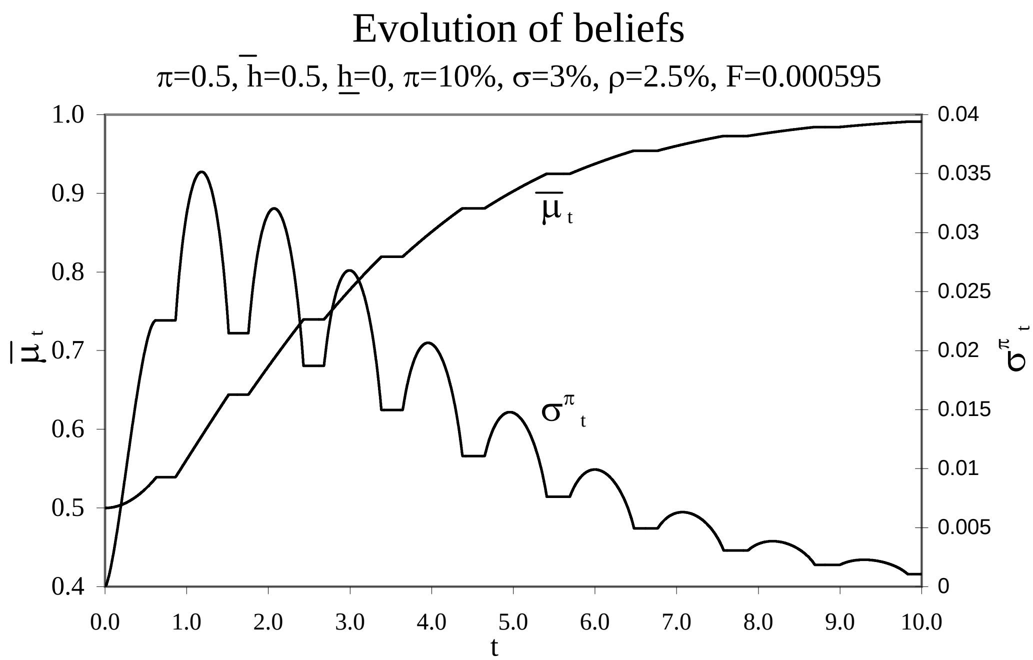
Evolution of beliefs
π=0.5, h̄=0.5, h=0, π=10%, σ=3%, ρ=2.5%, F=0.000595
1.0
0.9
0.8
0.7
0.6
0.5
0.4
μ̄ t
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
σ π t
μ̄ t
σ π t
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
t

Figure 9