Non-linear real exchange rate effects in the UK labour market

Gabriella Legrenzi

Keele University, UK
e-mail: g.d.legrenzi@keele.ac.uk

and

Costas Milas*

Keele University, UK
e-mail: c.milas@keele.ac.uk

Abstract

Using UK data over the 1973q1-2004q1 period, we find that the dynamics of the real exchange rate, real wages and unemployment vary both with large versus small real exchange rate disequilibria and rising versus falling unemployment regimes. The short-run real exchange rate adjusts only when large disequilibrium deviations occur. We report fast real exchange rate adjustment in periods of falling unemployment. This implies that prices and wages are more flexible when real output is high. When the real exchange rate is highly undervalued, workers respond to an improvement in domestic competitiveness by demanding and getting higher wages. Unemployment is reduced following gains in competitiveness when the real exchange rate is further away from equilibrium.

Keywords: Real exchange rate; unemployment; Smooth Transition Vector Error Correction Model.

JEL classification: C32; C51; C52

*Address for correspondence: Costas Milas, Department of Economics, Keele University, Keele, ST5 5BG, United Kingdom. Phone: +44 (0) 1782 583090. Fax: +44 (0) 1782 717577. Email: c.milas@keele.ac.uk.

We thank two anonymous referees for very useful comments and suggestions. We have also benefited from the comments of Christopher Martin (Brunel University), Philip Rothman (East Carolina University) and Peter N. Smith (University of York). Any remaining errors are ours.

1. Introduction

The use of non-linear models in explaining economic phenomena is motivated by the idea that the behaviour of economic variables depends on different states of the world or regimes that prevail at any point in time. Over the last few years, the Smooth Transition Autoregressive (thereafter STAR) methodology has been a popular way of introducing regime-switching behaviour in real exchange rate models, where the transition from one regime to the other occurs in a smooth way. [1] For instance, Baum *et al.* (2001) and Michael *et al*. (1997) model the real exchange rate (for a number of countries including the UK) as a stationary variable and estimate its dynamics based on Exponential STAR (ESTAR) models. Paya and Peel (2003) estimate an ESTAR model of the dollar–yen real exchange rate which incorporates a deterministic trend as a proxy for the equilibrium level. On the other hand, Sarantis (1999) finds that real exchange rates (including the UK one) are non-stationary and proceeds by estimating real exchange ESTAR and Logistic STAR (LSTAR) models in first differences. [2] A common feature of these papers is that they all estimate univariate real exchange rate models.

This marks a significant point of departure for our paper: while we follow Sarantis (1999) in treating the real exchange rate as a non-stationary variable, we find that the latter cointegrates with real wages and the unemployment rate. Acknowledging, however, the ongoing debate about the unit root properties of the real exchange rate, we also check the robustness of our main results by looking at the possibility that the real exchange rate is stationary.

We derive a linear real exchange rate equation within an economy producing traded and non-traded goods. In particular, we test whether the real exchange rate cointegrates with real wages and unemployment (another related paper is by Akram *et al*., 2005, who estimate a non-linear model of the Norwegian real exchange rate assuming that it cointegrates with the ratio of Norwegian to foreign gross domestic product). Noting that cointegration tests perform reasonably well when the adjustment process is non-linear (van Dijk and Franses, 2000), we then proceed by

[1] STAR models were introduced by Teräsvirta and Anderson (1992) in order to examine non-linearities over the business cycle, whereas their statistical properties were discussed in Granger and Teräsvirta (1993) and Teräsvirta (1994), among others.

[2] Empirical results in support of a stationary real exchange rate are quite mixed and depend e.g. upon the sample size selected or the definition of the price series used (for a recent survey see e.g. Baum *et al*., 2001). Studies using long data series (e.g. Taylor, 1996, Lothian and Taylor, 1996, and Michael *et al*., 1997) find that the real exchange rate is stationary but Engel (2000) argues that these tests may have reached the wrong conclusion. More recently, based on data over the last two centuries, Akram *et al*. (2005) treat the Norwegian real exchange rate as a non-stationary process.

employing a multivariate STAR framework to model the non-linear dynamics of the real exchange rate equation as part of a small system involving real wages and the unemployment rate. [3]

Modelling our system in a smooth transition framework contrasts with the Threshold Autoregressive (TAR; see e.g. Tong, 1990) and the Hamilton (1989) Markov regime-switching models, which assume that the transition between regimes occurs abruptly rather than smoothly. On economic grounds, STAR models seem to be more appropriate than TAR or Markov regime-switching models. Modelling the real exchange rate as a function of real wages and the unemployment rate implies that real exchange rate movements are affected by conditions prevailing in the production sector of the economy. In this case, a smooth transition rather than a sharp switch between regimes could be justified in terms of frictions in the product market due to product heterogeneity, government imposed barriers to trade, or labour market inflexibility distorting the rapid adjustment of wages.

According to our results, the dynamics of the real exchange rate, real wages and unemployment vary both with large versus small real exchange rate disequilibria and rising versus falling unemployment regimes. The short-run real exchange rate adjusts to disequilibrium deviations of the real exchange rate from its long-run level only outside an interval band. When regimes of rising versus falling unemployment are taken into account, fast real exchange rate adjustment occurs in periods of falling unemployment. This implies that prices and wages are more flexible when real output is high (see e.g. Weise, 1999 and references therein). Our results also suggest that when the real exchange rate is highly undervalued, workers respond to an improvement in domestic competitiveness by demanding and getting higher wages. We also find that unemployment is reduced following gains in competitiveness when the real exchange rate is further away from equilibrium.

The structure of the paper is as follows. The next section discusses briefly the theory of linearity testing within a multivariate STAR framework. Section 3 of the paper discusses the specification of a real exchange rate model, whereas Section 4 estimates the model. Section 5 presents a discussion of our findings and provides a robustness check by also looking at the possibility that the real exchange rate is a stationary rather than a non-stationary variable. Finally, section 6 provides some concluding remarks.

[3] Multivariate STAR models were recently discussed in Granger and Swanson (1996), Weise (1999), Rothman *et al*. (2001) and van Dijk *et al*. (2002) among others.

2. Specification of multivariate smooth transition models

Following Rothman *et al*. (2001), we write a k-dimensional Smooth Transition Vector Error Correction Model (STVECM) as:

$$\Delta y_t = \left(\mu_1 + \alpha_1 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j} \Delta y_{t-j} \right)(1 - G(s_t)) + \left(\mu_2 + \alpha_2 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{2,j} \Delta y_{t-j} \right) G(s_t) + \varepsilon_t, \qquad (2.1)$$

where y_t is a (k x 1) vector of $I(1)$ endogenous variables, $\varepsilon_t \sim iid(0,\Sigma)$, α_i, $i = 1, 2$, are (k x r) matrices, and $z_t = \beta' y_t$ for some (k x r) matrix β denote the error correction terms. $\Phi_{1,j}$ and $\Phi_{2,j}$, $j = 1,..., p-1$, are (k x k) matrices and μ_i, $i = 1, 2$, are ($k \times 1$) vectors. $G(s_t)$ is the transition function, assumed to be continuous and bounded between zero and one. The STVECM framework can be considered as a regime-switching model which allows for two regimes, $G(s_t) = 0$ and $G(s_t) = 1$, respectively, where the transition from one to the other regime occurs in a smooth way. Here we focus on three popular choices for the transition function $G(s_t)$. Other versions of these functions, including cubic polynomials and rational polynomials are discussed in Escribano (1986, 2004).

The first popular $G(s_t)$ choice is the 'logistic' function:

$$G(s_t;\gamma,c) = \{1 + \exp[-\gamma(s_t - c)/\sigma(s_t)]\}^{-1}, \gamma > 0, \qquad (2.2a)$$

where $\sigma(s_t)$ is the sample standard deviation of s_t. This model assumes asymmetric adjustment to positive and negative deviations of s_t relative to a parameter c. The latter is the threshold between the two regimes; $G(s_t)$ changes monotonically from 0 to 1 as s_t increases, and takes the value of $G(s_t) = 0.5$ at $s_t = c$. The parameter γ determines the speed of the transition from one regime to the other. When $\gamma \to 0$, the 'logistic' function equals a constant (i.e. 0.5), and when $\gamma \to +\infty$, the transition from $G(s_t) = 0$ to $G(s_t) = 1$ is almost instantaneous at $s_t = c$.

The second choice is the 'exponential' function:

$$G(s_t;\gamma,c) = 1 - \exp\{-\gamma(s_t - c)^2 / \sigma^2(s_t)\}, \gamma > 0. \tag{2.2b}$$

This model assumes asymmetric adjustment to small and large absolute values of s_t. The 'exponential' model above has the drawback that it becomes linear if either $\gamma \to 0$ or $\gamma \to +\infty$. This can be avoided by setting $G(s_t)$ equal to the 'quadratic logistic' function:

$$G(s_t;\gamma,c_1,c_2) = \{1 + \exp[-\gamma(s_t - c_1)(s_t - c_2)/\sigma^2(s_t)]\}^{-1}, \gamma > 0, \tag{2.2c}$$

as proposed by Jansen and Teräsvirta (1996). In this case, if $\gamma \to 0$, the model becomes linear, whereas if $\gamma \to +\infty$, $G(s_t)$ is equal to 1 for $s_t < c_1$ and $s_t > c_2$, and equal to 0 when $c_2 < s_t < c_1$.

Given the drawback of the exponential function as discussed by Jansen and Teräsvirta (1996), we focus on the transition functions (2.2a) and (2.2c) and choose the appropriate one based on statistical tests. Further, we assume that the possible candidates for the transition variable s_t are the r cointegrating relationships in $z_t = \beta' y_t$ as well as all other regressors in (2.1).

A test of linearity in model (2.1) using the transition function (2.2c), is a test of the null hypothesis H_0: $\gamma = 0$ against the alternative H_1: $\gamma > 0$. By taking a first-order Taylor approximation of $G(s_t)$ around $\gamma = 0$, the test can be done within the reparameterised model (see e.g. the discussion in Saikkonen and Luukkonen, 1988):

$$\Delta y_t = \mathrm{M}_0 + \mathrm{A}_0 z_{t-1} + \sum_{j=1}^{p-1} \mathrm{B}_{0,j} \Delta y_{t-j} + \mathrm{A}_1 z_{t-1} s_t + \sum_{j=1}^{p-1} \mathrm{B}_{1,j} \Delta y_{t-j} s_t + \mathrm{A}_2 z_{t-1} s_t^2 + \sum_{j=1}^{p-1} \mathrm{B}_{2,j} \Delta y_{t-j} s_t^2 + e_t, \tag{2.3a}$$

where e_t are the original errors ε_t plus the error arising form the Taylor approximation. Model

(2.3a) is a linear VECM augmented by additional cross-product regressors due to the Taylor expansion. Here, the null hypothesis of linearity is $H_0' : A_1 = A_2 = B_{1,j} = B_{2,j} = 0$, where $j = 1,..., p-1$. For each equation in the VECM, this is a standard variable addition Lagrange Multiplier (LM) which follows asymptotically the χ^2 distribution with $2r + 2k(p - 1)$ degrees of freedom. In small samples, the χ^2 test may be heavily oversized. Therefore, it may be preferable to use an *F* version of the test. Both the χ^2 and *F* versions of the *LM* statistic are equation specific tests for linearity which are computed from an auxiliary regression of the residuals from each equation in the linear VECM on all variables entering model (2.3a). To test the null hypothesis of linearity in all equations simultaneously, we need a system-wide test. Following Weise (1999), define Ω_0 and Ω_1 as the estimated variance-covariance residual matrices from the linear VECM and the augmented model (2.3a), respectively. The appropriate log-likelihood system-wide test statistic is given by $LR = T\{\log|\Omega_0| - \log|\Omega_1|\}$, where *T* is the size of the sample. Under the null hypothesis of linearity, the test follows asymptotically the χ^2 distribution with $2rk + 2k^2(p - 1)$ degrees of freedom.

On the other hand, a test of linearity in model (2.1) using the transition function (2.2a) involves using the reparameterised model:

$$\Delta y_t = M_0 + A_0 z_{t-1} + \sum_{j=1}^{p-1} B_{0,j} \Delta y_{t-j} + A_1 z_{t-1} s_t + \sum_{j=1}^{p-1} B_{1,j} \Delta y_{t-j} s_t + e_t \tag{2.3b}$$

In this case, for each equation in the VECM, the variable addition *LM* test follows asymptotically the χ^2 distribution with $r + k(p - 1)$ degrees of freedom, whereas the system-wide test statistic follows asymptotically the χ^2 distribution with $rk + k^2(p - 1)$ degrees of freedom. The equation specific *LM* tests and the system wide *LR* test are run for both transition functions and all possible s_t candidates. The decision rule is to select as the appropriate transition function and transition variable the combination for which the *p*-value of the test statistic is the smallest one. [4]

[4] An extension of the Saikkonen and Luukkonen (1988) linearity tests involves a second-order Taylor approximation of the transition function as suggested by Escribano and Jordá (1999, 2001). For model (2.3a), this involves adding cubic and fourth power terms. In the present paper, however, this is hardly practical to implement since we are faced with a small sample size. Further, as van Dijk *et al.* (2002) point out, neither one of the tests in Saikkonen and Luukkonen (1988) or Escribano and Jordá (1999, 2001) dominates in terms of power. Given that the tests are not exact but approximations, some caution is needed when using the rule of the minimum probability value in order to determine the appropriate transition variable.

3. Specification of a real exchange rate model

The theoretical framework discussed above will now be tested on a small model of the UK real exchange rate. We assume a model with traded (*T*) and non-traded (*NT*) goods. The model draws on Milas (1997). [5] It assumes perfect competition in the *T* sector with firms producing according to a CES production function, which is separable into capital and labour. The production function assumes constant returns to scale. For the *NT* sector, the model assumes profit maximising monopolistic competitive firms.

Assume that the domestic price, p, is a weighted average of the price in the traded sector, p_T, and the price in the non-traded sector, p_N (in log-form):

$$p = \tau p_T + (1-\tau) p_N , \tag{3.1}$$

where τ is the share of tradeables in domestic output ($0 < \tau < 1$). Due to perfect competition in the traded-goods sector, firms are assumed to take p_T as given. [6] On the other hand, the price of non-traded goods, p_N is set as a mark-up on unit labour costs:

$$p_N = w + \mu , \tag{3.2}$$

where μ is the mark-up. In (3.2), the wage rate, w, is assumed to be common for both sectors in the economy. In the long run, wages are common across sectors mainly due to competition for labour between firms in the tradeable and non-tradeable sectors. Common wages could also be seen as a struggle for factor income shares or wage relations between employees' organisations across sectors.

The mark-up, μ, is assumed to vary with deviations of output, y, from its full employment level, y_F (see e.g. Bean, 1994):

$$\mu = \psi (y - y_F) \tag{3.3}$$

[5] For other versions of price models with traded and non-traded goods see Martin (1997) and Alogoskoufis (1990).
[6] In this case, p_T could be proxied by international prices adjusted for exchange rate movements. However, as a referee pointed out, p_T refers to the price of domestically produced goods. We use the price in the UK manufacturing sector as a suitable proxy for p_T.

In (3.3), we write the mark-up as pro-cyclical (i.e. increasing in the level of activity). Indeed, Haskel *et al*. (1995) find that the mark-up is pro-cyclical for the UK, whereas Smith (2000) points out that the pro-cyclical behaviour of the UK mark-up is mainly driven by the 1980-1981 recession and goes on to show that it is hard to detect a stable relationship between the mark-up and the UK cycle. [7]

We also assume that Okun's law holds:

$$(y - y_F) = -\omega u \,, \tag{3.4}$$

where u is the unemployment rate in the economy. Using (3.2), (3.3) and (3.4), equation (3.1) can be re-written as:

$$p_T - p = -(1-\tau)(w - p_T) + (1-\tau)\psi\omega u \tag{3.5}$$

Equation (3.5) derives a measure of the real exchange rate $p_T - p$ as a negative function of real product wages in the traded goods sector, $w - p_T$. The sign on the unemployment rate, u, is positive, provided that the mark-up is pro-cyclical (in which case the elasticity $\psi > 0$). In our notation, an increase in $p_T - p$ is equivalent to a real depreciation or an improvement in the real competitiveness of the domestic economy. Following the notation in Section 2 of the paper, our model uses a set of $k = 3$ endogenous variables:

$$y_t = [p_T - p, w - p_T, u]'. \tag{3.6}$$

We use quarterly seasonally adjusted UK data over the period 1973q1-2004q1. The price of domestic tradeables, p_T, is given by prices in the UK manufacturing sector (as a proxy for traded goods), p refers to the GDP deflator, w is the average hourly wage rate in the manufacturing sector and u is the unemployment rate. All variables are in logs and are taken from the Office for National Statistics (ONS).

---

[7] The mark-up can also be counter-cyclical; for a recent discussion based on US data see e.g. Gali *et al.* (2002) and Rotemberg and Woodford (1999).

4. Empirical results

4.1 Long-run behaviour

Figure 1 plots the logs of the levels and the first differences of the $p_T - p$, $w - p_T$, and u series. Preliminary analysis using the ADF unit root tests suggested that at least for the sample period examined here, all series are $I(1)$ in levels and $I(0)$ in first differences. We estimate the linear VECM in levels using a lag length of $p = 4$ (based on the Akaike Information Criterion) and allowing for a drift parameter to enter the VECM unrestrictedly. Table 1 reports the eigenvalues, λ_i, and the λ-*max* and *trace* statistics (using the small sample correction discussed in Reimers, 1992) together with the 95 percent critical values (see Johansen, 1988). Both the λ-*max* and *trace* statistics indicate the existence of $r = 1$ cointegrating vector. For exact identification, we normalise the estimated vector on the real exchange rate, $p_T - p$. The resulting cointegrating vector is:

$$p_T - p = \underset{(0.017)}{-0.520}\,(w - p_T) \; \underset{(0.010)}{+0.041}\, u$$

where standard errors are given in parentheses below the estimated coefficients. The estimated cointegrating relationship looks like the theoretical real exchange rate equation (3.5) with the share of traded goods in total output (i.e. τ) estimated at 48 percent. This is comparable to the estimate of 40 percent in Batini *et al.* (2002) who use annual disaggregated UK data on 49 industrial categories over the 1972-1998 period. [8] We cannot identify ψ and ω separately, but the positive sign on unemployment points to a pro-cyclical mark-up, in line with Haskel *et al.* (1995). That said, the pro-cyclical finding should be treated with some caution, as it is not based on a direct measure of the mark-up.

4.2 Linearity testing and short-run estimates

Having estimated the long-run real exchange rate equation, we test for linearity in models (2.3a) and (2.3b) using the estimated cointegrating vector CV_{t-1} and all other RHS regressors as possible transition variables s_t. Linearity tests are run for a different number of lags of the transition variable $s_{t-d} = \{CV_{t-d}, (p_T - p)_{t-d}, (w - p_T)_{t-d}, u_{t-d}\}$, namely $d = 1$, 2, 3, and 4 lags. Then

---

[8] Using annual data over the 1952-1985 period, Alogoskoufis (1990) estimates τ between 31 percent and 39 percent.

the appropriate lag is selected as the one for which the linearity test is most strongly rejected. We report bootstrapped p-values instead of asymptotic p-values although our results are not sensitive to the above choice. To compute the bootstrapped p-values of the equation specific F tests and the system wide LR test reported in Table 2, we followed closely Weise (1999). First, we estimated the linear VECM equations, where there was evidence of heteroscedasticity. To control for this, the VECM residuals were regressed on all RHS variables entering the linear VECM as well as their squares, and the original residuals were transformed using the estimated coefficients from this auxiliary regression. Draws were taken from the transformed residuals and one thousand artificial data series were constructed. For each of these artificial series, F and LR statistics were constructed and then compared to the corresponding statistics from the actual data. The bootstrapped p-values were derived as the number of times the F and LR statistics from the artificial data exceeded the corresponding statistics from the actual data, divided by one thousand.

The system wide tests reported in Table 2a (for the quadratic logistic function) and in Table 2b (for the logistic function) suggest that linearity is strongly rejected for all transition variable candidates and both transition function specifications. Therefore, the system wide tests are not particularly informative on the most suitable transition function and transition variable. Next, we turn to the equation specific tests. Keeping in mind that (obviously) one cannot estimate every possible model for every possible transition function and transition variable, the results in Table 2a suggest that linearity is most strongly rejected for the quadratic logistic function across all three equations when $s_t = CV_{t\text{-}1}$, whereas the results in Table 2b suggest that linearity is most strongly rejected for the logistic function across all three equations when $s_t = \Delta u_{t\text{-}1}$. Therefore we proceed by estimating two STVECM specifications. The first one uses the quadratic logistic function with $CV_{t\text{-}1}$ as the appropriate transition variable. This model is particularly attractive from an economic point of view as it implies the existence of an interval band (c_1, c_2) outside which there is a strong tendency for the real exchange rate to revert to its equilibrium value. The second specification uses the logistic function with $\Delta u_{t\text{-}1}$ as the appropriate transition variable. This specification is also attractive in economic terms as it allows for the growth of the real exchange rate, wages and unemployment to vary at different points in the business cycle, that is, when unemployment is rising (when $\Delta u_{t\text{-}1}$ is above a threshold c) and when unemployment is falling (when $\Delta u_{t\text{-}1}$ is below a threshold c).

Before estimating the non-linear models, it is worth mentioning that Granger and Teräsvirta (1993) and Teräsvirta (1994) stress particular problems like slow convergence or overestimation associated with estimates of the γ parameter. For this reason, we follow their suggestion in (i) scaling the quadratic logistic function (2.2c) by dividing it by the variance of the transition variable $\sigma^2(CV_{t\text{-}1})$, and (ii) scaling the logistic function (2.2a) by dividing it by the standard deviation of the transition variable $\sigma(\Delta u_{t\text{-}1})$. This allows γ to become a scale-free parameter. We then use $\gamma = 1$, as a starting value for γ, values of $CV_{t\text{-}1}$ close to its minimum (which equals −0.11) and maximum (which equals 0.10) as starting values for the parameters c_1 and c_2, respectively, and values of $\Delta u_{t\text{-}1}$ close to its mean (which equals 0.00) as starting values for the parameter c. The estimates of the linear equations for $\Delta(p_T - p)_t$, $\Delta(w - p_T)_t$ and Δu_t are used as starting values for the remaining parameters in the STVECM equations (2.1). [9] The parsimonious STVECM equations are estimated simultaneously by non-linear least squares (see Tables 3a and 4a). In what follows, we report common threshold and speed of adjustment estimates for all three equations; separate estimates were virtually identical to each other.

5. Discussion of the results

5.1. STVECM specification using the quadratic logistic function and $s_{t\text{-}1} = CV_{t\text{-}1}$

5.1.1. Regime identification

The main parameters of interest in the non-linear models are the estimated values of the threshold parameters c_1 and c_2, and the speed of adjustment γ. The c_1 and c_2 estimates of -0.03 and 0.04, respectively, reported in Table 3a are statistically significant. The c_1 and c_2 estimates indicate the existence of two regimes; one characterised by small real exchange rate deviations from its equilibrium level (when $G(s_t)$ = 0) and an alternative one characterised by large deviations of the real exchange rate from its long-run equilibrium (when $G(s_t)$ = 1). The first regime has 90 observations within the (c_1, c_2) band whereas the second regime has 34 observations outside the (c_1, c_2) band. The estimate of 10.0 for the γ parameter is rather high indicating that the speed of the transition from $G(s_t)$ = 0 to $G(s_t)$ = 1 is rapid at the estimated thresholds c_1 and c_2. The rather high standard error of γ should not be interpreted as evidence of weak non-linearity. Accurate estimation of γ is not always feasible, as it requires many observations in the immediate neighborhood of the threshold parameters c_1 and c_2. Further, large changes in γ have only a small effect on the shape of the transition function implying that high accuracy in estimating γ is not necessary (see the discussion in van Dijk *et al.*, 2002). Figure 2 plots the speed of the transition between the two regimes. Given that the estimate of γ is large, the transition from one regime to the other at the estimated thresholds is quite rapid.

---

[9] To save space, the estimates of the linear models are not reported but are available by the authors on request.

Figure 3 plots the values of the transition function together with the transition variable CV_{t-1}. Movements of the disequilibrium error above (below) the zero line are associated with an undervalued (overvalued) real exchange rate. One can notice from Figure 3 (where the estimated threshold parameters $c_1 = -0.03$, and $c_2 = 0.04$ have been super-imposed), that the shift to the floating exchange rate regime has resulted in a highly volatile exchange rate during the start of our sample. We also see that during the 1980-1982 period, the real exchange rate is highly overvalued. Supported by the great appreciation of the dollar *vis a vis* the currencies of most of the industrialised countries during the first half of the 1980s (see e.g. Engel and Hamilton, 1990), the real exchange rate consequently reverts to its long-run equilibrium. In fact, it becomes highly undervalued in early 1985 after which the dollar witnesses a persistent depreciation against most foreign currencies. The real exchange rate is also undervalued during 1994-1996. It is notable that early 1980s, which captures the 1980-1981 economic recession, follows the second OPEC oil price hike (an increase in oil prices of around 15 percent in June 1979) and coincides with important changes in economic policies following the election of the Thatcher government in May 1979. In particular, 1979 saw the abolition of exchange rate controls, which was not aimed at any particular effect on the exchange rate, as well as public spending cuts and an increase in indirect taxation. The new government encouraged the use of cheaper labour, especially female labour, which led to more part-time employment. At the same time, a very tight monetary policy aiming at a rapid decrease in the rate of inflation, led to a more overvalued real exchange rate, a rapid increase in unemployment (see Figure 1) and a severe recession (see e.g. Britton, 1991; Mizon, 1995). Taking into account the slow adjustment of the real exchange rate reported in the previous section of the paper, it is not surprising that after the UK's exit from the ERM in September 1992, the real exchange rate experienced a path of persistent depreciation, which peaked between 1994 and 1996 and then again in 2001. Since then, the real exchange rate fluctuates within the band of thresholds.

5.1.2. STVECM estimates

Consider first the $\Delta(p_T - p)_t$ equation in Table 3a. The estimate of -0.397 for the disequilibrium error in the second regime (i.e. when $G(s_t) = 1$) implies that the short-run real exchange rate adjusts back to equilibrium only outside the estimated interval band of thresholds. Growth in real wage costs lowers the competitiveness of the domestic economy (i.e. coefficient on $\Delta(w - p_T)_{t-1}$ equals -0.538). The effect takes place when the real exchange rate is close to its equilibrium level (i.e. when $G(s_t) = 0$).

From Table 3a, short-run wages $\Delta(w - p_T)_t$ are affected by real exchange rate disequilibria outside the estimated interval band (i.e. coefficient on $CV_{t\text{-}1}$ is equal to 0.230 in the second regime $G(s_t)$ = 1). Hence, there is evidence that when the real exchange rate is highly undervalued, workers respond to an improvement in domestic competitiveness by demanding and getting higher wages. From Table 3a one can see that short-run unemployment Δu_t is reduced when the real exchange rate depreciates (i.e. coefficient on $\Delta(p_T - p)_{t\text{-}1}$ is equal to -0.435). This effect takes place only when the real exchange rate is further away from equilibrium (i.e. when $G(s_t)$ = 1). The effect is much higher than the estimate of 0.26 reported in Madsen (1998); however, Madsen does not distinguish between large and small real exchange rate disequilibria. [10] The error variance ratio of the non-linear relative to the linear models (i.e. $s^2_{NL}{}^Q/s^2_L$) is less than one, indicating that the non-linear models have a better fit. The models also pass the parameter constancy test (reported at the bottom of Table 3a). From Table 3b, the null hypothesis of no remaining non-linearity (in the form of an additional logistic specification) cannot be rejected (tests in the form of an additional quadratic logistic specification are not performed because the Taylor approximation resulted in lack of degrees of freedom).

5.2. STVECM specification using the logistic function and $s_{t\text{-}1} = \Delta u_{t\text{-}1}$

5.2.1. Regime identification

From Table 4a, the parameter γ is estimated at 2.0 whereas c is estimated at -0.001; the threshold estimate is statistically insignificant. There are 60 observations below and 64 observations above the estimated threshold. Figure 4 reports the speed of the transition between regimes. This is rather slow given the low γ estimate. Figure 5 plots the values of the transition function together with the transition variable $\Delta u_{t\text{-}1}$ and the estimated threshold c = -0.001. Large increases in unemployment are associated with values of the transition function close to $G(s_t)$ = 1, whereas large unemployment decreases are associated with $G(s_t)$ values close to zero. Notice that there are not many extreme values of $G(s_t)$ = 0 or $G(s_t)$ = 1; rather, more intermediate regimes occur.

[10] In Madsen (1998), an increase in the real exchange rate is equivalent to a real appreciation. His estimate is based on annual data over the 1960-1993 period for a panel of 22 OECD countries.

5.2.2. STVECM estimates

Consider first the $\Delta(p_T - p)_t$ equation reported in Table 4a. When unemployment is falling, the real exchange rate error corrects more than twice as much as when unemployment is rising. Indeed, the coefficient estimate on the disequilibrium error is equal to –0.502 when $G(s_t) = 0$, but only –0.224 when $G(s_t) = 1$. The slow real exchange rate adjustment back to equilibrium in the rising unemployment regime suggests that prices and wages are less flexible when real output is low. Further, growth in real wage costs lowers the competitiveness of the domestic economy (i.e. coefficient on $\Delta(w - p_T)_{t\text{-}1}$ equals -0.546). This effect takes place only when unemployment is falling.

From Table 4a, short-run wages $\Delta(w - p_T)_t$ increase in response to real exchange rate disequilibria. The effect is similar irrespectively of whether unemployment is falling or rising (i.e. coefficient on $CV_{t\text{-}1}$ equals 0.283 when $G(s_t) = 0$, whereas coefficient on $CV_{t\text{-}1}$ equals 0.203 when $G(s_t) = 1$). Short-run unemployment Δu_t goes down when the real exchange rate depreciates. In particular, when unemployment is falling, the combined effect from the $\Delta(p_T - p)_{t\text{-}2}$ and $\Delta(p_T - p)_{t\text{-}3}$ regressors is equal to -0.901 + 0.493 = -0.408. When unemployment is rising, the effect from the $\Delta(p_T - p)_{t\text{-}1}$ regressor is equal to –0.574. The reported negative real wage effect is consistent with earlier studies (see e.g. Manning, 1993). This effect takes place only when unemployment is falling.

The models pass the parameter constancy test (reported at the bottom of Table 4a). Comparing the $s^2_{NL}{}^Q / s^2_{NL}{}^L$ ratios in Table 4a, we see that the real exchange rate equation has a better fit when non-linearity is driven by past disequilibria rather than changes in unemployment, whereas the unemployment rate equation has a better fit when non-linearity is driven by unemployment changes rather than exchange rate disequilibria. The real wage equation works equally well irrespectively of the source of non-linearity. From Table 4b, the null hypothesis of no remaining non-linearity based on the equation specific and the system wide tests cannot be rejected.

5.3. Dynamic analysis

Following Teräsvirta (1994), the dynamic properties of the STVECM equations are examined by calculating the characteristic roots (eigenvalues) of the companion form representation of the system. Table 5a reports the most prominent roots for the STVECM specification (estimated in Table 3a), whereas Table 5b reports the most prominent roots for the STVECM specification (estimated in Table 4a). We report both the real and complex characteristics roots. Since none of the roots has modulus greater than one, the non-linear systems are stable. The presence of complex roots implies the existence of cycles with period lengths between 4.9 and 19.5 quarters. From Table 5a, the regime associated with large real exchange rate disequilibria has the longest cycle of 19.5 quarters. Bearing in mind that we are examining the dynamic behaviour of the whole system rather than the real exchange rate equation on its own, we notice from the estimates reported in Table 3a that this long cycle is driven by the inertia in the unemployment rate equation (which includes all Δu_{t-1}, Δu_{t-2} and Δu_{t-3} regressors). To verify this, we drop the Δu_{t-2} and Δu_{t-3} regressors from the unemployment rate equation in order to match the autoregressive characteristics of the unemployment rate equation in the other regime, $G(s_t) = 0$, and then re-do the dynamic analysis. This suggests the sole presence of real roots with values well below one.

5.4. What if the quadratic logistic function has a unique equilibrium $c_1 = c_2 = 0$?

We now compare the empirical estimates of Table 3a with those of a more restricted version of our model in which we impose a unique equilibrium condition, $c_1 = c_2 = 0$ in the quadratic logistic function (2.2c). We report the results in Table 6. Although the main message is the same as before, the estimates of the coefficients for both the real exchange rate and wage equations are in general higher than those reported in Table 3a, whereas the opposite seems to be true for the unemployment rate equation. The error variance ratio (i.e. $s^2_{NL}{}^{QRES}/\ s^2_{NL}{}^{Q}$) of these restricted models relative to the ones reported in Table 3a, suggest a slightly worse fit for the real exchange rate and the real wage equations when a unique equilibrium condition is imposed. Tests for no remaining non-linearity (not reported) are similar to those reported in Tables 3b.

5.5. What if the real exchange rate is stationary?

This paper has treated the real exchange rate as a non-stationary variable. Given the ongoing debate on its stationary properties, we checked the robustness of our results by estimating non-linear models where the demeaned real exchange rate $(p_T - p)_t$ is assumed to be stationary, in which case the Purchasing Power Parity holds as a long-run relationship. To save space, we only report a summary of our results. In brief, we still estimate the STVECM equation (2.1), with z_{t-1} given by $(p_T - p)_{t-1}$. Linearity tests using the quadratic logistic function favour $(p_T - p)_{t-1}$ as the most suitable transition variable. We do not find any significant feedback from $(p_T - p)_{t-1}$ in the non-linear equation for $\Delta(p_T - p)_t$. Noting that the non-linear regression of $\Delta(p_T - p)_t$ on $(p_T - p)_{t-1}$

is a type of non-linear unit root test, insignificance of the $(p_T - p)_{t\text{-}1}$ regressor provides some evidence against stationarity of the real exchange rate. On the other hand, the main results in Table 3a regarding the non-linear effect of the real exchange rate on unemployment and wages are reasonably robust to whether the real exchange rate is treated as a stationary or non-stationary variable.

6. Conclusions

This paper examined non-linearities in a multivariate model of the UK real exchange rate. According to our results, the dynamics of the real exchange rate, real wages and unemployment vary in a non-linear way with large versus small real exchange rate disequilibria and rising versus falling unemployment regimes. The short-run real exchange rate adjusts only when large disequilibrium deviations occur. Fast real exchange rate adjustment occurs in periods of falling unemployment, which implies that prices and wages are more flexible when real output is high. We also find that when the real exchange rate is highly undervalued, workers demand and get higher wages. Unemployment is reduced following gains in competitiveness when the real exchange rate is further away from equilibrium.

References

Akram, Q.F., Ø. Eitrheim, and L. Sarno (2005): "Non-linear dynamics in output, real exchange rates and real money balances: Norway, 1830-2003," forthcoming in Dick van Dijk, Costas Milas, and Philip Rothman, eds., *Nonlinear Time Series Analysis of Business Cycles*, Amsterdam: Elsevier Science.

Alogoskoufis, G. (1990): "Traded goods, competitiveness and aggregate fluctuations in the United Kingdom," *Economic Journal*, 100, 141-163.

Batini, N., B. Jackson, and S. Nickell (2002): "The pricing behaviour of UK firms", Bank of England External MPC Unit Discussion Paper No. 9.

Bean, C. (1994): "European unemployment: a survey," *Journal of Economic Literature*, 32, 573-619.

Baum, C.F., J.T. Barkoulas, and M. Caglayan (2001): "Nonlinear adjustment to purchasing power parity in the post-Bretton Woods era," *Journal of International Money and Finance*, 20, 379-399.

Britton, A.J.C. (1991): *Macroeconomic Policy in Britain 1974-1987*. National Institute of Economic and Social Research, Economic and Social Studies 36, Cambridge: Cambridge University Press.

Engel, C. (2000): "Long-run PPP may not hold after all," *Journal of International Economics*, 57, 243-273.

Engel, C., and J.D. Hamilton (1990): "Long swings in the dollar: are they in the data and do markets know it?," *American Economic Review*, 80, 689-713.

Eitrheim, Ø., and T. Teräsvirta (1996): "Testing the adequacy of smooth transition autoregressive models," *Journal of Econometrics*, 74, 59-75.

Escribano, A. (1986): "Nonlinear error correction, the case of money demand in the UK (1878-1970)," PhD dissertation, University of California San Diego.

Escribano, A. (2004): "Nonlinear error correction: The case of money demand in the UK (1878-2000)," *Macroeconomics Dynamics*, 8, 76-116.

Escribano, A., and O. Jordá (1999): "Improved testing and specification of smooth transition regression models," in Philip Rothman, ed., *Nonlinear Time Series Analysis of Economic and Financial Data*, Boston: Kluwer, 289-319.

Escribano A., and O. Jordá (2001): "Testing nonlinearity: Decision rules for selecting between logistic and exponential STAR models," *Spanish Economic Review*, 3, 193-209.

Gali, J., M. Gertler, and D. Lopez-Salido (2002): "Mark-ups, gaps, and the welfare costs of business fluctuations," National Bureau of Economic Research Working Paper No. 8850, Cambridge, MA.

Granger, C.W.J., and N.R. Swanson (1996): "Future developments in the study of cointegrated variables," *Oxford Bulletin of Economic and Statistics*, 58, 537-553.

Granger, C.W.J., and T. Teräsvirta (1993): *Modelling Nonlinear Economic Relationships*. Oxford: Oxford University Press.

Hamilton, J.D. (1989): "A new approach to the economic analysis of nonstationary time series and the business cycle," *Econometrica*, 57, 357-384.

Haskel, J., C. Martin, and I. Small (1995): "Price, marginal cost and the business cycle," *Oxford Bulletin of Economics and Statistics*, 57, 25-42.

Jansen, E.S., and T. Teräsvirta (1996): "Testing parameter constancy and super exogeneity in econometric equations," *Oxford Bulletin of Economics and Statistics*, 58, 735-768.

Johansen, S. (1988): "Statistical analysis of cointegration vectors," *Journal of Economic Dynamics and Control* 12, 231-254.

Lin, C-F.J., and T. Teräsvirta (1994): "Testing the constancy of regression parameters against continuous structural change," *Journal of Econometrics*, 62, 211-228.

Lothian, J.R., and M.P. Taylor (1996): "Real exchange rate behavior: The recent float from the perspective of the past two centuries," *Journal of Political Economy*, 104, 488-509.

Madsen, J. (1998): "General equilibrium macroeconomic models of unemployment: can they explain the unemployment path in the OECD?," *Economic Journal*, 108, 850-867.

Manning, A. (1993): "Wage bargaining and the Phillips curve: the identification and specification of aggregate wage equations," *Economic Journal*, 103, 98-118.

Martin, C. (1997): "Price formation in an open economy: theory and evidence for the United Kingdom, 1951-1991," *Economic Journal*, 107, 1391-1404.

Michael, P., A.R. Nobay, and D.A. Peel (1997): "Transactions costs and nonlinear adjustment in real exchange rates: an empirical investigation," *Journal of Political Economy*, 105, 862-879.

Milas, C. (1997): "Modelling employment policy, competitiveness and demand for traded and non-traded goods in a small open economy: The case of Greece," Unpublished PhD thesis, Department of Economics, University of Warwick, October 1997.

Mizon, G.E. (1995): "Progressive modeling of macroeconomic time series. The LSE methodology," in Kevin D. Hoover, ed., *Macroeconometrics: Developments, Tensions and Prospects*, Dordrecht: Kluwer, 107-180.

Paya, I., and D.A. Peel (2003): "Purchasing power parity adjustment speeds in high frequency data when the equilibrium real exchange rate is proxied by a deterministic trend," *The Manchester School*, 71, 39-53 (Supplement).

Reimers, H.E. (1992): "Comparisons of tests for multivariate cointegration," *Statistical Papers*, 33, 335-359.

Rotemberg, J.J., and M. Woodford (1999): "The cyclical behaviour of prices and costs," in John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics*, Vol 1B., Amsterdam: North-Holland, 1051-1135.

Rothman, P., D. van Dijk, and P.H. Franses (2001): "A multivariate STAR analysis of the relationship between money and output," *Macroeconomic Dynamics*, 5, 506-532.

Saikkonen, P., and R. Luukkonen (1988): "Lagrange Multiplier tests for testing non-linearities in time series models," *Scandinavian Journal of Statistics*, 15, 55-68.

Sarantis, N. (1999): "Modeling non-linearities in real effective exchange rates," *Journal of International Money and Finance*, 18, 27-45.

Smith, Peter N. (2000): "Output price determination and the business cycle," *Economic Modelling*, 17, 49-69.

Taylor, A.M. (1996): "International capital mobility in history: purchasing-power parity in the long run," National Bureau of Economic Research Working Paper No. 5472, Cambridge, MA.

Teräsvirta, T. (1994): "Specification, estimation, and evaluation of smooth transition autoregressive models," *Journal of the American Statistical Association*, 89, 208-218.

Teräsvirta, T., and H.M. Anderson (1992): "Characterizing nonlinearities in business cycles using smooth transition autoregressive models," *Journal of Applied Econometrics*, 7, S119-S136.

Tong, H. (1990): *Non-linear Time series: A Dynamical System Approach*. Oxford: Oxford University Press.

van Dijk, D., and P.H. Franses (2000): "Nonlinear error-correction models for interest rates in the Netherlands," in: William A. Barnett, David F. Hendry, Svend Hylleberg, Timo Teräsvirta, Dag Tjostheim, and Allan Würtz, eds., *Nonlinear Econometric Modelling in Time Series Analysis*, Cambridge: Cambridge University Press, 203-227.

van Dijk, D., T. Teräsvirta, and P.H. Franses (2002): "Smooth transition autoregressive models – a survey of recent developments," *Econometric Reviews*, 21, 1-47.

Weise, C.L. (1999): "The asymmetric effects of monetary policy: a nonlinear vector autoregression approach," *Journal of Money, Credit, and Banking*, 31, 85-108.

Table 1: Eigenvalues, test statistics and critical values

λ_i	*λ-max*				*Trace*			
	H_0	H_1	Stat.	95%	H_0	H_1	Stat.	95%
0.25	$r = 0$	$r = 1$	31.91	21.0	$r = 0$	$r \geq 1$	40.84	29.7
0.07	$r \leq 1$	$r = 2$	8.77	14.1	$r \leq 1$	$r \geq 2$	8.93	15.4
0.00	$r \leq 2$	$r = 3$	0.16	3.8	$r \leq 2$	$r = 3$	0.16	3.8

Notes: *r* denotes the number of cointegration vectors.

Table 2a: Linearity tests against a quadratic logistic STVECM representation

Transition variable s_t	Lagrange Multiplier F statistics for: $\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model	System wide test LR
$\Delta(p_T - p)_{t-1}$	0.021	0.002	0.000	0.000
$\Delta(p_T - p)_{t-2}$	0.136	0.001	0.000	0.000
$\Delta(p_T - p)_{t-3}$	0.020	0.001	0.000	0.000
$\Delta(p_T - p)_{t-4}$	0.096	0.494	0.001	0.000
$\Delta(w - p_T)_{t-1}$	0.005	0.000	0.000	0.000
$\Delta(w - p_T)_{t-2}$	0.186	0.000	0.000	0.000
$\Delta(w - p_T)_{t-3}$	0.010	0.000	0.000	0.000
$\Delta(w - p_T)_{t-4}$	0.081	0.060	0.003	0.000
Δu_{t-1}	0.080	0.008	0.000	0.000
Δu_{t-2}	0.033	0.026	0.000	0.000
Δu_{t-3}	0.001	0.006	0.000	0.000
Δu_{t-4}	0.297	0.326	0.092	0.010
CV_{t-1}	0.000*	0.000*	0.000*	0.000*
CV_{t-2}	0.178	0.432	0.006	0.000
CV_{t-3}	0.049	0.018	0.000	0.000
CV_{t-4}	0.846	0.540	0.001	0.002

Notes: The Table reports bootstrapped p-values. The p-values for the equation specific Lagrange Multiplier F statistics and the system wide LR test statistic are derived from bootstrapping with one thousand replications. CV is the transition variable: $CV = p_T - p + 0.520\ (w - p_T) - 0.041\ u$, in mean-corrected form. The null hypothesis is linearity. The alternative hypothesis is the STVECM equation (2.1) using the quadratic logistic function (2.2c). A * indicates strongest rejection of the null hypothesis of linearity across all three equations and the system.

Table 2b: Linearity tests against a logistic STVECM representation

Transition variable s_t	Lagrange Multiplier F statistics for: $\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model	System wide test LR
$\Delta(p_T - p)_{t-1}$	0.011	0.002	0.000	0.000
$\Delta(p_T - p)_{t-2}$	0.062	0.001	0.001	0.000
$\Delta(p_T - p)_{t-3}$	0.002	0.000	0.000	0.000
$\Delta(p_T - p)_{t-4}$	0.081	0.606	0.001	0.000
$\Delta(w - p_T)_{t-1}$	0.010	0.000	0.000	0.000
$\Delta(w - p_T)_{t-2}$	0.156	0.001	0.000	0.000
$\Delta(w - p_T)_{t-3}$	0.006	0.007	0.000	0.000
$\Delta(w - p_T)_{t-4}$	0.108	0.328	0.018	0.000
Δu_{t-1}	0.001*	0.000*	0.000*	0.000*
Δu_{t-2}	0.003	0.000	0.001	0.000
Δu_{t-3}	0.014	0.004	0.003	0.000
Δu_{t-4}	0.330	0.222	0.070	0.003
CV_{t-1}	0.600	0.367	0.012	0.000
CV_{t-2}	0.242	0.899	0.003	0.000
CV_{t-3}	0.035	0.026	0.000	0.000
CV_{t-4}	0.406	0.196	0.000	0.000

Notes: The Table reports bootstrapped p-values. The p-values for the equation specific Lagrange Multiplier F statistics and the system wide LR test statistic are derived from bootstrapping with one thousand replications. CV is the transition variable: $CV = p_T - p + 0.520\ (w - p_T) - 0.041\ u$, in mean-corrected form. The null hypothesis is linearity. The alternative hypothesis is the STVECM equation (2.1) using the logistic function (2.2a). A * indicates strongest rejection of the null hypothesis of linearity across all three equations and the system.

Table 3a: STVECM representation based on the quadratic logistic function

$$G(s_t;\gamma,c_1,c_2)=\{1+\exp[-\gamma(s_t-c_1)(s_t-c_2)/\sigma^2(s_t)]\}^{-1} \text{ and } s_t = CV_{t-1}$$

	$\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model
*Constant**(1-$G(s_t)$)	0.001 (0.002)	0.001 (0.002)	-0.001 (0.003)
$\Delta(p_T - p)_{t-3}$*(1-$G(s_t)$)	0.295 (0.092)	-0.344 (0.097)	
$\Delta(w - p_T)_{t-1}$*(1-$G(s_t)$)	-0.538 (0.094)	0.618 (0.099)	
Δu_{t-1}*(1-$G(s_t)$)			0.721 (0.006)
*Constant**$G(s_t)$	0.003 (0.004)	0.002 (0.004)	0.015 (0.006)
$\Delta(p_T - p)_{t-1}$*$G(s_t)$	0.390 (0.109)	-0.264 (0.105)	-0.435 (0.169)
Δu_{t-1}*$G(s_t)$			0.533 (0.116)
Δu_{t-2}*$G(s_t)$			0.376 (0.116)
Δu_{t-3}*$G(s_t)$			-0.250 (0.110)
CV_{t-1}*$G(s_t)$	-0.397 (0.067)	0.230 (0.069)	
c_1	-0.030 (0.006)		
c_2	0.040 (0.004)		
γ	10.0 (6.332)		
s_{NL}^Q	0.020	0.021	0.032
$s^2_{NL}{}^Q/s^2_L$	0.826	0.833	0.940
AR(5)	0.180	0.443	0.100
ARCH(4)	0.697	0.020	0.995
HET	0.131	0.121	0.265
NORM	0.190	0.010	0.008
PAR. CONSTANCY	0.121	0.230	0.141

Notes: Standard errors are given in parentheses. s_{NL}^Q: standard error of the non-linear quadratic logistic regression . s_L: standard error of the linear regression. AR(5): *p*-value of the F-test for up to 5th order serial correlation. ARCH(4): *p*-value of the 4th order Autoregressive Conditional Heteroscedasticity F-test.
HET: *p*-value of the F-test for Heteroscedasticity. NORM: *p*-value of the Chi-square test for normality. PAR. CONSTANCY: *p*-value of the parameter constancy F-test (see Lin and Teräsvirta, 1994, and Eitrheim and Teräsvirta, 1996).

Table 3b: Tests for no remaining non-linearity

Transition variable s_t	Lagrange Multiplier F statistics for: $\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model	System wide test LR
$\Delta(p_T - p)_{t\text{-}1}$	0.121	0.321	0.231	0.051
$\Delta(p_T - p)_{t\text{-}2}$	0.141	0.223	0.123	0.087
$\Delta(p_T - p)_{t\text{-}3}$	0.098	0.126	0.234	0.076
$\Delta(p_T - p)_{t\text{-}4}$	0.102	0.567	0.245	0.057
$\Delta(w - p_T)_{t\text{-}1}$	0.123	0.231	0.234	0.063
$\Delta(w - p_T)_{t\text{-}2}$	0.216	0.210	0.456	0.090
$\Delta(w - p_T)_{t\text{-}3}$	0.143	0.123	0.345	0.078
$\Delta(w - p_T)_{t\text{-}4}$	0.125	0.130	0.347	0.083
$\Delta u_{t\text{-}1}$	0.234	0.128	0.098	0.079
$\Delta u_{t\text{-}2}$	0.125	0.103	0.123	0.066
$\Delta u_{t\text{-}3}$	0.145	0.123	0.110	0.089
$\Delta u_{t\text{-}4}$	0.354	0.456	0.101	0.059
$CV_{t\text{-}1}$	0.219	0.231	0.367	0.078
$CV_{t\text{-}2}$	0.178	0.567	0.245	0.089
$CV_{t\text{-}3}$	0.049	0.182	0.110	0.100
$CV_{t\text{-}4}$	0.878	0.579	0.121	0.151

Notes: We test the null hypothesis of no remaining non-linearity in the STVECM estimates of Table 3a. The Table reports bootstrapped p-values. The p-values for the equation specific Lagrange Multiplier F statistics and the system wide LR test statistic are derived from bootstrapping with one thousand replications. CV is the transition variable: $CV = p_T - p + 0.520\ (w - p_T) - 0.041\ u$, in mean-corrected form.

Table 4a: STVECM representation based on the logistic function

$$G(s_t;\gamma,c) = \{1+\exp[-\gamma(s_t - c)/\sigma(s_t)]\}^{-1} \text{ and } s_t = \Delta u_{t-1}$$

	$\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model
*Constant**(1-$G(s_t)$)	0.005 (0.003)	-0.002 (0.003)	-0.046 (0.006)
$\Delta(p_T - p)_{t-2}$*(1-$G(s_t)$)			-0.901 (0.405)
$\Delta(p_T - p)_{t-3}$*(1-$G(s_t)$)	0.449 (0.150)		0.493 (0.202)
$\Delta(w - p_T)_{t-1}$*(1-$G(s_t)$)	-0.546 (0.155)	0.702 (0.160)	
$\Delta(w - p_T)_{t-2}$*(1-$G(s_t)$)			-0.912 (0.411)
Δu_{t-1}*(1-$G(s_t)$)			0.381 (0.010)
CV_{t-1}*(1-$G(s_t)$)	-0.502 (0.116)	0.283 (0.120)	
*Constant**$G(s_t)$	-0.004 (0.004)	0.010 (0.004)	0.049 (0.001)
$\Delta(p_T - p)_{t-1}$*$G(s_t)$	0.463 (0.106)		-0.574 (0.137)
Δu_{t-1}*$G(s_t)$			0.434 (0.119)
Δu_{t-2}*$G(s_t)$			0.265 (0.140)
Δu_{t-3}*$G(s_t)$			-0.326 (0.114)
CV_{t-1}*$G(s_t)$	-0.224 (0.099)	0.203 (0.100)	
c	-0.001 (0.110)		
γ	2.0 (1.501)		
s_{NL}^{L}	0.021	0.021	0.027
$s^2_{NL}{}^{L}/s^2_{L}$	0.911	0.833	0.711
$s^2_{NL}{}^{Q}/s^2_{NL}{}^{L}$	0.907	1.000	1.404
AR(5)	0.520	0.230	0.880
ARCH(4)	0.252	0.010	0.566
HET	0.126	0.001	0.240
NORM	0.178	0.020	0.005
PAR. CONSTANCY	0.162	0.218	0.321

Notes: Standard errors are given in parentheses. s_{NL}^{L}: standard error of the non-linear logistic regression. s_L: standard error of the linear regression. s_{NL}^{Q}: standard error of the non-linear quadratic logistic regression reported in Table 3a. AR(5): *p*-value of the F-test for up to 5th order serial correlation. ARCH(4): *p*-value of the 4th order Autoregressive Conditional Heteroscedasticity F-test.
HET: *p*-value of the F-test for Heteroscedasticity. NORM: *p*-value of the Chi-square test for normality. PAR. CONSTANCY: *p*-value of the parameter constancy F-test (see Lin and Teräsvirta, 1994, and Eitrheim and Teräsvirta, 1996).

Table 4b: Tests for no remaining non-linearity

Transition variable s_t	Lagrange Multiplier F statistics for: $\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model	System wide test LR
$\Delta(p_T - p)_{t-1}$	0.112	0.099	0.342	0.167
$\Delta(p_T - p)_{t-2}$	0.234	0.134	0.236	0.086
$\Delta(p_T - p)_{t-3}$	0.123	0.345	0.341	0.095
$\Delta(p_T - p)_{t-4}$	0.101	0.690	0.123	0.073
$\Delta(w - p_T)_{t-1}$	0.123	0.134	0.234	0.051
$\Delta(w - p_T)_{t-2}$	0.169	0.345	0.213	0.091
$\Delta(w - p_T)_{t-3}$	0.123	0.234	0.123	0.076
$\Delta(w - p_T)_{t-4}$	0.198	0.390	0.135	0.082
Δu_{t-1}	0.201	0.134	0.234	0.091
Δu_{t-2}	0.202	0.334	0.231	0.056
Δu_{t-3}	0.245	0.234	0.128	0.089
Δu_{t-4}	0.390	0.259	0.135	0.068
CV_{t-1}	0.909	0.401	0.243	0.077
CV_{t-2}	0.453	0.903	0.234	0.081
CV_{t-3}	0.379	0.234	0.231	0.090
CV_{t-4}	0.621	0.334	0.145	0.073

Notes: We test the null hypothesis of no remaining non-linearity in the STVECM estimates of Table 4a. The Table reports bootstrapped p-values. The p-values for the equation specific Lagrange Multiplier F statistics and the system wide LR test statistic are derived from bootstrapping with one thousand replications. CV is the transition variable: $CV = p_T - p + 0.520\,(w - p_T) - 0.041\,u$, in mean-corrected form.

Table 5a: Characteristic roots for the STVECM representation based on the quadratic logistic function

$$G(s_t;\gamma,c_1,c_2) = \{1+\exp[-\gamma(s_t - c_1)(s_t - c_2)/\sigma^2(s_t)]\}^{-1}$$

where $s_t = CV_{t-1}$, γ = 10.0, c_1 = -0.03 and c_2 = 0.04

Roots	Modulus	Period
$G(s_t)$ = 0 regime		
0.948	0.948	
0.721	0.721	
-0.161 ± 0.534i	0.557	4.918
-0.009	0.009	
$G(s_t)$ = 1 regime		
-0.646	0.646	
0.589 ± 0.197i	0.621	19.500
0.390	0.390	

Notes: Characteristic roots are calculated for the $G(s_t)$ = 0 regime (in which case CV_{t-1} is within the (c_1, c_2) band) and for the $G(s_t)$ = 1 regime (in which case CV_{t-1} is outside the (c_1, c_2) band).

Table 5b: Characteristic roots for the STVECM representation based on the logistic function

$$G(s_t;\gamma,c) = \{1+\exp[-\gamma(s_t - c)/\sigma(s_t)]\}^{-1}$$

where $s_t = \Delta u_{t-1}$, γ = 2.0 and c = -0.001

Roots	Modulus	Period
$G(s_t)$ = 0 regime		
0.765	0.765	
-0.382 ± 0.663i	0.765	5.995
0.702	0.702	
0.381	0.381	
$G(s_t)$ = 1 regime		
0.554 ± 0.419i	0.695	9.692
-0.674	0.674	
0.463	0.463	

Notes: Characteristic roots are calculated for the $G(s_t)$ = 0 regime (in which case Δu_{t-1} is below the c threshold) and for the $G(s_t)$ = 1 regime (in which case Δu_{t-1} is above the c threshold).

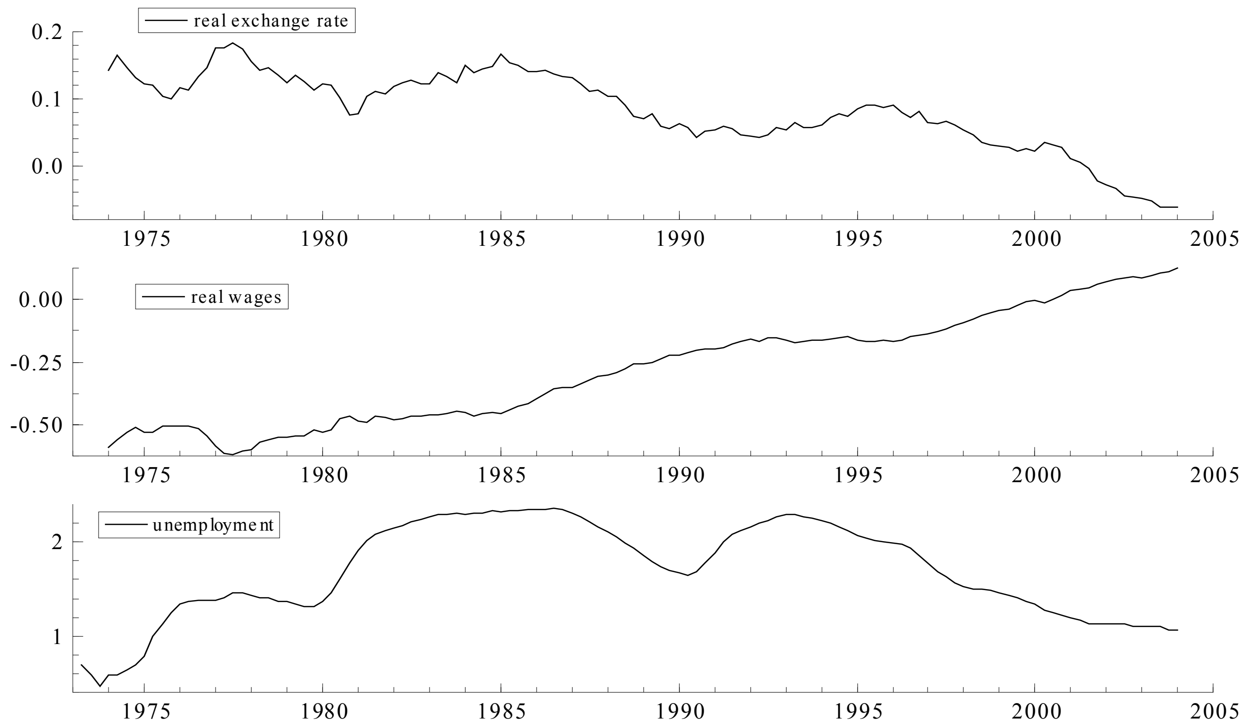
Table 6: STVECM representation based on the restricted quadratic logistic function $G(s_t;\gamma,c_1,c_2)=\{1+\exp[-\gamma(s_t-c_1)(s_t-c_2)/\sigma^2(s_t)]\}^{-1}$ and $s_t = CV_{t-1}$. The model assumes $c_1 = c_2 = 0$

	$\Delta(p_T - p)$ model	$\Delta(w - p_T)$ model	Δu model
$Constant*(1-G(s_t))$	0.017 (0.009)	-0.021 (0.009)	-0.011 (0.013)
$\Delta(p_T - p)_{t-3}*(1-G(s_t))$	0.729 (0.265)	-0.905 (0.269)	
$\Delta(w - p_T)_{t-1}*(1-G(s_t))$	-0.777 (0.332)	1.164 (0.339)	
$\Delta u_{t-1}*(1-G(s_t))$			0.201 (0.212)
$Constant*G(s_t)$	-0.003 (0.002)	0.008 (0.002)	0.002 (0.004)
$\Delta(p_T - p)_{t-1}*G(s_t)$	0.467 (0.083)	-0.366 (0.084)	-0.185 (0.114)
$\Delta u_{t-1}*G(s_t)$			0.744 (0.093)
$\Delta u_{t-2}*G(s_t)$			0.345 (0.102)
$\Delta u_{t-3}*G(s_t)$			-0.199 (0.090)
$CV_{t-1}*G(s_t)$	-0.336 (0.061)	0.203 (0.061)	
γ	10.2 (6.432)		
s_{NL}^{QRES}	0.021	0.022	0.032
$s^2_{NL}{}^{QRES}/s^2_{NL}{}^{Q}$	1.102	1.097	1.000
AR(5)	0.413	0.134	0.012
ARCH(4)	0.319	0.004	0.605
HET	0.106	0.090	0.110
NORM	0.368	0.132	0.000
PAR. CONSTANCY	0.133	0.151	0.123

Notes: Standard errors are given in parentheses. s_{NL}^{QRES}: standard error of the restricted non-linear quadratic logistic regression. s_{NL}^{Q}: standard error of the non-linear quadratic logistic regression reported in Table 3a. AR(5): *p*-value of the F-test for up to 5th order serial correlation. ARCH(4): *p*-value of the 4th order Autoregressive Conditional Heteroscedasticity F-test. HET: *p*-value of the F-test for Heteroscedasticity. NORM: *p*-value of the Chi-square test for normality. PAR. CONSTANCY: *p*-value of the parameter constancy F-test (see Lin and Teräsvirta, 1994, and Eitrheim and Teräsvirta, 1996).

Figure 1: Plots of the levels and the first differences of the series

Levels of the series



First differences of the series

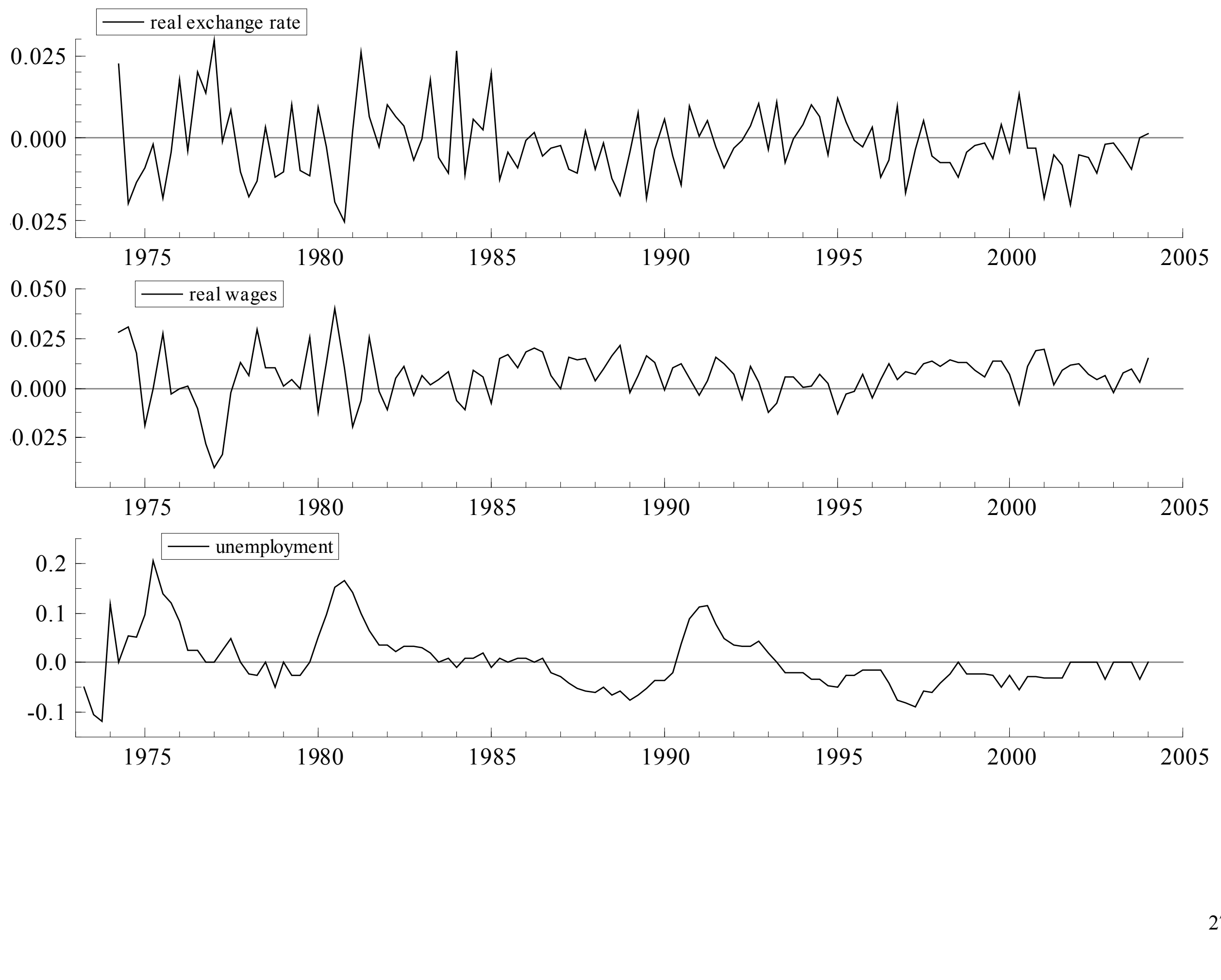
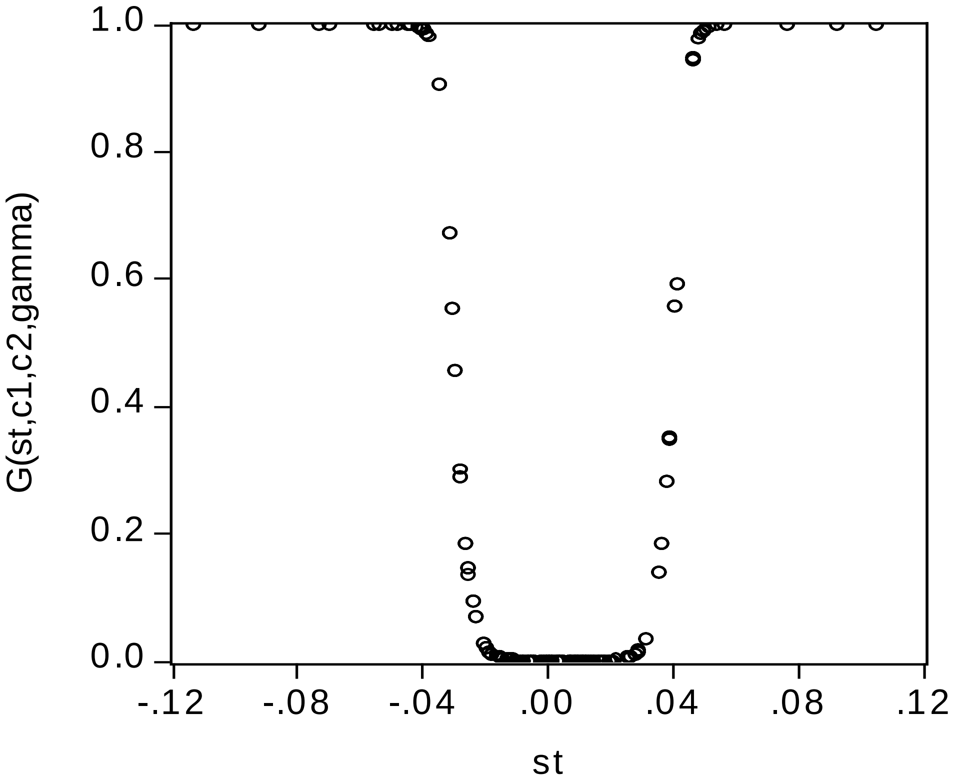
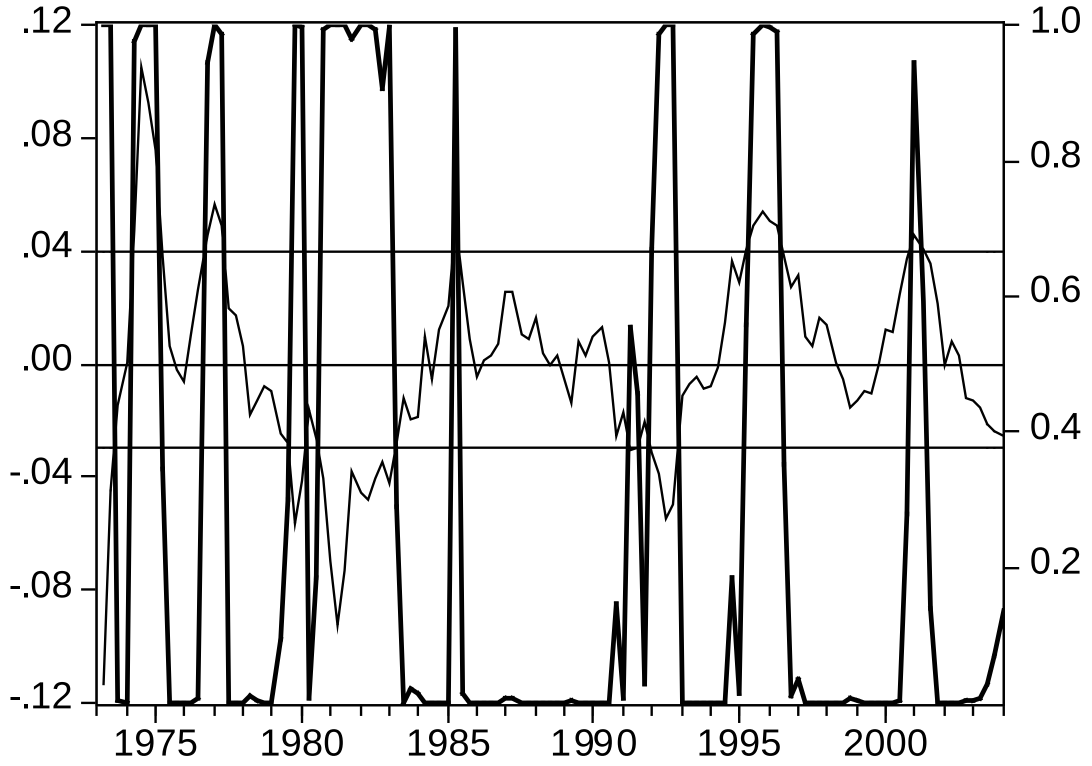


Figure 2: Estimated quadratic logistic transition function against $s_{t\text{-}1} = CV_{t\text{-}1}$



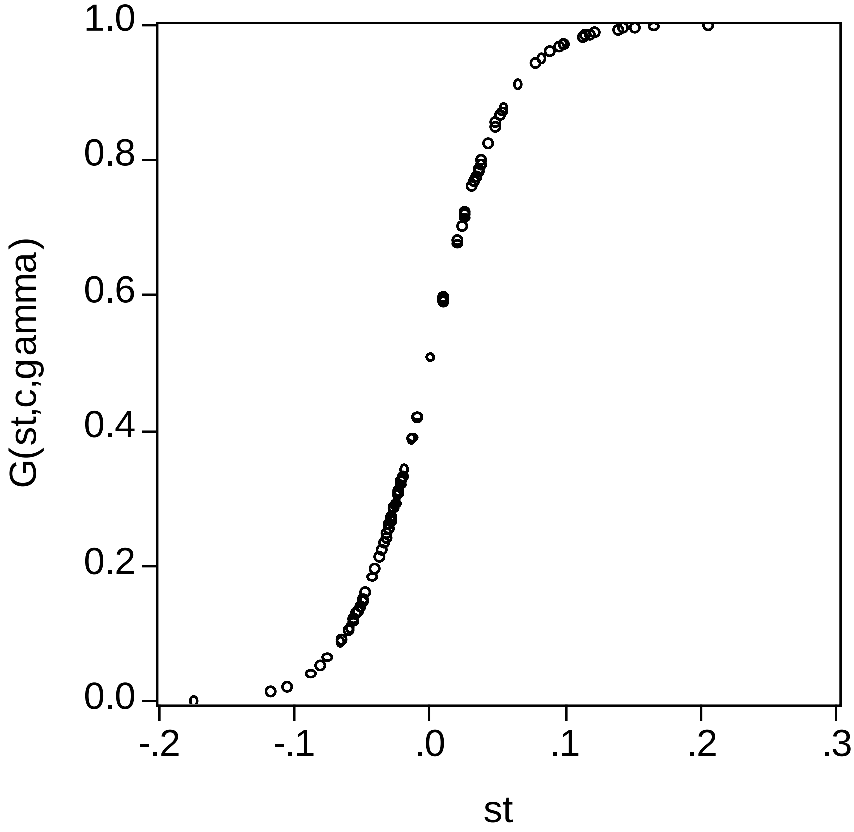
Notes: The transition function is $G(s_t;\gamma,c_1,c_2) = \{1+\exp[-\gamma(s_t - c_1)(s_t - c_2)/\sigma^2(s_t)]\}^{-1}$ where $s_t = CV_{t\text{-}1}$, $\gamma = 10.0$, $c_1 = -0.03$ and $c_2 = 0.04$.

Figure 3: Quadratic logistic transition function, $CV_{t\text{-}1}$, and estimated thresholds
c_1 = -0.03 and c_2 =0.04



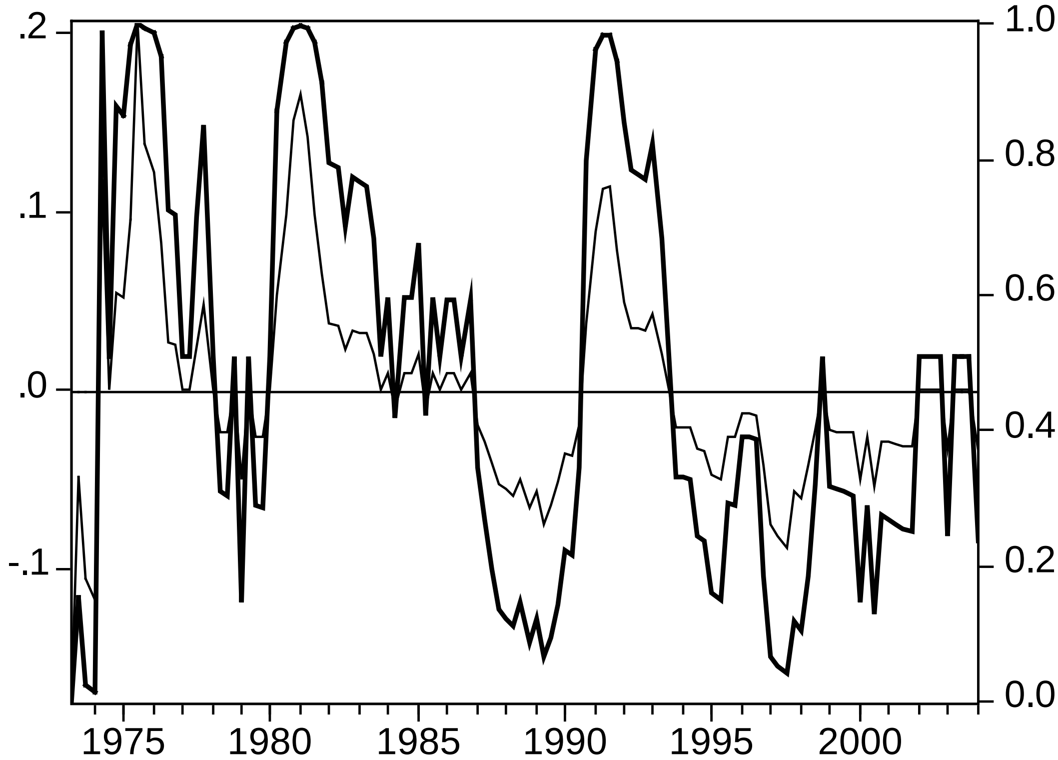
Notes: The solid line (right hand side axis) plots the values of the transition function $G(s_t;\gamma,c_1,c_2)=\{1+\exp[-\gamma(s_t-c_1)(s_t-c_2)/\sigma^2(s_t)]\}^{-1}$ where $s_t = CV_{t\text{-}1}$, $\gamma = 10.0$, c_1 = -0.03 and c_2 = 0.04. The transition variable $CV_{t\text{-}1}$ and the thresholds c_1 and c_2 are plotted on the left hand side axis.

Figure 4: Estimated logistic transition function against $s_{t-1} = \Delta u_{t-1}$



Notes: The transition function is $G(s_t;\gamma,c) = \{1+\exp[-\gamma(s_t - c)/\sigma(s_t)]\}^{-1}$ where $s_t = \Delta u_{t-1}$, $\gamma = 2.0$ and $c = -0.001$.

Figure 5: Logistic transition function, Δu_{t-1} and estimated threshold c = -0.001



Notes: The solid line (right hand side axis) plots the values of the transition function $G(s_t;\gamma,c)=\{1+\exp[-\gamma(s_t-c)/\sigma(s_t)]\}^{-1}$ where s_t = Δu_{t-1}, γ = 2.0 and c = -0.001. The transition variable Δu_{t-1} and the threshold c are plotted on the left hand side axis.